Photodissociation of iron nuclei in the collapse of a magnetic star

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(Submitted March 9, 1989)

Astron. Zh. 67, 326–333 (March–April 1990)

We consider the influence of a strong quantizing magnetic field on photodissociation of iron nuclei at high temperatures. The chemical potentials of nondegenerate nuclei and neutrons in the presence of a quantizing magnetic field are calculated. The concentration of disssociated iron nuclei and the density at which half of the matter (by mass) consists of iron nuclei and half of alpha particles and neutrons are determined as functions of the magnetic field strength and the temperature in a state of statistical equilibrium. The calculations are compared with a similar calculation in the absence of a magnetic field. We show that the influence of a strong quantizing magnetic field reduces to enhancement of the process of photodissociation of iron nuclei, which stimulates collapse of the star.

1. INTRODUCTION

According to our present understanding, the magnetic field at the surface of a neutron star may reach $\sim 10^{12}$-10^{13} G. This field considerably affects the structure of matter on the stellar surface.[1] Strong magnetic fields can also exist in the interior of a neutron star. In Ref. 2 it is noted that the deceleration of the pulsar (neutron star) in the Crab Nebula may be due to radiation from it due to the ellipticity of the star's figure. The ellipticity itself may be explained by internal magnetic fields with a strength $H \approx 10^{15}$ G, which correspond to internal fields of main-sequence stars with $H \approx 10^5$ G immediately before collapse. In Ref. 3 it was shown that the existence in nature of equilibrium degenerate configurations consisting of ^{56}Fe with a mass $M_{lim} \approx$ 800-2500 $M_{\odot}$ is possible if the strength of the magnetic field frozen into their interior reaches $H_c \approx 10^{15}$ G at the center. If the mass of a nondegenerate magnetic star is $M > M_{lim}$, then its collapse is possible. The collapse of the central part of the star, as is well known,[4] is stimulated by partial photodissociation of ^{56}Fe nuclei, as a result of which there is a decrease in temperature, and hence in pressure, at the center of the star.

The dispersal of the stellar envelope can be explained in the context of the magnetohydrodynamic rotational model of a supernova explosion,[5] in which the source of the explosion is the rapid rotation of a system consisting of a central superdense star and an outer envelope, and magnetic interaction is treated as the mechanism for transporting angular momentum. The magnetic field strength at the start of the explosion is $H \approx (5\text{-}15)\cdot 10^{16}$ G.

In the present communication we consider how a strong quantizing magnetic field affects the processes of photodissociation of ^{56}Fe nuclei. For this we determine the magnetic field strengths that have a quantizing effect on charged nuclei, we calcualte the chemical potentials of nuclei and neutrons in the presence of a strong magnetic field, and we then determine the dependence of the mass density on the magnetic field strength for which photodissociation of ^{56}Fe nuclei occurs.

2. QUANTIZING ACTION OF A MAGNETIC FIELD ON NONDEGENERATE NUCLEI

If the temperature of a contracting stellar core is very high, the equilibrium conditions shift abruptly from elements of the iron group to a mixture of α particles and neutrons. Dissociation of iron nuclei occurs in two stages. At $T \approx (3\text{-}12)\cdot 10^9$ K the isotope ^{56}Fe decays into α particles and neutrons,[4]

$$\gamma + {}^{56}\mathrm{Fe} \rightleftharpoons 13\alpha + 4n. \tag{1}$$

An energy Q_0 = 124.4 MeV is required for this process. At $T \approx (6\text{-}14)\cdot 10^9$ K, a ^{4}He nucleus decays into nucleons,

$$\gamma + {}^4\mathrm{He} \rightleftharpoons 2p + 2n. \tag{2}$$

An energy Q_0 = 28.3 MeV is required for this process.

A magnetic field of strength H will have a quantizing effect on nonrelativistic nuclei and protons at a temperature T if[6]

$$H_{\mathrm{qu}}^{\ i} \geqslant \frac{A_i^{\,2}}{Z_i}\frac{3kT}{m_i c^2} H_{\mathrm{cr}}^{\ p}, \qquad H_{\mathrm{cr}}^{\ p} = \frac{m_p^{\,2} c^3}{e\hbar} = 1{,}5\cdot 10^{20}\ \mathrm{G}, \tag{3}$$

where A_i, Z_i, and $m_i \approx A_i m_p$ are the mass number and charge of the nucleus and its mass, respectively, m_p is the proton mass, and H_{cr}^p is the so-called critical magnetic field for protons, at which the cyclotron quantum energy equals the proton rest energy.

From (3) we easily obtain

$$\frac{H_{\mathrm{qu}}^{\mathrm{Fe}}}{H_{\mathrm{qu}}^{\ \alpha}} = 1{,}077; \qquad \frac{H_{\mathrm{qu}}^{\mathrm{Fe}}}{H_{\mathrm{qu}}^{\ \mathrm{p}}} = 2{,}154; \qquad \frac{H_{\mathrm{qu}}^{\mathrm{He}}}{H_{\mathrm{qu}}^{\ \mathrm{p}}} = 2. \tag{4}$$

It is clear that if the magnetic field is quantizing for ^{56}He nuclei, then it is quantizing for α particles and protons. At T = 10^9 K, for example, $H_{qu}^p \gtrsim 4.14\cdot 10^{16}$ G for protons, $H_{qu}^{Fe} \gtrsim 8.91\cdot 10^{16}$ G for ^{56}Fe nuclei, and $H_{qu}^{\alpha} \gtrsim 8.27\cdot 10^{16}$ G for α particles. At T = 10^{10} K the quantizing magnetic fields for nuclei are $\approx 10^{18}$ G. Such magnetic fields change the binding energy of nuclei.[7]

3. CHEMICAL POTENTIAL OF NONDEGENERATE NUCLEI AND NEUTRONS IN A STRONG MAGNETIC FIELD

The energy levels of a nonrelativistic nucleus with a charge $Z_i e$ and a mass $m_i \approx A_i m_p$, where e is the elementary charge and m_p is the mass of a nucleon, in the presence of a permanent and uniform magnetic field with a strength H directed along the Z axis are determined by the equation[8]

$$\varepsilon_i(p_z, H)=m_ic^2+\frac{p_{zi}^2}{2m_i}+\beta_iH(2n+1)\pm\varkappa_i\beta_{nu}H, \quad (5)$$

where $\beta_i = Z_ieh/2A_im_pC$ is the magnetic moment of the nucleus originating from quantized motion in the plane perpendicular to the magnetic field direction, n = 0, 1, 2, ... is the number of the Landau quantum level, $\beta_iH(2n + 1)$ are the quantum levels of the transverse part of the kinetic energy of the charged particle, $p_{Zi}^2/2m_i$ is its unquantized longitudinal part, and κ_i is a factor that determines the intrinsic magnetic moment of the nucleus in Bohr nuclear magneton, $\beta_{nu} = e\hbar/2m_pe$.

For the ^{56}Fe nucleus $\kappa_{Fe} = 0$, for a α particle $\kappa_\alpha = 0$, and for a proton $\kappa_p = 2.793$.

The energy levels of a neutron in a magnetic field of strength H are

$$\varepsilon_n(H)=\frac{p_n^2}{2m_n}\pm\varkappa_n\beta_{nu}H, \quad (6)$$

where m_n is the neutron mass and $\kappa_n = -1.913$.

Following Ref. 9, with allowance for the fact that the multiplicity of degeneracy of an energy level of a charged particle is[8] $(Z_ieVH/3\pi^2h^2c)\,dp_z$, we obtain an expression for the chemical potential of nondegenerate nuclei in a quantizing magnetic field,

$$\mu_i(H)=A_im_pc^2 + kT\ln\left[n_i\left(\frac{2\pi\hbar^2}{m_ikT}\right)^{3/2}\right]Z_k^{-1}(H)\left[\frac{2\,\mathrm{sh}\left(\frac{Z_i}{A_i}\eta\right)}{2\frac{Z_i}{A_i}\eta}\right][2\,\mathrm{ch}\,\varkappa_i\eta]^{-1}, \quad (7)$$

where n_i is the concentration of the nuclei, $Z_k(H)$ is the statistical weight of a nucleus, and

$$\eta=\frac{\beta_{nu}H}{kT}=\frac{m_pc^2}{2kT}\frac{H}{H_{cr}^p}.$$

In Eq. (7) the factor that contains a hyperbolic sine is related to the quantized motion of a charged nucleus in a magnetic field; the factor that contains a hyperbolic cosine is related to the orientation of the intrinsic magnetic moment of the nucleus in the magnetic field.

For systems with internal degrees of freedom, the statistical weight $Z_k(H)$ is a function of the nuclear partition function,

$$Z_k(H)=\sum_k\omega_k\exp\left[-\frac{E_k(H)}{kT}\right], \quad \omega_k=2I_k+1, \quad (8)$$

where I_k is the spin of the k-th excited state and $E_k(H)$ is the energy of the excited state of the nucleus, which depends on the external magnetic field strength, as shown in Refs. 7 and 10.

In the absence of a magnetic field, for T $\lesssim$ $1.2\cdot10^{10}$ K we can set $\omega_\alpha = 1$ (the ground state, $I_\alpha = 0$), $\omega_n = 2$ ($I_n = 1/2$), and $\omega_{Fe} = 1.4$ (the ground state with $I_{Fe} = 0$ plus the excited states).[11]

For the chemical potential of a nondegenerate neutron gas of density n_n in a magnetic field of strength H, with allowance for Eq. (6) and the multiplicity of degeneracy of an energy level of a neutral particle, we find, following Ref. 9,

$$\mu_n(H)=m_nc^2+kT\ln\left[n_n\left(\frac{2\pi\hbar^2}{m_nkT}\right)^{3/2}\right][2\,\mathrm{ch}(\varkappa_n\eta)]^{-1}Z_k^{-1}(H). \quad (9)$$

For T $\gtrsim 10^9$ K and H $\lesssim 10^{16}$ G, we have $\eta \ll 1$. Then $\sinh\frac{Z_i}{A_i}\eta\approx\frac{Z_i}{A_i}\eta$, and $\cosh\chi_i\eta \approx 1$, in which case Eqs. (7) and (9) take the general form

$$\mu_i=A_im_pc^2+kT\ln\left[\frac{n_i}{2}\left(\frac{2\pi\hbar^2}{A_im_pkT}\right)^{3/2}\right], \quad (10)$$

which coincides with the corresponding equations for the chemical potentials of nuclei and nucleons for H = 0 (Ref. 7). Here we must allow for the fact that $A_i = 1$ for protons and neutrons and $n_e \approx m_p$.

4. EQUILIBRIUM BETWEEN THE CONCENTRATIONS OF α PARTICLES, NEUTRONS, AND ^{56}Fe NUCLEI

In the state of statistical equilibrium, following Ref. 11, for reaction (1) we have the following equation for the chemical potentials:

$$\mu_{Fe}(H)=13\mu_\alpha(H)+4\mu_n(H). \quad (11)$$

Substituting Eqs. (7)-(9) for the chemical potentials of α particles, ^{56}Fe nuclei, and neutrons in a quantizing magnetic field into (11), we obtain an equation of the Saha type that describes the equilibrium among the concentrations of thse particles,

$$A(H)=\frac{n_\alpha^{13}(H)n_n^4(H)}{n_{Fe}(H)} =\frac{\omega_\alpha^{13}\omega_n^4}{\omega_{Fe}}\left(\frac{m_\alpha^{13}m_n^4}{m_{Fe}}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{24}\exp\left[-\frac{Q(H)}{kT}\right]f(\eta), \quad (12)$$

where

$$f(\eta)=\left[\frac{\mathrm{sh}(0{,}5\eta)}{0{,}5\eta}\right]^{-13}\left[\frac{\mathrm{sh}(0{,}46\eta)}{0{,}46\eta}\right][2\,\mathrm{ch}(1{,}913\eta)]^4. \quad (13)$$

In (12), Q(H) is the binding energy of the ^{56}Fe nucleus, which depends on H, and m_α, m_n, and m_{Fe} are the masses of α particles, neutrons, and ^{56}Fe nuclei, respectively. For simplicity, we assume that ω_α, ω_n, and ω_{Fe} are the same as for H = 0. This should hold true at magnetic field strengths for which the nuclear binding energy does not depend on H.

In a weak magnetic field, in which $\eta \ll 1$, from (12) we have the corresponding expression for H = 0 (Ref. 11),

$$A_0=\frac{n_\alpha^{13}n_n^4}{n_{Fe}}\approx2^4\frac{\omega_\alpha^{13}\omega_n^4}{\omega_{Fe}}\left(\frac{m_\alpha^{13}m_n^4}{m_{Fe}}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{24}\exp\left[-\frac{Q_0}{kT}\right], \quad (14)$$

where Q_0 is the binding energy of ^{56}Fe nuclei that does not depend on H.

In Refs. 7 and 10, it has been shown that the nuclear binding energy in the liquid drop model depends on the magnetic field strength and is determined by the expression (the Weizsacker equation)

$$Q(H)=\alpha'A_i-\beta A_i^{2/3}-\gamma\frac{Z_i^2}{A_i^{1/3}}-\zeta\frac{\left(\frac{A_i}{2}-Z_i\right)^2}{A_i}-\delta A_i^{-3/4}-\sigma Z_iA_i^{2/3}, \quad (15)$$

TABLE I. Calculation of log ρ (H) as a Function of Magnetic Field Strength

η	H_{17}	log ρ(H)	η	H_{17}	log ρ(H)	η	H_{17}	log ρ(H)
10^{-1}	0,14	4,907	1,2	1,65	5,06	3,0	4,12	5,34
0,2	0,28	4,91	1,4	1,92	5,095	4,0	5,49	5,46
0,4	0,55	4,93	1,6	2,20	5,13	5,0	6,87	5,58
0,6	0,82	4,96	1,8	2,47	5,16	10,0	13,74	6,02
0,8	1,099	4,99	2,0	2,75	5,19	20,0	27,48	6,68
1,0	1,37	5,03	2,6	3,57	5,28			

T_9 = 5, log ρ_0 = 4.83.

where α' = 15.75 MeV, β = 17.8 MeV, γ = 0.71 MeV, ζ = 94.8 MeV, and |δ| = 34 MeV (Ref. 12), and the coefficient σ, a function of H, has the form

$$\sigma=\frac{e^2r_0^2}{20m_pc^2}H^2=1{,}076\cdot10^{-3}H_{17}\ (\text{MeV}). \tag{16}$$

Here e is the elementary charge, $r_0 = 1.5\cdot10^{-13}$ cm, and $H_{17} = 10^{-17}$ H is the magnetic field strength in units of 10^{17} G.

The correction σ was obtained in Refs. 7 and 10 under the assumption that a nucleus is incompressible and the proton density is constant. The last term in (15) is proportional to the surface area of the nucleus, in effect varying the surface tension in the Weizsacker equation.

For an iron nucleus in a magnetic field H = (1-15)H_{17}, the contribution of the factor multiplying $A_i^{2/3}$ in the last term in Eq. (15) lies in the range $2.798\cdot10^{-2}$-6.296 MeV. For H_{17} = 5, this factor is 0.697 Mev, much less than β = 17.8 MeV. As shown in Ref. 7, however, the nucleus is no longer a spherically symmetric drop in this case, but assumes an elongated dumbbell shape, which facilitates fission. For this reason, in calculating the concentration of ^{56}Fe nuclei in a quantizing magnetic field, we confine ourselves below to strength $H_{17} \le 2.5$. The energy Q(H) then varies quite little and we can take Q(H) ≈ Q_0. As for the quantities inside brackets in Eq. (12), they depend essentially on the strength of the quantizing magnetic field.

In concluding this section, we note that Eq. (12) has been discussed earlier in Ref. 13, but it has not been used to calculate the density as a function of H for which half the matter by mass consists of ^{56}Fe nuclei and half of α particles and neutrons.

5. DETERMINATION OF THE DENSITY AT WHICH HALF THE MAGNETIZED MATTER (BY MASS) CONSISTS OF IRON NUCLEI AND HALF OF α PARTICLES AND NEUTRONS

As in Ref. 11, we assume that ^{56}Fe nuclei are the most abundant heavy nuclei in the stellar matter. Then from reaction (1) we determine the concentrations

$$n_\alpha=13n_{\text{Fe}},\qquad n_n=4n_{\text{Fe}},\qquad n_n=\frac{4}{13}n_\alpha. \tag{17}$$

In this case, half the matter (by mass) consists of ^{56}Fe nuclei and half of α particles and neutrons. The mass density may therefore be determined to within 1% by the expression

$$\rho\approx112m_pn_{\text{Fe}}. \tag{18}$$

Taking into account conditions (17), from (18) and Eq. (12) we easily find that for $H_{17} \le 2.5$ and Q(H) ≈ Q_0,

$$\lg\rho(H)=11{,}62+1{,}5\lg T_9-\frac{39{,}15}{T_9}+\frac{1}{16}\lg f(\eta), \tag{19}$$

where density ρ is measured in g·cm^{-3} and T_9 in units of 10^9 K, and

$$\eta=\frac{m_pc^2}{2kT}\frac{H}{H_{\text{cr}}^p}=3{,}639\frac{H_{17}}{T_9}.$$

In weak magnetic fields (H ≪ H_{17}) we have η ≪ 1, f(η) ≈ 1, and from (19) we have log ρ(H) ≈ log ρ_0, where log ρ_0 is determined by the first three terms.[11]

In the absence of a magnetic field, at the temperature T_9 = 5 we have[11] log ρ_0 = 4.83. At the same temperature and with H ≥ $H_{\text{qu}}^{\text{Fe}}$, the function f(η) increases with increasing H, resulting in an increase in the density ρ(H) at which half the matter (by mass) consists of ^{56}Fe nuclei and half of α particles and neutrons. The results of a numerical calculation of log ρ(H) at T_9 = 5 as a function of magnetic field strength H are given in Table I. For η > 1.8 (H_{17} > 2.47), the values of ρ(H) should be higher than those in the table, since the numerator of the third term in Eq. (19) should be smaller than 39.15 due to the decrease in binding energy Q(H). This change may be determined to within 10-20 Mev using the semiempirical Weizsacker equa-

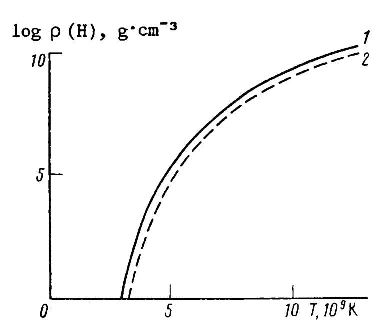

FIG. 1. Dependence of log ρ(H) on T_9 for H_{17} = 2.5.

tion. Moreover, not only the shape of the nucleus but also the filling of nucleon shells change in such strong magnetic fields. Such a calculation therefore seems unnecessary here. This problem requires special study and will be considered separately.

The calculations show that with an increase in magnetic field strength, photodissociation of ^{56}Fe nuclei increases, and magnetized matter consists half (by mass) of iron nuclei and half of α particles and neutrons at a higher density than for $H = 0$. This means that the influence of the magnetic field in the collapse of the central part of a magnetic star, like the influence the high temperature, results in the enhancement of photodissociation of iron.

In Fig. 1 we present the dependence of $\log \rho(H)$ on T_9 for a magnetic field strength $H_{17} = 2.5$ (curve 1). The dependence of $\log \rho_0$ on T_9 for $H = 0$ is presented in the same figure (dashed curve 2).[11] The region on the $\log \rho$, T_9 plane to the left of the curves corresponds to the equilibrium state of matter in which it consists almost entirely of iron, and the region to the right corresponds to matter that consists only of a mixture of α particles and neutrons. From the figure one can see that in the presence of a strong magnetic field, the catastrophic drop of the central part of the star to some density ρ at which half the matter (by mass) consists of ^{56}Fe nuclei and half of α particles and neutrons occurs at lower temperatures than in the case $H = 0$. One can see in the figure that for $H_{17} = 2.5$ the temperature decrease is $\Delta T = (0.2\text{-}0.4)T_9$.

At somewhat higher temperatures, but at the same densities, photodissociation of α particles into nucleons occurs by means of reaction (2). Somewhat lower temperatures are required for photodissociation of α particles for $H_{17} = 2.5$ than for $H = 0$, but the temperatures are higher than for photodissociation of iron nuclei. In other words, both for $H = 0$ and for $H_{17} = 2.5$ there is a temperature range in which iron nuclei dissociate into α particles but the α particles do not dissociate into free nucleons.

6. CONCLUSION

In this paper we have carried out a theoretical investigation of the influence of a strong quantizing magnetic field on the photodissociation of ^{56}Fe nuclei into α particles and neutrons. The magnetic field reduces the phase volume of the nucleus per quantum state, as a result of which the degree of dissociation also increases due to a decrease in nuclear binding energy in a strong magnetic field. The action of these two factors in the same direction results in the fact that, in the presence of a strong quantizing magnetic field, the catastrophic collapse of the central part of the star to some density ρ at which half the matter (by mass) consists of ^{56}Fe and half of α particles and neutrons occurs at lower temperatures than in the case with no magnetic field.

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Translated by Edward U. Oldham