

Orbital evolution studies of asteroids near the 5:2 mean motion resonance with Jupiter

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Abstract. The orbits of 29 mainbelt and 4 planet-crossing (1985 WA, 1986 DA, 1986 JK and 1987 QB) asteroids with present osculating semi-major axis close to the 5:2 resonance with Jupiter ($a = 2.824$ AU), plus 6 fictitious asteroids, have been integrated over 100,000 years into the future. A solar system model which includes the perturbations of all the planets from Venus to Neptune for the mainbelt objects and also including Mercury for the planet-crossers have been used. The RA15 integrator with variable step size was used in a similar approach as in Hahn & Lagerkvist, *Celest. Mech.* 43, 285–302, 1988.

For 15 mainbelt orbits a non-resonant behaviour was found with the critical arguments circulating and the eccentricities varying between 0.0 and 0.3. Out of the remaining 14 orbits, 8 became Mars-crossing, of those 3 Earth-crossing and 2 Jupiter-crossing. The latter showing similar orbital evolutions as the currently planet-crossing asteroids, all leading to chaotic, cometary orbits. Rapid orbital evolutions on timescales 10^4 years are typically found and eccentricities reaching values as high as 0.8 and 0.9.

A comparison with the results by Yoshikawa, *A&A* 213, 436–458, 1989, for the fictitious asteroids shows that the evolutions are very sensitive to the starting elements used but less dependent on the solar system model used for the integrations.

Key words: asteroids – numerical integrations – 5:2 mean-motion resonance with Jupiter – orbital evolution – asteroid-comet interrelationship

1. Introduction

The importance of commensurabilities of asteroid orbits has been realized since long ago. The discovery of gaps in the distribution of semi-major axes in the mainbelt (Kirkwood 1867), which correspond to low-order mean motion resonances with Jupiter revealed important clues to the dynamics of the asteroid belt. A thorough understanding of the dynamical evolution of orbits presently close to these resonances is important for many

cosmogonical and geophysical problems. For example, the knowledge about the detailed structure of the asteroid belt and the stability of the orbits over the age of the solar system can give us information of the processes and timescales which dominated this region of the planetary system (see e.g. Scholl 1987). On shorter timescales, the problem of the origin of planet-crossing asteroids is found to be connected to the asteroid belt via commensurable orbits and chaotic zones, (Wisdom 1982; Wetherill 1985, 1988), and the search for source regions and transport mechanisms of meteorites is another closely related subject. Finally, the size of the population of planet-crossing asteroids and its eventual time-dependence is directly correlated with the collision rates of these bodies with the terrestrial planets (Wetherill & Shoemaker 1982; Milani et al. 1990; Bailey 1990).

Following Poincaré (1902, 1905), important work on the motion of resonant asteroids has been done by Schubart (1964, 1966) and Scholl & Froeschlé (1974, 1975). While more recently, the motions of asteroids in commensurabilities with Jupiter have been studied – both semi-analytically and by numerical integrations – by Yoshikawa (1989, 1990). Šidlichovský & Melendo (1986) and Šidlichovský (1987, 1988) studied the 5:2 resonance using the mapping technique.

In the present paper we present results of an extended investigation on the stability of the orbits of both mainbelt and planet-crossing asteroids currently close to the 5:2 mean-motion resonance, based on numerical integration studies. Preliminary results have been reported in Hahn et al. (1990).

We considered objects with present osculating semi-major axes in the range between 2.80 and 2.85 AU. Figure 1 shows the distribution of orbital elements in the $a-e$ plane for all known asteroids (through March 1989, taken from the orbital element data base of the Minor Planet Centre) at their originally published epochs. Squares represent orbits of numbered or multiple-opposition asteroids (identified by an asterisk (*) in column 1 of Table 1), and triangles denote one-opposition asteroids. The filled symbols indicate the orbits actually studied here. Their osculating orbital elements are summarized in Table 1, where the different columns give the number and name (or the provisional designation), followed by the semi-major axis, eccentricity, inclination, longitude of ascending node, argument of perihelion and mean anomaly. The epoch is denoted (1) for JD 2446000.5 and (2) for 2447000.5, and the last two columns contain perihelion

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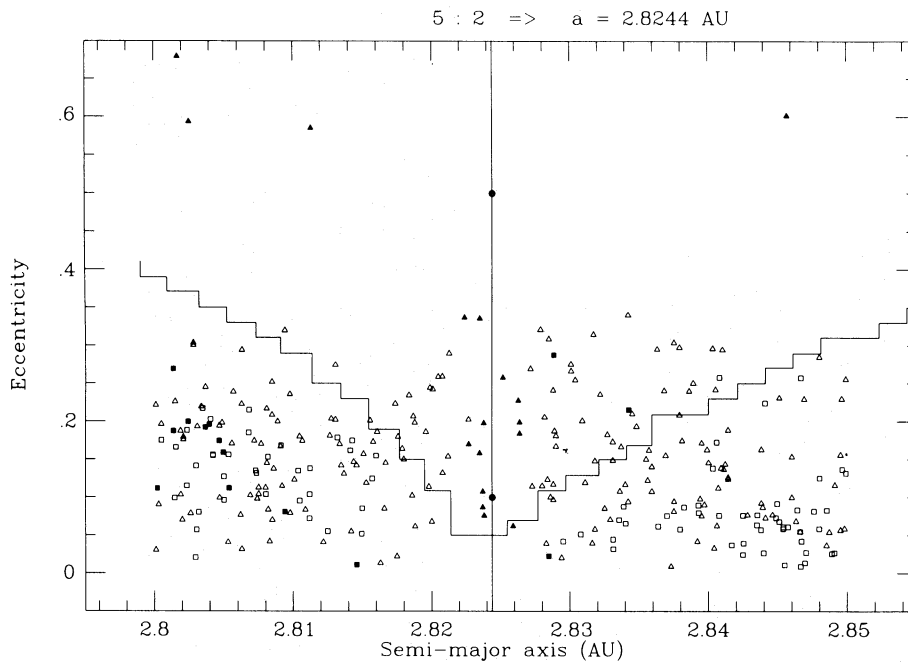


Fig. 1. Distribution of asteroid orbits around the 5:2 Kirkwood gap. Osculating eccentricity vs. semi-major axis. Squares represent numbered and multiple-opposition asteroids (high-quality orbits) and triangles one-opposition asteroids (low-quality orbits). Filled symbols designate those orbits actually studied here. The fictitious orbits are shown as filled circles. The borderline of the chaotic region is taken from Yoshikawa 1990. The vertical line marks the 5:2 resonance centre ($a_0 = 2.8244$ AU)

and aphelion distance. These elements were obtained by numerical integration from the original epochs to the chosen start-epochs, taken into account the perturbations by Mercury through Neptune.

In order to compare the results obtained by Yoshikawa (1989) on the 5:2 resonance, we have included six orbits of fictitious asteroids resembling closely the starting values in his paper (B1–B6). The location of these orbits is designated by filled circles in Fig. 1 and the starting elements are given in Table 1. The purpose of this comparison was to see whether the evolution of the orbits is dependent on the model of the solar system or the choice of other initial parameters. The boundary line of the resonance region as given in Fig. 2 of Yoshikawa (1990) is superimposed in Fig. 1. This can be compared with the chaotic region determined, using Liapunov characteristic exponents, for eccentricities up to 0.14 by Šidlichovský 1988, (see Fig. 6 of that paper).

One can see from Fig. 1 that most of the well-determined orbits are situated outside the resonance region as defined by Yoshikawa which is simply an other way of visualizing the existence of the Kirkwood gap. Nevertheless, there are quite some asteroids close to the boundary line which would deserve further study. The large number of one-opposition orbits in the gap might be due to the inclusion of spurious orbits, based on only very short arcs. Therefore it would be desirable to investigate further the possibility to improve some of these orbits in order to make sure whether there currently exist objects close to the 5:2 resonance. This is especially interesting because of the orbital evolutions found in our integrations (see below).

The transition from mainbelt to planet-crossing orbits via the resonance and the timescales involved is another aspect for our work. The results from earlier studies of the orbital evolution of planet-crossing asteroids (Hahn & Rickman 1985; Hahn & Lagerkvist 1988a, 1988b; Milani et al. 1989) showed the importance of mean-motion resonances and revealed often rapid and chaotic evolutions. Similarities with typical short-period comet

orbital behaviour suggest that some planet-crossing asteroids might be defunct cometary nuclei (Weissman et al. 1989). Recent studies of the orbital evolution of (2060) Chiron, or 'P/Chiron', showed links of that orbit to short-period comets and known planet-crossing asteroids, like 1986 JK, (Hahn & Bailey 1990), further emphasizing this interrelationship.

2. Calculations and data analysis

For our numerical integrations we used the same routines and solar system parameters as described in Hahn & Lagerkvist (1988). For the mainbelt asteroids the perturbations by Venus through Neptune were taken into account, for the planet-crossers also Mercury was included.

The 15th order RADAU integrator (RA15) (Everhart 1985) with variable stepsize was used. The calculations were performed over an interval of +100,000 years. Heliocentric rectangular coordinates X, Y, Z and velocities $\dot{X}, \dot{Y}, \dot{Z}$ were stored every 500 days. All close approaches to within 0.1 AU for the terrestrial planets and within 1.0 AU for the outer ones were recorded. In a separate post-processing routine the osculating elements $a, e, i, \omega, \Omega, q, Q$ were calculated at three and five times the sampling interval for the planet-crossers and mainbelt asteroids respectively and their evolution was plotted versus time. A resonance analysis programme was used to study the behaviour of the critical argument $\sigma = 5\lambda_1 - 2\lambda - 3\varpi$ and $\varpi - \varpi_1$, the difference of the longitude of perihelion of the asteroid and Jupiter; λ and λ_1 are the respective mean longitudes. In order to see whether there are other protection mechanisms at work which could prevent close encounters in the cases where the orbits evolved into planet-crossing orbits, the evolution of the distances to the mutual nodes with the respective planets was analysed. This, together with the evolution of the ω and Ω with respect to the planets, allowed us to check the recorded close encounters with the geometrical configuration at the time of close approach. The

Table 1. Osculating starting elements for epochs JD 2446000.5 (1) and JD 2447000.5 (2) as obtained by numerical integrations from the original epochs taking into account the perturbations by Mercury through Neptune. The angular elements i , Ω and ω refer to the ecliptic and equinox of 1950.0

Nr	Asteroid	a (AU)	e	i (°)	Ω (°)	ω (°)	M (°)	Ep	q (AU)	Q (AU)
236	Honoria	2.8013701	0.1872131	7.67786	185.75652	173.83919	225.05936	(1)	2.277	3.326
266	Aline	2.8049616	0.1591420	13.39533	235.57191	150.33072	348.80674	(1)	2.359	3.251
441	Bathilde	2.8093920	0.0810235	8.12918	253.38379	199.88080	287.81470	(1)	2.582	3.037
541	Deborah	2.8149645	0.0520849	5.99482	267.57611	357.00923	256.23802	(1)	2.668	2.962
670	Ottegebe	2.8036769	0.1921620	7.53440	174.42673	194.82152	268.37837	(1)	2.265	3.342
673	Edda	2.8145797	0.0116164	2.87628	226.66090	248.68172	134.19885	(1)	2.782	2.847
984	Gretia	2.8039732	0.1956968	9.10318	314.10123	55.11695	276.39970	(1)	2.255	3.353
1014	Semphyra	2.8024334	0.1995039	2.27410	251.64030	232.08849	183.26479	(1)	2.243	3.362
1022	Olympiada	2.8046891	0.1744417	21.05838	111.60178	125.22168	180.50639	(1)	2.315	3.294
1071	Brita	2.8002010	0.1117789	5.38140	52.28699	26.81558	267.90919	(1)	2.487	3.113
1693	Hertzprung	2.8013453	0.2688226	11.93212	69.80275	234.87499	8.12904	(1)	2.048	3.554
2353	1975 UD	2.8053742	0.1123515	4.79104	24.64695	297.87622	233.94127	(1)	2.490	3.121
2953	Vyshslavia	2.8284690	0.0226695	1.07260	250.98772	4.35361	1.07488	(1)	2.764	2.893
	1978 PU ₂	2.8248364	0.1672101	10.45741	146.99824	196.98343	102.01583	(1)	2.352	3.297
	1978 VU ₇	2.8235185	0.0790131	1.25638	332.94707	60.93392	119.43639	(1)	2.600	3.047
*	1980 DO	2.8153192	0.2939851	3.81897	121.26951	66.71346	329.68043	(1)	1.988	3.643
	1980 PV ₁	2.8227156	0.2036211	6.68351	285.98551	317.72007	35.46782	(1)	2.248	3.397
*	1981 WM ₄	2.8331436	0.2162027	10.74530	252.67985	119.11563	244.62108	(1)	2.221	3.446
	1982 JL ₂	2.8230002	0.3360172	6.67897	198.99916	127.36623	131.20480	(1)	1.874	3.772
	1983 ED	2.8264777	0.1996107	4.46141	350.71788	222.96685	89.36139	(1)	2.262	3.391
	1984 BH	2.8219446	0.3368601	8.38023	307.95966	204.19185	42.53745	(1)	1.871	3.773
	1987 QE ₃	2.8253725	0.2573483	8.62942	171.48925	278.13779	54.96417	(1)	2.098	3.552
	2447 T-3	2.8213261	0.2021858	5.10446	358.72931	346.24282	197.94049	(1)	2.251	3.392
	3097 P-L	2.8287165	0.2248645	7.02848	294.24822	354.30085	73.07705	(1)	2.193	3.465
	4551 P-L	2.8262000	0.0620654	2.79493	131.94750	231.09376	25.26016	(1)	2.651	3.002
	4752 P-L	2.8242206	0.1072405	2.98490	350.32740	318.72177	71.81763	(1)	2.521	3.127
	6018 P-L	2.8269637	0.1831913	6.30689	206.45598	175.14443	11.50789	(1)	2.309	3.345
	6337 P-L	2.8240473	0.0873246	1.78010	191.40889	164.55734	32.58792	(1)	2.577	3.071
	6510 P-L	2.8240609	0.1580512	13.83869	2.01897	13.22081	17.21547	(1)	2.378	3.270
	1985 WA	2.8463189	0.6017453	9.74718	43.16223	350.90873	129.29386	(2)	1.134	4.559
	1986 DA	2.8133605	0.5847893	4.29537	64.48830	126.84364	98.69438	(2)	1.168	4.459
	1986 JK	2.8000046	0.6795999	2.13959	62.23446	232.38519	81.44820	(2)	0.897	4.703
	1987 QB	2.8034843	0.5932952	3.46241	153.00079	156.03709	166.09065	(2)	1.140	4.467
	B-1	2.8244	0.1	5.0	190.5	5.2	94.75	(2)	2.542	3.107
	B-2	2.8244	0.1	5.0	190.5	5.2	72.75	(2)	2.542	3.107
	B-3	2.8244	0.1	5.0	190.5	5.2	49.75	(2)	2.542	3.107
	B-4	2.8244	0.1	5.0	190.5	5.2	4.75	(2)	2.542	3.107
	B-5	2.8244	0.1	5.0	190.5	185.2	2.75	(2)	2.542	3.107
	B-6	2.8244	0.5	5.0	190.5	185.2	2.75	(2)	1.412	4.237

computations have been performed on the Alliant FX/8 computer at the faculty of sciences of the Uppsala University, and the Apollo DSP10020 computer at the Astronomical Observatory in Uppsala.

3. Results

Our results are summarized in Table 2 which lists the maximum and minimum values of a , e , i reached during the integration interval together with the behaviour of σ and $\varpi - \varpi_j$.

Table 2. Summary of the orbital evolutions during 100,000 years: maximum and minimum values of semi-major axis, eccentricity and inclination. Behaviour of the critical argument σ and $(\varpi - \varpi_J)$; C-circulation, L-libration, vL-variable libration centre, sV-slow variation (period longer than 10^5 years). The type of orbital evolution as defined in the text is given as: NR-non-resonant; R1, R2, R3-three different types of resonant behaviour

Nr	Asteroid	a_{max}	a_{min}	e_{max}	e_{min}	i_{max}	i_{min}	σ	$\tilde{\omega} - \tilde{\omega}_J$	Type	Rem.
236	Honorio	2.807	2.796	0.20	0.10	10	5	C	C	NR	
266	Aline	2.811	2.800	0.19	0.07	17	12	C	C	NR	
441	Bathilde	2.812	2.803	0.14	0.04	11	7	C	C	NR	
541	Deborah	2.819	2.812	0.12	0.02	9	5	C	C	NR	
670	Ottegebe	2.811	2.799	0.21	0.11	9	5	C	C	NR	
673	Edda	2.819	2.813	0.09	0.00	6	2	C	C/L	NR	
984	Gretia	2.812	2.800	0.21	0.09	13	8	C	C	NR	
1014	Semphyra	2.815	2.796	0.28	0.16	6	1	C	C	NR	
1022	Olympiada	2.817	2.801	0.31	0.16	24	15	C	C	NR	
1071	Brita	2.806	2.798	0.15	0.06	7	2	C	C	NR	
1693	Hertzprung	2.814	2.794	0.31	0.17	14	8	C	C	NR	
2353	1975 UD	2.810	2.802	0.14	0.04	7	2	C	C	NR	
2953	Vyshslavia	2.831	2.825	0.10	0.00	4	0	C	C	NR	
	1981 WM ₄	2.844	2.828	0.32	0.08	14	9	C	C	NR	
	4551 P-L	2.830	2.824	0.07	0.00	4	0	L/C	C/L	NR	
	1980 DO	2.860	2.790	0.47	0.25	5	1	vL/C	C	R1	
	1980 PV ₁	2.847	2.800	0.33	0.15	11	5	C/vL	C	R1	
	1984 BH	2.865	2.785	0.51	0.30	13	6	C/L	C/L?	R1	
	1978 PU ₂	2.839	2.809	0.45	0.14	13	7	vL	L	R2	
	1978 VU ₇	2.838	2.810	0.46	0.06	5	0	vL	L	R2	
	1982 JL ₂	2.850	2.798	0.59	0.16	12	4	vL/L	L	R2	
	3097 P-L	2.85	2.80	0.55	0.20	12	5	vL/L	L	R2	
	4752 P-L	2.842	2.806	0.48	0.10	7	1	L	L	R2	
	6337 P-L	2.840	2.808	0.48	0.06	4	0	vL	L/C	R2	
	6510 P-L	2.840	2.808	0.45	0.15	19	12	L	L	R2	
	1983 ED	2.90	2.65	0.70	0.18	12	2	L/C	C/L	R3	
	1987 QE ₃	2.86	2.78	0.62	0.22	15	5	vL/L/C	sV/C	R3	
	2447 T-3	≈28	2.8	0.82	0.05	25	1	vL/L	L	R3	“comet”
	6018 P-L	2.88	2.77	0.66	0.14	13	3	L	sV/C	R3	
	1985 WA	>40	2.8	0.90	0.35	17	0	L/C	C	R3	“comet”
	1986 DA	3.15	2.8	0.73	0.33	12	1	L	C	R3	libr. 180°
	1986 JK	≈16	2.8	0.81	0.05	28	0	L/C	L/C	R3	“comet”
	1987 QB	2.9	2.6	0.81	0.36	10	0	L/C	L/C	R3	5:2 to 8:3
	B-1	2.836	2.812	0.50	0.10	9	4	vL	C	R2	
	B-2	2.836	2.812	0.50	0.10	9	3	vL	C	R2	
	B-3	8.3	2.6	0.77	0.10	30	3	vL/L/C	L/C	R3	
	B-4	2.838	2.809	0.19	0.08	7	3	C	C	NR	
	B-5	2.837	2.811	0.44	0.10	8	4	vL	L	R2	
	B-6	2.838	2.810	0.50	0.10	7	2	L	C	R2	

Preliminary results for the numbered mainbelt asteroids have been published in Hahn et al. (1990).

The orbital evolutions found can be divided into two types of behaviour:

(1) Non-resonant orbits – where the variations of the semi-major axis (a) never lead to a crossing of the resonance ($a_0 = 2.824$ AU) and therefore do not experience large variations in eccentricity (e).

(2) Resonant orbits – which have variations of a which at least during some interval of our integration timespan lead to resonance crossing and hence experience large variations in e .

The first group includes all the numbered asteroids in our sample, plus 1981 WM₄ and 4551 P-L. $\varpi - \varpi_J$ is circulating with periods varying between 20,000 and 30,000 years, and σ circulates with a short period of about 500 years, which is also reflected in the variations of a . For two asteroids, 673 Edda and 4551 P-L, $\varpi - \varpi_J$ alternates between libration around 0° and circulation. The variations in e are small, as can be seen from the first subset in Table 2 (labeled NR). Three typical evolutions of a , e vs. $\varpi - \varpi_J$, and σ are presented in Figs. 2a–c for (673) Edda, 3a–c for (1022) Olympiada and 4a–c for 4551 P-L.

The results for the resonant asteroids can be divided into the following subgroups:

(1) $\varpi - \varpi_J$ circulates and σ alternates between circulation and libration around 0° with a very large amplitude $\pm 150^\circ$. The resulting variations in e have a period corresponding to the circulation period of $\varpi - \varpi_J$ and reach maximum values between 0.4 and 0.5. 1980 DO, 1980 PV₁ and 1984 BH belong to this group which is summarized in the second subset of Table 2 (R1). Figures 5a–c illustrate the evolution for 1980 PV₁, showing the variations of a , e vs. $\varpi - \varpi_J$ and σ , respectively.

(2) $\varpi - \varpi_J$ librates around 0° and σ librates around a slowly varying libration centre with moderate amplitude $\pm 50^\circ$ – which is called ‘variable libration’ in the following. When σ librates around 0°, e increases and reaches values > 0.5 . The third subset in Table 2 (R2) contains the summarized results of the following asteroids: 1978 PU₂, 1978 VU₇, 1982 JL₂, 3097 P-L, 4752 P-L, 6337 P-L and 6510 P-L. In Figs. 6a–c the variations of a , e vs. $\varpi - \varpi_J$ and σ with time are shown for 1978 PU₂ and in Figs. 7a–c for 1982 JL₂, in addition in Fig. 7d the evolution of the distance to the mutual nodes with Mars is given and in Fig. 7e the distribution of the close encounters with Mars are presented.

(3) $\varpi - \varpi_J$ librates around 0° and/or revolves slowly ($P > 10^5$ years), σ librates ‘variably’ and then stabilizes around 0°. This leads to $e > 0.6$ and often to close approaches to Jupiter and consequently to a chaotic evolution. 1983 ED, 1987 QE₃, 6018 P-L and 2447 T-3 belong to this subset of Table 2 (R3). Figures 8a–c show the evolution for 1983 ED and Figs. 9a–c display the evolution of a , e and σ for 2447 T-3.

Those three types of orbital behaviour described above represent, presumably, different states on a longer evolution and transitions are bound to occur. Such trends are clearly visible for some of the asteroids studied here, see Figs. 7. Nevertheless, it is useful to isolate substates and compare the timescales involved for individual asteroids and/or group of orbits with similar starting values in the element space.

The four objects (1985 WA, 1986 DA, 1986 JK and 1987 QB) which are currently already in planet-crossing orbits are described separately, because their orbital evolution is strongly influenced by close encounters to the terrestrial planets and Jupiter.

As can be seen from Table 1 all orbits have large e , are of Apollo (1986 DA) or Amor type, and have aphelia near Jupiters perihelion distance. Close encounters with Jupiter are prevented because the critical argument is librating around 0°. But this protection is lost for all four objects after disruption of the libration for different reasons, as described below, leading to chaotic evolutions on time-scales between 30,000 and 90,000 years. In the orbital classification system defined by Milani et al.

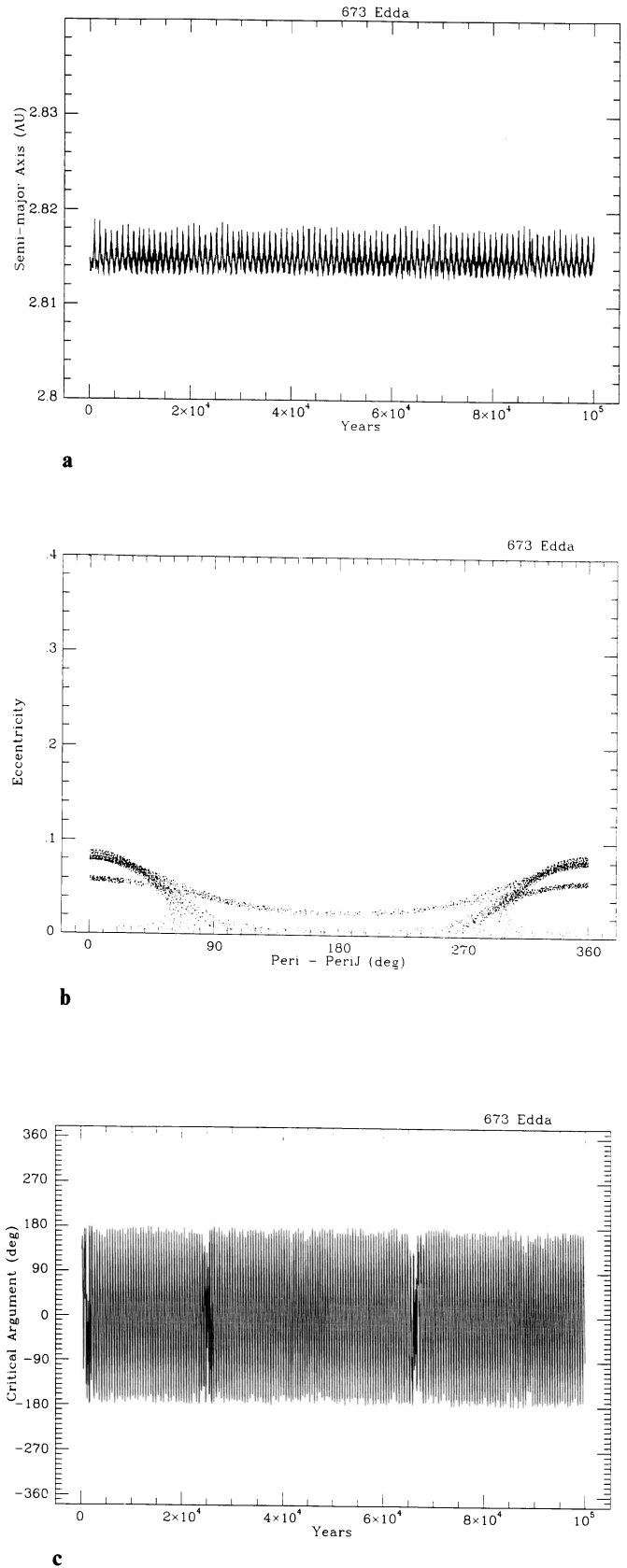
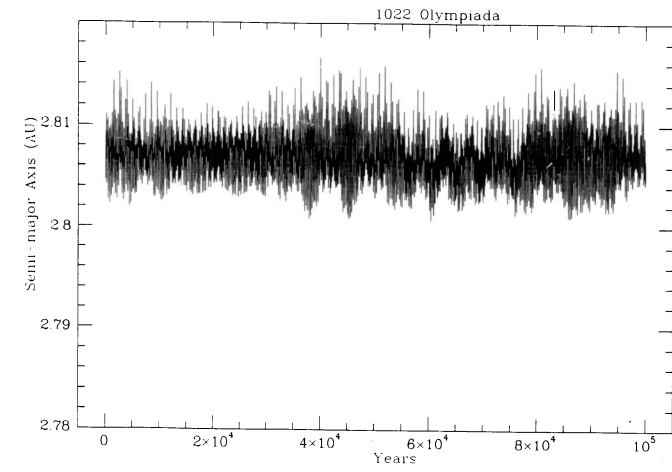
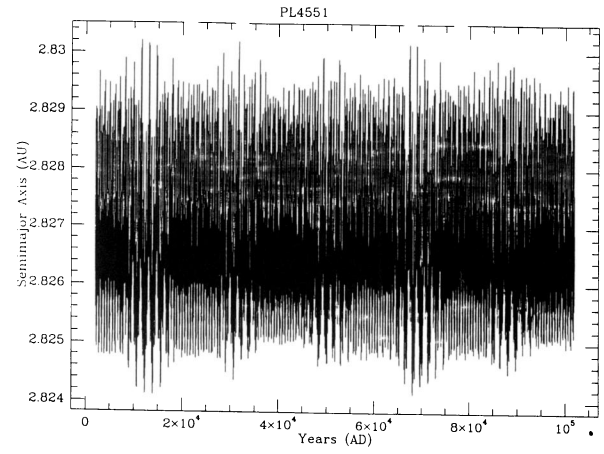


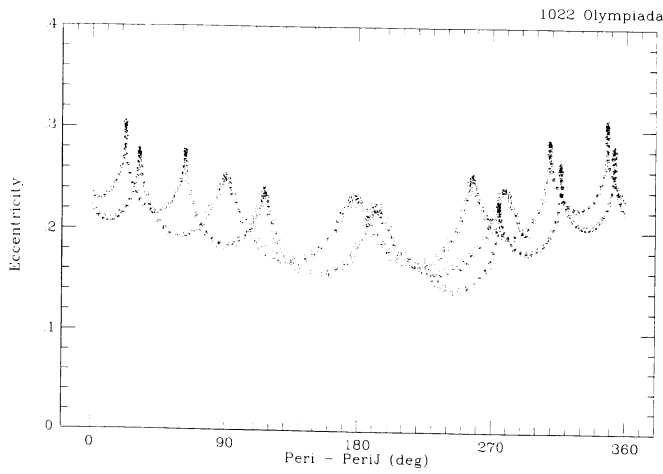
Fig. 2a–c. Time evolution of semi-major axis (a), variations of eccentricity versus $(\varpi - \varpi_J)$ (b), and critical argument (σ) (c) for the asteroid (673) Edda



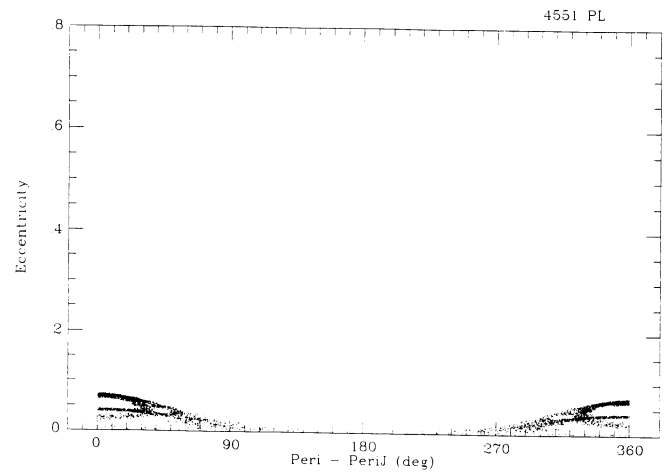
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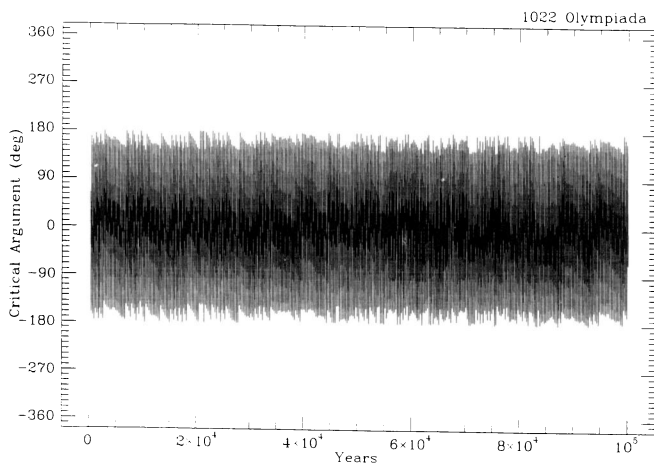
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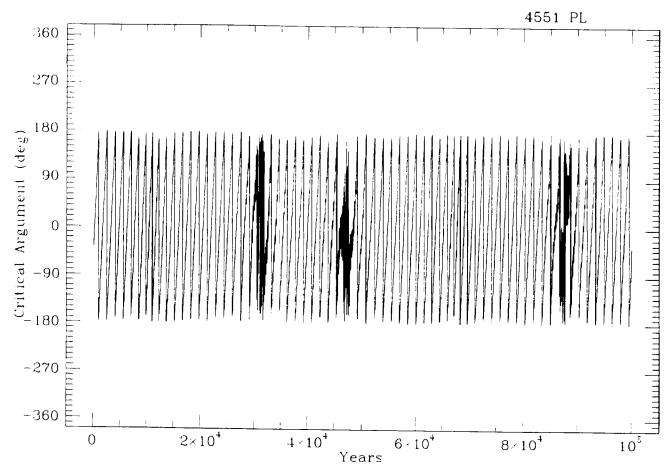
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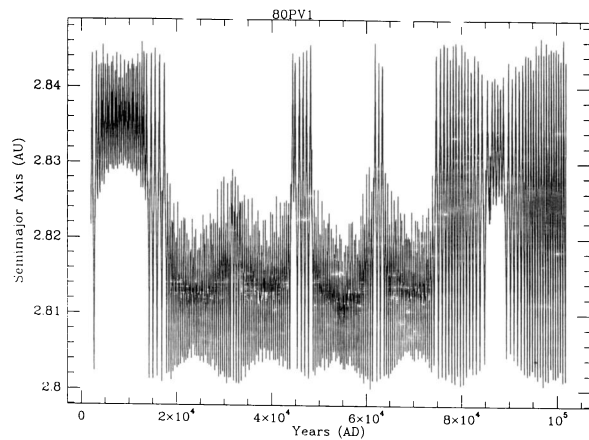
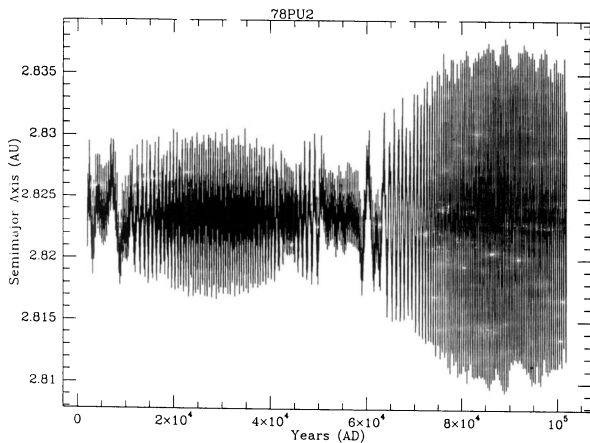
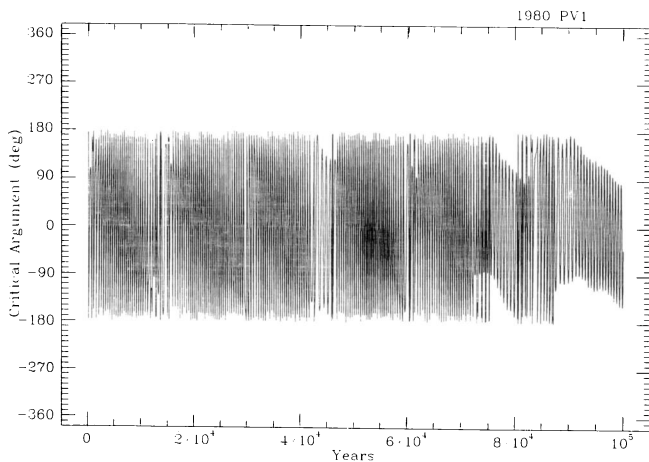
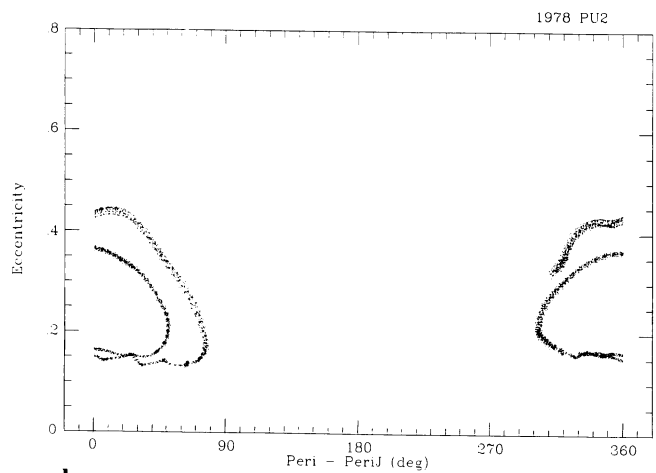
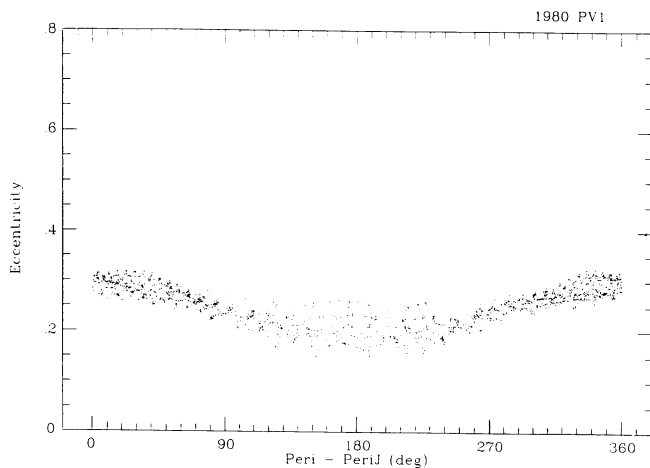
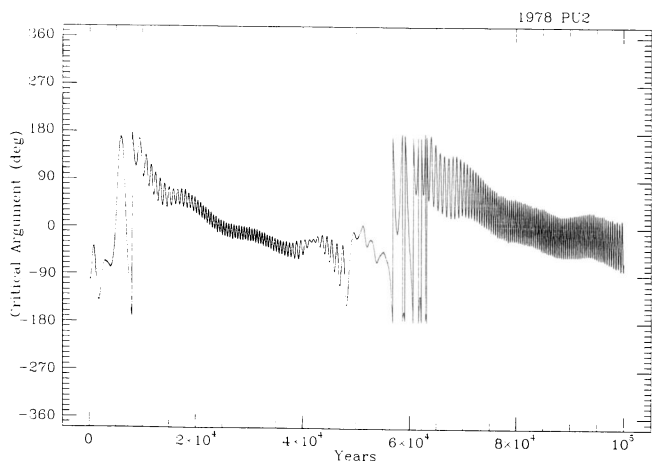
c



c

Fig. 3a–c. Same as in Fig. 2 but for (1022) Olympiada

Fig. 4a–c. Same as in Fig. 2 but for 4551 P-L

**a****a****b****b****c****c****Fig. 5a-c.** Same as in Fig. 2 but for 1980 DO**Fig. 6a-c.** Same as in Fig. 2 but for 1978 PU₂

(1989) the current orbits are typical for the Alinda-class, which is characterized by mean-motion resonances with Jupiter and their further evolution can be seen as transitions into the Comet-class, with the possible exception of 1986 DA. Therefore these objects are candidates for being extinct cometary nuclei, based on the similarity with orbits typical for short-period comets.

While the results from radar-observations of 1986 JK (Ostro et al. 1989) are compatible with a possible cometary origin, indicate the radar reflection properties found for 1986 DA (Ostro et al. 1987) a metallic surface quite incompatible with a cometary body. This latter finding is also confirmed by photometric and radiometric observations (Tedesco & Gradie 1987),

which lead to an M-type classification. Clearly, physical observations are important for the confirmation of dynamical indications concerning the origin of the planet-crossing asteroids and further emphasize the need of more powerful techniques (like radar) and large optical and IR telescope observations in order to get more data on these very faint objects.

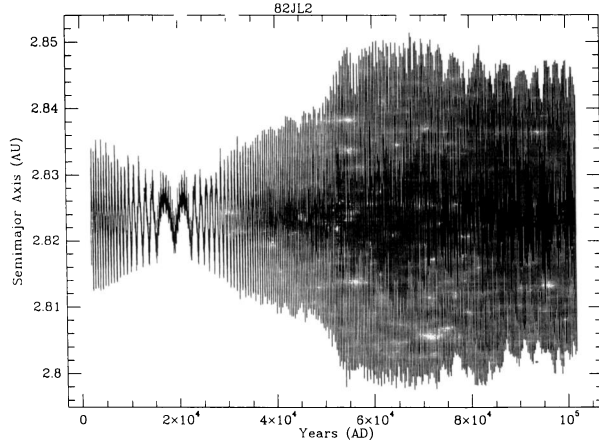
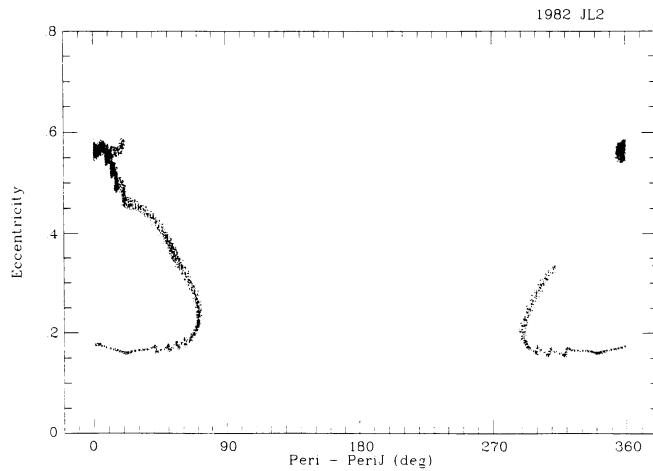
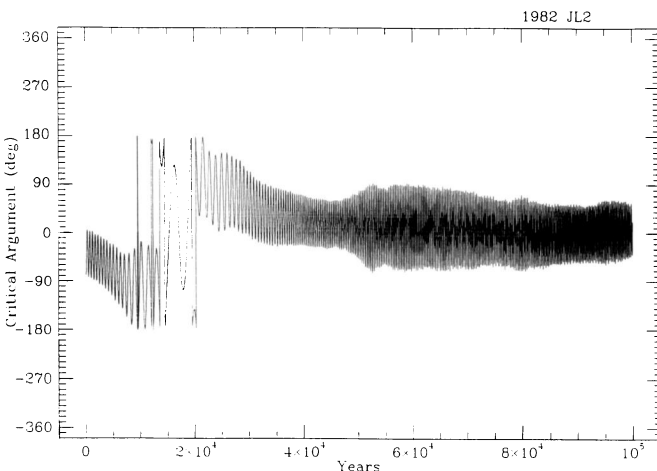
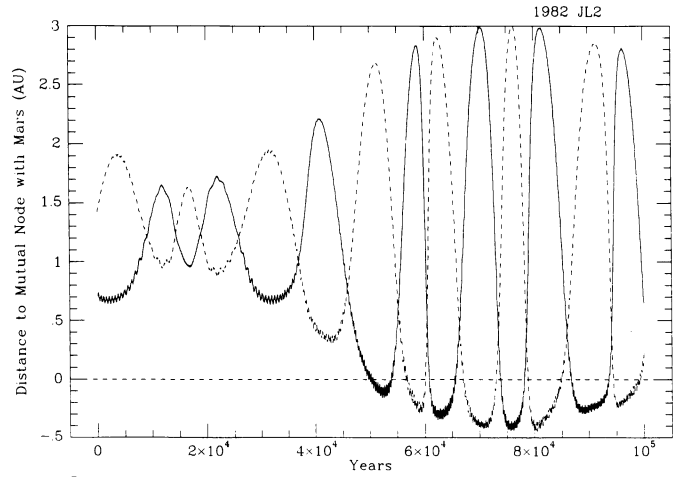
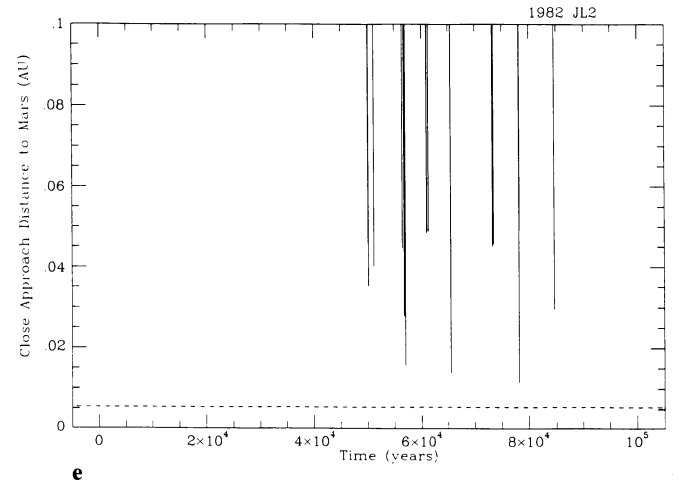
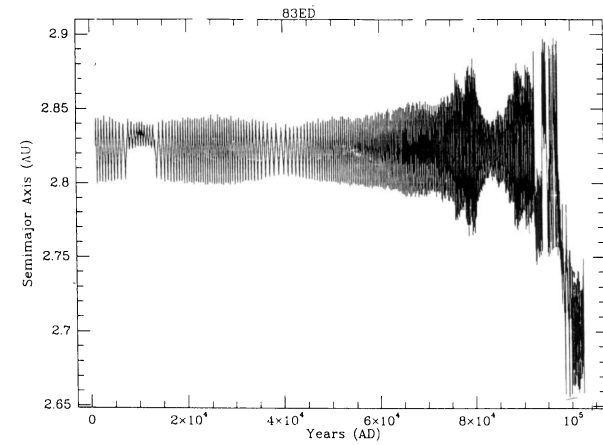
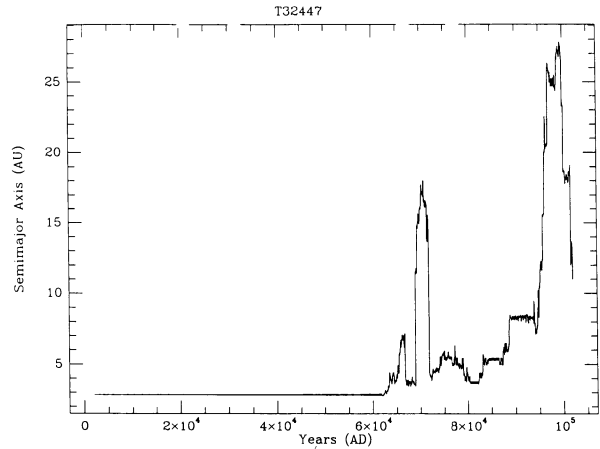
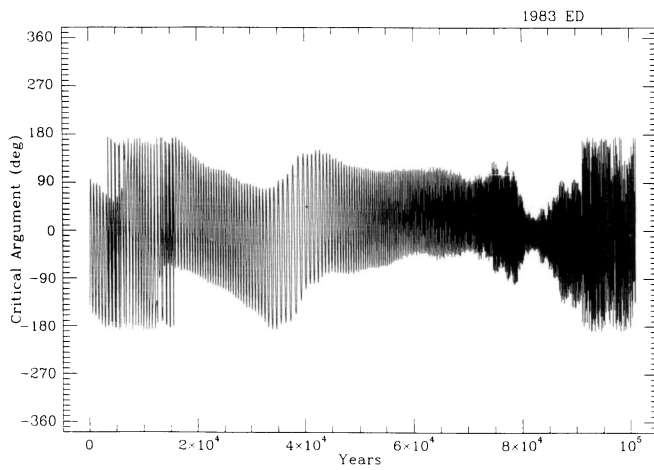
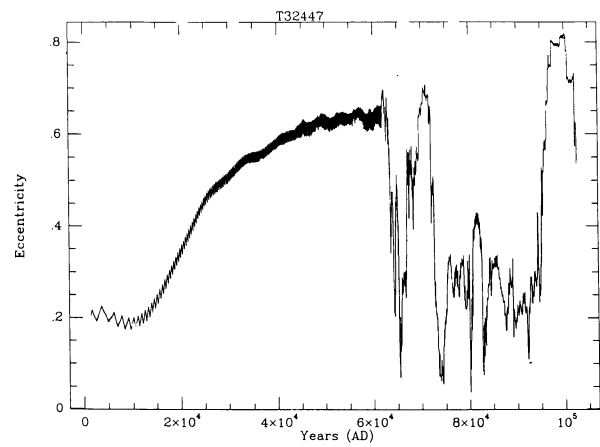
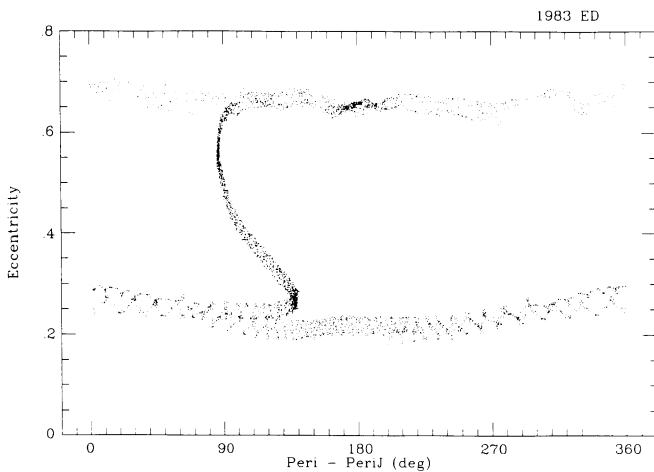
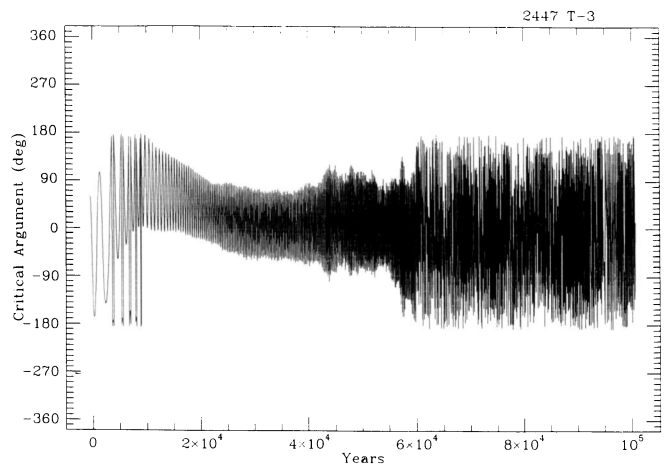
**a****b****c****d****e**

Fig. 7a–e. Same as in Fig. 2 but for 1982 JL₂. In addition, evolution of the distance to the mutual nodes with Mars, (dotted line=ascending node, solid line=descending node) (d), distribution of close encounters with Mars (e)

1985 WA – $\varpi - \varpi_J$ varies slowly and σ librates around $0^\circ \pm 40^\circ$, which increases e to 0.8. When $\varpi - \varpi_J$ passes through 0° , σ starts circulating and close encounters to Jupiter do occur which lead to a chaotic evolution. Figures 10a–d show the time evolution of a , e , σ and $\varpi - \varpi_J$, respectively.

1986 DA – turns out to be the most stable one of the four by staying in the resonance for almost the whole period of time (10^5 years). $\varpi - \varpi_J$ librates around 180° and σ around $0^\circ \pm 30^\circ$, which leads to a quasi-periodic variation of e between 0.3 and 0.6. A close encounter with the Earth disrupts the $\varpi - \varpi_J$ libration which leads to a larger amplitude of σ and a further pumping-up of e , which consequently allows Jupiter encounters. This occurs at the end of the integration interval, so no definite conclusions can be drawn from our data, but a transition to a chaotic orbital evolution after this event seems to be most likely. In Figs. 11a and b the variations of a and e are presented, while Fig. 11c shows the critical argument and Fig. 11d the variation of e vs. $\varpi - \varpi_J$.

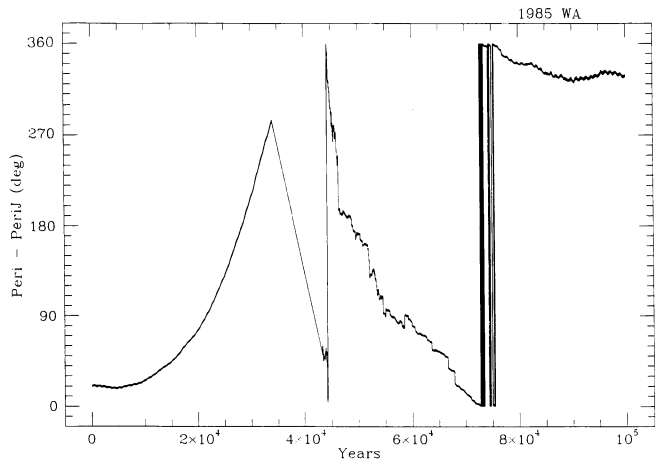
1986 JK – $\varpi - \varpi_J$ circulates and σ librates around 0° . e varies more or less periodically around 0.6 ± 0.08 but gets pumped-up

**a****a****b****b****c****c****Fig. 8a–c.** Same as in Fig. 2 but for 1983 ED**Fig. 9a–c.** Same as in Fig. 2 but for 2447 T-3

to 0.8 when $\varpi - \varpi_J$ passes from 180° to 360° and the amplitude of σ increases and finally leads to circulation. Close encounters to the Earth interrupt the σ -libration and lead to Jupiter encounters. Figures 12a–d show the variation of a , e , σ and $\varpi - \varpi_J$, respectively.

1987 QB – $\varpi - \varpi_J$ librates around 0° , σ librates ‘variably’ and then stabilizes around $0^\circ \pm 80^\circ$, giving rise to a regular e variation between 0.35 and 0.65. Again, $\varpi - \varpi_J$ starts to circulate, probably triggered by a close encounter to Earth and/or Mars, leading to an increase in the amplitude of the critical argument. σ starts to

circulate and moderately close (0.6 AU) encounters with Jupiter which remove the asteroid from the 5:2 resonance and bring it close to the 8:3 resonance ($a = 2.705$ AU). $\varpi - \varpi_J$ passes from 180° to 360° and $\sigma(8:3)$ circulates leading to a pumping of e to 0.8. Finally, the orbit stabilizes in the 8:3 resonance, $\sigma(8:3)$ starts librating and e varies quasi-periodically around 0.7. Close encounters with Jupiter do occur, however, and the further evolution has to be classified as chaotic. The evolution of a and e is shown in Figs. 13a and b, while the variation of the critical arguments can be seen in Figs. 13c (5:2) and 13d (8:3). Finally, Fig. 13e presents a plot e vs. $\varpi - \varpi_J$.

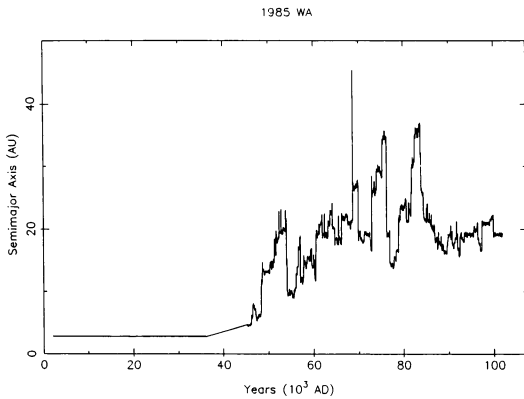


d

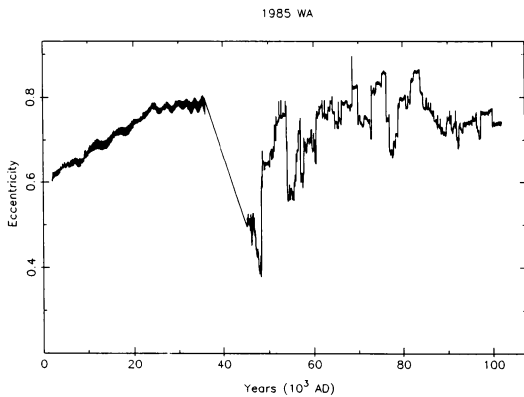
Fig. 10a–d. Variation of semi-major axis (a), eccentricity (b), critical argument σ (c) and $\varpi - \varpi_J$ (d) for 1985 WA

Due to the loss of a data file during the post-processing phase data-points for the above four planet-crossers are missing around 40,000 years. This is seen in the corresponding plots by the straight line joining the data points over this gap.

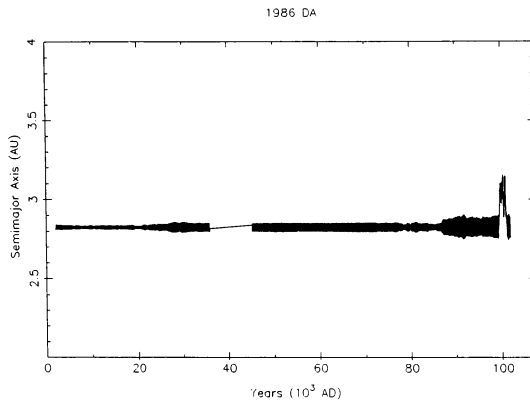
The fictitious orbits (B-1 to B-6), summarized in the last subset of Table 2, exhibit evolutions similar to all the above described types.



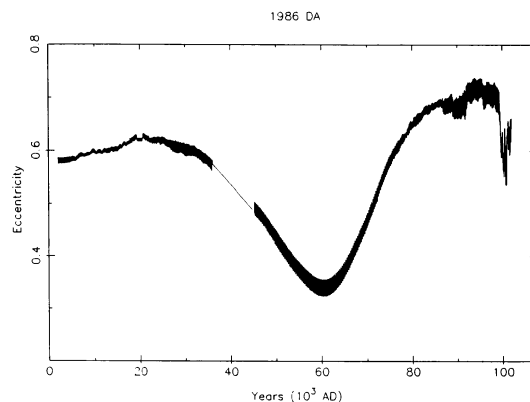
a



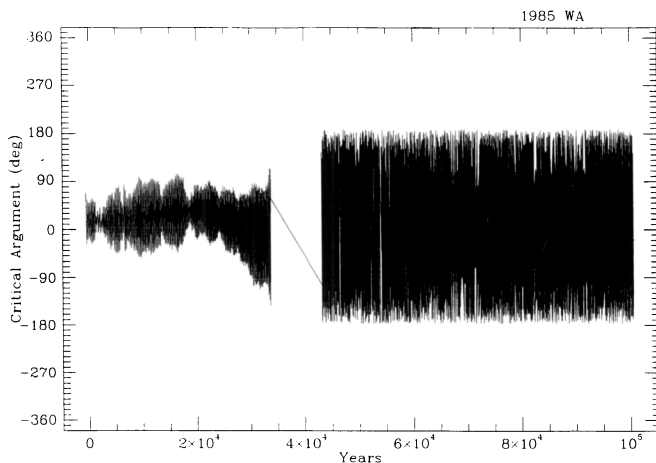
b



a



b



c

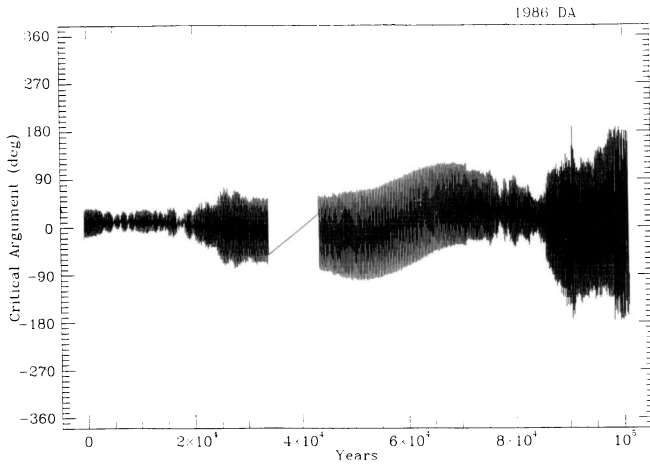
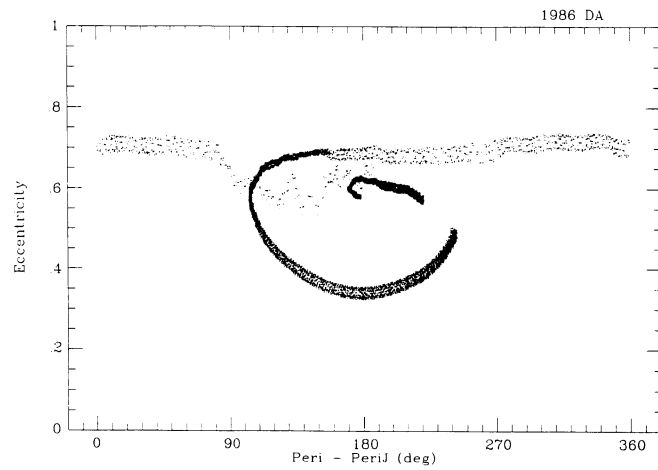
**c****d**

Fig. 11a–d. Variation of semi-major axis (a), eccentricity (b), critical argument σ (c) and eccentricity versus $(\varpi - \varpi_J)$ (d) for 1986 DA

B-1 and B-2 – evolve almost identically with $\varpi - \varpi_J$, circulating with a period of about 50,000 years and σ librating ‘variably’ with quite small amplitude; similar to type R2). In agreement with Yoshikawa (1989). Figures 14a and b show the evolution of a and e vs. $\varpi - \varpi_J$, and 14c that of σ .

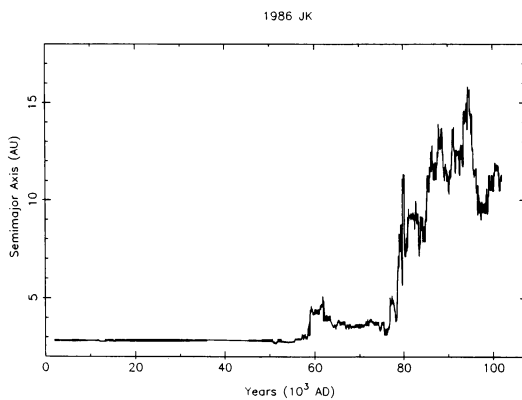
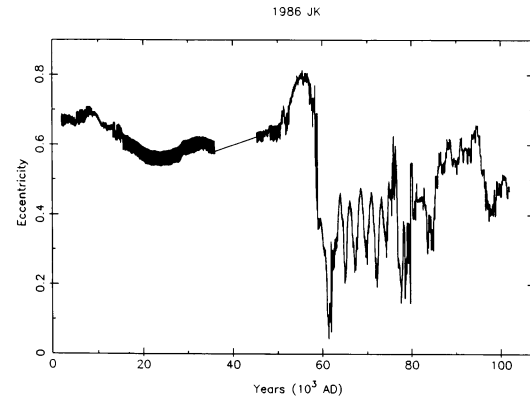
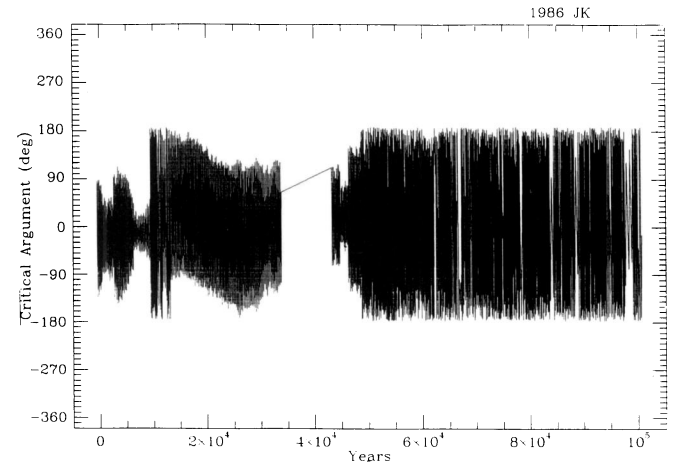
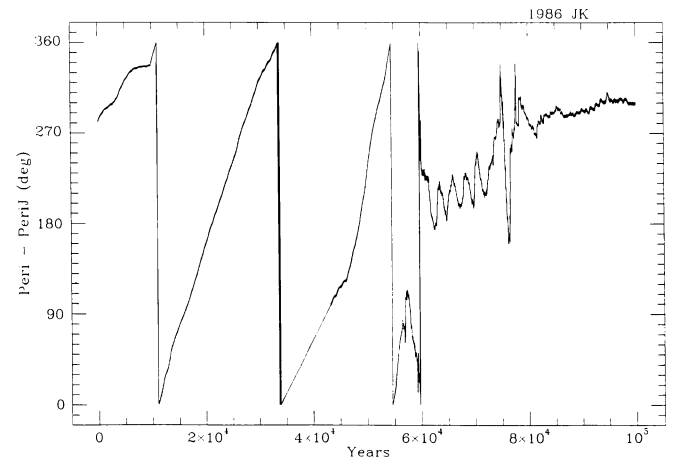
**a****b****c****d**

Fig. 12a–d. Same as in Fig. 10 but for 1986 JK

B-3 – $\varpi - \varpi_J$ librates around 180° and σ first librates ‘variably’ and then stabilizes around $0^\circ \pm 30^\circ$, which increases e to 0.8 and Jupiter encounters do occur (type R3). Different from Yoshikawa (1989). In Figs. 15a and b the time variations of a and e can be seen, and in Figs. 15c and d the evolutions of σ and e vs. $\varpi - \varpi_J$, are presented.

B-4 – Both $\varpi - \varpi_J$ and σ circulate; a crosses the resonance only once and stays then always above: non-resonant (NR)

behaviour. Different from Yoshikawa (1989). Figures 16a and b give the variations of a and e .

B-5 – $\varpi - \varpi_J$ librates around $0^\circ \pm 70^\circ$ and σ librates ‘variably’ with small amplitude (type R2). Different from Yoshikawa (1989). In Figs. 17a–c the evolution of e , σ and e vs. $\varpi - \varpi_J$ is shown.

B-6 – $\varpi - \varpi_J$ circulates with a period of 45,000 years and σ librates ‘variably’ with small amplitude; (type 2). A close encounter to Mars within the sphere of action leads to a very small but detectable change in σ , see Fig. 18a–c for the variations of e , σ and e vs. $\varpi - \varpi_J$, respectively.

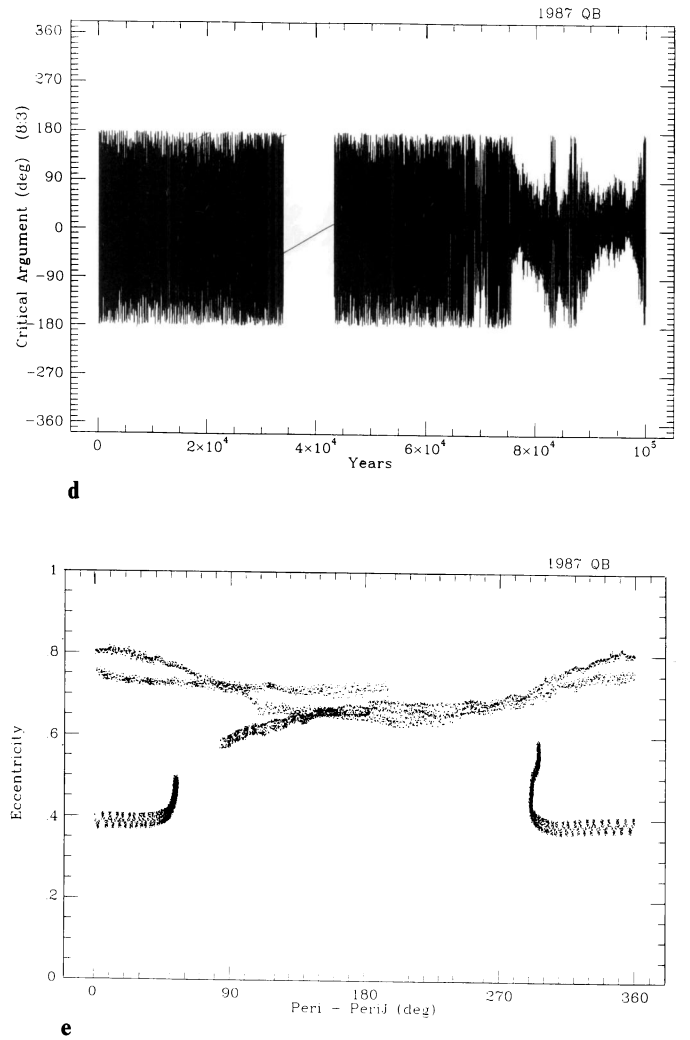
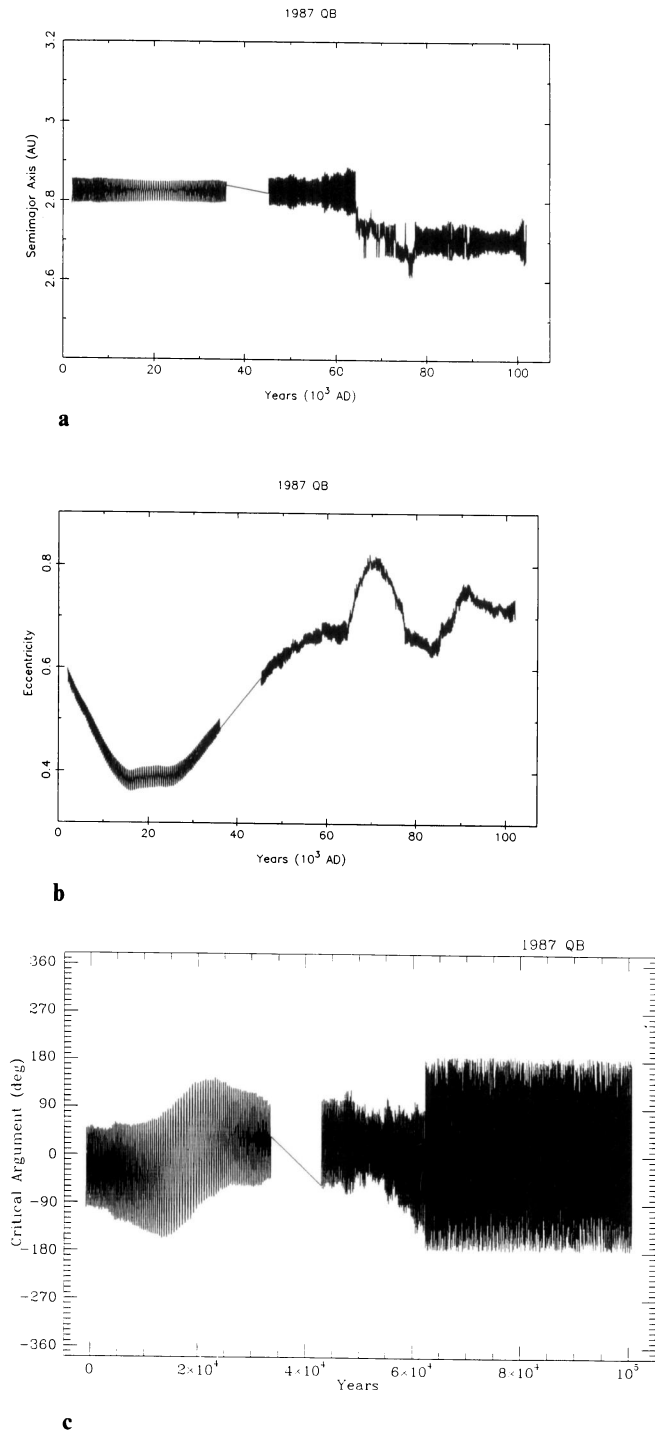


Fig. 13a–e. Variation of semi-major axis (a), eccentricity (b), and critical arguments $\sigma(5:2)$ (c) and $\sigma(8:3)$ (d), eccentricity versus $(\varpi - \varpi_J)$ (e) for 1987 QB

In general, one can say that our results compare quite well with those by Yoshikawa, and that the solar-system model used has only a minor influence as long as no planet-crossing behaviour is occurring. On the other hand, the outcome seems to be critically dependent on the starting elements, especially on the phase of σ and $\varpi - \varpi_J$ for a particular orbit.

3.1. Close encounters

In Table 3 we summarize all close encounters to the terrestrial planets and to Jupiter. The two columns for each planet contain the total number of encounters to within the chosen limits (0.1 AU and 1.0 AU, respectively) and the number of encounters to within the sphere of action as defined by Öpik (1976), given by the following formula

$$D_a = \left(\frac{1}{2\mu} \right)^{1/3} \text{ AU}$$

where μ is the mass of the planet in solar mass units.

The significance of Mars encounters for the orbital evolution during the time-span covered by our integrations is minute; in only one of the three deep encounters the effect was detectable (B-6), but still had no influence on the evolution. Longer interval (10^6 yrs) are needed to calculate the effect of Mars encounters and timescales involved. From theoretical investigations and Monte Carlo studies (see e.g. Greenberg & Nolan 1990) these effects should be of the order $\Delta a \pm 0.03$ AU, sufficient to remove the asteroid from the resonance.

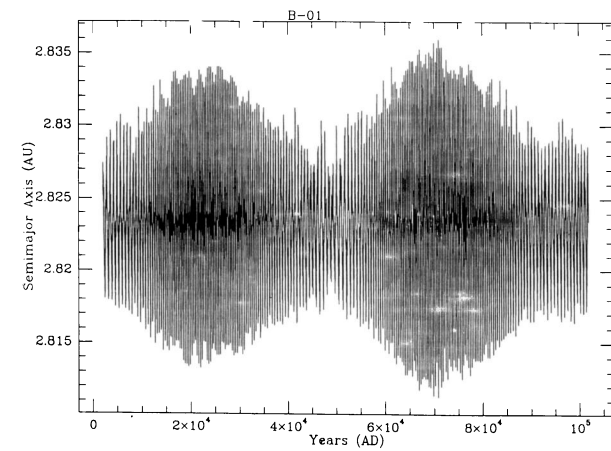
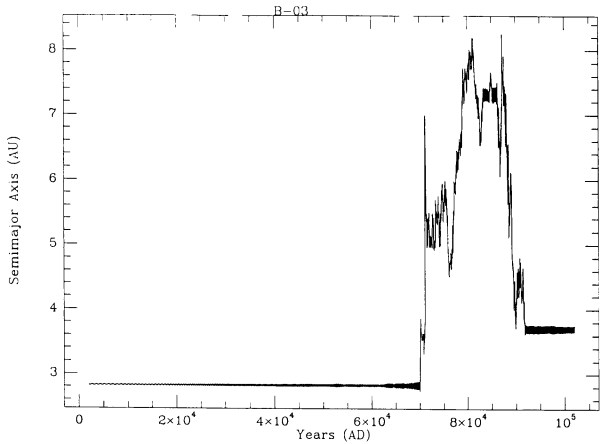
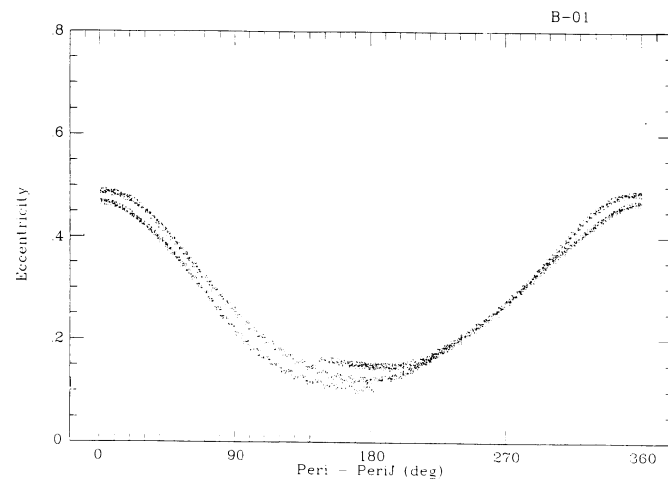
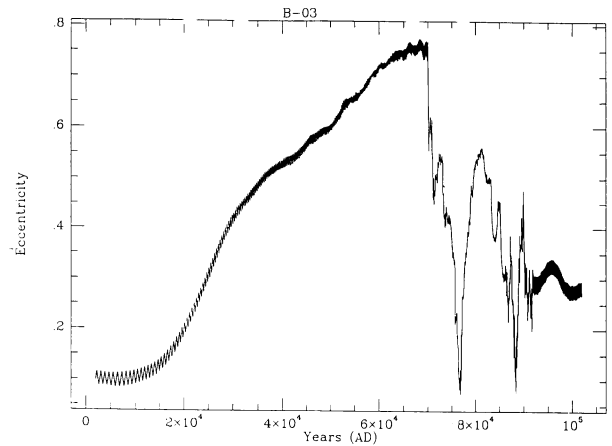
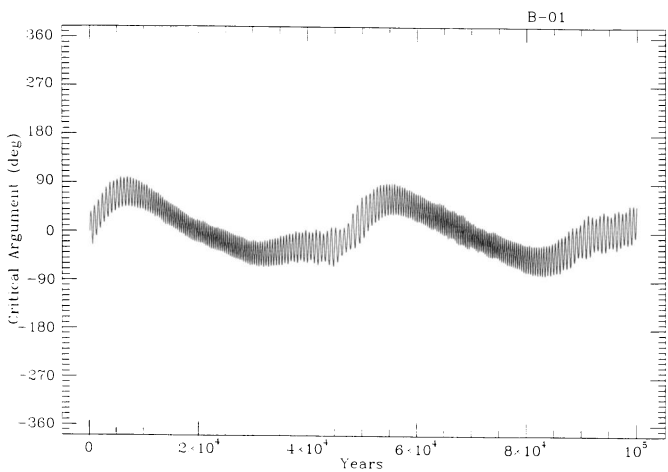
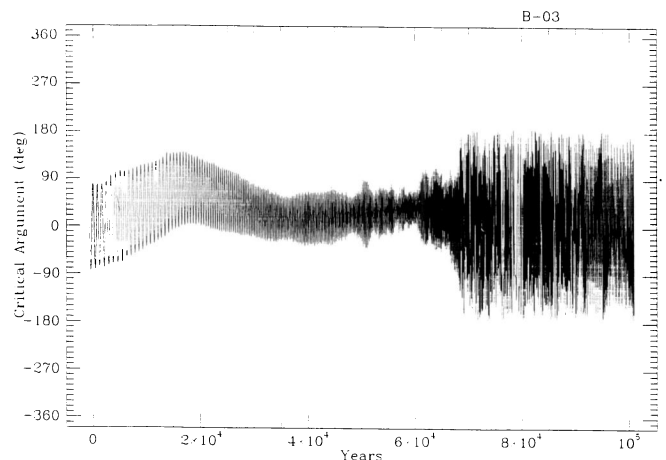
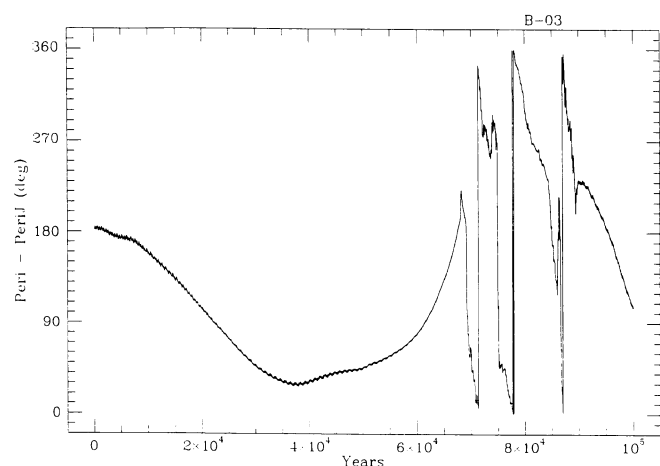
**a****a****b****b****c****c**

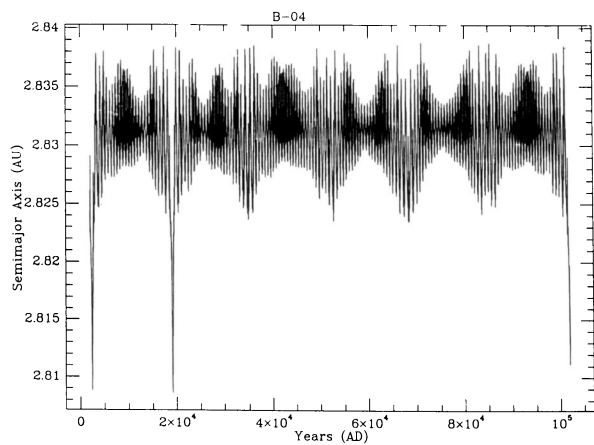
Fig. 14a–c. Time variation of semi-major axis (a), eccentricity versus $(\varpi - \varpi_J)$ (b), and critical argument σ (c) for B-1

From the 14 orbits which are resonant and not planet-crossing from the beginning (subsets 2–4 in Table 1), 8 become Mars-crossers (MC) and 3 Earth-crossers (EC); 2 also Jupiter-crossers (JC). Four of the six fictitious orbits become MC, one also VC, EC and JC.

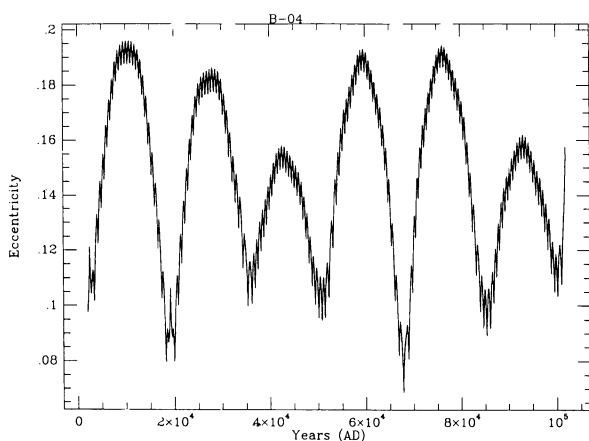


d

Fig. 15a-d. Time variation of semi-major axis (a) and eccentricity (b), critical argument σ (c) and $(\varpi - \varpi_J)$ (d) for B-3

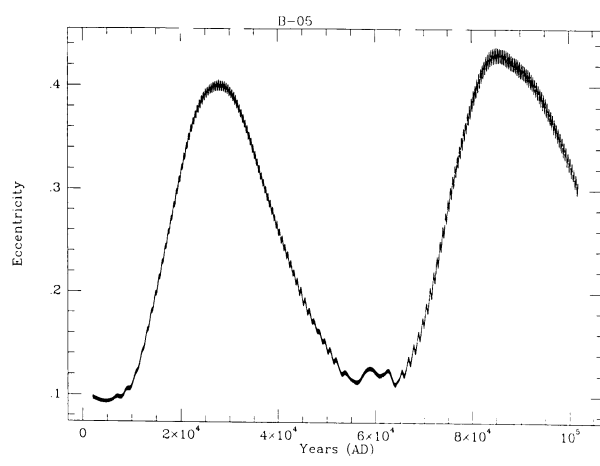


a

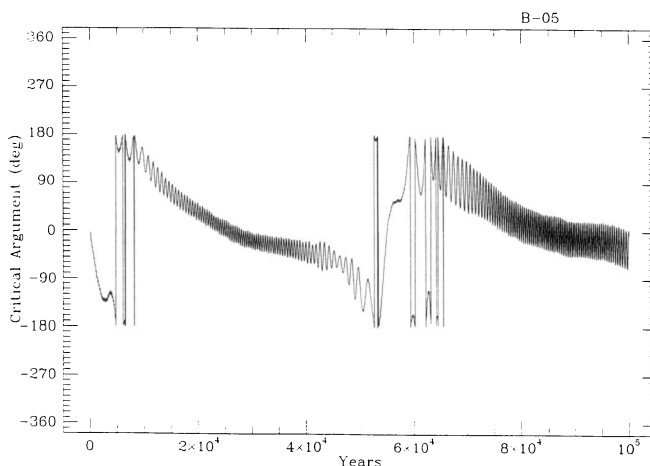


b

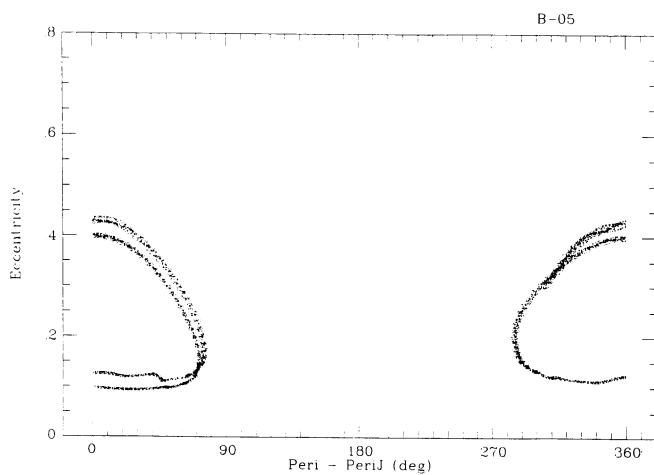
Fig. 16a and b. Time variation of semi-major axis (a) and eccentricity (b) for B-4



a



b



c

Fig. 17a-c. Time variation of eccentricity (a), critical argument σ (b) and eccentricity versus $(\varpi - \varpi_J)$ for B-5

For the four planet-crossers the importance of encounters with the terrestrial planets is much larger and significant effects has been detected in our data as mentioned above. Although little can be said about the individual orbital evolution of these objects – due to the chaotic nature of the trajectories – the rôle of close encounters is clearly shown to be a prominent one.

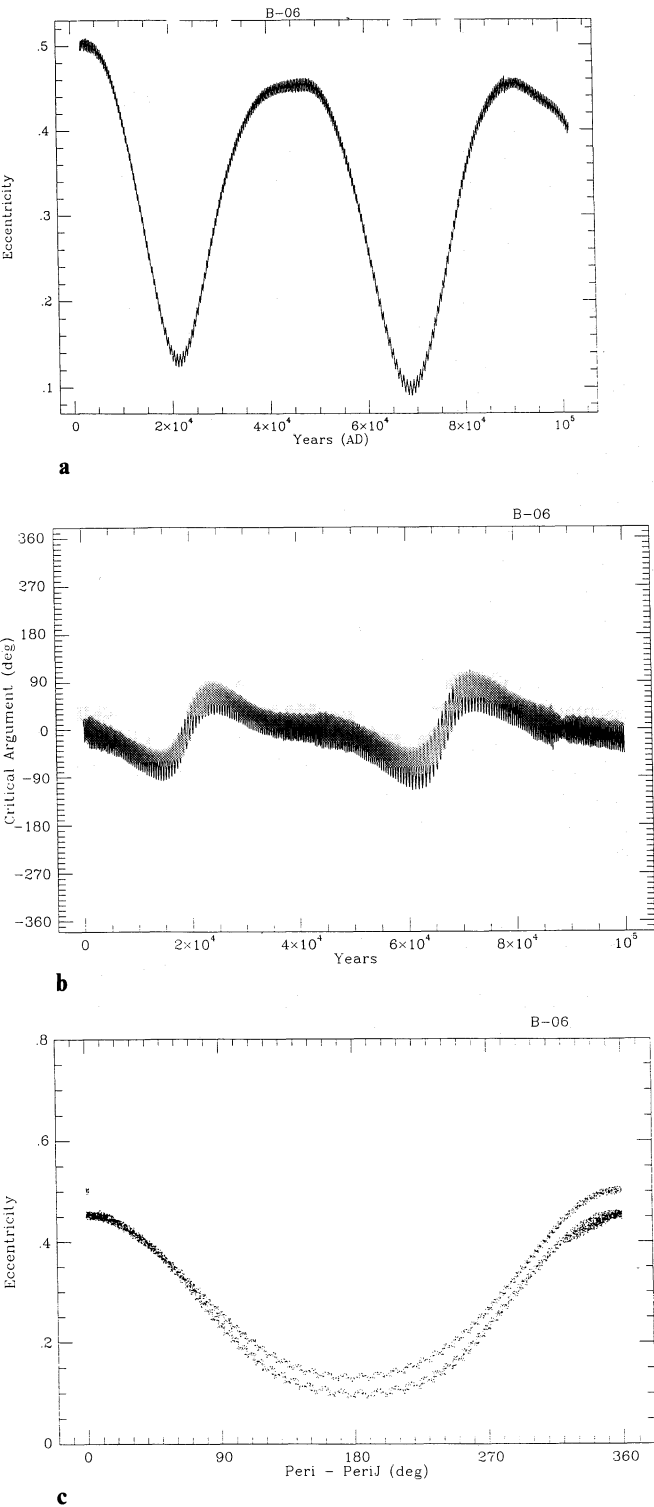


Fig. 18a–c. Same as Fig. 17 but for B6

4. Conclusion

Although a comparatively short interval of time has been covered by our integrations, a wide variety of orbital behaviour was found and one major conclusion from this study is that large orbital changes can occur on time-scales as short as 10^4 years for

Table 3. Statistic of close approaches to within 0.1 AU for the terrestrial planets and 1.0 AU for Jupiter. The second column for each planet gives the number of encounters within the sphere of action, as defined in the text

Asteroid	Venus		Earth		Mars		Jupiter	
1985 WA	47	2	45	3	7	-	341	2
1986 DA	2	-	81	1	59	-	172	-
1986 JK	4	-	157	9	60	1	295	1
1987 QB	141	6	107	4	37	1	353	-
1982 JL ₂	-	-	-	-	16	-	-	-
1983 ED	-	-	81	6	18	-	172	-
1987 QE ₃	-	-	-	-	8	1	-	-
3097 P-L	-	-	-	-	19	-	-	-
4752 P-L	-	-	-	-	3	-	-	-
6018 P-L	-	-	16	1	8	1	-	-
6337 P-L	-	-	-	-	10	-	-	-
2447 T-3	-	-	22	1	12	-	245	4
B-1	-	-	81	6	18	-	172	-
B-2	-	-	-	-	12	-	-	-
B-3	43	-	25	-	8	-	166	2
B-6	-	-	-	-	23	-	-	-

orbits in certain regions of the element space. The exact extension of this chaotic zone has to be explored further but it seems to be well represented by the borderline shown in Fig. 1, which is based on theoretical investigations.

Another interesting result is the very sensitive dependence of the strength of the suffered variation in e on the mutual phasing of the critical arguments σ and $\varpi - \varpi_J$. The amplitude and libration centre as well as the period of libration are decisive whether the change in e amounts to 0.4 or 0.6 or even higher and thereby governs the individual evolution on timescales of 10^4 years, the period of the revolution of the apsidal-line of the asteroids (10,000–20,000 years).

Close encounters to Mars are of minor importance during 10^5 years. Close encounters with the Earth and Venus can change the evolution noticeably.

If the orbit comes under the control of Jupiter the evolution is governed by the close encounters with that planet, resulting in a chaotic behaviour. This means that the timescales for becoming EC or even VC is comparatively short, but the time for removal from the inner solar system is also of the same order of magnitude. Close encounters with the terrestrial planets are important but it is Jupiter which governs the evolution. This means that the 5:2 resonance is not as effective as the 3:1 in transporting meteoritic material to the Earth as already pointed out by several investigators (see Greenberg & Nolan 1990). On the other hand, if meteorites can be traced back to such trajectories this would imply a rather recent injection of that material in such orbits.

The very fact that we are observing asteroids in such orbits at the present is therefore important for the understanding of recent events in the asteroid belt as well as for mutual collisions and other processes.

One simple explanation would of course be that these orbits are wrong and do not represent the real asteroids. But as can be seen from Fig. 1, there are quite a lot of such orbits and it would be worthwhile to try to improve some of these and clarify whether there currently exist asteroids that close to the 5:2 resonance, which could be indicative of a relatively recent collision or disruption event.

Acknowledgement. This study has been supported by the SERC.

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