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Effects of envelope overshoot on stellar models

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Abstract. We show that a certain amount of non-local overshoot at the base of the outer convective envelope of low mass stars, while climbing along the red giant branch (RGB) toward central He ignition, explains the shift of about 0.4 V mag required to bring the luminosity of the bump expected in the theoretical RGB luminosity function into agreement with the observational data. The extension of the envelope overshoot is about 0.7 Hp, where Hp is the local pressure scale height. In addition to this, we find that an equal amount of overshoot from the convective envelope in intermediate-mass stars greatly affects their subsequent evolution in the C-M diagram, producing extended loops even for models computed with significant overshoot from the convective core. This finding improves upon a weak point of models with core overshoot. In fact, while these models predict correctly the ratio of giant to main sequence stars observed in the C-M diagrams of well studied clusters in the Large Magellanic Cloud (LMC), they hardly fit the extension of the observed blue loops. It is worth recalling that classical models, in which core overshoot is neglected, although reproducing the morphology of the C-M diagrams (loop extension), fail in predicting the correct number of stars (ratio of lifetimes) in various phases across the C-M diagram. These results cast light on the role played by core and envelope overshoot. The former provides the correct H- and He-burning lifetimes for intermediate-mass stars, whereas the latter reproduces the observed morphology of young LMC clusters and the correct location of the bump in the RGB luminosity function of globular clusters. We propose that stellar models ought to be calculated taking into account both envelope overshoot, calibrated in the domain of globular clusters, and core overshoot, in turn calibrated in the range of young LMC clusters. New evolutionary sequences, to be presented elsewhere, incorporating both core and envelope overshoot, are found to agree with the observations much better than classical models (no overshoot of any kind) or models with core overshoot alone.

Key words: stars: evolution of – stars: Hertzsprung-Russell Diagram – stars: overshoot

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† It is with deep sorrow that we announce the death of Maurizio Alongi, who died at the age of 28, on Wednesday 25 July 1990, in Padova.

1. Introduction

Colour-magnitude (C-M) diagrams and luminosity functions (LF) are the classical tools by means of which the theory of stellar evolution is tested, and the age, distance and chemical composition of star clusters are determined. To this aim, the old, rich globular clusters and the young clusters in the Magellanic Clouds are ideal laboratories, as they allow the study of all evolutionary stages longer than a few hundred thousand years (e.g. Renzini & Fusi-Pecci 1988). In the last decade, these complementary tools of analysis have been improved very much with respect to the past both from the observational and theoretical point of view. As far as this latter is concerned, major attention has been devoted to the extension of convective cores in real stars since, among the key processes of stellar structure, convection is still far from being fully understood. More precisely, two main lines of thought have been developed. One considers the efficiency of convective mixing outside the unstable region to be marginal (hereafter the classical scheme), and the other (hereafter the non-local overshoot or core overshoot) ascribes the failure of classical models in explaining part of the observational information to neglecting this phenomenon. A concise discussion of the many points, in which the classical models fail to explain the observational data, and the limitations of the current theoretical models with non-local core overshoot is given in Sect. 2. Owing to the lack of a general theory of convection, the main drawback of all present day treatments of convective overshoot is the usage of free parameters (generally the mixing distance) to be fixed by comparing model results with observational data. As is well known, convection not only occurs in central cores but often also in external envelopes, which therefore are in principle affected by the phenomenon of convective overshoot. In this paper, we address the question of the extension of the envelope convection seeking to determine its maximum inward penetration by means of two observational constraints, namely the bump in the luminosity function of red giant branch (RGB) stars in globular clusters, and the extension of the blue loop of intermediate-mass stars. We endeavour to show (Sect. 3) that the observed luminosity of the bump strongly indicates an efficient penetration of the convective envelope into the stable region underneath, thus constituting a probe, perhaps unique, of overshoot from a convective region. The bump originates from the H-burning shell when, moving across the chemical profile left out by the previous central H-burning, it reaches the discontinuity in hydrogen abundance produced by the early, maximum inward penetration of the envelope convection (Iben 1968).

Classical models predict that the bump should occur at a well defined luminosity which, however, turns out to be too high with respect to the value indicated by the observations. In this section we examine how the luminosity of the bump depends on the amount of mixing at the bottom of the convective envelope. This type of mixing is hereafter referred to as the "envelope overshoot". In Sect. 4, in addition to revising the efficiency of core overshoot, we deal with the effect of envelope overshoot on intermediate-mass stars. In particular, assuming for intermediate-mass stars the same amount of overshoot as for RGB stars, we find that the core He-burning models in this mass range may exhibit extended loops in the C-M diagram. This finding eliminates the major drawback of the too narrow loops in the C-M diagram in models of intermediate-mass stars calculated with overshoot from the central convective core, which on the other hand are found to reproduce the number ratio of main sequence to evolved stars much better than the classical models. Finally, a few concluding remarks are made in Sect. 5. In particular, we make the point that, in the absence of a self-consistent theory of convection, stellar models ought to be calculated taking into account both envelope overshoot, calibrated on the bump in the RGB luminosity functions of globular clusters, and overshoot from the convective core, calibrated on the C-M diagrams and luminosity functions of young, rich clusters.

2. Convective overshoot versus classical mixing

Since a thorough discussion of the properties of models with overshoot from the convective regions (usually the core) is beyond the aims of this paper, and the vast majority of studies deal with core overshoot, we will outline the main differences with respect to the classical models limited to models with core overshoot alone. In the classical models, the size of a convective region is defined by a local stability analysis, usually the Schwarzschild inequality between the local adiabatic and radiative temperature gradients. This includes also those models, in which (during the central He-burning phase) a perturbation of the chemical composition in the stable region surrounding the convective core leads to the violation of the local stability condition, and eventually to the total (local overshoot) or partial (semi-convection) mixing of these layers into the core. On the contrary, in models with a non-local scheme of overshoot a certain amount of matter, stable against the onset of convection, is supposed to be totally mixed with elements overshooting from the unstable region because of their positive kinetic energy. Generally, the efficiency of convective overshoot is parameterized either by a fraction Λ of the local pressure scale height Hp (Shaviv & Salpeter 1971; Maeder 1975; Bressan et al. 1981; Bertelli et al. 1985; Maeder & Meynet 1989) or a diffusive scale height (Cloutmann & Whitaker 1980; Cloutmann 1987; Stothers & Chin 1981; Xiong 1985, 1986, 1989).

Models incorporating core overshoot possess a larger convective core and differ more and more from the classical ones as they evolve from the zero age main sequence. They run at a higher luminosity and live longer during the central H-burning phase, possess a bigger H-exhausted core and a higher luminosity while burning He in the core, and finally approach the double shell phase with a larger He exhausted core (Bertelli et al. 1985, 1986). Compared to those obtained from classical models, isochrones and synthetic C-M diagrams based on models with core overshoot exhibit the following characteristics (Chiosi et al. 1988; Maeder & Meynet 1989; Chiosi et al. 1989; Bertelli et al. 1990):

(i) much wider main sequence band;

(ii) older ages when determined from turn-off, red star clump (He-burning) and asymptotic giant branch (AGB) luminosities;

(iii) greater number of main sequence stars per given number of post main sequence objects, reflecting the different ratios of lifetimes;

(iv) higher luminosity of the core He-burning models but much narrower loops, which hardly fit the well observed blue loops of intermediate age star clusters;

(v) absence of luminous C and M stars in cluster with turn-off masses greater than $M_{UP} \simeq 5\,M_\odot$, (instead of $M_{UP} \simeq 8$–$9\,M_\odot$ of classical models);

(vi) lower turn-off mass in the mass (age) range which sees the transition from clusters exhibiting a well developed RGB to those which do not. For a solar composition, this transition occurs at $M_{Hef} \simeq 1.6\,M_\odot$ (instead of $M_{Hef} \simeq 2.2\,M_\odot$ of classical models).

The larger the size of the overshoot region is, the more these features stand out. In principle, this would allow the calibration of the parameter Λ by means of the careful comparison of model predictions with observations of real clusters. In practice, since other effects are present both in the theory and the observations, the fine tuning of Λ is hardly feasible.

The presence of binary stars may broaden the main sequence band in the C-M diagram and, for a mass ratio of the binary components close to one, increase the turn-off luminosity by more than half a magnitude.

As far as the effects upon the age determination are concerned, nothing can be said unless independent estimates of the ages are available. Indeed, ages determined through the analysis of stars contracting toward the lower main sequence in young clusters seem in good agreement with those obtained from fitting the cluster C-M diagram with isochrones accounting for core overshoot (Mazzei & Pigatto 1989).

The number ratios of main sequence to evolved stars in clusters have been examined by Chiosi et al. (1989), Vallenari et al. (1990a,b) and Bressan (1990). The results can be summarized by saying that in well studied young or intermediate age clusters, the observed number of main sequence stars per given number of red giant stars is greater by more than a factor of two than the value predicted by canonical models, and agrees with the predictions from models with overshoot. Taking the $5\,M_\odot$ star as the typical mass at the turn-off of the intermediate age clusters, from classical models we get the ratio of He- to H-burning lifetimes equal to about 0.25, whereas from models with core overshoot we obtain 0.10 to 0.06 when the overshoot parameter Λ varies from 0.5 to 1, respectively. These lifetime ratios reflect onto the theoretical integrated luminosity function (ILF) of the main sequence stars, normalized to the number of He-burning stars (Chiosi et al. 1989). In the case of models with overshoot, the ILF runs steeper than in the case of classical models. Only for the former case, the ILF has been seen to reproduce the observed luminosity function of well studied LMC clusters (Chiosi et al. 1989; Vallenari et al. 1990a,b). If those findings are confirmed by further observations, it will be hard to provide an alternative to the simple explanation offered by models with convective overshoot.

The narrower extension of the blue loops in models with overshoot from the convective core will be discussed in more detail below, whereas the absence of bright AGB stars in intermediate age clusters of the Magellanic Clouds (MC) can be explained by the effects either of mass loss by stellar wind or core overshoot or a combination of both (Chiosi et al. 1986; Frogel et al. 1990).

Finally, the last topic of the above list refers to the observations of old open clusters in the Galaxy or of coeval clusters in MC. The luminosity functions of red giant stars observed in several clusters indicate a transition mass between models igniting He quietly and those undergoing core helium flash, which is lower than the value given by classical models and in agreement with the expectations from models with substantial overshoot during the central H-burning phase (Barbaro & Pigatto 1984; Maeder & Meynet 1989). However, the poor statistics on old open clusters together with the uncertainties introduced by the indetermination of the distance, age and composition, makes the above arguments very weak, even if recent studies of C-M diagram of old open clusters (Twarog, private communication) indicate excellent fits with models incorporating core overshoot.

In spite of the good results obtained with models accounting for convective overshoot from the central cores, classical models are often preferred, partly because of their simple physical description of the extension of convection (no free parameter is required), and partly because models with overshoot hardly reproduce the morphology of the blue loops of young clusters of the MC (Brocato & Castellani 1988; Chiosi et al. 1989). The first reason is an inevitable consequence of the lack of a satisfactory theory of convection in stellar interiors so that as soon as the simple Schwarzschild condition is abandoned, free parameters like the mixing distance Λ come in, which eventually should be determined by comparing model results with observation. The second one is a true weak point of models with core overshoot that was already noticed by Matraka et al. (1982). While the higher luminosity of the loops of models with core overshoot, during the helium burning phase, would offer an elegant way out of the old discrepancy between pulsational and evolutionary mass of Cepheid stars (Matraka et al. 1982; Bertelli et al. 1985), the fact that, even for metal abundances as low as $Z = 0.004$, the loops are much narrower than in classical models, very much limits the usage of models with overshoot in studies of the C-M diagrams of young clusters (see the discussion in Sect. 4). It has been suggested that the loop morphology is less of a problem as it is known to depend on a number of details in the input physics, such as the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate (Iben 1967), the mixing length parameter in the outer envelope (Huang & Weigert 1983) or the amount of mass lost along the Hayashi line (Bertelli et al. 1985), so that a fine tuning of the above parameters could yield the desired extension. However, numerous evolutionary calculations have clearly demonstrated that none of these effects can reinstate extended loops in models with core overshoot.

Since star counts (luminosity functions) constrain stellar evolution theories much more strongly than the loop morphology, instead of abandoning the models with core overshoot in favour of the classical ones, we looked for an ingredient still missing in the input physics and, as we endeavor to show in the sections below, we find that allowing for a small amount of overshoot at the bottom of the convective envelope, produces very extended loops even in models with core overshoot. This idea is not as ad hoc as it may seem, since first it is consistent with the treatment of convection in the core, and second it is strongly indicated by the location of the "bump" in the luminosity functions of the RGB stars in globular clusters.

3. The bump in the RGB star luminosity function of globular clusters: a hint for overshoot in convective envelopes

Only recently, observations of RGB stars in globular clusters (King et al. 1985; Hesser et al. 1987; Fusi-Pecci et al. 1990) have confirmed the existence of a bump, whose origin was first pointed out by Iben (1968), in the differential luminosity function of these stars. Since the RGB luminosity function maps the hydrogen profile established during the central H-burning phase (Renzini & Fusi-Pecci 1988), the luminosity of the bump identifies the mass-coordinate of the bottom of the homogeneous envelope and, in turn, the maximum depth reached by the external convection.

Fusi-Pecci et al. (1990), revisiting the relationship between the absolute visual magnitude of the horizontal branch (HB) stars and the metallicity, first detect the RGB bump for eleven globular clusters, and second determine its luminosity. They find that the observed luminosity of the bump is $0.415 \pm 0.07\, M_V$ fainter than the value predicted by current stellar models.

Analyzing the causes of the discrepancy, they indicated three main possibilities related to the input physics of the models: (i) the usage of old opacities (Cox & Stewart 1970a,b), which are lower than the most recent ones for the same metal content (Huebner et al. 1977); (ii) the extension of the envelope convection; (iii) finally, the initial abundance of He.

They attributed 0.2 mag of the luminosity discrepancy to the usage of the old (lower) opacities and the remaining 0.2 mag to the He abundance, for which they indicated the value $Y = 0.20$ instead of the classical, mean value of 0.23 derived by Buzzoni et al. (1983) from an analysis of 15 globular clusters, which is in better agreement with the predictions for the primordial nucleosynthesis. As far as the use of the classical scheme of convection in the models at the base of their analysis is concerned, they estimated that in order to extend the convective envelope downward in mass by the amount (about 0.02–0.03 $M_\odot$) that is indicated by the desired shift in the bump luminosity, overshoot should take place across about 5 Hp, which seems too high compared to what is usually found in other astrophysical contexts (Bertelli et al. 1985; Maeder & Meynet 1989). However, the following remarks can be made against their conclusions. On one hand, the shift in magnitude with the more recent opacities is estimated for a solar metallicity, while the correction is expected to be negligibly small going to much lower metallicities like those holding for globular clusters. On the other hand, the negligible opacity effect at lower metallicities implies that an even lower abundance of primordial He should be invoked to account for the required magnitude shift. On the contrary, the indication found from the use of the new opacities in models of the Sun goes the other way around. In fact, with the new opacities the solar abundance of He turns out to be higher (about 0.27 instead of 0.25) than with the old opacities (VandenBerg 1983). Furthermore, the suggested lower abundance of He has a contradictory effect that will be discussed in more detail below.

At the light of these arguments we would like to explore further the point already made by Fusi-Pecci et al. (1990) that the bump in the RGB star luminosity function may act as a powerful probe of mixing processes in stellar interiors. To this purpose, we have computed evolutionary sequences with masses and chemical composition characteristic of the RGB stars of globular clusters, and various prescriptions for the extension of the envelope convection, which go from the classical formulation to including downward overshoot from the base of the convective envelope at different efficiencies.

3.1. Physical assumptions and mixing schemes

Evolutionary models are presented for stars of initial mass of $0.8\,M_\odot$, helium abundance of $Y = 0.23$, and two extreme metal abundances, namely $Z = 0.001$ and $Z = 0.0001$. In these calculations we adopt the new opacity library of Huebner et al. (1977) supplemented by the molecular contribution (Alexander 1975; Alexander et al. 1983) according to the prescription by Bessell et al. (1989) and the revision by Wood (1990, private communication).

Finally, care is paid to imposing that our evolutionary code matches the age and position of the Sun in the C-M diagram. Assuming the age of $4.6\,10^9$ yr, we obtain $Z = 0.017$, $Y = 0.27$ and a mixing length parameter in the envelope $\alpha = 1.5$, in agreement with other computations (VandenBerg 1983). Maeder & Meynet (1989) using their evolutionary code, the same opacity, and the same age of the Sun adopted in the present paper, find that to account for the solar radius a value $\alpha = 1.9$ is necessary. For the arguments given by Pedersen et al. (1990) the disagreement is more apparent than real and likely due to small differences in the treatment of the molecular opacity, in the particular version of the mixing length theory that has been adopted in the outermost convective layers, and many other computational details like the interpolation technique of the opacity tables, boundary conditions at the photosphere, etc. As argued by Pedersen et al. (1990), α is a "local" parameter whose calibration on the solar properties is sufficient to get internal consistency and correct RGBs of globular clusters. However, models will be presented calculated with $\alpha = 2$ in order to test the sensitivity of the inner structure, the maximum inward penetration of the external convection in particular, to variations of this parameter. As expected, the effect will turn out to be negligible.

As a preliminary step, we analyze the results for the classical mixing scheme (without overshoot of any kind) at the base of the convective envelope (hereafter indicated as $\Lambda_{en} = 0$). For this particular case we also investigate the effect of a lower He content, namely $Y = 0.20$. Then we proceed to test a local-overshoot scheme, which is an extension of the classical convective treatment suggested by Castellani et al. (1971) for the core He-burning stars. In this scheme a shell of matter surrounding the unstable region, is first temporary homogenized and then definitely mixed if, with the new composition, it turns out to be unstable. We apply this algorithm to the bottom of the convective envelope in the model of $M = 0.8\,M_\odot$, $Z = 0.001$, $Y = 0.23$ and $\Lambda_{en} = 0$. However, since no appreciable additional mixing is obtained, this scheme is abandoned. Finally, we study the effects of non-local overshoot in a simple manner. This phenomenon is described and parameterized by mixing a fraction Λ_{en} of the pressure scale height below the classical boundary, determined by the Schwarzschild criterion. The following cases of envelope overshoot are investigated: $\Lambda_{en} = 0.7$ and $\Lambda_{en} = 1.0$. It is worth clarifying that the parameter Λ_{en} has a different meaning from the parameter Λ in the formalism by Bressan et al. (1981) used to describe overshoot from convective cores (see Sec. 4).

3.2. The results

Table 1 summarizes our results for the luminosity of the bump in the RGB luminosity function together with other properties of the computed models. The meaning of the various quantities listed in Table 1 is as follows. The first column indicates the efficiency of envelope overshoot. The second column gives additional information on the models: (FP) indicates that the values of the absolute magnitude of the bump is obtained from the relationships (1) and (2) in Fusi-Pecci et al. (1990), (Y02) labels the sequences computed with lower helium abundance ($Y = 0.20$), while all the others have the He content indicated in the heading of the table, finally $\alpha_{ML} = 2.0$ labels the sequence computed with a higher mixing length parameter. L_b, T_{eff_b}, M_V(YA), M_V(BG) and M_V(BK) are the logarithm of the luminosity in solar units, the logarithm of the effective temperature, three different values of the absolute visual magnitude of the bump, respectively. These latter are given for different sources of bolometric corrections, which are Green et al. (1989), otherwise known as the Yale conversions, Bell & Gustafsson (1978) and Gustafsson & Bell (1979), and Buser & Kurucz (1989), indicated by YA, BG and BK, respectively. L_{tip}, $T_{eff_{tip}}$, $M_{V_{tip}}$(YA), $M_{V_{tip}}$(BG) and $M_{V_{tip}}$(BK) are the corresponding quantities at the RGB tip. Finally, m_d is the

Table 1

Notes	L_b	T_{eff_b}	M_{v_b}(YA)	M_{v_b}(BG)	M_{v_b}(BK)	m_d	m_c	L_{tip}	$T_{eff_{tip}}$	$M_{v_{tip}}$(YA)	$M_{v_{tip}}$(BG)	$M_{v_{tip}}$(BK)
$M = 0.8\,M_\odot$ $Z = 0.001$ $Y = 0.23$												
FP	1.97			0.28								
$\Lambda_{en} = 0.0$	1.95	3.67	0.25	0.44	0.34	0.300	0.450	3.13	3.60	−2.00	−2.04	−2.03
$\Lambda_{en} = 0.7$	1.76	3.68	0.75	0.85	0.75	0.282	0.452	3.15	3.60	−2.03	−2.05	−2.06
$\Lambda_{en} = 1.0$	1.68	3.69	0.84	1.05	0.95	0.275	0.453	3.15	3.60	−2.03	−2.05	−2.06
Y02	1.89	3.67	0.45	0.55	0.54	0.297	0.456	3.13	3.60	−2.00	−2.01	−2.03
$\alpha_{ML} = 2.0$	1.95	3.70	0.09	0.25	0.19	0.300	0.450	3.12	3.63	−2.45	−2.26	−2.30
$M = 0.8\,M_\odot$ $Z = 0.0001$ $Y = 0.23$												
FP	2.27			−0.48								
$\Lambda_{en} = 0.0$	2.25	3.69	−0.53	−0.44	−0.44	0.352	0.449	2.96	3.65	−2.05	−2.02	−2.06
$\Lambda_{en} = 0.7$	2.07	3.69	−0.08	0.02	−0.05	0.331	0.451	2.98	3.65	−2.10	−2.07	−2.11
$\Lambda_{en} = 1.0$	2.02	3.70	0.00	0.11	0.04	0.325	0.452	2.98	3.65	−2.10	−2.07	−2.11
Y02	2.19	3.69	−0.38	−0.28	−0.36	0.348	0.456	2.98	3.65	−2.10	−2.07	−2.11

mass of the maximum inward extension of the convective envelope and m_c is the mass of the core at He flash, both expressed in solar units. To compare our results with those obtained from classical models we use the relationships (1) and (2) together with the data of Table 5 by Fusi-Pecci et al. (1990), which are derived from the theoretical models of Crocker & Rood (1990). Those relationships provide the bolometric luminosity and the absolute visual magnitude of the bump as functions of various parameters.

Looking at the results reported in Table 1, we see that the predicted bump luminosity of models without envelope overshoot is almost coincident with the one obtained from Fusi-Pecci's et al. (1990) relationships, thus clarifying that the novel opacities have little effects, at least in the range of metallicities we are considering. The corresponding M_V's depend on the adopted bolometric corrections. At the luminosity and effective temperature of the bump, the conversions by Green et al. (1987) give M_V magnitudes that are about 0.2 brighter than those obtained from using Bell & Gustafsson (1978) and Gustafsson & Bell (1979) data, whereas there is a little difference at the RGB tip. Since also Fusi-Pecci et al. (1990) have used the latter tables, the difference between their results and ours may be ascribed to marginal differences in luminosity and effective temperature.

The mass of the layer reached by the bottom of the convective envelope (the quantity m_d of Table 1) clearly indicates that the net effect of envelope overshoot is to extend the mixed region by a small quantity. In fact, comparing m_d of models without and with envelope overshoot, the mixed region increases only by about 0.018 $M_\odot$ and 0.025 $M_\odot$ more than in the classical scheme, for $\Lambda_{en} = 0.7$ and $\Lambda_{en} = 1.0$, respectively. This almost independently of the metal content. Looking at the run of the hydrogen abundance and pressure as a function of the mass, shown in Fig. 1 for the model with $\Lambda_{en} = 1.0$ at the stage of maximum inward penetration of the convective envelope, we see that the estimate made by Fusi-Pecci et al. (1990) of about 5 pressure scale heights required to mix the same amount of material turns out not to be correct. This was likely caused by having considered the stage when the outward shift of the H-burning shell has already stiffened the run of the thermodynamical variables at the chemical discontinuity.

The effect of envelope overshoot on the luminosity of the bump is shown in Figs. 2 through 5. Figures 2 and 3 display the differential luminosity functions (in arbitrary units) obtained for the two chemical compositions (neglecting the slight effect of the IMF) and with the Buser & Kurucz (1989) bolometric corrections, whereas Figs. 4 and 5 show the corresponding integrated luminosity functions, where the bump is visible as a jump. At increasing efficiency of the envelope overshoot, the bump shifts to fainter luminosities and becomes more prominent. The latter feature bears some interest in disentangling among the various explanations that have been invoked for the effect of envelope convection. Within the range of Λ_{en} considered here and the bolometric corrections either by Buser & Kurucz (1989) or Bell & Gustafsson (1978) and Gustafsson & Bell (1979), we can express the magnitude shift ΔM_V as a function of the efficiency of envelope overshoot by means of the following relation:

$$\Delta M_V = 0.6\Lambda_{en} \quad (1)$$

which is almost independent of the metal content, whereas relation (1) is no longer linear if Green's et al. (1987) bolometric corrections are used. Therefore, assuming that relation (1) is holding good, the magnitude shift of about 0.415 M_v in the bump of the RGB luminosity functions invoked by Fusi-Pecci et al. (1990), can be explained by envelope overshoot alone with $\Lambda_{en} = 0.7$, a value compatible with the amount of overshoot adopted in other

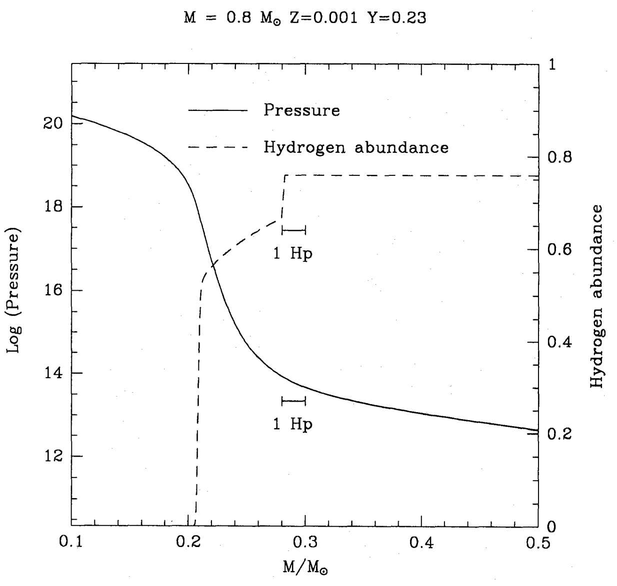

Fig. 1. The run of the pressure and the hydrogen profile as a function of the internal mass at the stage along the RGB of maximum inward penetration of the convective envelope in the star of 0.8 $M_\odot$ and $Z = 0.001$, $Y = 0.23$. The horizontal bar shows the extension of the region interested by envelope overshoot with efficiency of 1 Hp

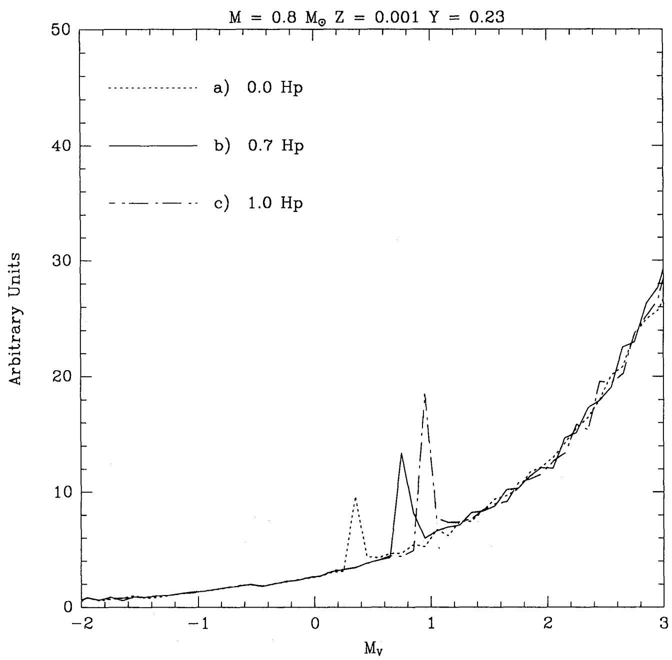

Fig. 2. Differential luminosity functions (in arbitrary units) vs. the absolute visual magnitude M_V for the RGB stars of the sequences with $M = 0.8\,M_\odot$ and $Z = 0.001$, $Y = 0.23$ with $\Lambda_{en} = 0$, $\Lambda_{en} = 0.7$ and $\Lambda_{en} = 1.0$ as indicated. Λ_{en} gives the amount of envelope overshoot in units of pressure scale height. The absolute visual magnitudes have been obtained by means of the Buser & Kurucz (1989) bolometric corrections (see the text for more details)

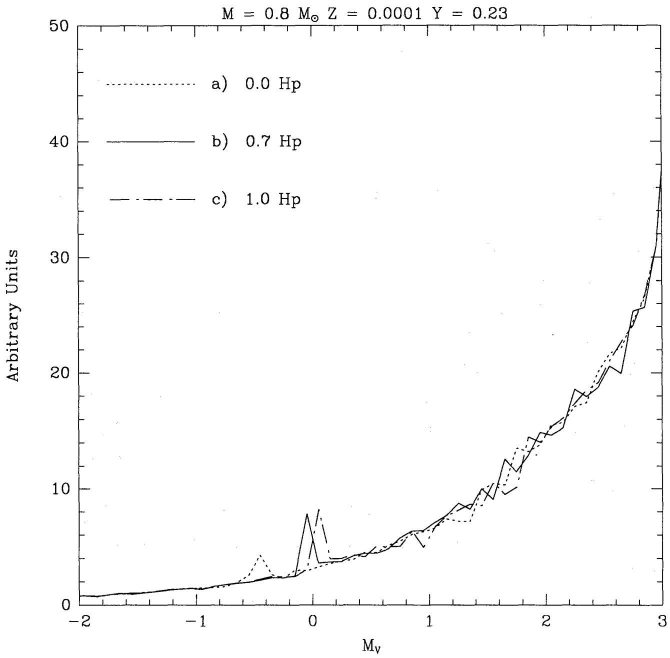

Fig. 3. The same as in Fig. 2 but for the chemical composition $Z = 0.0001$, $Y = 0.23$

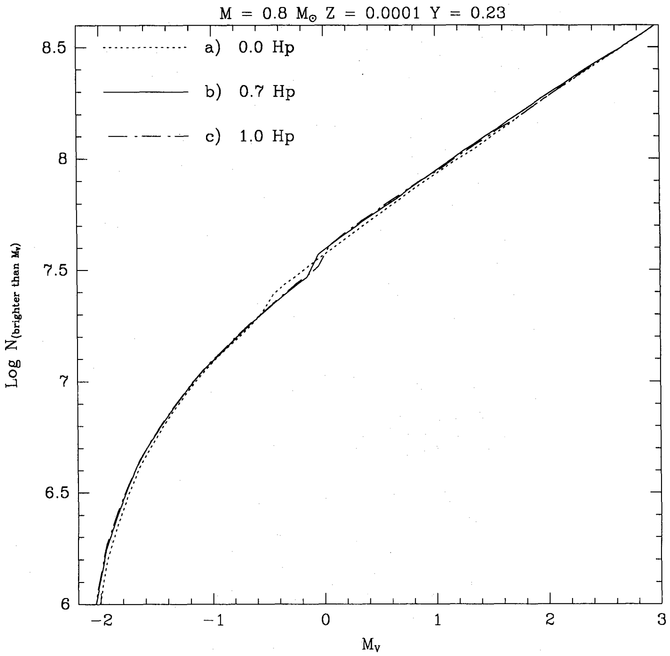

Fig. 5. Integrated luminosity functions in arbitrary units as a function of the absolute visual magnitude for the three models of Fig. 3

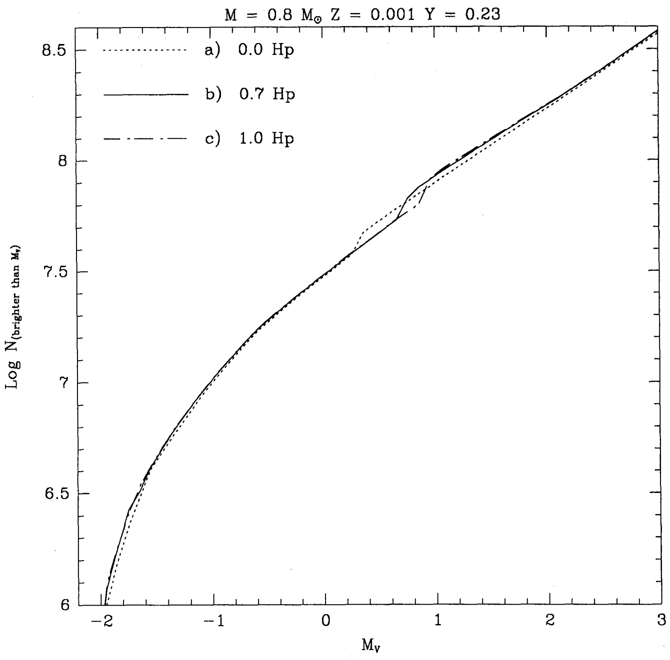

Fig. 4. Integrated luminosity functions in arbitrary units as a function of the absolute visual magnitude for the three models of Fig. 2

astrophysical circumstances (Bressan et al. 1981; Bertelli et al. 1985; Chiosi et al. 1989; Maeder & Meynet 1989).

As far as the effect of a different value of He content is concerned, Table 1 shows that lowering the initial abundance of this element by 0.03 decreases the brightness of the bump by about 0.15 M_v, as already noticed by Fusi-Pecci et al. (1990). However, this way of reconciling the theoretical luminosity of the bump with the observational determination is not very safe. In fact, not only it disagrees with the He abundance indicated by the other properties of globular clusters and primordial nucleosynthesis, but also it implies a higher turn-off mass if the C-M diagram of a given cluster is fitted by isochrones calculated with such a lower value of Y. As shown by Eq. (2) of Fusi-Pecci et al. (1990), this fact would compensate for the effect of a lower He abundance on the bump luminosity.

Finally, we analyze the effect of a different choice for the mixing length parameter (α). To this purpose, an evolutionary sequence with $Z = 0.001$ and $Y = 0.23$ is calculated assuming $\alpha = 2.0$, a value almost identical to the one adopted by Maeder & Meynet (1989). We would like to recall here that, on the contrary, the calibration of our evolutionary code on the properties of the Sun gave $\alpha = 1.5$. The results of Table 1 (row labeled by $\alpha_{ML} = 2.0$) indicate that in spite of the different mixing length, no appreciable differences in the maximum inward penetration of the convective envelope and correspondingly in the luminosity of the bump of the RGB luminosity function are found. The RGB locus as a whole shifts by $\Delta \log T_{\rm eff} \simeq 0.03$ which renders the bump slightly brighter in the visual by $\Delta M_v \simeq -0.15$ or -0.19, depending on the adopted bolometric correction scale. Therefore, the use of a higher mixing length parameter affects the absolute visual magnitude of the bump only through the different bolometric corrections and, in such a case, acts in the opposite sense, giving rise to a brighter bump.

3.3. General remarks

Our evolutionary models show that under the occurrence of mild envelope overshoot (about 0.7 pressure scale heights or less below the classical limit) it would be possible to shift downward the bump luminosity, as it is required to match the observed M_v of HB stars [see Fusi-Pecci et al. (1990) for all details]. Such an achievement is entirely due to the response of the outermost layers to the deeper extension of the convective envelope since, as indicated by the results contained in Table 1, other important quantities, like the core mass and the He-flash luminosity remain unchanged. We would like to emphasize that the above results have been obtained by allowing convective envelope to penetrate deeper by a very small amount of mass and in turn the surface abundance of He to increase by a tiny fraction. In fact, in the case of $\Lambda_{en} = 1.0$, the abundance of He in the envelope is increased by overshoot only by about 0.002 with respect to the classical scheme. We would also like to remark that it cannot be a mere coincidence the fact that the extra-mixed region roughly corresponds to the one we would have obtained by considering that elements possess enough kinetic energy to travel a mixing length across the boundary of the unstable region.

Our models do not confirm the need of significant variations in the opacity, whereas they cannot rule out lower abundances of initial He. However this latter alternative either looks unlikely or even acts in the opposite sense if one considers the dependence of the bump luminosity on the initial mass of the stars.

Finally we would like to comment on the fact that Fusi-Pecci et al. (1990) have looked at the theoretical luminosity of the bump in the RGB luminosity functions of globular clusters as a way of calibrating their distances. Unfortunately, it is a shame that this route to the distance scale passes through the wild forest of convection. We rather prefer to turn the problem around and use the high sensitivity of the bump to convection to calibrate this latter.

In conclusion, the observations of the RGB luminosity functions in globular clusters strongly suggest that full mixing takes place below the formal border of convective envelope over a significant fraction (0.7–0.5) of the local pressure scale height. In the next section, we will show how such an "envelope overshoot" can greatly affect the extension of the loops of intermediate-mass stars even in models accounting for overshoot from their convective cores, thus alleviating a point of drawback in this type of model.

4. Core and envelope overshoot in intermediate-mass stars

As already recalled in Sect. 1, Bertelli et al. (1985), Chiosi et al. (1989a), Vallenari et al. (1990a,b), comparing the main sequence star luminosity functions of well studied clusters of the Large Magellanic Cloud (LMC) with their theoretical counterparts, favour models accounting for non-local overshoot from the convective core, against those in which the boundary of the central convective region is determined by the simple Schwarzschild criterion. The main reason of this is the fact that evolved stars in real clusters are much fewer than predicted by the classical models. Looking at the prototype cluster NGC 1866, the number ratio of evolved to main sequence stars, these latter counted in the luminosity range interested by the main sequence phase of all stars whose mass is compatible with the mass of the evolved stars, is of the order of 0.10. This is also confirmed by the run of the integrated main sequence star luminosity function normalized to the number of giants (see Chiosi et al. 1989a for more details). It is worth recalling that these star counts are made using good quality CCD photometric data (Chiosi et al. 1989b), which include analyses for photometric completeness, crowding, and foreground stars contamination, and supersede the previous data by Robertson (1974). However even with these latter, the difficulty stands out as originally indicated by Becker & Mathews (1983). Since the ratio of giants to main sequence stars simply reflects the ratio of core He- to H-burning lifetimes, in order to decide whether or not evolutionary models agree with the observational information, an inspection of current lifetimes and lifetime ratios may be instructive. These are summarized in Table 2 for the stellar masses typical of the turn-off of young clusters (5 and 7 $M_{\odot}$) together with information on the type of mixing in the convective core and opacities in usage. The lifetime ratio t_{He}/t_{H} implied by the observational data for NGC 1866, whose turn-off mass is about 5 $M_{\odot}$, is about 0.1. Limiting the comparison to the case of 5 $M_{\odot}$ star in Table 2, we can evaluate separately the effect of opacities and overshoot on the lifetimes and their ratios. In classical models (i.e. models in which the border of the convective core is determined either by the Schwarzschild criterion and/or local analyses like semiconvection) the ratio t_{He}/t_{H} lowers from 0.43 to 0.25 going from old to recent opacities (molecular contribution included). This effect is mainly due to the increase in t_{H}, while there are smaller effects attributable to differences in chemical compositions and details of the input physics. As soon as the local treatment is abandoned and non-local convective overshoot is included, almost independently of the opacity and chemical composition, the ratio is decreased to the range 0.10 to 0.13. The lifetimes of this type of models are obtained adopting the so-called mild overshoot corresponding to $\Lambda = 0.5$ in the formalism of Bressan et al. (1981). The lower ratios are mainly due to a 20% increase in t_{H} and to a decrease in t_{He} by more than a factor of two, this latter effect being the consequence of the bigger H-exhausted core. The only exception to this scheme are the models by Maeder & Meynet (1989), who find the ratio $t_{He}/t_{H} = 0.28$ like the classical models. While their H-burning lifetime is similar to that obtained by other authors using new opacities and mild overshoot, their He-burning lifetimes are significantly longer. The lifetimes of Table 2 together with the available observational data suggest that agreement is possible only if a certain amount of overshoot from the convective core is considered.

Although the data of Table 2 and the above discussion support the conclusion reached by Chiosi et al. (1989a) and Vallenari et al. (1990a,b) a deeper analysis is required since those authors have made use of models calculated with old opacities and nuclear reaction rates, not fully appropriate chemical compositions, and more efficient core overshoot such as given by $\Lambda = 1$ in the Bressan et al. (1981) formalism, whereas a lower value for this parameter is favoured at present (Maeder & Meynet 1989). Furthermore, although these models possess lifetimes of the main burning phases that are in agreement with the star counts in rich young clusters of LMC, they are unable to reproduce the observed morphology of the loops in the C-M diagrams even for the low metallicities appropriate to LMC clusters. This despite the fact that those models are calculated with the enhanced rate of the $^{12}C(\alpha, \gamma)^{16}O$ reaction (Kettner et al. 1982; Langanke & Koonin 1982; Caughlan et al. 1985 – CFHZ85 –), which is known to favour large loops (Iben 1967; Brunish & Becker 1990). However, recent determinations (Caughlan & Fowler 1988 – CF88 –) of the cross section of the $^{12}C(\alpha, \gamma)^{16}O$ reaction have lowered the rate almost to the 1975 value (Fowler et al. 1975). Therefore, we present here models computed with the new opacities, both the new (CF88) and the enhanced (CFHZ85) rate of the $^{12}C(\alpha, \gamma)^{16}O$ reaction, adopting either the classical treatment of core convection (semiconvection during central He-burning) or the non-local overshoot from the convective core all over the main nuclear phases, and finally we investigate the effect of envelope overshoot. By doing this, we would be able to test several

Table 2

$M/M_{\odot}$	t_H	t_{He}	t_{He}/t_H	Convection	Opacity
Brunish & Becker 1990:					$Z=0.020$ $Y=0.280$
5	65.5	22.5	0.34	Classical	Old
7	33.3	7.0	0.21	Classical	Old
Matraka et al. 1982:					$Z=0.044$ $Y=0.354$
5	48.7	20.9	0.43	Classical	Old
7	24.9	7.3	0.29	Classical	Old
5	62.8	8.5	0.13	Ov. mod.	Old
7	31.1	4.1	0.13	Ov. mod.	Old
Castellani et al. 1989:					$Z=0.020$ $Y=0.270$
5	87.5	22.5	0.26	Classical	LA
7	41.3	8.0	0.19	Classical	LA
Lattanzio 1989:					$Z=0.020$ $Y=0.280$
5	83.3	25.2	0.30	Classical	LA
7	39.9	8.6	0.21	Classical	LA
This work:					$Z=0.020$ $Y=0.280$
5	84.9	21.0	0.25	Classical	LA
7	40.5	7.1	0.18	Classical	LA
5	100.9	9.6	0.10	Ov. mod.	LA
7	47.8	3.7	0.08	Ov. mod.	LA
Maeder & Meynet 1989:					$Z=0.020$ $Y=0.280$
5	98.8	27.1	0.28	Ov. mod	LA
7	47.3	9.7	0.21	Ov. mod	LA

Notes to Table 2: Lifetimes in units of 10^6 yr.
Classical: Schwarzschild criterion (with or without semiconvection).
Ov. mod.: moderate overshoot.
Old: opacities before the update by Huebner et al. 1977.
LA: opacities updated by Huebner et. al. 1977.

aspects of the input physics in a homogeneous fashion (same evolutionary code). In particular, we would clarify the effects of the different treatments of convection and new opacity on the He/H lifetime ratios, and of the general physical input on the morphology of the loops.

Having in mind the properties of star clusters in LMC we calculate complete evolutionary sequences for stars of initial mass of 4,5,6,7,9 $M_{\odot}$, adopting the metallicity suggested by Russel & Bessel (1989), $Z=0.008$, and He abundance $Y=0.25$. The He abundance is chosen assuming for globular clusters the canonical value, i.e. $Y=0.23$, and scaling it according to the relation $\Delta Y=\gamma\,\Delta Z$ with γ in the range 2 to 3. Moreover, to analyze the dependence of the morphology of the C-M diagram on the chemical abundance, we calculate a few evolutionary sequences with $Z=0.02$ and $Y=0.28$. Since a detailed description of the evolutionary models is beyond the scope of this paper, we will present the most salient results limited to the 5 and 7 $M_{\odot}$ stars, whose evolution is representative of the stellar content of young and intermediate age clusters.

4.1. *The results*

Table 3 contains a few relevant quantities of the evolutionary tracks for the 5 $M_{\odot}$ and 7 $M_{\odot}$ stars. These are the H- and He-burning lifetimes (in units of 10^6 yr) and lifetime ratios, the effective temperature of the hottest point of the loops, and annotations identifying the input physics: Λ and Λ_{en} label the efficiency of convective overshoot from the core and the envelope, respectively, and finally CF88 and CFHZ85 refer to the new and enhanced $^{12}C(\alpha,\gamma)^{16}O$ reaction rate, respectively. Figures 6 and 7 show the path in the HR diagram for the sequences calculated with $Z=0.008$ and various assumptions about the amounts of both core and envelope overshoot, and rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction. Lower case letters labelling the various sequences in Figs. 6 and 7 identify the models of Table 3.

4.1.1. Models without envelope overshoot

Looking at the extension of the loops in the HR diagram, we notice that passing from the composition typical of the solar vicinity ($Z=0.02$ and $Y=0.28$) to the one suitable to young LMC clusters ($Z=0.008$ and $Y=0.25$), the loops of classical models remain almost unchanged, while the loops of models with mild core overshoot, surprisingly get much narrower. This effect being more pronounced at increasing mass of the star and slightly dependent on the rate for the $^{12}C(\alpha,\gamma)^{16}O$ reaction. This implies that the main factors determining the extension of loops, namely the $^{12}C(\alpha,\gamma)^{16}O$ rate, the metal content, the amount of convective overshoot, the external mixing length (Iben 1967; Matraka et al. 1982; Bertelli et al. 1985; Brunish & Becker 1990), do not always act in a linear fashion, so that any prediction about the loop extension may be grossly in error. Furthermore, the behaviour of models with mild core overshoot alone points out that if loops

Table 3

t_H	t_{He}	t_{He}/t_H	$\log T_{eff_{(loop)}}$	Λ	$^{12}C(\alpha,\gamma)^{16}O$	Λ_{en}	
$M = 7\,M_\odot$ $Z = 0.020$ $Y = 0.28$							
40.54	7.10	0.175	4.000	0	CFHZ85	0	
47.83	3.58	0.075	3.909	0.5	CF88	0	
47.83	3.67	0.077	3.945	0.5	CF88	0.7	
$M = 7\,M_\odot$ $Z = 0.008$ $Y = 0.25$							
40.29	7.14	0.177	3.945	0	CFHZ85	0	a)
49.96	4.19	0.084	3.710	0.5	CF88	0	b)
49.96	4.07	0.081	3.821	0.5	CFHZ85	0	c)
49.96	3.81	0.076	3.952	0.5	CF88	0.7	d)
49.96	3.84	0.077	4.016	0.5	CFHZ85	1.0	e)
49.96	3.83	0.077	4.016	0.5	CF88	1.0	f)
58.12	2.69	0.046	3.795	1.0	CF88	0.7	g)
$M = 5\,M_\odot$ $Z = 0.020$ $Y = 0.28$							
84.87	21.05	0.248	3.832	0	CFHZ85	0	
100.87	9.12	0.090	3.769	0.5	CF88	0	
100.87	9.62	0.095	3.787	0.5	CF88	0.7	
$M = 5\,M_\odot$ $Z = 0.008$ $Y = 0.25$							
80.16	20.01	0.250	3.846	0	CFHZ85	0	a)
101.61	8.98	0.088	3.717	0.5	CF88	0	b)
101.61	9.25	0.091	3.923	0.5	CF88	0.7	c)
101.61	9.35	0.092	3.932	0.5	CF88	1.0	d)

Notes to Table 3: Lifetimes in units of 10^6 yr.
Letters refer to the figures below.

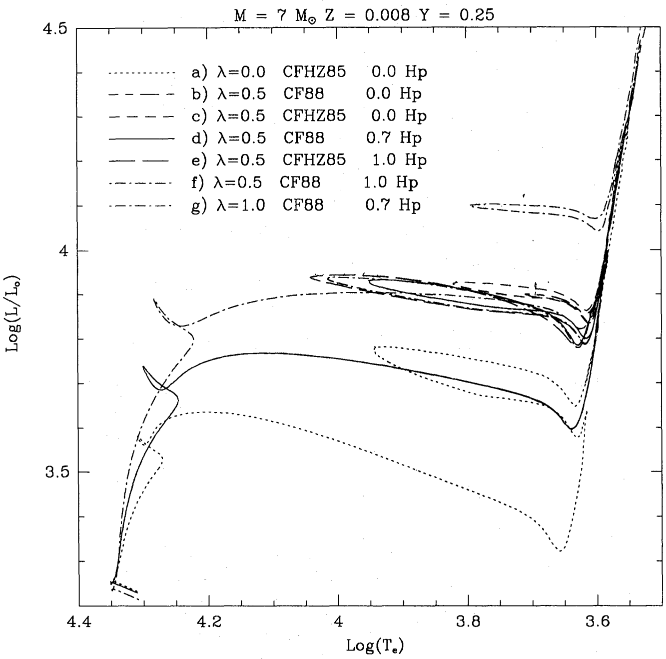

Fig. 6. Evolutionary tracks in the HR diagram of the models with $M = 7\,M_\odot$ and $Z = 0.008$, $Y = 0.25$. Lower case letters (a) to (g) label the sequences according to the nomenclature given in Table 2

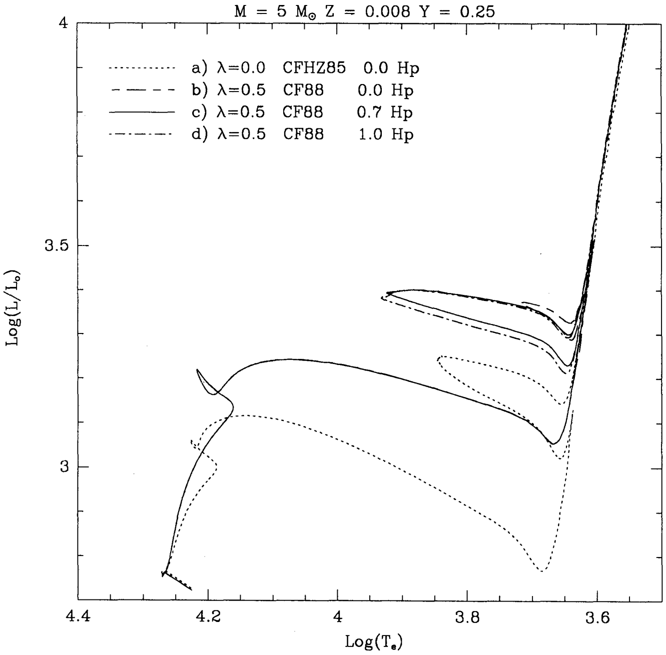

Fig. 7. Evolutionary tracks in the HR diagram of the models with $5\,M_\odot$ and $Z = 0.008$, $Y = 0.25$. Lower case letters (a) to (d) label the sequences according to the nomenclature given in Table 3

extended enough to account for the properties of galactic clusters can be obtained with the solar vicinity composition (see also Maeder & Meynet 1989), this is no longer possible for the less metal-rich LMC clusters, whose C-M diagrams show much more extended loops. Therefore, despite the fact that models with core overshoot may account for the relative numbers of stars in the luminosity functions of LMC star clusters, they seem to be unable to fit the properties (loops) of the C-M diagram.

4.1.2. Models with envelope overshoot

Since the early work of Lauterborn et al. (1971) it is known that many details of the input physics affect the extension of the loops of intermediate mass stars, often giving the impression of an erratic behaviour. Among other causes, Huang & Weigert (1983), analyzing the effect of changing the mixing length parameter in the external convection, pointed out that the extension of the blue loop correlates with the maximum penetration of the envelope convection along the Hayashi line. The deeper the convective envelope penetrates into the star, the bluer are the loops. At the light of the Huang & Weigert (1983) analysis and the results presented in Sect. 3 for the bump in the luminosity function of RGB stars in globular clusters, we include envelope overshoot in the models of intermediate mass stars, following the same procedure adopted for the low mass stars. Evolutionary tracks for the $7\,M_\odot$ star are computed with envelope overshoot given by $\Lambda_{en} = 0.7$ and low rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction (track d in Fig. 6), envelope overshoot of $\Lambda_{en} = 1$ and both high and low rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction (tracks e and f in Fig. 6, respectively), finally with high efficiency of core overshoot $\Lambda = 1$, low rate and low efficiency ($\Lambda_{en} = 0.7$) of envelope overshoot (track g in Fig. 6). In the case of the $5\,M_\odot$ star, models are computed assuming the low rate for the $^{12}C(\alpha,\gamma)^{16}O$ reaction, and envelope overshoot of $\Lambda_{en} = 0.7$ and 1 (tracks c and d in Fig. 7, respectively).

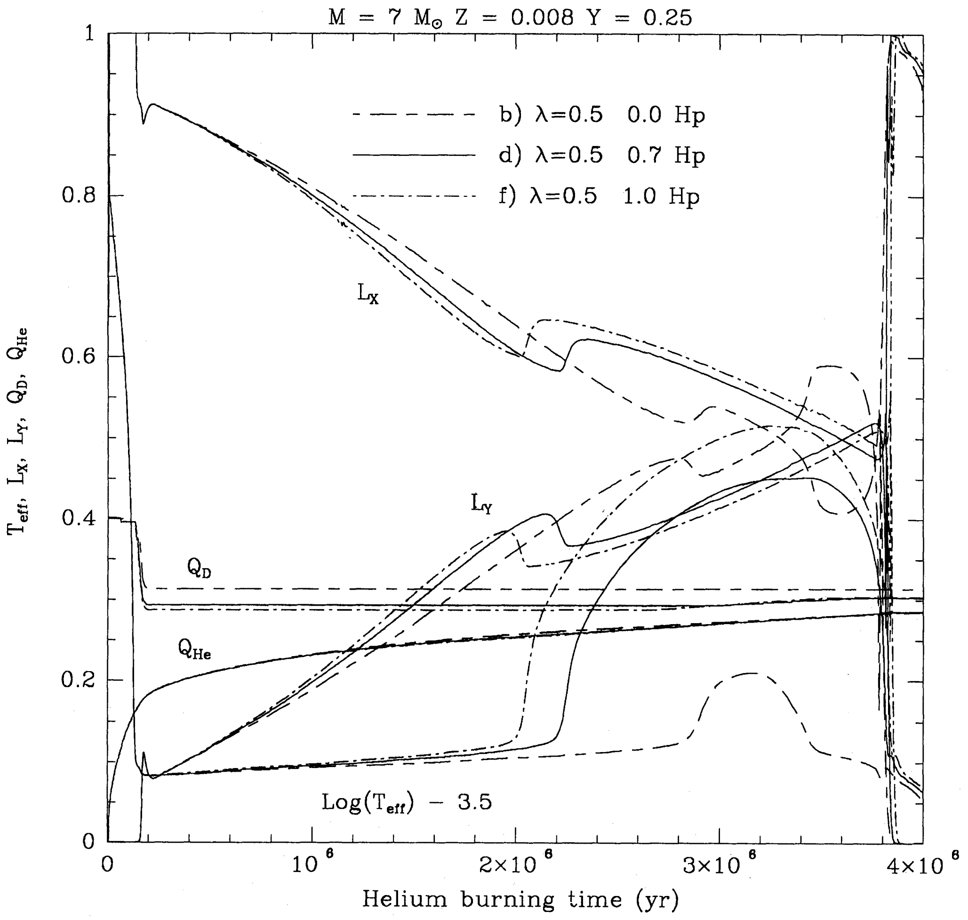

Fig. 8. The run of several quantities versus time for the sequences (b), (d) and (f) of the $7\,M_\odot$ star during the central He-burning phase, whose input parameters are specified in Table 2. $T_{\rm eff}$ is the logarithm of the effective temperature minus 3.5, L_x the ratio of the hydrogen to the total luminosity, L_y the ratio of the helium to the total luminosity, $Q_{\rm D}$ is the fractionary mass of the bottom of the hydrogen rich envelope, and $Q_{\rm He}$ is the fractionary mass of the hydrogen depleted core. Each line is labelled according to the quantity and the input parameters it refers to

In order to understand better the effects of envelope overshoot on stellar models, Fig. 8 displays as a function of the He-burning lifetime the variation of a few basic quantities for the $7\,M_\odot$ star models (b), (d) and (f). These are the effective temperature expressed as log $T_{\rm eff}$-3.5, the ratio L_x of the H-burning shell to total luminosity, the ratio L_y of the He-burning core to total luminosity, the mass $Q_{\rm D}$ at the bottom of the hydrogen-rich envelope, and the mass $Q_{\rm He}$ of the hydrogen depleted core, both normalized to the total mass of the star. In Fig. 8, each quantity is shown for the models (b), (d) and (f) of Table 3 using different symbols to facilitate the identification.

The rapid inward penetration of the convective envelope across the hydrogen profile is visible from the rapid variation of $Q_{\rm D}$ decreasing from about 0.40 to either 0.32 in absence of envelope overshoot, or 0.29 in presence of envelope overshoot with $\Lambda_{\rm en}=1$. Indeed this 3% decrease in the mass of the region between the inner core and the homogeneous envelope, in which about 10% of the total mass is stored, has dramatic effects on the structure of the envelope, as it can be seen from the variation of log $T_{\rm eff}$ with time and in turn location in the HR diagram. With the metallicities adopted here, in all models calculated with mild core overshoot, no matter of the efficiency of the envelope overshoot and rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction, extended loops are possible. On the contrary, in models with full core overshoot ($\Lambda=1$) no extended loops are found [track labelled (g) in Fig. 6]. This is mainly due to the higher evolutionary rate of theses latter models during central He-burning. In any case extended loops are recovered for metallicities as low as $Z=0.001$ independently of the efficiency of core overshoot (Bertelli et al. 1990).

The relative lifetime spent by the models at the red and blue side of the loop depends on the amount of envelope overshoot. Unfortunately, since this ratio is also affected by the efficiency of the $^{12}C(\alpha,\gamma)^{16}O$ reaction and by the adopted chemical composition, the observed numbers of blue and red giant stars cannot be safely used to constrain the efficiency of envelope overshoot in the domain of intermediate-mass stars.

Finally, the data of Table 3 show that adding some overshoot at the base of the convective envelope does not alter the core He- to H-burning lifetime ratio. In fact, this ratio is controlled by the helium core mass, which remains unchanged. This is mainly determined by the amount of core overshoot during central H-burning.

5. Discussion and conclusions

In this paper we endeavour to infer from observations of the bump in the RGB luminosity function of the globular clusters, and the luminosity function and morphology of the C-M diagram of young LMC clusters, that an efficient mixing outside the classical boundaries of convective (internal as well as external) regions ought to take place.

In RGB stars, we find that material in the region extending across about half pressure scale height below the Schwarzschild border ought to be mixed with the convective envelope in order to explain the shift in the bump luminosity invoked by Fusi-Pecci et al. (1990). This result cannot be obtained choosing a larger value for the mixing length in the outer convection, as no appreciable differences in the maximum inward penetration of the convective envelope and correspondingly in the luminosity of the bump of the RGB luminosity function are found at varying this parameter. The reason for it is that the region containing the gradient in molecular weight (hydrogen) is very stable against the onset of convection. In addition to this, variations in the mixing length would also affect the effective temperatures of the RGB stars. On the contrary, with the mixing length currently in use, the effective temperatures of these stars agree with the observational data. Different opacities and/or He contents are found not to improve upon the discrepancy in the bump luminosity. Therefore, we are inclined to attribute the discrepancy in question to inadequacies of the classical treatment of convection, which likely underestimates the region actually mixed by a non negligible amount.

In the hypothesis that envelope overshoot is entirely responsible of the required magnitude shift in the bump, $0.415\,M_V$ according to Fusi-Pecci et al. (1990), relation (1) gives $\Lambda_{\rm en} = 0.7$. Although the main trend is already given by the present results, definitive conclusions will be possible only by comparing the observational difference between the magnitude of the bump and the magnitude of the HB stars together with the RGB star luminosity function with their theoretical counterparts using a homogeneous set of models encompassing all evolutionary phases. Work is in progress to this aim.

The present analysis indicates first that the amount of *envelope overshoot* required in RGB stars is compatible with the value for *core overshoot* in intermediate-mass stars, and second that the same amount of *envelope overshoot* together with *mild core overshoot* in these latter stars greatly improves upon the interpretation of the properties of the C-M diagrams and luminosity functions of young star clusters. Our study suggests that, with the new opacities and the low $^{12}C(\alpha,\gamma)^{16}O$ rate, the efficiency of core overshoot in intermediate-mass stars is given by $\Lambda = 0.5$, whereas values of $\Lambda = 1$ can be excluded. This however holds good *if and only if* envelope overshoot is also taken into account so that all available constraints (C-M diagrams and luminosity functions) can be fulfilled at the same time.

It is worth noticing that if instead of mixing over a fraction of the local pressure scale height below the limit of the unstable region, envelope overshoot is modeled with the aid of the Bressan et al. (1981) formalism, a mean free path of convective elements greater than one pressure scale height should be adopted. However, if the same formalism and efficiency of convective overshoot is applied to the cores of intermediate mass stars, too big H-exhausted cores would result, thus making the blue loops in the C-M diagrams of young LMC clusters very hard to explain. On the contrary, our analysis indicates for core overshoot a value of the parameter Λ which is about half of the value adopted for envelope overshoot, this latter being constrained by both the luminosity of the bump in the RGB luminosity functions and the ability of intermediate-mass stars to perform extended loops.

Moreover, if our explanation of the magnitude shift of the bump in terms of envelope overshoot is correct, the results show that convection can efficiently erode a discontinuity in molecular weight. This finding has strong implications for the applicability of non-local overshoot in the convective cores of low mass stars with mass in the range 1 to $1.5\,M_\odot$ during the H-burning phase. In this range of masses, the sudden increase of the convective core, caused by the CNO cycle reaching full efficiency toward the end of central H-burning, together with the further extension given by convective overshoot according to the Bressan et al. (1981) formulation, produce stellar models whose lifetimes in the subgiant and red giant phase are at variance with star counts in old open clusters like King 2 and King 11 (Aparicio et al. 1990a,b). To cope with this difficulty, Aparicio et al. (1990) suggested that the presence of a discontinuity in molecular weight would act as a barrier against the penetration of convective motions. However, the present results somewhat weaken their conclusion and indicate that a major revision of the overshoot scheme by Bressan et al. (1981) is required. In brief, since this algorithm adopts the mixing length theory of convection and rests on the mean free path of the elements, the estimated overshoot distance is likely to be correct if much shorter than the typical size of the convective region (as in the case of convective envelopes or convective cores in more massive stars), whereas it is likely inadequate if the mixing distance is comparable or even greater than the size of the unstable region itself, as in the case of stars in the above mass range. In fact, one pressure scale height corresponds to about 5% of the mass of the convective envelope of a red giant star, grows to about 10% of the core mass of a supergiant star, reaches about 100% of the core mass of intermediate-mass star, and finally it is more than 200% of the mass of the H-burning convective core of a low mass star. This clearly indicates that the straight use of the pressure scale height to model overshoot all across the HR diagram may be grossly in error.

As a conclusion, since there are observational evidences that in real stars the mixed regions are likely larger than the unstable ones, and at the same time a simple theory predicting the size of the convective regions is still missing, we tried to calibrate envelope overshoot by means of the bump in the RGB luminosity function, and independently infer the efficiency of core overshoot from the comparison of the theoretical lifetime ratios with the observed star counts in the C-M diagram of LMC clusters. We would like to emphasize once more that, while in the former case the choice of a fixed fraction of the pressure scale height may be suitable for the convective envelope of all stars, in the latter it should be limited to range of stellar masses under examination, and it could turn out to be wrong going to other mass intervals. Hints on the real efficiency of both core and envelope overshoot, may be given by other yet unsettled questions of stellar astrophysics. For instance, the number frequency-period distribution of galactic Cepheid stars together with the long lasting question of the discrepancy between evolutionary and pulsational mass of these stars, for which we expect differences with respect to the classical models to be caused by models with core overshoot (Chiosi et al. 1990). See also Matraka et al. (1982) and Bertelli et al. (1985) for preliminary discussions of these topics. Furthermore, envelope overshoot may play a role in dredging-up fresh carbon during the double shell phase (Hollowell & Iben, 1989), or may modify the $^{12}C/^{13}C$ and other isotopic ratios of RGB stars predicted by the classical models. Work is in progress to cast light on these topics (Alongi et al. 1991).

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