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ASTRONOMY AND ASTROPHYSICS

Dynamics of flaring loops

I. Thermodynamic decay scaling laws

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Abstract. We derive a simple analytical approximate solution for the hydrodynamic equations describing the decay of a flaring loop. We find that, independent of chemical composition of the plasma, the entropy per particle at the top of the loop undergoes an initial phase of linear decay, with a slope related to the initial loop conditions. The characteristic decay time is shorter than conductive or radiative times for typical solar flare conditions. We compare our analytical solution with numerical solutions of the full set of hydrodynamic equations for loop flares, and show that its validity extends over a large fraction of the decay. We also relate the decay times of temperature, density and pressure to the entropy decay time.

Key words: hydrodynamics – Sun, corona – Sun, flares – stars, flare

1. Introduction

According to the picture derived from X-ray observations, the plasma of the solar corona is confined in loop-like magnetic flux tubes anchored in the photosphere. In addition to providing a channel for heat deposition, the magnetic field acts both by confining plasma motions along its direction and suppressing thermal conduction across the field. It thus provides a means of insulating one loop structure from another. The solar corona, therefore, is a collection of loops, of different sizes and thermodynamical plasma parameters.

The constraints provided by the magnetic field, together with the condition that these plasma structures be stationary over the time scales indicated by solar observations, require that only two parameters are independent among the linear dimension of the loops (L), their base pressure (p), maximum temperature ($T_{\max}$) and heating flux (F_H). We have thus two relationships among these four parameters which, for loops shorter than the pressure scale height and uniform heating, are known as the RTV (Rosner et al. 1978) scaling laws: $T^3_{\max} \propto pL$, and $E_H \propto p^{7/6} L^{-5/6}$ (where $E_H = F_H/L$ is the volumetric heating rate).

After the discovery by the Einstein observatory of the widespread X-ray emission by normal stars (Vaiana et al. 1981), this picture of the solar corona has been extended to stellar coronae. The evidence for this extension is strong although indirect. For example, the existence of coronae with temperature $\sim 10^7$ K in red giants requires their confinement; the escape velocity from these stars is in fact smaller than the thermal velocity (e.g. Maggio et al. 1989). Loop models of stellar coronae have been developed and confronted with observations (Giampapa et al. 1985, Schmitt et al. 1985, Landini et al. 1985, Stern et al. 1986, Reale et al. 1988, Schrijver et al. 1989) giving this picture a more solid foundation.

One dimensional hydrodynamic models have been applied both to solar and stellar flares (Peres et al. 1987, Reale et al. 1988). They have shown a remarkable success in modeling the sequence of events characteristic of a flare following the sudden switching of an impulsive heating mechanism. Moreover, Jakimiec et al. (1986) have shown that loop models are very useful in understanding the evolution of temperature and density in solar flares, and in ascertaining whether energy deposition continues during the decay phase. However, despite the insight we may gain from models, because of the complications of the numerical hydrodynamics involved in these calculations, a simpler approach would be highly desirable.

We follow here an approach to confined solar and stellar flares complementary to that of numerical hydrodynamics, i.e. an approximate analytical approach. In the present paper we shall investigate, in particular, the early decay phase of single loop flares. Since this requires approximations, we shall use the results of numerical calculations as a guide and as a way of verifying their validity. In particular, we shall study the slow evolution of a flaring loop following the peak phase.

In Sect. 2 of our paper we shall summarize the framework of the hydrodynamic treatment of loop flares, giving the relevant equations. We shall give in Sect. 3 the derivation of a simple approximate solution to the equations, valid during the decay phase of the flaring plasma. We shall compare the analytical results with results of numerical solutions of the fully hydrodynamic equations in Sect. 4, both to justify the approximations made and to validate our treatment. Section 5 will summarize and discuss our results.

In a forthcoming series of papers we shall use results of numerical calculations of the full set of hydrodynamic equations relaxing some of the approximations to investigate in more

Send offprint requests to: S. Serio[1] details the decay phase of loop flares (Paper II), and shall compare our results to a representative sample of SMM observations (Paper III).

2. Plasma dynamics in a flux tube

We model the flaring loop as a hydrodynamic process in a rigid tube, i.e. we consider a flaring loop in which there are no appreciable changes in magnetic field geometry. For simplicity we consider a constant cross section loop anchored in the chromosphere, with axis along a plane passing through the center of the Sun. The shape of the flux tube enters into the equations by means of g_s, i.e. the component of gravity along the local direction of the tube. We assume that the magnetic field acts only to confine plasma motions and heat conduction along the loop, and therefore our problem is effectively 1 dimensional. Whenever we shall give results of the full hydrodynamic equations, we shall use a semicircular geometry and, by virtue of the assumed symmetry, solve our equations only in half loop (i.e. a quarter of circle).

The basic equations for the evolution along the tube coordinate s (reckoned from the tube basis) of plasma density n, velocity v, and internal energy density ε, taking into account ionization (β is the ionization fraction), viscosity (μ is the viscosity coefficient), are written as follows:

$$\frac{dn}{dt} = -n\frac{\partial v}{\partial s} \tag{1}$$

$$nm_H\frac{dv}{dt} = -\frac{\partial p}{\partial s} + nm_H g_s + \frac{\partial}{\partial s}\left(\mu\frac{\partial v}{\partial s}\right) \tag{2}$$

$$\frac{d\varepsilon}{dt} + (p+\varepsilon)\frac{\partial v}{\partial s} = E_H(s,t) - n^2\beta P(T) + \mu\left(\frac{\partial v}{\partial s}\right)^2 - \frac{\partial}{\partial s}F_c. \tag{3}$$

To these equations we add the equation of state, and the relationship between internal energy density and pressure:

$$p = (1+\beta)nkT, \tag{4}$$

$$\varepsilon = \frac{3}{2}p + n\beta\chi, \tag{5}$$

where k is Boltzman constant, and χ is the hydrogen ionization potential.

The term $E_H(s,t)$ in Eq. (3) describes a source term for coronal heating, both steady state and dynamic, and can, in principle, be specified by different physical models. As discussed elsewhere (for example Peres et al. 1987), while the detailed evolution depends on the physical model for E_H, its general characteristics will depend only on the rate of energy input, because of the extremely high thermal conductivity in the corona that redistributes any heat regardless of where and how it is being deposited.

As apparent from Eq. (2), our equations are written for a hydrogen plasma (m_H is the hydrogen mass); however, we take into consideration radiation losses from a solar abundancy plasma. The second term in the right hand side of Eq. (3) does in fact describe radiation losses by the optically thin plasma. In the following, the function $P(T)$ will be parametrized, as in RTV, with power laws with different indexes in different temperature ranges, and a simple thermal bremsstrahlung extension in the region $T > 10^7$ K.

$$\frac{P(T)}{[\mathrm{erg\,cm^3\,s^{-1}}]} = \begin{cases} 10^{-21.85}; & 10^{4.3} < T(\mathrm{K}) < 10^{4.6} \\ 10^{-31}\,T^2; & 10^{4.6} < T(\mathrm{K}) < 10^{4.9} \\ 10^{-21.2}; & 10^{4.9} < T(\mathrm{K}) < 10^{5.4} \\ 10^{-10.4}\,T^{-2}; & 10^{5.4} < T(\mathrm{K}) < 10^{5.8} \\ 10^{-21.94}; & 10^{5.8} < T(\mathrm{K}) < 10^{6.3} \\ 10^{-17.73}\,T^{-2/3}; & 10^{6.3} < T(\mathrm{K}) < 10^{7.0} \\ 10^{-22.40}\,T^{1/2}; & 10^{7.0} < T(\mathrm{K}). \end{cases} \tag{6}$$

Finally, the last term in Eq. (3) is the conduction term; the conductive flux is

$$F_c = -\kappa_0\,T^{5/2}\frac{\partial T}{\partial s}, \tag{7}$$

where κ_0 is Spitzer's coefficient of thermal conductivity. This term is effective in redistributing excess heat all over the loop. Because of its presence, the actual dependence of $E_H(s,t)$ on plasma parameters (p, T) and on s is almost irrelevant to the general evolution of the flare.

Notice that the condition of symmetry at the top of the loop implies $F_c(s_{max}, t) = 0$. For the stationary phase, the small value of F_c at $T \sim 2\,10^4$ K and the necessary condition $T(L) = T_{max}$ are discussed in RTV.

Peres et al. (1982) have discussed how important is to consider the chromosphere, together with the corona, in systems modeled by Eqs. (1)–(5). The chromosphere is in fact essential for a proper treatment of coronal evolution because it acts as a reservoir providing the flux of plasma particles necessary to maintain a dynamical equilibrium. Therefore, in the standard implementations of the numerical solution of Eqs. (1)–(5), we shall follow the prescription of Peres et al. (1982), describing the radiation losses by means of an effectively thin $P(T)$, down to the chromospheric temperature minimum. The effective $P(T)$ in the region below $2\,10^4$ K can be derived from empirical chromospheric models, and we have used the VAL solar atmospheric model (Vernazza et al. 1980, however, see Reale et al. 1988, for a discussion of the little importance of a detailed chromospheric model).

Under Eqs. (1)–(5), the evolution of a flare can be computed by prescribing initial conditions (e.g. a solution of the static equations, with $v=0$ and $d/dt=0$), and a heating term $E_H(s,t) = E_t(s,t) + E_0$, i.e. a transient term in addition to the steady state heating term E_0 (which can be assumed as independent on s, as stated above, because of the high conduction efficiency). We emphasize here that the boundary condition $F_c \sim 0$ at $T \sim 2\,10^4$ K is only used in deriving the solution of the static equation which constitutes the starting point of any dynamical solution, and that, in addition to the condition of symmetry at the top of the loop, we use as boundary conditions at the base $v=0$ and $n=$ constant.

3. Thermodynamics of the flare decay phase

Solutions of the system of Eqs. (1)–(5) have been discussed by Peres et al. (1982, 1987), Pallavicini et al. (1983), Reale et al. (1988), Antonucci et al. (1987). In response to an impulsive heat source in corona, the coronal plasma increases its temperature, while a conduction front travels down to the transition region. As soon as it penetrates the transition region, it ablates it, allowing evaporating material to be shot up in the corona. The evaporation phase lasts a few tens of seconds; if the impulsive heating term is active for a time significantly longer than the evaporation phase, then there is generally enough time for a quasi-stationary state to be reached in corona before the heating is switched off. In these cicumstances we can describe the phenomenology of the decay as due both to the cooling of the hot ($T > 10^7$ K) coronal plasma and to the "bouncing back" of the transition region, which starts traveling upward as soon as the impulsive heating is switched off, and might even reach near the top of the loop.

We shall now show that, if we are only interested in describing high temperature phenomena, i.e. those accessible to X-ray observations, the decay of the flare can be described in a somewhat simplified way.

As a first approximation, it is easily shown that the viscosity terms in Eqs. (2) and (3) can be neglected. They act only to smooth shock fronts, and have little influence on average parameters such as integrated emission in a band or a line.

It is also easy to see that we can disregard the term in χ and set $\beta = 1$ in Eq. (5), if we limit our attention to the region of the loop where $kT \gg \chi$[1]. Using these two approximations we can now rewrite Eq. (3) in terms of the entropy per particle $S = k \ln(T^{5/2}/p)$:

$$p \frac{d}{dt} S/k = E_H(s, t) - n^2 P(T) - \frac{\partial}{\partial s} F_c. \tag{8}$$

We now set ourselves to analyze our system under the condition that the initial conditions describe a static state with spatially uniform heating, i.e. d/dt and $v = 0$, $E_H(s, t) = E_H$, and that its evolution after $t = 0$ is driven by a sudden decrease of heating. A similar description is appropriate to the initial decay of a flare, from its peak phase.

The possibility of describing plasma conditions in the loop at flare peak as "static" needs some qualifications. It is in fact obvious that the system will deviate from staticity because of the disturbances going back and forth along the loop, remaining from the highly dynamic rise phase; moreover, it is well known that different plasma parameters peak at different times, making the definition of the initial state ambiguous. We shall investigate in Paper II how and when a Quasi Steady State (QSS) phase may be reached. However, for simplicity, we shall assume here that the system is initially in a steady state.

It is easy to see that, if we start from a steady state ($d/dt = 0$), a sudden change in $E_H(s, t)$ will result in the switching on of the d/dt term in Eq. (8). Under the effect of switching E_H off, therefore, since the radiative and conductive terms in Eq. (8) will initially be balanced by E_H, the result is just

$$p \frac{dS/k}{dt} = -E_H. \tag{9}$$

[1] The validity of this approximation would appear to be obvious, where it not for the possibility of exchange between coronal plasma and plasma in the low temperature region, where the effect of ionization in regulating the energy balance is large, and its effect on the amount and velocity of evaporating plasma might be large as well. However, we can safely adopt this approximation, if we are only interested in describing the high temperature coronal plasma. During the decay phase, the ionization terms will be important only in the description of the upward shift of the transition region.

If we limit our consideration to the region near the top of the loop, where we have $d/dt \sim \partial/\partial t$ being $v \sim 0$ for symmetry, Eq. (9) can be integrated to give

$$S \sim S_0 - \frac{kt}{\tau}, \tag{10}$$

$$\frac{T^{5/2}}{p} \sim \frac{T_0^{5/2}}{p_0} e^{-t/\tau}, \tag{11}$$

where

$$\tau = \frac{p_0}{E_H}, \tag{12}$$

and T_0, p_0 and S_0 are, respectively, the initial temperature, pressure and entropy at the top of the loop. Obviously, Eqs. (10) and (11) hold only during the initial phase of the decay of the flare, because, in due time, the pressure p in Eq. (8) will change, making (10) deviate from the true solution. Moreover, the radiative and conductive terms in Eq. (8) might not keep their initial distribution during the advanced phases of decay.

Equation (12) gives, however, the characteristic time for the decay of entropy near the top of the loop during the initial decay phase of a flare. Assuming that the heating rate during decay is completely switched off[2] during the decay phase, and using the RTV scaling laws and cgs units, the thermodynamic decay time τ can also be written as

$$\tau = \frac{10^{-5} L^{5/6}}{p_0^{1/6}}, \tag{13}$$

or else,

$$\tau = \frac{3.7\, 10^{-4} L}{\sqrt{T_0}} \sim 120 \frac{L_9}{\sqrt{T_7}}, \tag{14}$$

where T_7 is the initial temperature at the top of the loop in units of 10^7 K, and L_9 the length of the loop in units of 10^9 cm. Notice that the dependence expressed by Eq. (12) is independent of the details of the radiative losses (i.e. on the chemical composition of the plasma), while the dependence of the decay time on L and T_0 is a consequence of the RTV scaling law, and therefore, of the details of the conduction term and the radiative loss function.

4. Comparison with detailed numerical results

We want now to check the validity of our approach to the decay phase of loop flares by comparing the predictions of Eq. (10) with decays computed from numerical solutions of the complete set of Eqs. (1)–(5) by means of the Palermo-Harvard numerical code (a complete description of the code is in Peres & Serio 1984).

We have computed a grid of steady state model loops, as discussed in Sect. 3 above (see Table 1 for the parameters of the models), and have let them evolve by switching off completely the heating term. As an example, we show in Fig. 1 the evolution of temperature, density, pressure and entropy at the top of the loop

[2] Actually, if the system has to decay to a pre-flare condition typical of non-flaring active region loops, the heating term should not be allowed to go to zero, but rather to an appropriate value. In all practical cases, however, this baseline value is negligible with respect to the flare E_H, and we shall neglect it.

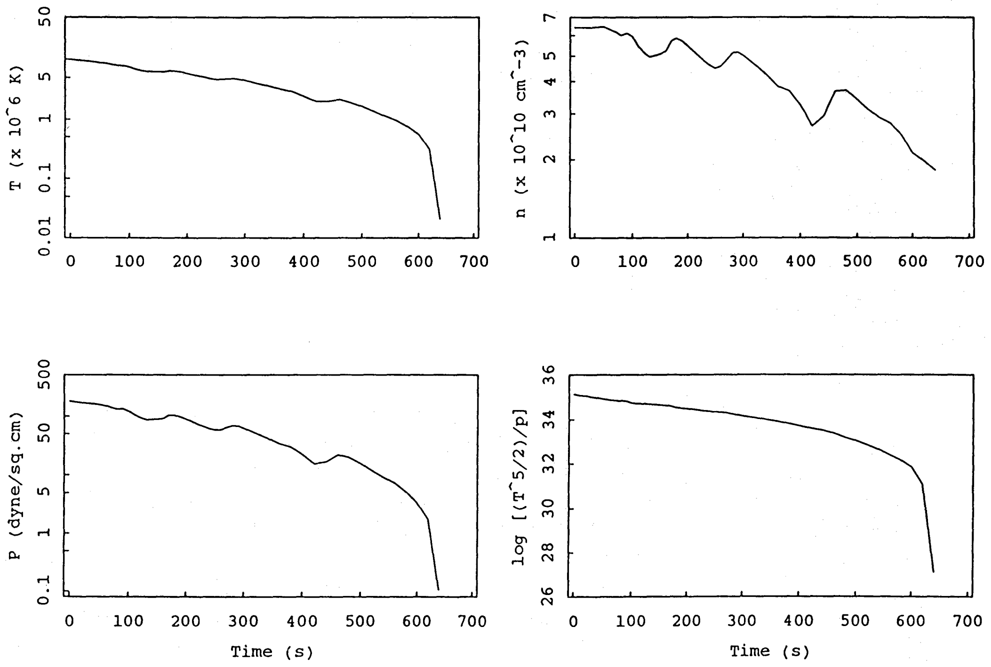

Fig. 1. Decay curves for temperature (top left), density (top right), pressure (bottom left) and entropy (bottom right) at the top of the flaring loop of model B_1 in Table 1

Table 1. Models of decaying flaring loops

Model	L[a]	T_0[b]	p_{base}[c]	τ_{th}[d]	τ_{sim}[e]	$\Delta t_{\mathrm{lin}}/\tau_{\mathrm{sim}}$[f]
A_1	0.5	1	1000	58	62	1.45
B_1	2	1	185	234	279	1.72
C_1	5	1	75	585	759	1.71
D_1	10	1	37	1170	1557	1.99
A_2	0.5	2	7200	41	41	1.59
B_2	2	2	1458	165	208	1.63
C_2	5	2	580	414	548	1.64
D_2	10	2	290	827	1110	1.76

[a] Length of semiloop from footpoint to apex, in units of 10^9 cm
[b] Initial temperature at the loop apex in units of 10^7 K
[c] Initial pressure at the base of the corona in dyne cm^{-2}
[d] Entropy decay time (s) as computed from Eq. (14)
[e] Entropy decay time (s) derived from calculations (cf. Fig. 2). The values shown are derived by means of linear regressions starting at $t=0$ and terminating at the time at which the correlation coefficient drops below 0.99 after initial fluctuations
[f] Time (in units of the characteristic decay times τ_{th}) of termination of the linear regression for the entry in the previous column. Over this time decay of entropy can be considered linear

for model B_1 (Table 1), and in Fig. 2 the decay of entropy for all models studied.

Notice that the behaviour of the entropy is initially linear in time, as predicted. The abrupt drop at late times corresponds to the thermal instability caused by having set the heating term equal to zero (see footnote 2). A small residual heating function would, instead, allow the loop to eventually reach a steady state at coronal conditions.

In Table 1 we also show the values of the entropy decay times as obtained from Eq. (12), and the corresponding values obtained from our numerical calculations, together with the range of validity of the linear decay phase. To obtain these values we have computed the linear regression of entropy versus time, and the corresponding correlation coefficient, at progressive times after the beginning of the decay phase. After a few initial oscillations, the correlation coefficient stabilizes at a value close to unity, starting, thereafter, a slow decrease; we stop the regression when this coefficient becomes smaller than 0.99. The coefficient of the regression provides the decay time, and the stopping time the range of the validity of our linear solution. We shall refer to the duration of this regime as the *linear phase* of flare decay.

As can be seen in Table 1, this linear phase lasts, generally, approximately 1.5–2 decay time scales, and, therefore, it describes the decay of the entropy by a factor ~ 5.

5. Discussion and conclusions

It is interesting to compare the thermodynamic decay time given by Eq. (14) with the conductive (τ_c) and radiative (τ_r) times that are sometimes used in the literature to estimate the dimensions of flaring regions. These parameters, derived by the crude assumptions that the decay is driven only by the conduction or the radiative term in Eq. (3), are defined according to:

$$\tau_c = \frac{3nkL^2}{\kappa_0 T_0^{5/2}} \tag{15}$$

$$\tau_r = \frac{3kT_0}{nP(T_0)}. \tag{16}$$

It is easy to show, by using the RTV scaling laws on the initial decay state assumed as static, as we have done in Sect. 3, that $\tau_c = 1.5\tau$, and $\tau_r = 6.7\tau/T_7$, where T_7 is the temperature in units of 10^7 K (we have assumed here that the initial flare temperature is $>10^7$ K and, therefore, $P(T) = 10^{-22.4}\,T^{1/2}$).

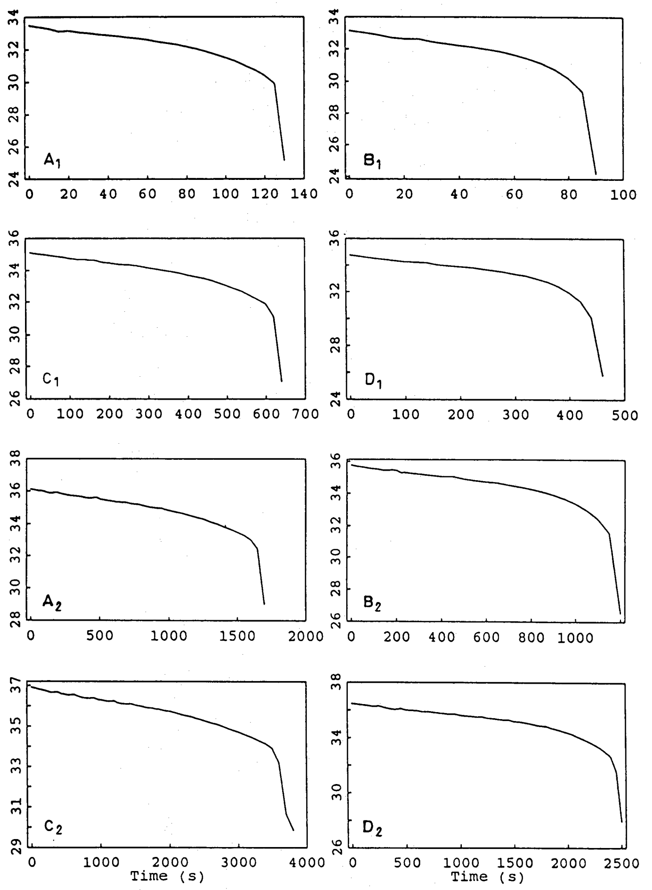

Fig. 2. Decay curves for log $T^{5/2}/p$ (in units of $K^{-5/2}$ dyne^{-1} cm^2) for the models reported in Table 1. From top to bottom and from left to right: models A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2

We see, that the thermodynamic decay time scales with T_0 and L as the conductive time, although it is shorter by a factor 2/3; it is comparable to the radiative cooling time only for initial temperature near $6\,10^7$ K. Notice, however, that the thermodynamic time acts as the decay time constant only for the specific combination of temperature and pressure appearing in the expression for entropy, i.e. $T^{5/2}/p$, and not for other thermodynamic or observable variables. As can be seen in Fig. 1, the thermodynamic variables T, n, and p, in fact, show similar behaviors (i.e. exponential, although with some more pronounced fluctuations) over the linear decay phase, but on different time scales.

We have derived the decay time corresponding to each of the three variables above for each of the models in Table 1, by means of independent simple regressions over the linear phase (as previously defined). The results are presented in Table 2 which lists, for each model, the decay time for temperature (τ_T), pressure (τ_p), and density (τ_n), in units of the entropy decay time.

It is easy to verify that the values so derived are compatible with the two relationships that should hold among the four decay times, i.e. $\tau_p^{-1}=\tau_T^{-1}+\tau_n^{-1}$, and $\tau^{-1}=2.5\,\tau_T^{-1}-\tau_p^{-1}$. We see, therefore, that the temperature at the top of the loop decays with essentially the same time scale as entropy, while the decay of density is approximately twice slower, reflecting the fact that both gravity and pressure forces are essentially zero near the top

Table 2. Temperature, density, and pressure decay times

Model	τ_T[a]	τ_n[b]	τ_p[c]
A_1	0.97	1.82	0.63
B_1	0.95	1.72	0.61
C_1	1.03	2.20	0.70
D_1	1.06	2.39	0.73
A_2	0.85	1.32	0.51
B_2	1.01	2.06	0.68
C_2	1.05	2.35	0.73
D_2	1.03	2.19	0.70

[a] Temperature decay time derived from a best fit in the linear region (s)
[b] Density decay time (as above)
[c] Pressure decay time (as above)

of the loop and, therefore, rather inefficient in driving the evacuation of the structure. We notice also that the emission measure ($\propto n^2$) will decay on the same timescale as temperature and entropy. It is also interesting to note that the factor of 2 difference in the decay times of density and temperature, will force a $T\propto n^2$ relationship during the decay phase. This relationship will be investigated in Paper II of this series. We wish to point out here that these relationships among decay times appear to be only of an empirical and approximate nature, since we have been unable to find a compelling physical reason for them to hold. We notice, moreover, that while the linear decay of the entropy can be closely verified (Fig. 2) in all our calculations, the decays of temperature, density and pressure do show pronounced deviations from pure exponential decays.

A comment is of order, with respect to the meaning, in terms of observable quantities, of our analytically derived time scale τ, and of the time scales for temperature, pressure, and density. Our solution is, in fact, valid only for the top point of the loop, while quantities observed with limited resolution instruments should in general be represented by some kind of average over the loop. However, since temperature and density in most of the loop appear to be reasonably close to their value near the top, as can be ascertained by the numerical calculations, we can argue that our findings can be easily extended to observable, average thermodynamical variables. This extension, and the detailed behavior of temperature, density and pressure during the decay phase will be the subject of Papers II and III of this series.

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