

The beaming model and Hubble diagram of BL Lacertae objects

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Abstract. Based on the orientation of relativistic beaming of BL Lacertae objects, we show that the observed flux density is enhanced by a factor $\delta^{3+\alpha}$, where δ is the bulk relativistic Doppler factor. Thus, the observed apparent magnitude m_v^b must be corrected for the Doppler effect. For 33 BL Lac objects given by Burbidge & Hewitt (1987), the corrected apparent magnitude $m_v^{\text{corr}} - \log z$ diagram (Fig. 1) shows a good correlation.

Key words: BL Lac objects – relativistic beaming – corrected Hubble diagram

1. Introduction

In the standard cosmological model, we have the well-known relation (Weinberg 1972),

$$m_v - M_v = 25 - 5 \log H_0 (\text{km s}^{-1} \text{Mpc}^{-1}) + 5 \log cz (\text{km s}^{-1}) + 1.086(1 - q_0)z, \quad (1)$$

where m_v and M_v are, respectively, the apparent and absolute magnitudes of the object, and z the redshift. If a group of objects have the same absolute magnitude, then we can expect a good statistical relation between magnitude and redshift. Thus, one can find q_0 by fitting the $m_v - z$ relation (1) to the observed $m_v - z$ data.

Can all BL Lac objects be regarded as a category with similar physical properties? Would they have nearly the same absolute magnitude? If so, then we expect a good statistical relation when we plot their magnitudes against their redshifts. In 1987, Burbidge & Hewitt plotted 28 BL Lac objects on a single apparent magnitude-redshift diagram, and they found a very large scatter of points in their Fig. 1 (Burbidge & Hewitt 1987). If we apply linear regression analysis to these samples without modifying the data, we find that the correlation coefficient γ between m_v and $\log z$ is only $\gamma = 0.510$, a very low value. The explanation of Burbidge & Hewitt is that the BL Lac objects are not objects of one single type: eight of the 28 BL Lac objects that show emission or absorption line systems with large redshifts

typical of QSOs should be redefined as QSOs and removed from the BL Lac category (Burbidge & Hewitt 1987). On the other hand, a referee (see Burbidge & Hewitt 1987) has suggested to Burbidge & Hewitt that such a reclassification may be misleading, since it would result in two fundamentally different populations: BL Lac objects and QSOs. We note that, in studies based on the relativistic beaming model of BL Lac objects and accounting for the orientation of the beaming, the observed flux density is enhanced by a factor $\delta^{3+\alpha}$, where δ is the bulk relativistic Doppler factor and α the spectral index, δ is defined in terms of the ratio of velocity to the speed of light, β at an angle θ towards the observer by

$$\delta = (1 - \beta^2)^{1/2} / (1 - \beta \cos \theta), \quad (2)$$

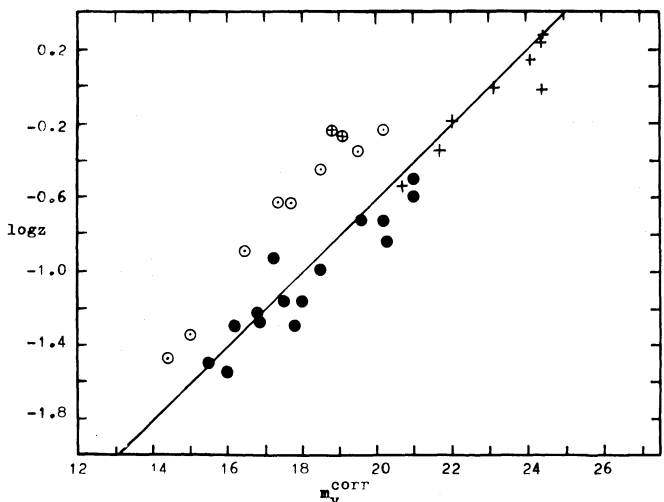


Fig. 1. Plot of $\log z$ against m_v^{corr} for 33 BL Lac objects. The 15 objects with redshift z_{gal} and with rapid flux variability are indicated by the symbol (●). The eight objects with redshifts z_{em} and with rapid flux variability are indicated by the symbol (+). Two objects with redshift z_{em} , but with no rapid flux variability are indicated by the symbol (⊕). The eight objects with redshifts z_{gal} , but with no rapid flux variability are indicated by the symbol (○). The m_v^{corr} is the corrected apparent magnitude for the Doppler effect. These values are given in Table 1. A Hubble relation of the form $m_v^{\text{corr}} = 5 \log z + K$ is included

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θ is the angle between the bulk motion and the line of sight. $\theta=0$ for BL Lac objects. Therefore, the observed apparent magnitude m_v^{ob} must be corrected for the Doppler effect. The corrected apparent magnitude is indicated by m_v^{corr} . If the Doppler effect is indeed an important effect for BL Lac objects, then we will obtain a good statistical relation between m_v^{corr} and $\log z$.

2. Observational data from BL Lac objects

To avoid selection effects and to compare with the results of Burbidge & Hewitt (1987), we will discuss mainly their samples in this paper. Table 1 lists, in order of right ascension, 33 BL Lac objects given by Burbidge & Hewitt (1987). The first column gives the coordinate designation, the second column the name, the third column the observed apparent magnitude given by Burbidge & Hewitt (1987) and designated m_v^{ob} (BH). The faintest apparent magnitude m_v^{ob} (min) of each object is shown in column four and its reference in column five. The redshift z of each object is shown in column six, where an asterisk indicates a redshift

based on the detection of broad emission lines of the type seen in QSOs, and the relevant references are listed in column seven. The bolometric luminosity of each object and the relevant references are listed in columns eight and nine, respectively. The minimum variability timescale Δt_{min} of each object and relevant references are listed in columns ten and eleven, respectively. The spectral index of each source and its reference are shown in columns 12 and 13, respectively, while η (ob) and δ are shown in columns 14 and 15, respectively. Column 16 gives the corrected magnitude m_v^{corr} (BH). Column 17 gives the corrected values of m_v^{ob} (min), designated m_v^{corr} . We define the observed value of Δt_{min} as the minimum variability timescale reported for a given object, regardless of the measurement waveband. This timescale corresponds to a nominal change of flux of 50% or more in the waveband under consideration. The bolometric luminosity of the nucleus, defined as the sum of the infrared, optical, ultraviolet and X-ray emissions, has been used as the source luminosity. With the observed values of infrared, optical, ultraviolet, X-ray monochromatic fluxes, and associated spectral indices, we inte-

Table 1

Coordinate designation (1)	Name (2)	m_v^{ob} (BH) (3)	m_v^{ob} (min) (4)	Ref. (5)	Redshift (6)	Ref. (7)	$\log L$ (erg s^{-1}) (8)	Ref. (9)
0138–097	PKS	17.5	19.5	3	0.44	3		
0215+015	PKS	18.33	19.5	3, 37	1.715 ^a	1	48.3	21, 29
0219+428	3C 66A	15.5	16.6	5	0.444 ^a	1	47.18	6
0235+164	Ao	15.5	19.5	7	0.94 ^a	1	47.77	6
0317+185	IE	18.12	18.8	8	0.190	1	45.53	18
0323+022	H	17.5	18.4	9	0.147	1	46.86	20, 21
0521–365	PKS	16.0	16.8	10	0.061	1	44.91	6
0548–322	PKS	15.5	15.5	1, 2	0.069	1	45.62	6
0754+100	PKS	14.5	16.3	11	0.66 ^a	36	47.49	32
0846+513	WI	17.0	21.0	34	1.86 ^a	1	47.37	6
0851+202	OJ 287	14.0	16.1	8	0.306 ^a	1	46.39	6
0954+658		16.7	18.5	10	0.368	1		
1101+384	Mkn 421	13.5	14.1	5	0.0308	1	45.67	6
1133+704	Mkn 180	14.49	15.0	3	0.0458	1		
1207+397	W 4	20.3	20.3	1, 2	0.59	1		
1218+304	RS 4	16.5	16.5	1	0.13	1	45.76	6
1219+285	WCom	17.0	17.0	1	0.102	1	45.31	18, 32
1235+632	IE	18.52	20.7	4	0.297	1	45.58	24
1308+326	B 2	19.0	19.0	1, 3	0.996 ^a	1	47.66	6
1400+162		17.0	17.7	12	0.245	1		
1413+135	PKS	20.0	21.0	3	0.26	1		
1415+259		16.0	17.4	13	0.237	1		
1514–241	ApLib	16	16.7	10	0.0486	1	44.51	6
1652+398	Mkn 501	14.4	14.4	1	0.034	1	45.31	6
1727+502	IZW 187	16.9	16.9	5	0.055	1	44.46	6
1807+698	3C 371	14.81	15.1	14	0.050	1	44.67	6
2032+107	PKS	18.6	18.8	3	0.601 ^a	1		
2131–021		18.67	19.0	10	0.557 ^a	1		
2155–304		14.2	14.2	1	0.117	1	46.76	6
2200+420	BL Lac	16.2	16.5	3	0.0688	1	44.95	6
2201+044		15.2	16.0	15	0.028	1		
2223–052	3C 446	17.9	18.6	16	1.404 ^a	1	47.13	6
2254+074	PKS	16.5	17.33	5, 8	0.19	28, 36	45.96	5, 25, 27

Table 1 (continued)

Coordinate designation	$\log \Delta t_{\min}(\text{s})$ (10)	Ref. (11)	α_{op} (12)	Ref. (13)	$\eta(\text{ob.})$ (14)	δ (15)	$m_{\text{V}}^{\text{Corr}}(\text{BH})$ (16)	$m_{\text{V}}^{\text{Corr}}$ (17)
0138–097							17.5	19.5
0215+015	4.93	17, 38	1.8	33	11.72	2.56	23.23	24.40
0219+428	3.66	8, 23, 32	1.3	33	16.56	2.99	20.61	21.71
0235+164	4.45	5	2.5	33	10.45	2.19	20.45	24.45
0317+185	4.13	18	1.0 ^b		0.13	1.20	18.92	19.6
0323+022	4.92	19	1.0 ^b		0.435	1.54	19.38	20.3
0521–365	5.41	6	1.5	33	0.002	1.0	16	16.8
0548–322	3.57	4	0.3	33	0.56	1.75	17.5	17.5
0754+100	3.68	32	1.1	33	32.3	3.56	20.15	21.95
0846+513	4.78	6	2.83	34	1.95	1.71	20.39	24.39
0851+202	2.95	30, 31	1.0	33	13.8	3.08	18.88	20.98
0954+658							16.7	18.5
1101+384	3.95	23	0.9	33	0.26	1.40	14.9	15.53
1133+704							14.49	15.0
1207+397							20.3	20.3
1218+304	5.48	6	0.9	33	0.01	1.0	16.5	16.5
1219+285	3.58	18, 22, 32	1.6	33	0.27	1.35	17.0	18.5
1235+632	4.82	4, 22	1.0 ^b		0.03	1.0	18.52	20.7
1308+326	4.64	25	1.5	33	5.24	2.33	23.13	23.13
1400+162							17.0	17.7
1413+135							20.0	21.0
1415+259							16.0	17.4
1514–241	2.95	6	1.5	33	0.18	1.26	17.14	17.85
1652+398	5.40	6	0.65	33	0.004	1.0	14.4	14.4
1727+502	5.61	6	0.87	33	0.0004	1.0	16.9	16.9
1807+698	3.15	26	1.5	33	0.17	1.24	15.87	16.2
2032+107							18.6	18.8
2131–021							18.67	19.0
2155–304	4.23	6	0.9	33	1.69	2.05	17.24	17.24
2200+420	3.23	6, 23	2.3	33	0.26	1.30	17.71	18.01
2201+044							15.20	16.0
2223–052	3.48	6	2.0	33	22.33	2.76	23.42	24.12
2254+074	3.60	18	2.6	35	1.15	1.61	19.38	20.21

^aEmission line redshift.^bAverage spectral indices.*References to Table 1*

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|-------------------------------|-----------------------------|--------------------------------|------------------------------|
| 1. Burbidge & Hewitt (1987) | 11. Smith et al. (1987) | 21. Treves et al. (1986) | 31. Valtaoja et al. (1985) |
| 2. Véron-Cetty & Véron (1987) | 12. Miller et al. (1978) | 22. Xie et al. (1988a) | 32. Xie et al. (1991) |
| 3. Urry (1984) | 13. Impey & Tapia (1988) | 23. Xie et al. (1988b) | 33. Ghisellini et al. (1986) |
| 4. Stocke et al. (1985) | 14. Miller (1975) | 24. Maraschi et al. (1986) | 34. Arp et al. (1979) |
| 5. Moles et al. (1985) | 15. Bolton et al. (1968) | 25. Impey et al. (1982) | 35. Stein et al. (1976) |
| 6. Bassani et al. (1983) | 16. Stoke et al. (1983) | 26. Staubert et al. (1986) | 36. Persic (1984) |
| 7. Spinrad & Smith (1975) | 17. Brindle et al. (1986) | 27. Madejski & Schwartz (1983) | 37. Gaskell (1982) |
| 8. Xie et al. (1990a) | 18. Xie et al. (1989) | 28. Stick et al. (1988) | 38. Webb et al. (1988) |
| 9. Filippenko et al. (1986) | 19. Feigelson et al. (1986) | 29. Allen et al. (1982) | |
| 10. Preston et al. (1985) | 20. Doxsey et al. (1983) | 30. Carrasco et al. (1985) | |

grate each waveband assuming a power-law distribution of flux. Radio data have not been considered, because of the difficulty in isolating the nuclear component from the surrounding radio regions. The bolometric luminosity is calculated in the rest frame of each galaxy, assuming a Friedman cosmology with $q_0=1$ and $H_0=50 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

3. Arguments for relativistic beaming

The inferred efficiency of the conversion of accreted matter into energy for a spherical, homogeneous, non-relativistically beamed region is given as (Fabian & Rees 1979),

$$\eta \gtrsim 5.0 \cdot 10^{-43} \Delta L / \Delta t_{\min}. \quad (3)$$

For pure accretion, the value of η is usually ≤ 0.1 . If it is found to be greater than 0.1, relativistic beaming is a possible conclusion. Using the observed data in Table 1 and $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, one can estimate η .

4. Corrected apparent magnitude–redshift relation

Since $\Delta L(\text{observed}) = \delta^{3+\alpha} \Delta L(\text{intrinsic})$ and $\Delta t_{\min}(\text{observed}) = \delta^{-1} \Delta t_{\min}(\text{intrinsic})$ (Worrall 1986; Frank et al. 1985), if we let

$$\eta(\text{observed}) = 5.0 \cdot 10^{-43} \Delta L(\text{observed}) / \Delta t_{\min}(\text{observed})$$

relation (3) gives

$$\eta(\text{intrinsic}) \gtrsim 5.0 \cdot 10^{-43} \Delta L(\text{intrinsic}) / \Delta t_{\min}(\text{intrinsic})$$

and thus

$$\delta \gtrsim (\eta(\text{observed}) / \eta(\text{intrinsic}))^{1/(4+\alpha)}. \quad (4)$$

Using Eq. (3), we can estimate $\eta(\text{observed})$. If we have a method to get $\eta(\text{intrinsic})$, using Eq. (4), we can estimate δ . It is known that the efficiency of the conversion in the nuclear reaction is 0.007, and that for pure accretion the value of η is smaller than 0.1. Thus, we have

$$0.007 < \eta(\text{intrinsic}) < 0.1.$$

This range is rather narrow, and its median value of ~ 0.05 is almost equal to the value 0.057 of the thick accretion disk theory for the Schwarzschild metric (Paczynski et al. 1980). Thus, we can use this median value $\eta(\text{intrinsic}) = 0.05$ to replace the real $\eta(\text{intrinsic})$ to estimate δ , if $\eta(\text{observed}) > 0.1$. Conversely, let $\delta = 1$ if $\eta(\text{observed}) < 0.1$, since in this case the sources do not require relativistic beaming. Since the observed flux density of a relativistically moving mass is enhanced by a factor $\delta^{3+\alpha}$, we have,

$$m_v^{\text{Corr}} = m_v^{\text{ob}}(\text{min}) + \Delta m \quad (5)$$

$$m_v^{\text{Corr}}(\text{BH}) = m_v^{\text{ob}}(\text{BH}) + \Delta m \quad (6)$$

where Δm is the Doppler correction of the apparent magnitude. Using δ , α_{opt} , $m_v^{\text{ob}}(\text{min})$ and $m_v^{\text{ob}}(\text{BH})$ of Table 1, from Eqs. (5) and (6) we can obtain m_v^{Corr} and $m_v^{\text{Corr}}(\text{BH})$, respectively, these results are listed in Table 1.

To compare with the results of Burbidge & Hewitt (1987), we first analyse the correlation between the Doppler-corrected apparent magnitudes $m_v^{\text{Corr}}(\text{BH})$ and $\log z$ of the 28 samples given in their Fig. 1. If we apply linear regression analysis for these samples, we obtain a correlation coefficient between $m_v^{\text{Corr}}(\text{BH})$ and $\log z$ of $\gamma = 0.834$. This is a highly significant value, much larger than the $\gamma = 0.510$ obtained before Doppler correction. This result implies that the Doppler correction is indeed a very important effect for BL Lac objects. Next, we extend the number of samples and again determine the observational minimum apparent magnitudes. The relevant data of 33 samples are listed in Table 1, of which 23 objects with redshifts z_{gal} from spectrum lines which can reasonably be considered to indicate the presence of a galaxy and 10 objects with redshifts z_{em} based on the detection of broad emission lines of the seen in QSOs. We will analyse the correlation between m_v^{Corr} and $\log z$ of these 33 samples in detail. The $m_v^{\text{Corr}} - \log z$ diagram is shown in Fig. 1. Where we have fitted by eye a Hubble relation with a slope of 5. It can be seen that they fit the normal Hubble relation very well, the correlation coefficient between m_v^{Corr} and $\log z$ is $\gamma = 0.899$. For

$H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 1$, the average absolute magnitude is $M_v = -21.5$. This is a typical value for the brightest E galaxies (Weinberg 1972). There are three objects that are noticeably brighter than the others. They are 1415+209, 2032+107, 2131–021. They lie about 2.5 mag above the Hubble line. It can be seen that their observed apparent magnitudes m_v^{ob} have not been corrected for the Doppler effect, because their observed data are very few and the Δt_{\min} have not been observed. Therefore, these sources should be primary monitoring targets for future observation.

Applying the modified Hubble relation (1) and multivariate linear regression analysis to the 33 samples, we obtain a correlation coefficient between m_v^{Corr} and z of $\gamma = 0.889$, with a deceleration parameter $q_0 = 1.99$ and $M_v = -21.4$. These results are very satisfying.

5. Discussion

It is well known that rapid flux variability at radio, IR, visual and X-ray wavelengths is a very important physical property of BL Lac objects, according to their original definition.

From Fig. 1, we see that the 23 BL Lac objects whose minimum variability timescales Δt_{\min} have been observed fit the Hubble relation very well, and we note with great satisfaction that eight of the 23 BL Lac objects have large emission line redshifts z_{em} .

If we apply multivariate linear regression analysis to the 23 samples, we obtain a very high correlation coefficient between m_v^{Corr} and z , $\gamma = 0.962$.

For $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, we have $q_0 = 0.534$ and $M_v = -21$. Obviously, these results are very good. Therefore, the search for short variability timescales of BL Lac objects is a very important task for future observations.

On the other hand, it can be seen from Fig. 1 and Table 1 that the redshifts of 3 BL Lac objects that lie about 2.5 Mag above the Hubble line in Fig. 1 can be divided into two categories:

- (a) 1415+259 with redshift of type z_{gal} .
- (b) 2032+107 and 2131–021 with emission line redshifts z_{em} .

The above discussion suggests that:

- (a) The scatter of points in the $m_v^{\text{Corr}} - \log z$ is dependent on the Doppler effect and independent of the category of redshifts of BL Lac objects.
- (b) Search for short variability timescales of BL Lac objects is a very important task for future observations.
- (c) Detail studies of BL Lac objects, according to the original definition, are probably required.

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