

# Magnetic fields in the Milky Way neighbourhood as deduced from warps in spiral galaxies

E. Battaner, J.L. Garrido, M.L. Sanchez-Saavedra, and E. Florido

Departamento de Física Teórica y del Cosmos, Facultad de Ciencias, Universidad de Granada, E-18002 Granada, Spain

Received January 2, accepted March 15, 1991

**Abstract.** It is shown that warps of spiral galaxies are not randomly oriented in the Milky Way neighbourhood. By adopting a previous model, in which warps are produced by intergalactic magnetic fields, and considering all northern hemisphere warped edge-on NGC spiral galaxies, an analysis of the intergalactic magnetic field in the  $\sim 100$  Mpc neighbourhood of our Galaxy is carried out. At the 100 Mpc scale the magnetic field is still rather homogeneous, having a direction given by ( $\alpha = 289^\circ$ ,  $\delta = 8^\circ$ ), but a characteristic scale of about 25 Mpc is found, inside which the dispersion is very low. The region containing the Virgo Cluster has a direction of the magnetic field different from the direction found in adjacent regions.

**Key words:** Galaxies: spiral – intergalactic medium – magnetic field – Universe (the): structure of

## 1. Introduction

Several recent measurements of high magnetic fields in clusters and intercluster regions may indicate that they play an important role in the large structure of the Universe. Kronberg (1987) detected  $0.5 \mu\text{G}$  in Coma; Vallée et al. (1986) and Vallée et al. (1987),  $0.2 \mu\text{G}$  in A2319; Kim et al. (1989),  $0.3\text{--}0.6 \mu\text{G}$  in a region between the Coma Cluster and A1367, in the Coma Supercluster; Vallée (1990a),  $1.5 \mu\text{G}$  in the Virgo Supercluster. Despite these large values at the cluster and supercluster scales, the field seems to be very low on a cosmological scale: less than  $10^{-12}$  G (Vallée 1990b).

In fact, diffuse non-thermal radio emissions clearly indicate the presence of large magnetic fields in some clusters, probably as the result of hydromagnetic dynamos in the intraclusters gas (Jaffe 1980; Roland 1981; Ruzmaikin et al. 1988; Ruzmaikin et al. 1989). The stripping of galactic gas from active galaxies, remnants of old radio galaxies and primordial magnetic fields (Perola & Reinhardt 1972; Rees 1987; Rees & Reinhardt 1972) could provide the seed field for the dynamo (Ruzmaikin et al. 1989). Ruzmaikin & Sokoloff (1977) estimated an upper limit of  $5 \cdot 10^{-10} \Omega^{-1}$  G for a uniform cosmological field, but if the field were chaotic, as noted by Rees & Reinhardt (1972), then Ruzmaikin et al. (1989) found an upper limit of  $10^{-6}$  G. Perola & Reinhardt (1986) also interpreted the absence of X-ray inverse Compton scattering as evidence of magnetic fields above  $0.1 \mu\text{G}$  in clusters (see Vallée et al. 1986).

Send offprint requests to: E. Battaner

Another indirect theoretical argument favouring large intergalactic magnetic fields comes from the work of Battaner & Sánchez-Saavedra (1986) and Battaner et al. (1988). They integrated the MHD equations for the interstellar magnetic field in several nearby galaxies, taking boundary conditions at the centre.  $\mathbf{B}$  was then calculated; the asymptotic value of the field modulus for large radii should not be very far from the intergalactic field modulus. Despite the fact that very different profiles were obtained for low and intermediate radii,  $\mathbf{B}(\infty)$  was for the considered galaxies of the order of the values quoted above.

Battaner et al. (1990) proposed a theoretical model in which intergalactic magnetic fields are responsible for warps. Magnetic strengths of the order of  $10^{-8}$  G may bend the disks, and values of about  $1 \mu\text{G}$ , as reported by Vallée (1990a), must produce a substantial deformation in the peripheric orbiting plasma. It was also claimed that the distribution of warps is not random and the possibility of obtaining the direction of the intergalactic magnetic field in the Milky Way neighbourhood was then explored, though from a very short sample of warped galaxies. Other observational tests which could support this theoretical model were considered by Battaner et al. (1990b).

Sánchez-Saavedra et al. (1990) presented a list of all warped northern NGC highly inclined spirals. This complete larger sample (42 optical warps) provides a better possibility of obtaining the direction of the intergalactic magnetic field, and perhaps its gradient, characteristic scale, etc. Warps have also been detected in SO (Galleta 1990) and ellipticals, but here we restrict ourselves to spiral galaxies.

The objectives of the present paper are: a) adopting the Battaner et al. (1990) model and the sample of Sánchez-Saavedra et al. (1990), to obtain the intergalactic magnetic field in the Milky Way neighbourhood. b) to obtain the ratio of the random and the uniform field component at different scales. The distances considered here are those of the most distant galaxies in the sample (typically  $4000 \text{ km s}^{-1}$ ,  $z \simeq 0.01$ ,  $r \simeq 50$  Mpc).

This study could provide some clues about the origin of intergalactic magnetic fields. In particular the hypothesis of the turbulent dynamo which effectively generates chaotic magnetic fields and which has been reviewed by Ruzmaikin et al. (1989), could be explored from an additional observational point of view.

## 2. Theory and sample

Following Battaner et al. (1990), once a warped edge-on spiral is observed the direction of the intergalactic magnetic field producing the distortion can be approximately determined. Let us

call  $B_i$  the “individual” magnetic field strength “around” the  $i$ th galaxy. The direction of  $B_i$  should approximately lie in the plane of the sky (perpendicular to the line of sight) and form an angle of  $45^\circ$  with the rotation axis (i.e. also  $45^\circ$  with the mean galactic plane). There are however 4 possible directions at  $45^\circ$  from the mean galactic plane. Only 2 of them can coincide with the direction of  $B_i$ ; these are the vectors 1 and 3 in Fig. 1, those which are “closing the rising warps”. An ambiguity remains, as if  $b_i$  is the unitary vector in the direction of  $B_i$ ,  $-b_i$  is also a possible direction. The modulus of  $B_i$  is not determined here. The amount of warping is surely a function of this modulus, but the exact relation would require a more sophisticated model, which is beyond the scope of this paper. Here we consider only the direction of  $B_i$  as specified by the unitary vector  $b_i$ .

Simple spherical trigonometric derivations provide the coordinates  $(\alpha, \delta)$  of  $b_i$  as a function of the coordinates  $(\alpha_0, \delta_0)$  of the  $i$ th galaxy and the angle  $\beta$  (the position angle of  $b_i$ ). See Fig. 1.

$$\sin \delta = \cos \delta_0 \cos \beta$$

$$\sin(\alpha_0 - \alpha) = \frac{\sin \beta}{\cos \delta}.$$

$b_i$  may not be exactly in the plane of the sky. The error introduced when this approximation is used is evaluated in Appendix A. Also  $b_i$  may not be exactly at  $45^\circ$  with the rotation axis. The error introduced when this approximation is used is evaluated in Appendix B.

The sample of galaxies, adopted from Sánchez-Saavedra et al. (1990), is shown in Table 1. We give in the different columns the following quantities:

1: NGC name.

2: O, E – The first figure “O” is a parameter given by Sánchez-Saavedra et al. (1990) to roughly give the degree of warp in the O-plates of the POSS (0, no warp detected; 1, barely perceptible warp; 2, clear warp; 3, very clear warp). The second figure “E” is similar but for the E-plates of the POSS.

3 & 4: Galaxy coordinates (de Vaucouleurs et al. 1976).

5: Radial velocity (Huchra et al. 1990; Palumbo et al. 1983).

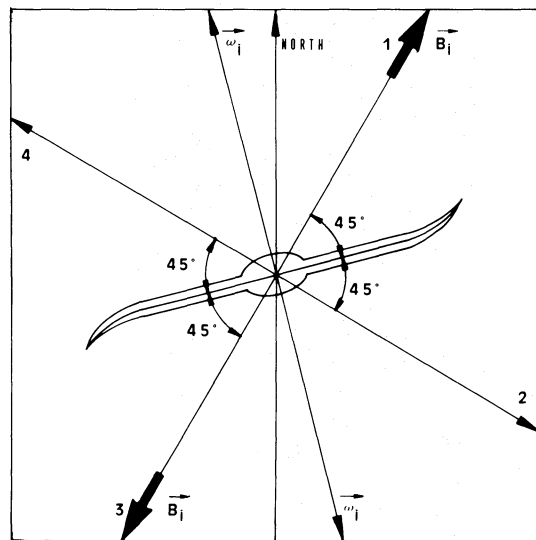


Fig. 1. Possible directions of the intergalactic magnetic field for an edge-on warped spiral

6 & 7: Components of  $B_i$ .

8: Group- The galaxies lie in very different zones in space, so they were classified in groups defined in Table 2.

The interval  $6^h < \alpha_0 < 18^h$  used in defining those groups seems too large, but in practice nearly all galaxies in the sample lie between  $\alpha = 9^h$  and  $\alpha = 15^h$ , with very few exceptions, and those exceptions have a right ascension very close to either 9 hours or 15 hours. Note also that all considered galaxies belong to the northern hemisphere. Group 1 contains the Virgo Supercluster ( $\alpha = 12^h 48^m$ ,  $\delta = 12^\circ 40'$ ;  $b = 74^\circ$ ,  $l = 284^\circ$ ;  $r = 16$  Mpc for  $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). Together with groups 2 and 3, it has the largest number of galaxies. Other groups with very few galaxies cannot be considered in a statistical analysis, but are too far away to be merged with groups 1, 2, or 3. However a calculation was also carried out for these poorer groups.

### 3. Mathematical treatment

We know  $b_i$ , the unitary vectors with the direction of  $B_i$  around each galaxy, and must obtain the average direction for each group and all galaxies. The problem is that we do not know whether  $b_i$  or  $-b_i$  represents the actual direction of  $b$ , or in other words, the “polarity” remains unknown. We define the “polarity” as having the value 1 if the true  $B_i$  points towards the northern hemisphere, and 0 if  $B_i$  points towards the southern hemisphere. The numerical procedure for determining the mean direction of the magnetic field must provide a way of determining the polarities for each galaxy. It is clear that the mean direction will in turn have a polarity, which will remain unknown. Figure 2 represents an ideal set of  $b_i$ . For a given polarity of the mean direction the individual polarities can be determined, but the polarity of the mean direction itself, cannot. Three numerical methods have been developed:

#### 3.1. Least squares

Let us call  $p_i$ , the polarity matrix. We calculate the mean direction  $b$  for every value of the polarity matrix, i.e. for every combination

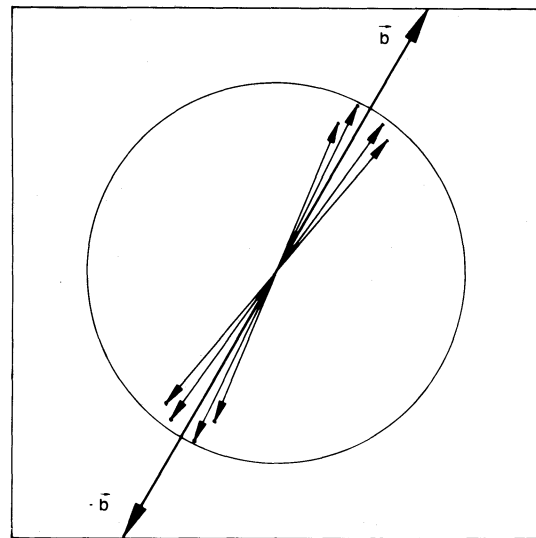


Fig. 2. Ideal plot showing that  $b$  and  $-b$  can be determined but not which of them is the actual direction of  $B$

**Table 1.** Galaxies and groups of galaxies considered in this paper

NGC	O, E	Galaxy coordinates		$v$	$B_i$		Group
		$\alpha_0$	$\delta_0$		$\alpha$	$\delta$	
2683	2, 1	8:49.58	33:36.5	415	−51:7.8	56:20.46	1
3044	1, 1	9:51.10	1:49.0	1335	53:17.18	67:55.44	1
3245A	2, 1	10:24.21	28:53.8	1358	−51:14.51	58:9.57	1
3365	2, 1	10:43.7	2:5	986	−103:39.54	68:54.5	1
3454	1, 1	10:51.83	17:36.7	1153	31:48.18	64:19.22	1
3510	1, 1	11:1.02	29:9.3	710	−59:46.54	51:42.59	1
3600	1, 1	11:13.0	41:52	719	−68:1.51	31:46.29	1
3628	3, 3	11:17.66	13:52.1	847	60:31.11	52:40.53	1
3692	1, 1	11:25.8	9:41	1766	71:54.7	44:11.22	1
4206	1, 0	12:12.74	13:18.2	701	−73:44.8	43:28.59	1
4565	2, 1	12:33.86	26:15.6	1227	12:58.45	63:40.39	1
5560	3, 2	14:17.56	4:13.3	1733	117:55.44	56:45.43	1
2591	1, 0	8:30.71	78:11.9	1331	−57:18.42	11:46.7	2
2654	3, 1	8:45.18	60:24.4	1355	−71:23.20	27:39.16	2
2820	1, 1	9:17.71	64:28.3	1574	−72:8.6	22:8.48	2
4013	3, 3	11:55.96	44:13.7	835	−106:0.56	−13:21.3	2
4010	2, 1	11:56.05	47:32.4	905	−104:35.34	−14:11.20	2
5229	2, 0	13:31.98	48:10.3	365	−14:36.45	35:16.55	2
5301	1, 1	13:44.36	46:21.4	1503	7:4.39	42:2.48	2
5907	1, 1	15:14.61	56:30.4	650	−43:0.56	−1:6.13	2
2634A	1, 0	8:43.15	74:7.4	2269	−64:45.16	15:19.39	3
3735	1, 1	11:33.06	70:48.5	2696	−89:9.53	2:37.23	3
4256	1, 0	12:16.35	66:10.6	2577	−81:16.43	2:1.5	3
5529	2, 2	14:13.47	36:27.4	2878	−41:8.50	19:52.23	3
5777	2, 0	14:50.0	59:10	2126	−51:47.46	−2:33.37	3
5981	2, 1	15:36.86	59:33.2	2611	−34:55.22	0:30.24	3
2424	1, 1	7:37.27	39:21.0	3223	−113:28.33	39:18.16	4
2752	1, 0	9:2.89	18:32.4	4022	−139:20.14	−15:8.59	4
3221	2, 2	10:19.59	21:49.1	4085	−85:13.21	51:7.58	4
4617	3, 2	12:38.8	50:42	4639	61:47.57	26:36.25	5
5965	3, 1	15:32.8	56:52	3416	47:12.8	32:59.28	5
2830	1, 1	9:16.65	33:56.9	6237	−0:51.28	48:45.28	6
4892	1, 1	12:57.6	27:10	5898	−59:5.4	28:59.0	6
6045	2, 1	16:2.88	17:53.5	9913	−41:24.57	−33:5.0	7
7640	1, 1	23:19.72	40:34.2	369	−60:10.34	−36:46.13	8
1560	2, 1	4:27.06	71:46.2	0	−104:49.5	18:2.57	9
0522	1, 1	1:22.14	9:44.0	2809	−68:25.25	−5:54.48	10
0876	1, 0	2:15.17	14:17.4	3860	−53:26.0	−10:39.21	10
7463	2, 0	22:59.38	15:42.8	2445	−87:12.56	−47:30.55	10

of  $p_i$ 's – for instance (1, 1, 0, 1, 0, ...) – as

$$\mathbf{b} = \frac{\sum_i \mathbf{b}_i}{|\sum_i \mathbf{b}_i|}$$

and find out which of all combinations gives the minimum value of the functions  $S$

$$S = \sum_i (\cos^{-1}(\mathbf{b} \cdot \mathbf{b}_i))^2.$$

The solution is therefore  $\mathbf{b}$  and the polarity matrix which renders  $S$  minimum. It is then necessary to repeat the same calculation  $2^N$  times,  $N$  being the number of galaxies. Actually not all  $2^N$  possibilities need to be explored, because of the indetermination of the polarity of  $\mathbf{b}$  ( $-p_i$  gives  $-\mathbf{b}$  if  $p_i$  gives  $\mathbf{b}$ ). Just half of these possibilities need be taken into account, say  $2^{N-1}$ . For large  $N$  no computer is able to carry out this calculation. In our case, we were able to calculate  $\mathbf{b}$  by this method for each group, but not for the complete set of the galaxies in the sample.

### 3.2. Convergence of planes

The method follows the following steps:

3.2.1. Any plane is adopted initially (for instance the equator), the final result must be irrespective of this initial choice, and indeed it was. Also arbitrarily, we take one of the resulting hemispheres as the “upper” one, (for instance, the northern hemisphere if the equator was chosen). We initially set all individual polarities so that all  $\vec{b}_i$  point to the “upper” hemisphere. (For instance, if the equator and the northern hemisphere were chosen, we set  $p_i = 1$ ,  $\forall i$ ). See Fig. 3. We then calculate a first value of  $\vec{b}$ , which is called  $\vec{b}(0)$ .

3.2.2. As a new plane, we adopt the plane perpendicular to  $\vec{b}(0)$ . The “upper” hemisphere will now be the hemisphere containing  $\vec{b}(0)$ . We reset the polarities so that now, all polarities point to the “upper” hemisphere. A new mean direction is calculated and called  $\vec{b}(1)$ . The iterative process is continued obtaining  $\vec{b}(2)$ ,  $\vec{b}(3)$ , ... the final solution being  $\vec{b}(\infty)$ . In practice, convergence was obtained in just 3 or 4 steps and the results did coincide with those obtained with the least squares method. In some cases

however,  $\vec{b}(\infty)$  was not completely independent on the initial plane, and two close values were obtained.

### 3.3. Non-warped galaxies

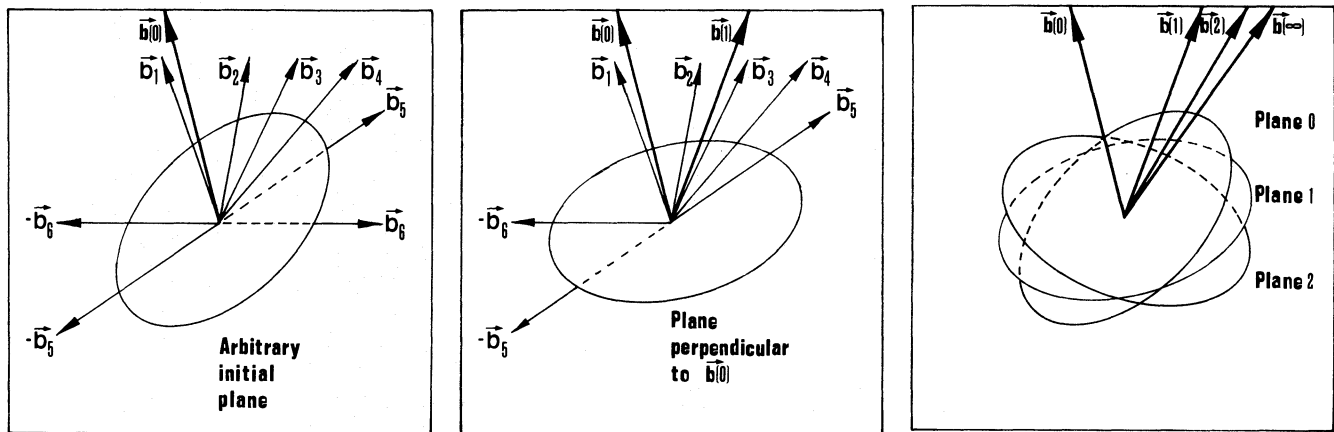
It is shown in Appendix C that the average direction of the position vectors of all non-warped galaxies gives the direction  $\vec{b}$ , if the galaxies in the sample (warped and non-warped) are uniformly distributed in the sky. If this condition is not met, then the zone of highest non-warped/warped galaxies ratio contains the vector  $\vec{b}$ . This method has lower precision and was not used. It is described in Appendix C and mentioned here because it explains the fact (already noticed by Sánchez-Saavedra et al. 1990) that non warped galaxies are not distributed in the sky as are warped galaxies.

## 4. Results: mean direction

The convergence of planes method for all galaxies gave ( $\alpha = 289^\circ$ ,  $\delta = 8^\circ$ ). Let us remember that ( $\alpha = 289^\circ + 180^\circ$ ,  $\delta = -8^\circ$ ) is also a solution. The “least squares” method applied to each group gave

**Table 2.** Geometrical definition of the groups. (1) NGC 5529 as an exception has  $\delta_0 = 36.5^\circ$ , but this value is close to  $45^\circ$  and it was therefore decided not to isolate it in its own group

Group	$\alpha_0$	$\delta_0$	$v$	Number of galaxies
1	$6^h < \alpha_0 < 18^h$	$< 45^\circ$	$< 2000$	12
2	$6^h < \alpha_0 < 18^h$	$> 45^\circ$	$< 2000$	8
3	$6^h < \alpha_0 < 18^h$	$> 45^\circ$ <sup>(1)</sup>	$2000 < v < 3000$	6
4	$6^h < \alpha_0 < 18^h$	$< 45^\circ$	$3000 < v < 5000$	3
5	$6^h < \alpha_0 < 18^h$	$> 45^\circ$	$3000 < v < 5000$	2
6	$6^h < \alpha_0 < 18^h$	$< 45^\circ$	$5000 < v < 7000$	2
7	$6^h < \alpha_0 < 18^h$	$< 45^\circ$	$> 9000$	1
8	$18^h < \alpha_0 < 6^h$	$< 45^\circ$	$< 2000$	1
9	$18^h < \alpha_0 < 6^h$	$> 45^\circ$	$< 2000$	1
10	$18^h < \alpha_0 < 6^h$	$< 45^\circ$	$2000 < v < 4000$	3



**Fig. 3.** The method of “convergence of planes”

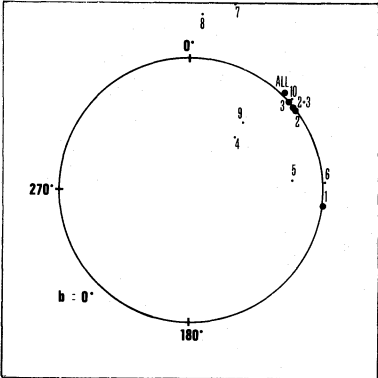
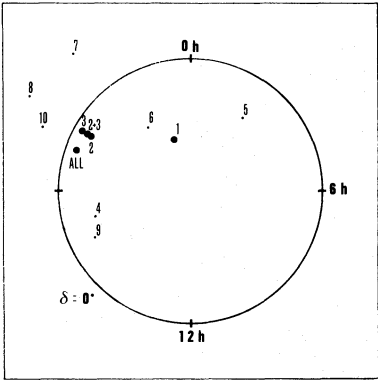
**Table 3.** Direction of the magnetic field in each group and standard deviation. Those values in the box are not meaningful because these groups contain few galaxies

Group	<i>b</i>		Standard deviation	Galactic coordinates	
	$\alpha$ (°)	$\delta$ (°)		<i>b</i> (°)	<i>l</i> (°)
1	342	54	31	−5	106
2	299	13	40	−8	52
3	299	6	21	−12	47
2 & 3	299	10	32	−10	50
4	244	23	35	43	39
5	54	29	7	20	84
6	326	38	24	−11	87
7	319	−33	—	−44	13
8	300	−37	—	−30	4
9	255	18	—	32	38
10	293	−20	22	−18	47
all	289	8	40	−2	43

the results shown in Table 3. Figures 4a and 4b are polar diagrams ( $\alpha, \delta$ ) and ( $l, b$ ) with  $b$  for each group and  $b_{all}$ . Figure 5 shows a projection onto the plane ( $\alpha=0^h, \alpha=12^h$ ) of the space surrounding the Milky Way, taking the radial velocity as a radial coordinate. In each group's zone the coordinates of  $b$  are plotted. Note that  $b_1$  forms just  $18^\circ$  with the  $0^h$ -plane, whilst  $b_2$  forms  $61^\circ$  with this plane. The angle between  $b_1$  and  $b_2$  is  $126^\circ$ . Note also that similar results are obtained for the two adjacent groups 2 and 3.

5. Results: standard deviation

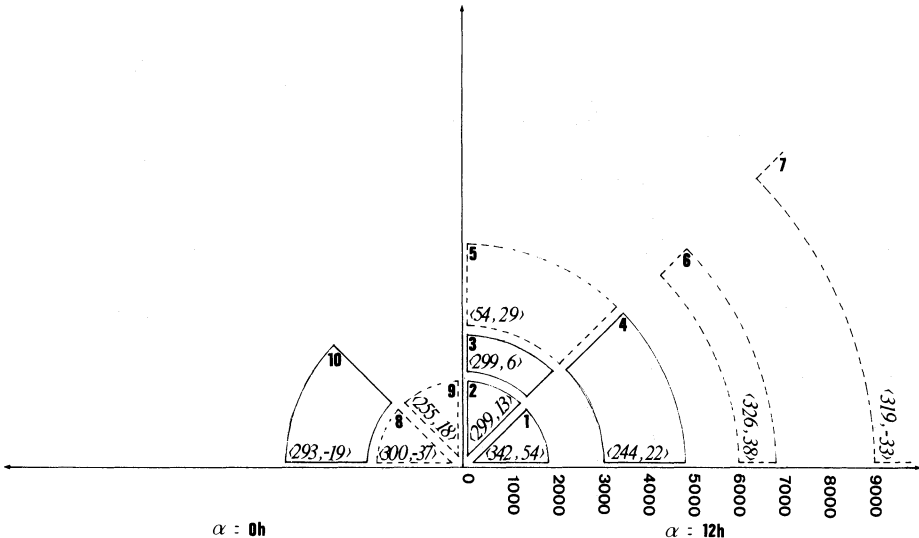
This analysis is important for two main reasons: First, if the standard deviation is low enough we may conclude that the  $b$ -



**Fig. 4a and b.** Plot of the magnetic field vectors for each group (Points 1, . . . 10), group 2 & 3, all galaxies (point “all” with the method of convergence of planes). **a** ( $\alpha, \delta$ ) polar diagram. **b** ( $l, b$ ) polar diagram. Points 4, 5, . . . 10 are less confident

distribution is not random, an interesting cosmological result. Second, the random/uniform magnetic field ratio can be obtained, which has obvious interest.

Standard deviations for each group and for all galaxies are also shown in Table 3. They are of the order of  $30^\circ$ . The value of



**Fig. 5.** Projection of the sky in the ( $\alpha=0^h, \alpha=12^h$ ) plane showing the different spatial regions of the different groups of galaxies. In each region, in parentheses, the direction of the magnetic field



30° is very low indeed and means that the warps are not randomly oriented. Suppose that the  $\mathbf{b}_i$  were uniformly distributed in the sphere, i.e. that all possible directions of  $\mathbf{b}_i$  were equally probable. Then a value of 65.35° would be obtained for the standard deviation. This is demonstrated in Appendix D. No further statistical test is therefore required to conclude that the different warps are not randomly distributed in space.

However, as the directions of the individual magnetic fields  $\mathbf{b}_i$  are related to the directions of the rotation axes  $\omega_i$ , it might be suspected that the coherent alignment of the  $\mathbf{b}_i$  is a consequence of the coherent alignment of the  $\omega_i$ . This could be an intrinsic dynamic property in clusters. Coherent orientation effects of galaxies and clusters have been studied (e.g. Hawley & Peebles 1975; Djorgowski 1986). In this case the mean direction  $\mathbf{b}$  would coincide with  $\omega$ , the mean direction of the  $\omega_i$ 's.

$\mathbf{b}_i$  is obtained from  $\omega_i$  by the formula  $\mathbf{b}_i = \omega_i + \mathbf{A}_i$ , where  $\mathbf{A}_i$  forms an angle of 45° with  $\omega_i$  and is a function of the galaxy coordinates and of the angle  $\beta_i$ . Suppose that the  $\omega_i$ 's are not random, with a mean  $\omega$  and a standard deviation  $\sigma_\omega$  and that  $\mathbf{A}_i$  takes all possible values matching the condition angle  $(\mathbf{A}_i, \omega_i) = 45^\circ$ . By taking averages in the above formula

$$\mathbf{b} = \omega.$$

Table 4 shows that this is not the case for our data. On the other hand  $\sigma_b$  (standard deviation of  $\theta_i$ ,  $\theta_i$  being the angle between  $\mathbf{b}_i$  and  $\mathbf{b}$ ) would become very large. The somewhat complicated calculation is not included here. In the best case, where the distribution of  $\omega_i$ 's is a Dirac's  $\delta$  function (see Fig. 6) it is confirmed that  $\mathbf{b} = \omega$  and that  $\sigma_b = 45^\circ$ . For larger values of  $\sigma_\omega$ , higher values of  $\sigma_b$  are obtained. So  $\sigma_b$  is much larger than the values found here of the order of 30°.

A coherent orientation of rotation axes was actually found in our sample, and will be reported elsewhere, but it is concluded that the non-random orientation of warps is a fact independent of the non-random orientation of axes.

The relative importance of the random component of the magnetic field  $\mathbf{R}_i$  with respect to its uniform component  $\mathbf{U}$  is given by the ratio

$$x = \frac{\langle R_i^2 \rangle}{U^2}.$$

In  $\mathbf{B}_i = \mathbf{U} + \mathbf{R}_i$ , if averages are taken,  $\langle \mathbf{B}_i \rangle = \mathbf{U}$ . This defines both  $\mathbf{U}$  and  $\mathbf{R}_i$ . One of the objectives of this paper is to examine if  $x$  is scale-dependent. Suppose that all moduli  $B_i$  are equal (we only know the directions).  $\mathbf{U}$  has the direction of  $\mathbf{b}$  and  $\theta_i$  is the

**Table 4.** Coordinates ( $\alpha, \delta$ ) obtained as averages of rotation axes and individual magnetic fields, and the angle between both

Group	$\omega$		$\mathbf{b}$		Angle ( $\omega, \mathbf{b}$ )
	$\alpha$ (°)	$\delta$ (°)	$\alpha$ (°)	$\delta$ (°)	
1	-83	11	342	54	117
2	-16	-37	299	13	131
3	-3	22	299	6	79
All	-60	22	289	8	155

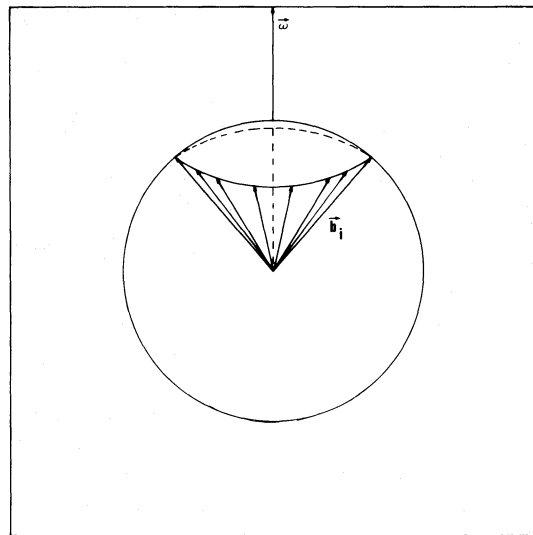
angle between  $\mathbf{b}$  and  $\mathbf{b}_i$ , i.e. between  $\mathbf{U}$  and  $\mathbf{B}_i$ . We further assume that the modulus of  $\mathbf{U}$  is also the same as  $\mathbf{B}_i$ . Then

$$R_i^2 = B_i^2 + U^2 - 2B_i U = 2U^2 - 2U^2 \cos^2 \theta_i = 2U^2 \sin^2 \theta_i.$$

Therefore

$$x = 2 \langle \sin^2 \theta_i \rangle.$$

Results are given in Table 5, where  $x$  is given for each group and for all galaxies, together with some theoretical limiting values for comparison.  $x_{45}$  is the equivalent  $x$  ratio due to the systematic error of always taking the angle  $(\omega_i, \mathbf{b}_i) = 45^\circ$ . It is calculated in Appendix B.  $x_p$  is the equivalent  $x$  ratio due to the systematic error of always taking  $\mathbf{b}_i$  perpendicular to the line of sight. It is calculated in Appendix A.  $x_R$  is the  $x$  ratio if all  $\mathbf{b}_i$  were randomly distributed. It is calculated in Appendix D.



**Fig. 6.** When the distribution function of  $\omega_i$  is a Dirac function, then  $\sigma_b = 45^\circ$

**Table 5.** Values of  $x$ , the random to uniform field ratio, obtained for each group and for all galaxies, compared with  $x_{45}$ ,  $x_p$  and  $x_R$ .  $x_{45}$  is an equivalent ratio  $x$  due to the error of taking always the angle  $(\mathbf{b}, \omega)$  and  $45^\circ$ .  $x_p$  is an equivalent ratio  $x$  due to the error of taking always  $\mathbf{b}$  in the plane of the sky.  $x_R$  is the theoretical value of the ratio  $x$  that would be obtained if all warps were randomly oriented

Group	$x$
1	0.56
2	0.72
3	0.26
2 & 3	0.53
4	0.60
10	1.73
All	0.99
$x_{45}$	0.24
$x_p$	0.25
$x_R$	1.3

## 6. Conclusions

( $x_i < x_R$ ): Indicates that at the typical length scale of a group, about 25 Mpc, there is a well ordered magnetic field. This is in partial contrast, but not in conflict, with some theoretical works (e.g. Jaffe 1980; Ruzmaikin et al. 1989) assuming that magnetic fields in clusters are highly tangled and created by dynamo action in the turbulent wakes behind galaxies moving supersonically through the intergalactic medium. They predict a characteristic scale of about 25 kpc, much shorter than the scale found here. These short scale fields could produce the warps of our galaxies, but then we would have obtained  $x_i \simeq x_R$ . Other sources of magnetic fields may be involved, like those in X-ray clusters. In particular, a tangled field is to be expected in X-ray clusters, because otherwise the high density of about  $2 \cdot 10^{-3} \text{ cm}^{-3}$  would produce a Faraday rotation measure of about  $2 \cdot 10^3 \text{ rad m}^{-2}$ . Probably we are dealing here with lower densities.

Rather interestingly, as a comparison, Vallée (1990a), in order to study the Faraday rotation in the Virgo cluster, divided the radio-sources into zones which are essentially our group 1 and our group 2 (or may be 2 & 3). He also found a different magnetic field in groups 1 and 2. The field strength was found larger in group 1 (the Virgo group), but this cannot be compared with our results, just devoted to directions. The agreement between Vallée's results and those presented here is noticeable. A value of  $1.5 \mu\text{G}$  in Virgo was obtained by this author, assuming that  $\mathbf{B}$  forms an angle of  $30^\circ$  with the line of sight. The angle is  $72^\circ$ . Then, a larger field strength of about  $4 \mu\text{G}$  is required to produce the same rotation measure. It is therefore possible that the magnetic field in the Virgo cluster is still larger than the large value of Vallée.

( $x_{\text{all}} < x_R$ ): This would mean that even at this scale (about 100 Mpc) the magnetic field remains uniform. A chaotic cosmological magnetic field is expected (Rees & Reinhardt 1972) but this must be matched at still higher scales than the typical distances considered here.

( $x_i \simeq x_{4.5}$ ,  $x_i \simeq x_P$ ): At the 25 Mpc scale no random field is detected, at least within the errors inherent in these methods, mainly represented by  $x_{4.5}$  and  $x_P$ .

( $x_i < x_{\text{all}}$ ): The turbulent spectrum of the magnetic field must present a characteristic scale at about 25 Mpc (or perhaps, less). Lengths smaller than these cannot be explored with this technique, because of systematic error limitation and because we would require a richer sky region. In fact the short scale 25 kpc is beyond our scope. A direct confrontation with the dynamo theory (Ruzmaikin et al. 1988) is therefore not possible.

It is interesting to contrast these results with those of Vallée (1991), who has carried out a similar analysis to the one presented here. At the large scale of 100 Mpc he finds no homogeneous fields. We claim however that it is observable, even if the random component is as large as the uniform component, and is therefore close to the limit of detectability. At a cell size of 19 Mpc (close to our values 25 Mpc) he finds no clear evidence of ordered magnetic fields, this possibility not being rejected, however. Its limit value, S/N 3, is to be compared with the inverse of our value  $x$  = random/uniform, in the range 0.5–0.7 (values lower than 0.25 are undetectable because of systematic errors inherent to the method). Just a direct inspection of Table I, indicates a noticeable degree of order which was indeed supported by the mathematical analysis made.

To summarize, regions, with typical sizes of 25 Mpc with a

coherent orientation of warps have been detected. Following the model of Battaner et al. (1990) this would be the characteristic scale of the intergalactic magnetic field. The region containing the Virgo cluster would have magnetic properties differing from those in other adjacent regions.

## Appendix A: Systematic errors due to the assumption that the magnetic field is in the plane of the sky

Suppose that there is only a uniform field  $\mathbf{U}$ , but we just observe its projection on the plane of the sky  $\mathbf{P}_i$ , depending on the galaxy coordinates. Now  $\mathbf{P}_i = \mathbf{U} + \mathbf{R}_i$ ,  $\mathbf{P}_i = \mathbf{U} \cos \varphi_i$ , where  $\mathbf{R}_i$  and  $\varphi_i$  are defined in Fig. 7. One has:

$$\frac{R_i^2}{U^2} = \sin^2 \varphi_i.$$

We define

$$x_P = \frac{\langle R_i^2 \rangle}{U^2} = \langle \sin^2 \varphi_i \rangle$$

and  $\langle \sin^2 \varphi_i \rangle$  is calculated by

$$\langle \sin^2 \varphi_i \rangle = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \sin^2 \varphi \cos \varphi \, d\varphi \, d\alpha \, p(\varphi)$$

where  $p(\varphi)$  is the probability of observing a warp in an edge-on galaxy. If  $\varphi = 90^\circ$  the warp is not produced. It is assumed that  $p(\varphi) = K \cos \varphi$ ,  $K$  being a constant calculated with the normalization condition

$$\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \cos \varphi \, d\varphi \, d\alpha \, K \cos \varphi = 1.$$

Hence

$$K = \frac{1}{\pi^2}$$

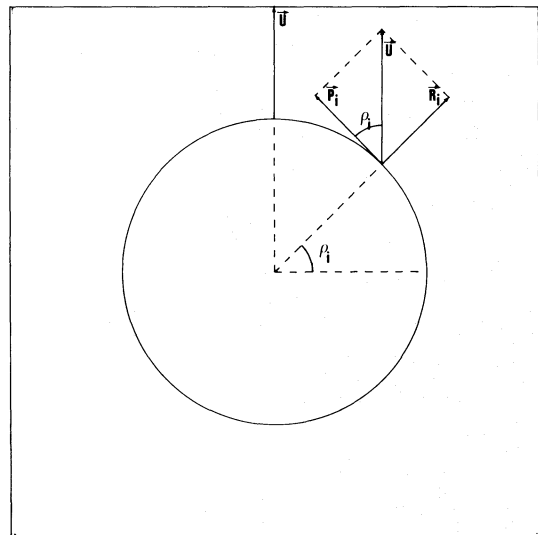


Fig. 7

and therefore

$$x_p = 2\pi \left[ \frac{1}{\pi^2} \right] \int_{-\pi/2}^{\pi/2} \sin^2 \varphi \cos^2 \varphi d\varphi = \frac{1}{4}.$$

#### Appendix B: Systematic error due to the assumption that the magnetic field and the rotation axis form an angle of $45^\circ$

The value of  $x$  was calculated to be  $2\langle \sin^2 \theta_i \rangle$ , assuming that the angle  $(\mathbf{b}_i, \boldsymbol{\omega}_i) = 45^\circ$ . Now we suppose that there is a uniform component  $U$  forming  $45^\circ$  with  $\boldsymbol{\omega}_i$ , and  $\theta_i$  with  $\mathbf{b}_i$ . Not all possible directions of  $\mathbf{b}_i$  are equally probable. The warping force is proportional to the product  $B_y B_z$  (Battaner et al. 1990),  $B_y$  and  $B_z$  being the components of  $\mathbf{B}$  along the observed mean plane of the warped edge-on galaxy and in the direction perpendicular to this plane (both in the plane of the sky). Let us call  $\varepsilon$  the angle between  $\mathbf{B}$  and the observed mean plane. Then the warping force is proportional to

$$B_y B_z = B^2 \sin \varepsilon \cos \varepsilon = \frac{1}{2} B^2 \sin 2\varepsilon.$$

To estimate  $\langle \sin^2 \theta \rangle$  we calculate the average for all values of  $\theta$  (i.e. for all values of  $\varepsilon$ ) taking  $\sin^2 \varepsilon$  as a weight

$$x_{45} = 2\langle \sin^2 \varepsilon \rangle = 2 \frac{\int_0^{\pi/2} \sin 2\varepsilon \left( \varepsilon - \frac{\pi}{4} \right)^2 d\varepsilon}{\int_0^{\pi/2} \sin 2\varepsilon d\varepsilon} = 2 \left( \left( \frac{\pi}{4} \right)^2 - \frac{1}{2} \right) = 0.24.$$

#### Appendix C: Non-warped galaxies method

Suppose first that we have a homogeneous distribution of edge-on spiral galaxies (either warped or not). Suppose that there is only one uniform component of the mean field  $\mathbf{B} \equiv U$ . Let  $\mathbf{r}_i$  be the (unitary) position vector of the galaxy, and  $\mathbf{p}_i$  the (unitary) vector along the position angle. Then the projection of  $\mathbf{B}$  on the plane of the sky is  $\mathbf{B} - (\mathbf{B} \cdot \mathbf{r}_i) \mathbf{r}_i$  and  $\boldsymbol{\omega}_i = \mathbf{r}_i \times \mathbf{p}_i$ . The condition for a galaxy to be observed as non-warped is that both these vectors be either perpendicular or parallel

$$(\mathbf{B} - (\mathbf{B} \cdot \mathbf{r}_i) \mathbf{r}_i) \cdot (\mathbf{r}_i \times \mathbf{p}_i) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  means either 1 or 0. It is easily derived that

$$\mathbf{p}_i \cdot (\mathbf{B} \times \mathbf{r}_i) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

For instance, if  $\mathbf{B} \parallel \mathbf{r}_i$  no warp is produced (or observed). If we consider any slice perpendicular to  $\mathbf{B}$  (see Fig. 8),  $\mathbf{B} \times \mathbf{r}_i$  is tangential to the circumference of the slice, whilst  $\mathbf{p}_i$  for a galaxy in the slice will have any direction. Then the non-warped galaxies will be homogeneously distributed in the slice. Therefore the average value of  $\mathbf{r}_i$  for all slices will also give the direction of  $\mathbf{B}$

$$\Sigma_{\text{non-warped}} \mathbf{r}_i \parallel \mathbf{B}.$$

The probability of finding a warped galaxy will surely decrease for increasing values of  $\varphi$  (as shown in Appendix A) and will become 0 for  $\varphi = 90^\circ$ , but this does not modify the result.

The problem is however that we have a distribution of edge-on spirals which is not homogeneous at all. The above formula must be replaced by another requirement: the zone of the sky with

the highest frequency of non-warped galaxies contains the direction of  $\mathbf{B}$ . For calculating frequencies it is necessary to consider wide zones and the procedure loses sensitivity.

To apply this method, the northern hemisphere was divided into 8 parts (4 quadrants and  $>45^\circ$  or  $<45^\circ$ ) and the relative frequencies shown in Fig. 9 were obtained. A tentative direction of  $\mathbf{B}$  ( $\alpha = 225^\circ$ ,  $\delta = 40^\circ$ ) was then derived. We consider this method less dependable than the other two. But this Appendix explains the fact that non-warped galaxies have a different distribution than warped ones, as already noticed by Sánchez-Saavedra et al. (1990).

#### Appendix D: Variance of directions of individual magnetic fields if they were randomly distributed

Suppose that  $\mathbf{B}_i$  are completely random. Then, the mean  $\mathbf{B}$  will be null. But as we calculate directions and not the modulus of  $\mathbf{B}$ , a certain mean direction (arbitrary and meaningless) will be obtained with both the “least squares” and the “convergence of planes” methods. The standard deviation will become too large, and will be found as shown below. The ratio  $x$  should also become

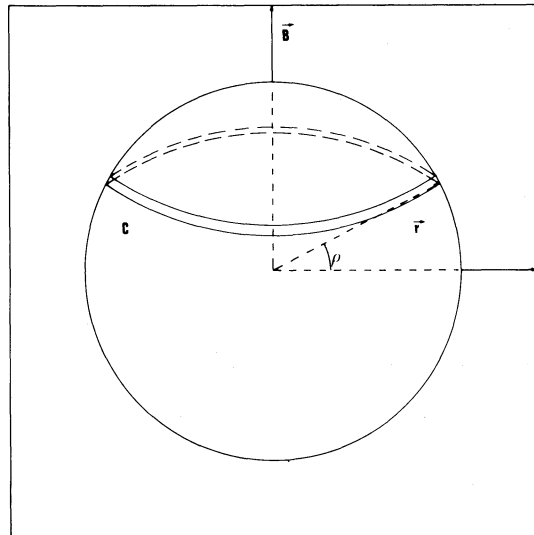


Fig. 8

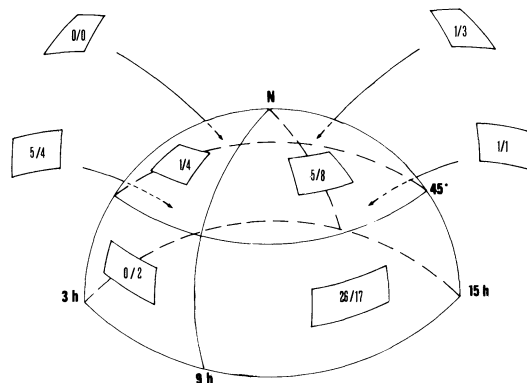


Fig. 9. Relative non-warped/warped frequencies ratio in 8 regions of the sky



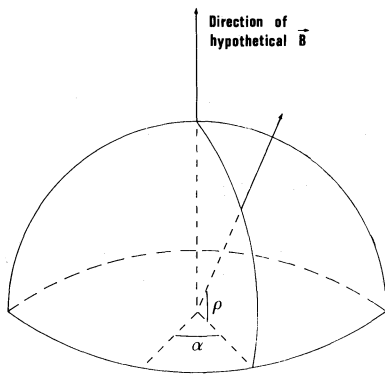


Fig. 10

$\infty$ , as  $U$  is null, but when calculated with  $2\langle \sin^2 \theta \rangle$  another value will be obtained characterizing the randomness of  $b_i$ . This is because both methods place all  $b_i$  in the same hemisphere (see Fig. 10). Then the variance will be

$$\sigma^2 = \frac{\int_0^{2\pi} \int_0^{\pi/2} \left( \frac{\pi}{2} - \varphi \right)^2 \cos \varphi \, d\varphi \, d\alpha}{\int_0^{2\pi} \int_0^{\pi/2} d\varphi \cos \varphi \, d\alpha} = \pi - 2.$$

Therefore  $\sigma = 61^\circ$ . If we now replace  $(\frac{\pi}{2} - \varphi)$  by  $\sin(\frac{\pi}{2} - \varphi) = \cos \varphi$  we will obtain half  $x_R$

$$x_R = 2 \int_0^{\pi/2} \cos^3 \varphi \, d\varphi = \frac{4}{3} = 1.3.$$

## References

- Battaner E., Sánchez-Saavedra M.L., 1986, *ApJ* 302, 450  
 Battaner E., Florido E., Sánchez-Saavedra M.L., 1988, *ApJ* 331, 116  
 Battaner E., Florido E., Sánchez-Saavedra M.L., 1990, *A&A* 236, 1  
 Battaner E., Florido E., Sánchez-Saavedra M.L., 1991, in: *Warped Disks and Inclined Rings around Galaxies*, eds. S. Cassertano, F. Briggs, P. Sackett, Cambridge University, Cambridge  
 Djorgovski S., 1986, in: *Nearly Normal Galaxies*, ed. S.M. Fabler, Springer, Berlin Heidelberg New York, p. 277  
 Florido E., Prieto M., Battaner E., Mediavilla E., Sánchez-Saavedra M.L., 1991, *A&A* 242, 301  
 Galleta G., 1990, in: *Warped Disks and Inclined Rings around Galaxies*, eds. S. Cassertano, F. Briggs, P. Sackett, Cambridge University, Cambridge  
 Hawley D., Peebles J., 1975, *ApJ* 80, 477  
 Huchra J.P., 1991, Radial velocity catalogue (in preparation)  
 Jaffe W., 1980, *ApJ* 241, 925  
 Kim K.T., Kronberg P.P., Giovanini G., Venturi T., 1989, *Nat* 341, 720  
 Kronberg P.P., 1987, in: *Interstellar Magnetic Fields*, ed. R. Beck, R., Graves, Springer, Berlin Heidelberg New York, p. 86  
 Palumbo G.G.C., Tanzella-Nitti G., Vettolani G., 1983, *Catalogue of radial velocities of galaxies*, Gordon and Breach, New York  
 Perola G.C., Reinhardt M., 1972, *A&A* 17, 432  
 Rees M.J., 1987, *Q. Jl. R. Astr. Soc.* 28, 197  
 Rees M.J., Reinhardt M., 1972, *A&A* 42, 151  
 Roland J., 1981, *A&A* 93, 407  
 Ruzmaikin A.A., Shukurov A.M., Sofoloff D.D., 1988, *Magnetic Fields of Galaxies*, Kluwer, Dordrecht  
 Ruzmaikin A.A., Sokoloff D.D., 1977, *A&A* 58, 247  
 Ruzmaikin A.A., Sokoloff D.D., Shukurov A.M., 1989, *MNRAS* 241, 1  
 Sánchez-Saavedra M.L., Battaner E., Florido E., 1990, *MNRAS* 246, 458  
 Vallée J.P., 1990a, *AJ* 99, 459  
 Vallée J.P., 1990b, *ApJ* 360, 1  
 Vallée J.P., 1991, *A&A* 251, 411  
 Vallée J.P., MacLeod J.M., Broten N.W., 1986, *A&A* 156, 386  
 Vallée J.P., MacLeod J.M., Broten N.W., 1987, *Astrophys. Lett. Commun.* 25, 181  
 de Vaucouleurs G., de Vaucouleurs A., Corwin H.G. Jr., 1976, *Second Reference Catalogue of Bright Galaxies*, Texas Press, Austin