

## CHAOS IN A GALAXY MODEL WITH NUCLEUS AND BULGE COMPONENTS

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## ABSTRACT

A galaxy model, with a disk halo and a spherical nucleus, is used to study the transition from regular to chaotic motion for stars moving in the  $r$ - $z$  plane. Stars with values of angular momentum  $L_z$  less than or equal to a critical value  $L_{zc}$ , moving near the galactic plane, are scattered to the halo upon encountering the nuclear region, and subsequently display chaotic motion. A linear relationship exists between the critical angular momentum and the mass of the nucleus  $\mathcal{M}_n$ . Some theoretical arguments are presented to explain this behavior. Similar results are obtained if we use the same model with a spherical bulge instead of a nucleus. A comparison is made between  $L_{zc}$  and the circular angular momentum  $L_{z0}$  at different distances from the center of the galaxy. Our results suggest that, for disk galaxies with prominent nuclei, the stars in an associated bulge may be in predominantly chaotic orbits.

## 1. INTRODUCTION

A convenient method for the study of the orbits of stars in a galaxy, is to represent the galaxy by a simple dynamical model having about the same dynamical properties as the galaxy. A simple model for an axially symmetric galaxy is the disk-halo potential

$$V(r, z) = -\mathcal{M}/R, \quad (1)$$

with

$$R^2 = \left( a + \sum_{i=1}^3 \beta_i (z^2 + h_i^2)^{1/2} \right)^2 + b^2 + r^2, \quad (2)$$

where  $r, z$  are the usual cylindrical coordinates,  $\mathcal{M}$  is the mass,  $a$  is the scale length of the disk,  $h$  corresponds to the disk scale height, and  $b$  is the core radius of the halo component.  $\beta_1, \beta_2, \beta_3$  represent three disk components and sum to 1 (see Clutton-Brock *et al.* 1977; Carlberg & Innanen 1987). The symbol  $\Sigma$  denotes summation over these three components. To this potential we add a spherically symmetric nucleus

$$\varphi(r, z) = -\mathcal{M}_n / (c^2 + r^2 + z^2)^{1/2}, \quad (3)$$

where  $\mathcal{M}_n$  is the mass and  $c$  is the scale length of the nucleus.

Carlberg & Innanen (1987), using a galaxy model with disk halo, nucleus, bulge, and dark-halo components, among other matters, confirmed that low angular momentum stars moving near the galactic plane, when passing near the nucleus, are scattered to much higher scale heights. The corresponding orbits are chaotic. In this article, we use the same model as described above but with only two components: the disk halo, and the spherically symmetric nucleus or bulge.

The purpose of the present work is: (1) to find a relationship between the critical value of the angular momentum  $L_{zc}$  (that is the maximum value of the angular momentum  $L_z$ , for which, stars scattered to the halo display chaotic motion, for a given value of  $\mathcal{M}_n$ ) and  $\mathcal{M}_n$ ; (2) to explain this relation theoretically; and (3) to compare the critical value  $L_{zc}$  with the value of the circular angular momentum  $L_{z0}$  at dif-

ferent distances from the center of the galaxy. The relation between  $L_{zc}$  and  $\mathcal{M}_n$  is found in Sec. 2. In the same section a theoretical explanation for this behavior is given. In Sec. 3 we do similar calculations using a bulge instead of a nucleus. We also try to compare the differences in the behavior of the orbits, produced by substituting the total vertical force  $F_z$  near the center of the galaxy, by that of an harmonic oscillator. Finally, in Sec. 4, we present a discussion and the conclusions of this research.

## 2. CHAOTIC MOTION AND THE CRITICAL VALUE OF THE ANGULAR MOMENTUM

In order to study the behavior of the orbits, the  $r$ - $u_r$  ( $z = 0, u_z > 0$ ) surface of section was used, where  $u_r, u_z$  are the radial and vertical velocity, respectively. In the system of galactic units used in this paper, the unit of length is 1 kpc, the unit of time is  $0.97748 \times 10^8$  yr and the unit of mass is  $2.325 \times 10^7 \mathcal{M}_\odot$ . The velocity unit is 10 km/s while  $G$  is equal to unity. The values of the parameters used are  $a = 3$  (kpc),  $b = 8$  (kpc),  $\mathcal{M} = 12\,000$ ,  $(\beta_1, \beta_2, \beta_3) = (0.4, 0.5, 0.1)$  and  $(h_1, h_2, h_3) = (0.325, 0.090, 0.125)$  kpc. The mass and the scale of the nucleus are treated as parameters. The numerical calculations were made by means of a Bulirsch-Stoer method in double precision and the accuracy of the calculations was checked by the constancy of the energy integral, which was conserved up to the twelfth significant figure. The time scale for the calculation of the orbits in Figs. 2 and 3 was several times  $10^9$  yr while the time scale for each surface of section was several times  $10^{10}$  yr.

Orbits were started at  $r = 8.5$  kpc,  $z = 0$  with initial radial and vertical velocities less than 50 km/s. The numerical experiments show that the relationship between  $L_c$  and  $\mathcal{M}_n$  is nearly a straight line which is shown in Fig. 1. Note that the slope of the straight line is higher when  $c$  is larger. Each line divides the  $[L_c - \mathcal{M}_n]$  plane in two parts. Orbits with starting conditions on the left part (which also contains the line) are chaotic while orbits with starting conditions on the right part are regular. It seems interesting that the slope of the line is different for very low values of  $L_z$ . We shall come to that point later on in this section.

Figure 2 shows a chaotic orbit when  $L_z = 20$ ,  $\mathcal{M}_n = 400$  while Fig. 3 shows a regular orbit when  $L_z = 35$ ,  $\mathcal{M}_n = 400$ .

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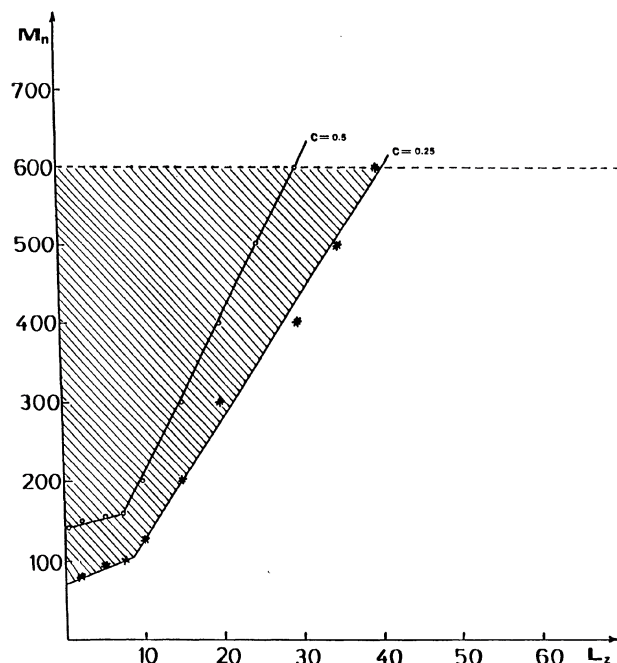


FIG. 1. Relationship between  $L_z$ - $M_n$ . The unit for  $L_z$  is 10 (km/s) kpc while the unit for the mass is given in the text.

The corresponding  $r-u_r$  surfaces of section are shown in Figs. 4 and 5, respectively. We observe, that the consequents of the chaotic orbit form a stochastic layer while those of the regular orbit form five well-defined islands. This can be clearly seen in Fig. 5(a) which is a part of Fig. 5 showing one of the islands near the  $r$  axis. The value of the energy integral for both orbits is  $E = -1030$  while  $c = 0.25$  kpc.

An explanation for the linear relationship between  $L_z$  and  $M_n$  will now be provided using numerical evidence together with some elementary theory. Our numerical experiments show that, near the nucleus, the total  $F_z$ -force component is dominant (it is about 20 times greater than that at  $r = 8.5$  kpc, for the same value of  $z$ ) and in fact is nearly

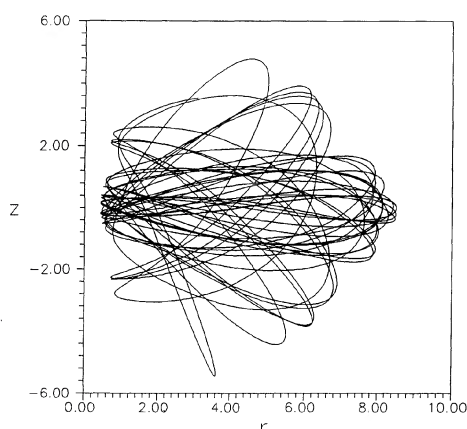


FIG. 2. A chaotic orbit when  $L_z = 20$ ,  $M_n = 400$ , and  $c = 0.25$  kpc.

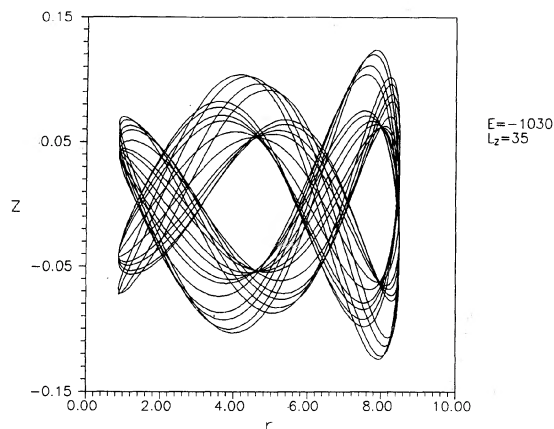


FIG. 3. A regular orbit when  $L_z = 35$ ,  $M_n = 400$ , and  $c = 0.25$  kpc.

equal to that produced by the nucleus itself. Even when  $z$  is of order of unity, about 80% of the total  $F_z$  force is due to the nucleus. Furthermore, it is the nuclear force that causes the scattering to the halo producing chaotic motion. In fact, on approaching the nucleus, there is a change in the star's momentum in the  $z$  direction given by

$$\mathcal{M} \Delta u_z = \langle F_z \rangle \Delta t, \quad (4)$$

where  $\mathcal{M}$  is the mass of the star,  $\langle F_z \rangle$  is the total (i.e., nuclear) average  $z$  force while  $\Delta t$  is the duration of the encounter. Here we observe on a completely empirical basis that the star's rise into the halo proceeds in each case cumulatively, a little more with each successive pass by the nucleus, rather than with a single "dramatic" encounter. We assume that the star goes to the halo when the total change in the momentum in the  $z$  direction, after  $n$  ( $n > 1$ ) encounters, is of order of  $\mathcal{M} u_\phi$ , where  $u_\phi$  is the tangential velocity of the star near the nucleus. Therefore, we shall assume that

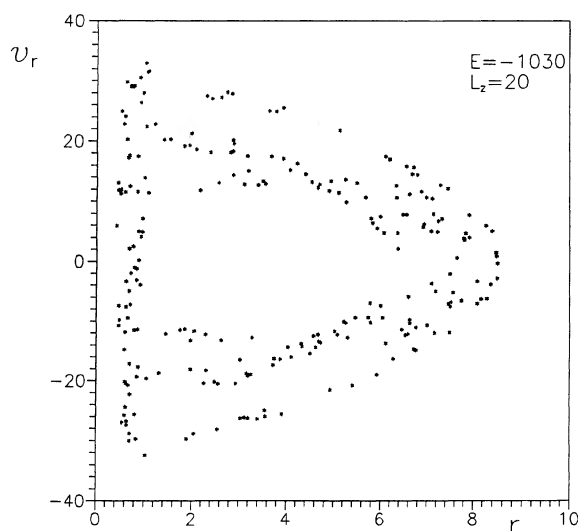


FIG. 4. The  $r-u_r$  surface of section corresponding to the chaotic orbit of Fig. 2.

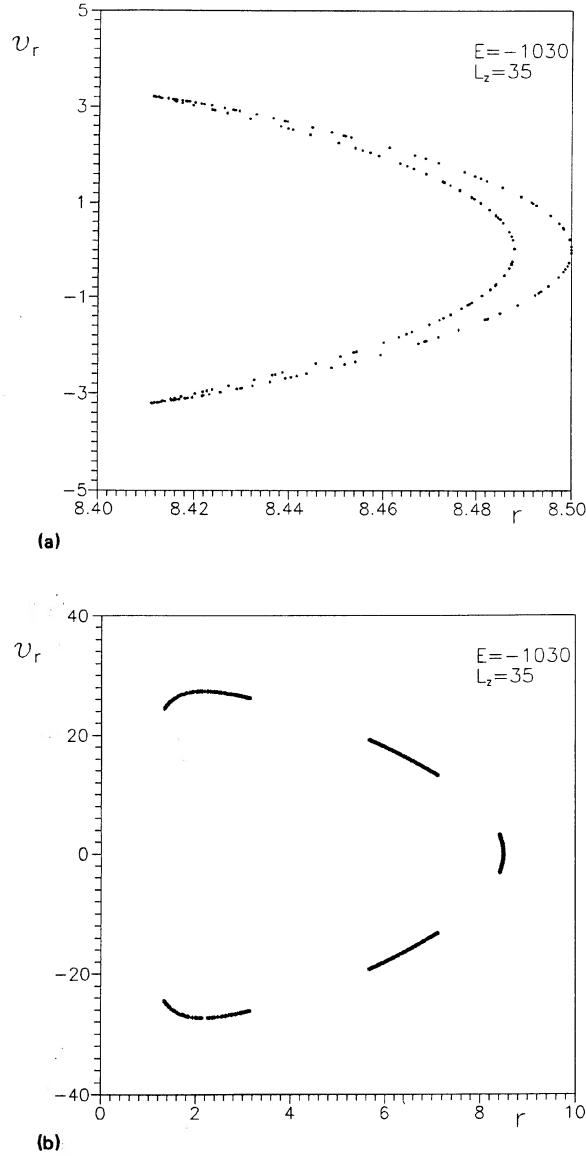


FIG. 5. (a) The  $r-u_r$  surface of section corresponding to the regular orbit of Fig. 3. (b) Part of (a) showing one of the five islands.

$$\mathcal{M} \sum_{i=1}^n \Delta u_{zi} \approx \langle F_z \rangle \sum_{i=1}^n \Delta t_i. \quad (5)$$

Setting  $\mathcal{M} \sum_{i=1}^n \Delta u_{zi} = \mathcal{M} u_{\varphi} = \mathcal{M} L_{zc}/r$ ,  $\sum_{i=1}^n \Delta t_i = T_c$ ,  $\langle F_z \rangle / \mathcal{M} = \langle F_n \rangle$ ,  $\mathcal{M} = 1$  in Eq. (5), we find

$$L_{zc}/r \approx \langle F_n \rangle T_c. \quad (6)$$

For a given value of  $r$  (the particular value is irrelevant as soon as  $r$  is of order of unity) the quantitative behavior of  $F_z$  is shown in Fig. 6. For small values of  $z$ ,  $F_z$  behaves as an harmonic oscillator force while for larger values of  $z$  falls as  $\mathcal{M}_n/z^{3/2}$ . Expanding the potential of the nucleus around the point  $r=r_0$ ,  $z=0$ , the square of the frequency of the harmonic motion in the  $z$ -direction is  $\mathcal{M}_n/(c^2 + r_0^2)^{3/2}$ . Substi-

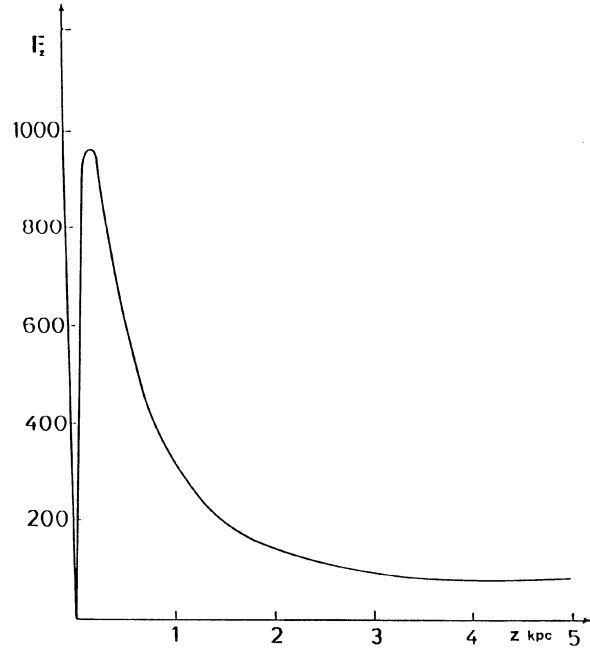


FIG. 6. A plot of  $z-F_z$  when  $r=c=0.25$  kpc,  $\mathcal{M}_n=300$ .

tuting the harmonic motion law in Eq. (6) with  $r=r_0=z=c$  we find

$$\mathcal{M}_n \approx c L_{zc} \sqrt{8/T_{ca}}, \quad (7)$$

where, for convenience, we have written  $T_{ca}$  instead of  $T_c$ . If we set  $\langle F_z \rangle = \mathcal{M}_n/z^{3/2}$ ,  $r=c$ ,  $z=kc$  ( $k \geq 2$ ) into Eq. (6) we obtain

$$\mathcal{M}_n \approx L_{zc} \sqrt{ck^3/T_c}. \quad (8)$$

Formulas (7) and (8) give the observed linear relationship between  $L_{zc}$  and  $\mathcal{M}_n$ . Relation (8) also explains the dependence of the slope on the scale of the nucleus. The larger is  $c$  the higher is the slope. This is because  $T_c$  is observed to be about the same for the same scale of the nucleus. On the other hand, when  $L_{zc}$  is very small the star goes much closer to the nucleus, so that relation (7) must be used. Note that for the nucleus we always have  $c < 1$  and, because of the smaller mass of the nucleus  $T_{ca} > T_c$ . Therefore, a lower slope results for the line in the region of very low angular momenta or, what is equivalent, for very low nuclear masses (see Fig. 1).

A large number of orbits were computed, using different starting distances from the center of the galaxy, with the same low values of the initial radial and vertical velocities as was mentioned before. Orbits starting at the same distance  $r$ , are considered to belong to the same group. We have chosen this empirical alternative because the alternative, a self-consistent treatment with the energy as an additional parameter, in a scattering setting and in a galaxy potential, is beyond the scope of this paper. Some of these ideas may indeed be incorporated into future work. Our numerical results suggest that, for each group of orbits, there is a linear relationship between  $L_{zc}$  and  $\mathcal{M}_n$  similar to that shown in Fig. 1.

The orbits in each group have the same (for a fixed mass of the nucleus) or about the same value of the energy integral.

The small change in the energy is required in order to cover a certain range of values of  $\mathcal{M}_n$ , a certain range of values of  $L_z$  and also to keep the initial radial and vertical velocities less than 50 km/s. As an example, let us consider the group of orbits with  $r = 8.5$  kpc,  $c = 0.25$  (see Fig. 1). In order to cover the range  $100 \leq \mathcal{M}_n \leq 600$ ,  $10 \leq L_z \leq 45$  we must use a range of energies from  $E = -1000$  to  $E = -1038$ . This means a change in the energy less than 4%. Therefore the orbits in each group can be considered as isoenergetic.

It is also of interest to compare  $L_{zc}$  with the circular angular momentum  $L_{z0}$ . Such a comparison, for three values of the mass of the nucleus when  $c = 0.25$  kpc, is made in Table 1. Orbits start at three different distances from the galactic center and, according to the above, belong to three different groups of orbits. It is important to observe, that the ratio  $L_{zc}/L_{z0}$  increases with decreasing  $r$ . This means that more stars (initially in the plane) are leaving the galactic plane resulting from chaos in the central region than for larger values of  $r$ . This phenomenon could be taken as a first step for the explanation of the existence of a chaotic bulge in most disk-shaped galaxies or, perhaps, the presence of chaotic orbits "contaminating" some more conventional stellar dynamical bulge.

### 3. CHAOS PRODUCED BY THE BULGE

The preceding analysis applies to the transition from regular to chaotic motion resulting from the presence of the galactic nucleus. It is natural then to ask what happens if a galaxy model with a bulge component instead of a nucleus is used. The results for the same disk-halo potential with a spherical bulge with a scale length  $c_b = 1$  kpc, without a nucleus are shown in Fig. 7. We see again a straight line relation between  $L_{zc}$  and  $\mathcal{M}_b$ . Thus, it is evident, that the bulge produces similar results to those produced by the nucleus with one significant difference: in order to observe chaotic motion the scale length of the bulge must be of the order of unity. Numerical experiments show that when  $c_b \geq 1.3$  kpc, no chaotic behavior is seen. This result seems to be independent of the mass of the bulge. Therefore, we can say, that chaos is produced by a bulge only with a relatively high mass concentration.

For orbit calculations in the central regions of galaxies, it is usual to use a perturbed harmonic oscillator potential (see Contopoulos & Zikides 1981; Caranicolas & Barbanis 1982; Caranicolas & Gousidou-Koutita 1987; Caranicolas 1989; Contopoulos & Barbanis 1989; Barbanis 1990; Caranicolas 1990; Deprit 1990). Of course this approximation gives only the local dynamical properties of the galaxy in the central region. As an experiment we decided to replace the total  $F_z$  force, near the center of our galaxy model, with the force of an equivalent harmonic oscillator. The harmonic motion

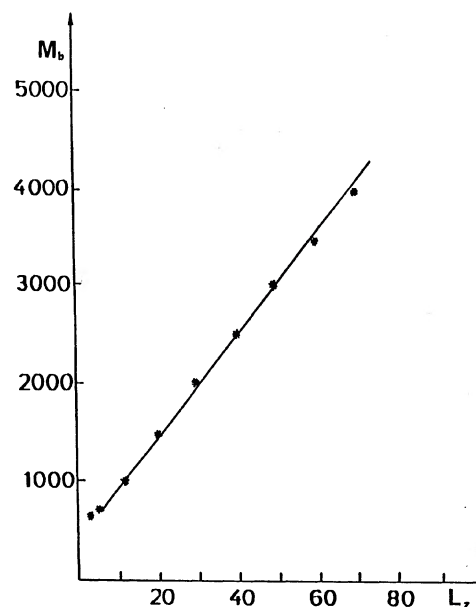


FIG. 7. Relationship between  $L_{zc} \sim \mathcal{M}_b$  when  $c_b = 1$  kpc.

was introduced in the numerical integration when  $d = (r^2 + z^2)^{1/2} \leq 0.75$  kpc. The results, for the adopted disk-halo potential with a nucleus or bulge component, show that the orbits look very similar for several revolutions while the general behavior of orbits (regular or chaotic) is the same in both cases. We conclude that this analytical approximation may be useful in some cases.

### 4. DISCUSSION

It is evident that there is a well-defined transition from regular to chaotic motion in our galaxy model, for a given mass of the disk-halo component. This transition is nearly a straight line connecting points on the  $L_z \sim \mathcal{M}_n$  plane. The slope of this line increases with the scale of the nucleus. The physical interpretation of this result is that, a nucleus of the same mass but with greater concentration (smaller  $c$ ), scatters stars with higher values of the angular momentum. It is also evident that the above straight-line relation can be derived using simple analytical calculations. For this derivation, we have assumed that the deflection into a vertical chaotic orbit happens after  $n$  passes with correlated deflections. This choice was made by observing the behavior of the orbits in our numerical experiments and also by taking into account the smoothness of the  $F_z$  force (see Fig. 6).

Similar results were obtained when a bulge instead of a nucleus was used. The only difference was that, in order to observe chaotic motion, a dense bulge with  $c_b \leq 1.3$  was necessary.

A comparison of the critical angular momentum  $L_{zc}$  to the local circular angular momentum  $L_{z0}$ , shows that the ratio  $L_{zc}/L_{z0}$  increases as we approach the center of the galaxy. For a galaxy with a nearly flat rotation curve, this variation mainly reflects the variation of the galactocentric distance  $r$ . Therefore stars moving near the center can move easily go to higher  $z$ . This could be connected with the pres-

TABLE 1. A comparison between  $L_{zc}$  and  $L_{z0}$  for different distances from the galactic center.

| $\mathcal{M}_n$ | $r = 8.5$ |          |                 | $r = 5$  |          |                 | $r = 2.5$ |          |                 |
|-----------------|-----------|----------|-----------------|----------|----------|-----------------|-----------|----------|-----------------|
|                 | $L_{zc}$  | $L_{z0}$ | $L_{zc}/L_{z0}$ | $L_{zc}$ | $L_{z0}$ | $L_{zc}/L_{z0}$ | $L_{zc}$  | $L_{z0}$ | $L_{zc}/L_{z0}$ |
| 200             | 15        | 190      | 0.08            | 15       | 80       | 0.19            | 8         | 31       | 0.26            |
| 300             | 20        | 194      | 0.10            | 20       | 85       | 0.24            | 12        | 37       | 0.31            |
| 400             | 30        | 196      | 0.15            | 25       | 90       | 0.28            | 15        | 40       | 0.38            |

ence of bulges in the central regions of disk galaxies with prominent nuclei and suggests that such galactic bulges may be chaotic, i.e., with isotropic velocity dispersions.

Our numerical experiments show that, near the center, the total  $F_z$  force is entirely due to the nucleus or the bulge component and behaves like that of an harmonic oscillator. This

is an indication that, under certain conditions, the harmonic oscillator approximation can give good results for the motion in the central regions of real galaxies.

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