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Lunar visibility and the crucifixion

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SUMMARY

The Christian Bible and historical evidence imply that Jesus Christ's crucifixion occurred on a Friday the fourteenth or fifteenth of the Jewish lunar month of Nisan between the years AD 28 and 33. One of the primary uncertainties in identifying a specific date is that the first dates of Jewish months were based (in ancient times) on the first visibility of the thin lunar crescent soon after new moon. The calculation of these dates is a difficult astronomical problem. Recently, however, a new algorithm has been developed which improves the accuracy of lunar visibility predictions by over a factor of two when compared with the best previous algorithm. For this reason, I have re-examined the question of the visibility of the young crescent moon from Jerusalem for the first four solar months during the reign of Pontius Pilate. The new results do not substantially alter the conclusions obtained with the previous best algorithm. I also examine the visibility of the partial lunar eclipse of 3 April AD 33, which has been associated with the crucifixion. The eclipsed moon would not have been visible at the time of moonrise and because the umbral portion of the eclipse finished minutes later, any naked-eye effects on the appearance of the moon would have been relatively subtle and probably not detectable even to an experienced observer. I find that any 'blood colour' associated with the eclipse would not be visible to the unaided eye. However, the rising eclipsed moon would have an amber colour from atmospheric absorption, just like any other time when the moon is low on the horizon.

INTRODUCTION

The exact date of the crucifixion of Jesus Christ has long held a fascination for scholars. Part of the reason lies in the fundamental importance of the crucifixion in Christian theology and part in the fact that sufficient information may be available to yield a specific date for the event. Among modern researchers (Maunder 1911, Fotheringham 1924, Ogg 1962, Maclean & Grant 1963, Finegan 1964, Doggett 1976, Humphreys & Waddington 1983), there is virtually universal agreement concerning some basic constraints on the date of the crucifixion. The Bible claims that the crucifixion occurred during the time when Pontius Pilate was the procurator of Judea (AD 26–36). Other historical and Biblical evidence indicates that this possible range of years can at least be reduced to the years AD 28 to 33. All four gospels place the death of Jesus Christ on a Friday afternoon. Three of the gospels place the date as that of the first day of the Jewish feast of Passover, that is, the fifteenth of the Jewish month of Nisan. The gospel of John places the crucifixion one day before the Passover feast, 14 Nisan. If the correlation between the day of the week and the day of the month is known, then enough information may be available to determine a unique date.

This basic programme has been used by many researchers (e.g. Maunder 1911, Fotheringham 1924, Finegan 1964, Humphreys & Waddington 1983) to date the original Easter. Many dates have been suggested (including Sir Isaac Newton's 23 April AD 34), with the most serious candidates being 18 March AD 29, 7 April AD 30 and 3 April AD 33. The latter date is also special because the moon rose during a lunar eclipse.

This programme runs into two difficulties (Doggett 1976). First, there are a number of uncertainties regarding ancient Jewish calendrical practices: we assume that the Jewish sabbath at the time of Christ fell on Saturday, and that there is an unbroken cycle of day names until modern times. It is also possible that non-calendric intercalations of extra months in the calendar were made to adjust the lunar year with the seasonal year. Another uncertainty lies in the methodology and accuracy of the determination of the equinox. Finally, the first day of each lunar month was nominally the day of first visibility of the lunar crescent, but was this calculated or observed or some mixture of the two? Each of the above points means that additional dates must be considered.

The second big difficulty comes in the calculation of the first day of the lunar month. Lunar visibility algorithms were extensively studied by the ancient Babylonians, many famous Islamic astronomers from AD 700 to 1300, and in modern times by Fotheringham (1910), Maunder (1911) and Ilyas (1984). Each of these incorporates a simplistic empirical criterion based on a relatively small observational data base that is valid for only one set of observing conditions. One indication of the accuracy of these criteria is that the predictions based upon modern criteria (which are all derived from one set of observations) mutually disagree with each other and with the observations by over 105 degrees of longitude for a given latitude (Schaefer 1988). Another indication of their accuracy is that the same criteria are applied to all observing locations ranging from the Amazon basin to the top of Mauna Kea. In an effort to break away from the inevitably poor empirical algorithms, Bruin (1977) pioneered a technique based on mathematically modelling the various physical phenomena involved. This theoretical approach has the advantage that its results can be applied to the prevalent local conditions. This one advantage would greatly increase the reliability of the predictions. Bruin did not successfully carry his programme to completion. Recently, I have completed a theoretical model of lunar visibility which incorporates accurate mathematical models for various physical, meteorological and physiological effects (Schaefer 1989a). I explicitly include atmospheric clarity, which is calculated from the site's altitude, latitude, relative humidity and the time of the year. The resulting set of equations has been combined with a lunar and solar ephemeris to yield a computer program that will predict the date of first crescent sighting from any location. Schaefer (1988) has collected 201 observations of lunar visibility from the astronomical literature and finds the new theoretical algorithm to have a total uncertainty in the predicted longitude of first visibility (for a given latitude) of 47 degrees of longitude. Doggett, Seidelman & Schaefer (1988), Doggett & Schaefer (1989) and DiCicco (1989) report on three campaigns involving roughly two thousand observers where the new theoretical algorithm is the only algorithm which successfully predicted the observations.

With a greatly improved means for calculating the starting day of the lunar month, it is worthwhile re-examining the question of lunar visibility around the time of the crucifixion. In addition, the same physical principles can be used to understand the appearance of the lunar eclipse on 3 April AD 33.

JERUSALEM'S EXTINCTION COEFFICIENT

A fundamental parameter for calculating lunar visibility near the horizon is the clarity of the atmosphere, which is quantified as the extinction coefficient. This quantity is defined as the light loss (in magnitudes) suffered by a beam of light that vertically traverses our atmosphere (that is, the beam travels through one airmass of air) and has units of magnitudes per airmass. Our applications are to cases involving a relatively bright sky and hence the eye's day vision, so that the extinction should be calculated for an effective wavelength of 0·55 μm (similar to the astronomical V filter).

The extinction coefficient can be divided into three components, caused by Rayleigh scattering from common gas molecules, absorption by ozone in the stratosphere and Mie scattering from aerosols. The first two components can be reliably calculated. Hayes & Latham (1975) derive a sea-level extinction of 0·1066 mags per airmass with an exponential fall off with altitude for a scale height of 8200 m. Hence, at the altitude of Jerusalem (450 m above sea-level), Rayleigh scattering will contribute 0·100 mags per airmass. The second component is 0·033 mags per airmass for a typical ozone equivalent depth of 3·0 mm (Allen 1973). At the latitude of Jerusalem, for the springtime, the ozone layer will be equal to 3·0 mm (Bower & Ward 1982). Therefore the extinction coefficient will be 0·133 mags per airmass plus the aerosol component. This aerosol component can be estimated for ancient Jerusalem by three separate methods.

The first method is to use established correlations to adjust the world mean aerosol component to the special circumstances of Jerusalem. I have collected daily extinction coefficient measures for 189 sites worldwide, and find that the average pollution-free site (corrected to sea-level, on the equator, with zero relative humidity and near the equinox) has an aerosol component of 0·088 mags per airmass. This world average should be multiplied by 0·74 so as to correct to the altitude of Jerusalem, because the aerosol component falls off exponentially with a scale height of 1500 m (Hayes & Latham 1975). The latitudinal extinction difference between the equator and Jerusalem (based on my analysis of the 189 sites as well as work by Patterson 1982) corresponds to a correction factor of 1·14. The relative humidity in the early evening is typically a third of the way from the daily minimum to the daily maximum, so the spring evening relative humidity for Jerusalem is roughly 50 per cent (Pearce & Smith 1984). I have calculated the necessary Mie scattering integrals (along with the particle size growth function of Hanel 1984) and find that the correction factor from zero relative humidity to 50 per cent is 1·66. No correction factor for the time of the year is needed since we are concerned with times near the equinox. So the total corrections for Jerusalem yield an aerosol extinction of 0·123 mags per airmass. The average day-to-day value of this number has a one sigma variation of 44 per cent (based on my analysis of the data from the 189 sites).

The second method is to evaluate the extinction in Jerusalem based on measured extinctions at nearby sites. I have located substantial amounts of data from three sites within 300 km of Jerusalem. The three sites are the Wise Observatory near Mitzpeh Ramon (Brosch 1988, private communication and Vidal, Brosch & Livio 1978), Mt St Katherine in the Sinai peninsula (Abbot 1908) and the Kottamia Observatory near Suez (Jones 1966). The average extinction coefficients near the equinox for these sites are 0·25, 0·13 and 0·36 mags per airmass, so the aerosol components are 0·118, 0·019 and 0·226 mags per airmass, respectively. When corrected to the altitude of Jerusalem, the aerosol components should be 0·131, 0·080 and 0·231 mags per airmass, respectively, for an average estimate of $0{\cdot}147 \pm 0{\cdot}063$ mags per airmass.

The third method uses 11022 actual measurements taken from Jerusalem itself during the years 1930–4 and 1961–8 (Joseph & Manes 1971). The early epoch data are substantially uncontaminated by modern pollution sources such as the automobile and worldwide industrial pollution. Their early epoch data show the April aerosol component to be 0·184 with a one sigma scatter of 0·098 mags per airmass.

The three methods give values of $0{\cdot}123 \pm 0{\cdot}054$, $0{\cdot}147 \pm 0{\cdot}063$ and $0{\cdot}184 \pm 0{\cdot}098$ mags per airmass for the aerosol component in ancient Jerusalem near the equinox. I will adopt the average value of $0{\cdot}15 \pm 0{\cdot}07$ mags per airmass for the aerosol component. This gives a total extinction coefficient of $0{\cdot}28 \pm 0{\cdot}07$. The variation about the mean is highly skewed so that much of the variance is caused by a small number of days with high extinction, so it is highly improbable that the extinction will be greatly lower than the mean.

For observations near the horizon, care must be taken to calculate separately the optical pathlength through each of the three components, because each component is differently distributed in the Earth's atmosphere. The pathlength of the ozone component is calculated geometrically as if all the ozone were concentrated in a thin layer 20 km above sea-level. The pathlengths for the Rayleigh and aerosol components are calculated as by Schaefer (1989b) for the appropriate scale heights. For a source on the horizon, the pathlengths will be 37, 12·6 and 121 airmasses for the Rayleigh, ozone and aerosol components, respectively. At an apparent altitude of 3·8 degrees, the corresponding values are 14, 10 and 16 airmasses.

For the expected case with a total extinction coefficient of 0·28 mags per airmass, the total extinction in the direction of the horizon will be 22·3 mags, while at 3·8 degrees apparent altitude the light loss will be 4·2 mags. For the most optimistic case with 0·21 mags per airmass, the total horizon extinction will be 13·8 mags, while the line of sight at 3·8 degrees altitude will suffer 3·0 mags of light loss.

LUNAR VISIBILITY

I have used my algorithm (Schaefer 1989a) to calculate the visibility of the young crescent moon (see Table I). The calculations are for the years AD 26–36 for which Pontius Pilate was the procurator of Judaea (Ogg 1962, Maclean & Grant 1963, Humphreys & Waddington 1983), even though most scholars think that this time range can at least be reduced to the years from AD 28 to 33 (Ogg 1962, Maclean & Grant 1963, Finegan 1964, Doggett 1976,

TABLE I

The visibility of the young crescent moon, calculated for dates between AD *26 and 36. See text for a description of columns 4–9.*

Number	Y	M	D	Age	ARCL	ARCV	DAZ	LAG	*R* *DR*
1	26	1	8	1·0	12·0	11·6	4·1	48	0·5±0·3
2	26	2	7	1·5	17·8	17·2	5·5	70	2·1±0·2
3	26	3	8	1·0	12·3	11·5	5·1	48	0·3±0·4
4	26	4	6	0·6	8·3	6·3	6·1	33	−2·0±0·4
5	26	4	7	1·7	21·3	19·7	8·7	84	2·6±0·1
6	27	1	27	1·1	12·9	12·0	5·4	53	0·6±0·3
7	27	2	26	1·5	18·3	17·0	7·3	72	2·1±0·2
8	27	3	27	1·0	12·9	11·1	7·2	51	0·2±0·4
9	27	4	25	0·7	9·1	6·8	6·6	36	−1·8±0·4
10	27	4	26	1·7	21·5	18·9	10·5	88	2·6±0·2
11	28	1	16	1·0	12·0	10·4	6·7	51	0·1±0·4
12	28	2	15	1·2	15·2	13·5	7·5	61	1·3±0·3
13	28	3	15	0·6	9·1	7·1	6·2	36	−1·7±0·4
14	28	3	16	1·6	19·7	18·1	8·2	78	2·3±0·2
15	28	4	14	1·1	14·4	12·6	7·3	58	0·9±0·3
16	29	1	4	0·9	11·7	9·2	7·7	50	−0·3±0·4
17	29	1	5	1·9	23·4	19·7	13·0	101	2·8±0·1
18	29	2	3	1·2	15·2	13·1	8·0	63	1·3±0·3
19	29	3	4	0·6	8·1	6·0	6·0	32	−2·2±0·4
20	29	3	5	1·5	18·4	16·9	7·7	72	2·1±0·2
21	29	4	3	0·9	11·3	9·9	6·1	45	−0·2±0·4
22	29	4	4	1·8	21·9	20·7	7·7	86	2·8±0·1
23	30	1	23	1·1	14·0	11·5	8·4	59	0·6±0·4
24	30	2	21	0·5	7·3	5·0	5·9	29	−2·6±0·4
25	30	2	22	1·4	17·9	16·4	7·5	71	2·0±0·2
26	30	3	23	0·8	10·7	9·5	5·6	42	−0·6±0·4
27	30	3	24	1·7	21·3	20·4	6·6	82	2·6±0·2
28	30	4	22	1·1	14·1	13·2	5·4	56	1·0±0·3
29	31	1	12	0·7	9·3	6·1	7·6	39	−1·9±0·4
30	31	1	13	1·8	22·1	18·7	12·0	94	2·7±0·2
31	31	2	11	1·2	15·3	13·6	7·5	62	1·3±0·3
32	31	3	12	0·7	9·2	8·0	5·2	36	−1·3±0·4
33	31	3	13	1·7	20·3	19·6	6·0	78	2·5±0·2
34	31	4	11	1·1	13·5	13·1	4·4	52	0·9±0·3
35	32	1	1	0·5	6·8	2·4	6·9	25	−4·1±0·7
36	32	1	2	1·6	19·9	15·2	13·1	86	2·1±0·2
37	32	1	31	0·7	9·5	6·9	7·0	39	−1·6±0·4
38	32	2	1	1·8	22·2	20·4	9·3	91	2·8±0·1
39	32	3	1	1·3	16·4	15·6	5·5	63	1·7±0·2
40	32	3	30	0·9	10·8	10·6	3·3	40	−0·2±0·4
41	32	3	31	1·9	22·6	22·4	4·2	83	2·8±0·1
42	32	4	29	1·4	17·1	16·9	3·5	64	1·9±0·2
43	33	1	20	1·1	13·9	11·4	8·3	59	0·6±0·3
44	33	2	18	0·7	8·9	7·8	4·9	36	−1·3±0·4
45	33	2	19	1·8	21·9	21·3	5·8	83	2·8±0·1
46	33	3	20	1·4	17·3	17·2	3·1	62	2·0±0·2
47	33	4	18	1·0	12·2	12·2	2·6	42	0·5±0·3
48	34	1	10	1·5	18·8	16·0	10·3	81	2·1±0·2
49	34	2	8	1·0	12·9	12·2	5·0	51	0·7±0·3
50	34	3	9	0·7	8·1	8·0	2·6	28	−1·4±0·4
51	34	3	10	1·8	21·8	21·8	2·8	76	2·8±0·1
52	34	4	8	1·4	17·4	17·4	−2·6	59	2·1±0·2
53	34	12	30	1·1	13·4	9·9	9·4	58	0·2±0·4
54	35	1	29	1·5	18·0	17·3	5·7	71	2·1±0·2
55	35	2	27	1·0	12·9	12·9	2·6	44	0·8±0·3
56	35	3	28	0·6	8·4	8·3	−3·1	24	−1·4±0·4
57	35	3	29	1·8	21·8	21·7	−2·7	73	2·7±0·1
58	35	4	27	1·4	17·5	17·5	2·7	63	2·1±0·2
59	36	1	18	1·2	14·7	13·9	5·4	60	1·3±0·3
60	36	2	16	0·6	7·7	7·7	−2·6	25	−1·5±0·4
61	36	2	17	1·6	19·3	19·3	2·7	67	2·4±0·1
62	36	3	17	1·1	14·0	13·9	−3·1	43	1·1±0·3
63	36	4	15	0·7	9·3	8·9	−3·5	25	−0·9±0·4
64	36	4	16	1·8	21·8	21·8	2·7	77	2·8±0·1

Humphreys & Waddington 1983). The visibility is given for new moons occurring in the first four solar months of each year, because it is not clear which lunation was identified as Nisan, because of uncertainties in the equinox and intercalations. For each lunation, the lunar visibility is reported for days of potentially uncertain visibility. In practice, I included those dates on which the moon had an age at the time of best visibility during the evening twilight of between 0·5 and 2·0 d.

The required input for the algorithm includes the following: (1) The latitude and longitude of Jerusalem are 31·8 degrees North and 35·2 degrees East. (2) The extinction coefficient as calculated in the previous section. (3) The positions of the sun and moon are taken to an accuracy of 0·1 degree from interpolation of the values given by Tuckerman (1964). The time chosen was the time of best visibility, which is typically 40 minutes after sunset. (4) The dates for calculation were determined from Tuckerman, and are given in the Julian Calendar.

My program calculates several quantities that relate to the visibility of the moon, and are given in Table I. The age of the moon (i.e the time elapsed since the instant of new moon) is given to the nearest tenth of a day. The column labelled ARCL gives the arc of light, which is the angular distance between the centres of the sun and moon. ARCV is the arc of vision, that is, the vertical distance between the centres of the sun and moon. Both the arc of light and the arc of vision are tabulated with the corrections for parallax and refraction removed to facilitate comparison with other calculations and prediction algorithms. DAZ is the difference in azimuth between the sun and the moon with a negative sign indicating that the moon was more northerly along the horizon. The various angular distances are expressed in degrees. The preceding four quantities were calculated for the time during twilight when the moon was best visible. The column labelled LAG gives the lag time from sunset to moonset in minutes. The final two columns give the visibility parameter *R* and its uncertainty as calculated from my model.

The *R* parameter is defined as the logarithm of the ratio of the apparent brightness of the moon to the minimum brightness required to distinguish the moon against the twilight sky evaluated at the time of best visibility. *R* is calculated such that an adult with average eyesight will have a 50 per cent probability of detecting the moon in a cloudless sky for the given conditions for an *R* value of zero (Schaefer 1989a). A negative value implies a prediction that the moon will not be visible. Positive values indicate that the moon should be visible. If *R* is less than 1, I would characterize the moon as being difficult to spot, while a value greater than 2 means easy visibility.

The *DR* parameter is a roughly one sigma uncertainty for *R* and is calculated by comparing the *R* value for the expected observing conditions with the *R* evaluated for observing conditions changed by one sigma. The ratio of *R* to *DR* will be indicative of whether Jerusalem was inside the zone of uncertainty. Schaefer (1988) found that if the magnitude of *R*/*DR* is greater than 2·5, then the criterion gives correct predictions for all 201 observations and a similar conclusion results from the roughly two thousand moonwatch observations (Doggett *et al.* 1988, Doggett & Schaefer 1989, DiCicco 1989). The closer *R*/*DR* is to zero the more uncertain will be the prediction. As an example, the first entry in the table (for 8 January AD 26) has a value of R equal to 0·5 and R/DR equal to 1·7. In this case, the best prediction is that the moon will be visible with difficulty, however, there is a small probability that conditions may be exceptionally poor so that even a serious and experienced observer would not sight the moon. The value of this probability can be calculated as the likelihood that a Gaussian variable will exceed the average by 1·7 sigma, so there is a 5 per cent probability that the prediction will be wrong. As a second example, the crescent on 7 February AD 26 would certainly be readily apparent to even a casual observer.

These results can be compared with similar calculations using other algorithms. Maunder (1911) claims that the moon would not have been visible on the evenings of 8 March 26, 27 March 27, 3 April 29, 23 March 30 and 30 March AD 32. For the first two dates, it is likely that the moon actually was visible (with difficulty). For all five cases, my calculations show that the visibility in Jerusalem is not certain, so the start of the Jewish month would depend on the seriousness of the observer and on whether unusual local conditions existed. I find excellent agreement between my results and the calculations of Humphreys & Waddington (1983). The one minor exception is that I would expect the moon to be barely not visible on the night of 30 March AD 32 since the R value is (barely) negative.

LUNAR ECLIPSE CIRCUMSTANCES

The partial lunar eclipse of 3 April AD 33 has been widely associated with the crucifixion (Link 1969, Humphreys & Waddington 1983). Humphreys & Waddington (1983) consider the prophecy of Joel (Joel 2:31 and Acts 2:20) concerning the moon turning into blood to be a reference to an actual event near the time of Christ's death. In this view, the moon turning to blood could be a reference to a lunar eclipse during which the moon often has a reddish hue inside the umbra. This is further suggested by a possible reference to a solar eclipse ("The sun shall be turned into darkness") in the same verse. If this view is accepted, the 3 April AD 33 eclipse would be corroborating evidence for the crucifixion on one of the most likely dates as deduced from the previously discussed calendric evidence (Humphreys & Waddington 1983).

I would like to examine how this eclipse would have appeared from Jerusalem. The position and (ephemeris) times of the moon's passage through the earth's shadow can be accurately calculated. The eclipse was only a partial one in which the moon had at most 59 per cent of its diameter inside the umbra. The primary uncertainty for predicting the visibility from Jerusalem concerns the altitude of the moon when it leaves the umbra. This difficulty arises because the time of moonrise is calculated in local or Universal time, which is tied to the earth's rotation, whereas the times of contacts are calculated in terms of ephemeris time, which is tied to the earth's revolution around the sun. These two times do not advance uniformly with respect to each other because the earth's rotation and the moon's orbit have slight irregularities that add up to a significant offset over the centuries. This offset can be determined for ancient times only by the examination of ancient eclipse records and is known only to an accuracy of several minutes.

For calculations concerning the visibility of the eclipse from Jerusalem, the vital parameter is the apparent altitude of the moon when the end of the (umbral) eclipse happened. The calculation of ancient eclipse circumstances is a complicated problem, one which I am not qualified to answer. To overcome this problem, I have consulted Fred Espenak (NASA/Goddard Space Flight Center), John Bangert (U.S. Naval Observatory), Jean Meeus and Hermann Mucke (Meeus & Mucke 1979) and Graeme Waddington (Humphreys & Waddington 1983 and Waddington 1988, private communication). The results from their calculations are presented in Table II. Espenak performed calculations for a variety of different assumptions, for which two are presented here. Bangert performed calculations using two independent ephemerides with the results given.

In Table II, the first two lines give the source for the solar and lunar ephemerides used. The third line gives the assumed difference in time between ephemeris time and universal time as discussed above. The lunar acceleration is the rate that the moon's motion is speeding up due to tidal interaction with the earth, a value which depends on the ephemeris and ΔT value adopted. The umbral enlargement parameter is to take account for the size of the Earth's shadow being apparently larger than deduced from geometric considerations alone. The eclipse magnitude for a partial lunar eclipse is the fraction of the moon's diameter covered by the umbra at the time of maximum eclipse. The umbral duration is the total time that any portion of the moon is within the Earth's umbral shadow. The geocentric altitude of the moon is the altitude of the centre of the moon's disk above an ideal horizon with no parallax or refraction corrections. The topocentric altitude is the apparent altitude of the moon's centre as viewed by an observer, and is the geocentric altitude with parallax and refraction corrections.

In this tabulation, the results of Humphreys and Waddington stand out from the other five calculations concerning the eclipse magnitude and duration. The reason for this difference is not clear, since these two parameters have only slight dependences on the assumed ΔT and lunar acceleration.

The primary result of these calculations is that the altitude of the moon at the time of the umbral contact is between 2·0 and 5·65 degrees. I believe that these differences are indicative of fundamental uncertainties which are not likely to be overcome in the near future. While it may be that the actual altitude is outside this range, I will follow the best modern calculations which indicate that at umbral contact the moon had an altitude of 3·8 degrees altitude (with a possible error of 1·9 degrees).

The geocentric zenith distance of the sun (Z_S, in degrees) is given by

$$Z_S = 180 - R(Z_M) + \pi(Z_M) - S\cos(A) - Z_M,$$

where R is the change in the apparent zenith distance of the moon (Z_M) caused by refraction, π is the lunar parallax, S is the angular distance between the shadow's centre and the moon's centre and A is the angle from the vertical to the line joining the moon's centre and the shadow's centre. Near the end of the umbral part of the eclipse in Jerusalem, S was 0·95 degrees, A was 140 degrees and π was 0·95 degrees. At the start of moonrise ($Z_M = 90{\cdot}25$ degrees), R was 0·55 degrees so that the sun had a geocentric zenith distance of 90·9 degrees. At the end of the umbral eclipse, Z_M was 86·2 degrees and R was 0·2 degrees so that Z_S was 95·2 degrees.

TABLE II

3 April AD 33 lunar eclipse circumstances

	Humphreys & Waddington	Espenak	Espenak	Bangert	Bangert	Meeus & Mucke
Solar ephemeris	Newcomb	Newcomb	Newcomb	DE102 to DE200	VSOP82	Newcomb
Lunar ephemeris	ILE[a]	ILE[a]	ILE[a]	LE51 to LE200	ELP2000-85	ILE
ΔT (sec)	9534	9425·6	9534	8830	8830	9124
Lunar acceleration (″/c²)	−26	−25·10	−26	−23·9	−23·895	−22·44
Umbral enlargement (%)	0	2	2	2	2	1·5
Eclipse magnitude	0·60	0·587	0·586	0·582	0·582	0·580
Umbral duration (min)	190	168	172	171·1	171·0	170
Geocentric altitude[b] (deg)	6·45[c]	2·6	5·8	5·6	5·6	2·9
Topocentric altitude[b] (deg)	5·65	2·0	5·1	4·7	4·7	2·3

[a] Morrison's corrections for the moon have been applied. [b] For the time of umbral contact. [c] This is calculated from the geocentric altitude of the centre of the moon when the top is at the ideal horizon (with a geocentric altitude of 0·15 degrees) plus the altitude change over the 30 min claimed to elapse from moonrise to umbral contact.

The altitude of the moon (and hence the sun) can be calculated for other times from the usual trigonometry. However, for practical purposes, it is of sufficient accuracy to assume that the rate of change of the moon's geocentric altitude was the moon's angular velocity times the sine of the angle that the moon rises from a flat horizon. The moon's angular velocity was slightly less than one degree every four minutes and it rose at an angle 58 degrees from the horizontal, so that the moon gained altitude at a rate of 0·21 degrees per minute.

ECLIPSE VISIBILITY

One uncertainty concerning the appearance of the moon is the apparent altitude of the local horizon from Jerusalem. The old city is somewhat hilly, so it may be possible for observers to have had a horizon ranging several degrees in altitude above or below an ideal horizon. This uncertainty will not affect the conclusions below because a high horizon will merely delay first visibility even more than dictated by atmospheric conditions, while a low horizon will in no way improve the lunar visibility. Similarly, an observer in the vicinity of Jerusalem cannot have observed from a sufficiently higher or lower elevation so as to change significantly the extinction coefficient calculated above.

The primary uncertainty regarding the visibility of the partially eclipsed moon is the clarity of the air. I will perform the calculations below for two values of the extinction coefficient, first the expected value as calculated above, 0·28 mags per airmass, and second, the lowest reasonable value of 0·21 mags per airmass.

The sun is always close to the horizon whenever an eclipsed moon is rising, so daylight or twilight may strongly interfere with the visibility of the moon. The eastern horizon is only a tenth of the surface brightness of the western sky when the sun is 3 degrees below the horizon, so this twilight will strongly interfere with lunar visibility. The sky brightness as a function of the sun's position can be found in the paper by Koomen *et al.* (1952) for two sites. The extinction coefficients for the two sites can be calculated by the same three methods as used above to calculate the extinction from Jerusalem. The sky brightness as a function of extinction coefficient is given in equation 2 of Schaefer (1986). For an extinction coefficient in Jerusalem of 0·28 mags per airmass, the sky brightnesses at Sacramento Peak should be multiplied by 2·33. At the time of moonrise ($Z_S = 90{\cdot}0$ degrees), the sky brightness near the moon was 130 milliLamberts. When the moon left the umbra ($Z_S = 95{\cdot}2$ degrees), the sky brightness was 1·5 milliLamberts. For the highly optimistic value of 0·21 mags per airmass, the two sky brightnesses just calculated must be multiplied by 0·75.

The surface brightness of the full moon is roughly 1500 milliLamberts. This is reduced by five orders of magnitude within the umbra. In the penumbra, the reduction in surface brightness is the fraction of the sun covered by the earth, as viewed by an observer on the moon. Near the centre of the moon at the time of an external umbral contact, the surface brightness is reduced by roughly a factor of three. Along the outer edge, the reduction in surface brightness is only 10 per cent or so. The earth's atmosphere will also dim the moonlight. For an extinction coefficient of 0·28 mags per airmass, the surface brightness is reduced by a factor of 8×10^{8} when the moon is rising. Hence, the average apparent brightness of the rising moon was 6×10^{-7} milliLamberts (with a maximum of 2×10^{-6} milliLamberts on the bright edge). At the last umbral contact, the extinction reduced the brightness by a factor of 48, for an average brightness of 10 milliLamberts. At the same time, for an optimistic extinction coefficient, the moon's average surface brightness will be 31 milliLamberts.

The contrast ratio is the brightness of the moon divided by the brightness of the sky. It averaged 5×10^{-9} at moonrise, increasing to 6 when the moon left the umbra. For the optimistic extinction coefficient, the contrast ratio would be 2×10^{-5} at moonrise and 28 when the moon left the umbra. Numerous physiological studies (e.g. Siedentopf 1941) show that a uniformly illuminated disc with the same angular size as the moon can be seen against an illuminated background only if the contrast ratio is greater than 0·01 (for the background illumination at moonrise) or 0·02 (at the last umbral contact). Similar detailed calculations for different altitudes show that this critical contrast ratio should be realized for altitudes slightly smaller than two degrees. For the optimistic case of atmospheric clarity, the critical altitude will be lower by 0·3 degrees.

This calculation is predicated on the assumption that the observer knows exactly where and when to look. If a source is at an unknown position, an observer can easily overlook the object despite it being significantly brighter than threshold (Minnaert 1954). The reason is that the eye's photopic sensitivity is highly peaked at the fovea, so unless the source is looked at exactly, the effective sensitivity of the eye is greatly reduced. This is the reason that Venus is almost never spotted in the daytime sky, whereas an observer who knows exactly where to look will usually find Venus to be trivially visible (Minnaert 1954). Similarly, the visibility of the first star in the evening twilight is governed not by the sky brightness, but by the time when the eye first happens to chance across the exact position of a bright star. For the case of the crescent moon, approximately 50 participants of the various moonwatches (Doggett *et al.* 1988, Doggett & Schaefer 1989, DiCicco 1989) commented that the moon was not sighted until binoculars were used to locate the moon whereupon immediately afterwards the moon was easily visible to the unaided eye, because they knew where to look. In the case of a rising moon, the time, the altitude and the azimuth would have been only known approximately to a watcher in Jerusalem. In practice, this means that the moon will be first sighted several minutes after the theoretically calculated earliest time.

The above calculations should be tested against real observations. The partial lunar eclipse of 1988 August 27 provided an opportunity to observe an eclipse where the visibility conditions were comparable to those of the 'crucifixion eclipse'. The morning of 27 August was unusually clear for my observing site for that time of year and I estimate the extinction coefficient to be roughly 0·25 mags per airmass. The partial umbral eclipse started at 10:08 UT when Z_M was 86·2 degrees and Z_S was 95·4 degrees. These visibility conditions are quite close to those deduced for the 'crucifixion eclipse' (the extinction was $0{\cdot}28 \pm 0{\cdot}07$ mags per airmass, Z_M was $86{\cdot}2 \pm 1{\cdot}9$ degrees and Z_S was $95{\cdot}2 \pm 1{\cdot}9$ degrees) so that the visibility should be closely comparable.

I observed with Martha Schaefer from the roof of a building in Greenbelt, Maryland. We first detected penumbral shadings roughly 25 min before the first umbral contact, however, the shading was noticeable only because we knew what to look for. From 10:08 to 10:13 UT the moon rapidly faded to invisibility at an altitude of 2·8 degrees (with $Z_S = 94{\cdot}4$ degrees). Fred Espenak observed from nearby Bowie, Maryland, with similar results.

Several observations relating to the visibility of the moon should be mentioned. (1) During the last three minutes before the moon vanished (corresponding to 0·6 degrees of altitude), I was able to locate the moon only by finding it in binoculars with relation to horizon features so that I would know exactly where to look. This difficulty would have been much greater for the 'crucifixion eclipse' because the moon was rising and the Jews had no binoculars to let them anticipate where and when the moon would first appear. (2) The moon disappeared at a higher altitude than I would have calculated. This difference cannot be due to a haze layer or poorly estimated atmospheric clarity, because the extinction coefficient would have to be 0·75 mags per airmass for the observation to agree with the calculation. Any discrete clouds would have appeared as edges across the moon, and these were neither observed nor photographed. (3) I observed and Martha Schaefer photographed the brightness distribution of the penumbral shadings to be approximately offset by the differential extinction so that the vertical brightness distribution was uniform across most of the disk. However, the horizontal brightness distribution was noticeable (when looked for) because the moon was not directly below the anti-sun direction. (4) The penumbral shadings were apparent to me for 25 min before the moon entered the umbra. However I am experienced at detailed observations of lunar eclipses, I knew exactly what to look for and I had the positive knowledge that the shadings existed. I estimate that the penumbral shadings would have been detected as an anomaly by an observer with no previous knowledge of the eclipse only for a time within five to ten min from contact. During this short interval, it would have been difficult for an observer to decide whether an eclipse was occurring or merely that a haze layer was covering the moon. (5) At no time was any colouration observed inside the umbra either through binoculars or photographically. However, the visible portion of the moon appeared as a pale magenta colour when it was setting low over the horizon.

The theoretical calculation results in a figure of roughly 1·9 degrees as the minimum altitude that the 'crucifixion eclipse' moon could possibly be visible. The observational evidence results in a figure of 2·8 degrees for the altitude of minimum possible visibility. It is difficult to decide which value to accept, however, I would tend to accept the results of a simple observation over the results of a complex calculation. The minimum possible altitude of visibility will not be the altitude at which the Jews in Jerusalem first sighted the moon, because they did not know exactly where or when to look. My observations of the 1988 lunar eclipse show that the moon needs to be 0·6 degrees or more above the minimum altitude. So I conclude that the eclipsed moon would be first spotted when it had an altitude of 3·4 degrees or higher.

So the moon would be noticed only after the umbral shadow either had completely left the moon or had a small (less than 1 per cent) fraction of the moon's surface in eclipse. With the visible umbral eclipse being so small in magnitude and so brief in duration (if it occurred at all), the anomaly of the moon's shape could easily be ascribed to clouds by any naked eye observer.

More than several minutes after leaving the umbra, the shading will not be readily noticeable to any observer not expecting to see an eclipse. Even if the penumbral darkening had been spotted, it would be easy and natural to ascribe it to clouds or a haze layer. When the moon is several degrees above the horizon, the differential extinction across the face of the moon will offset the shadowing from the penumbra. For example, when the upper limb has an altitude of 3 degrees, it will be brighter by half a magnitude than the lower limb. This effect is comparable to, and works against, the penumbral shading, tending to mask the presence of an eclipse. Hence, the penumbral shadings were unlikely to be recognized, and even if they were, it is more likely for an observer to ascribe them to a haze layer or clouds. In any case, the subtle shadings would not have been spectacular enough for Peter's audience (Acts 2) to have remembered the doubtful event from seven weeks earlier as claimed by Humphreys & Waddington (1983).

The visibility of the lunar eclipse of 3 April AD 33 from Jerusalem would have been difficult. During the umbral portions of the eclipse, the moon would have been so low in a bright twilight sky that it could not have been seen with the naked eye. The moon would be first spotted just after leaving the umbra, at a time when the differential extinction would to a great extent mask the penumbral shadings. By the time the moon rose high enough for the differential extinction to be insignificant, the deepest shadows of the penumbra would have left the moon. In summary, at no time during the eclipse would even a serious and experienced observer in Jerusalem have realized that an eclipse was occurring.

ECLIPSE COLOURATION

There is also a difficulty concerning the visibility of any reddish colouration caused by the eclipse. This reddish light is caused by refraction of sunlight through the earth's upper atmosphere, where normal scattering will prevent blue light from penetrating. But this refracted light is much less intense than direct light from even a small portion of the sun. So the reddish colour can be seen only in the umbra where no direct yellow sunlight swamps out the red colour. The large atmospheric scattering when the moon is low on the horizon will make the umbral light even fainter and more difficult to view.

When the umbra covers only a small fraction of the lunar disk, the close proximity of the much brighter penumbral parts will make any reddish hue invisible to a naked eye observer. The red light in the umbra has a surface brightness that is 0·00001 of the normal full moon brightness and a total area of, say, 20 per cent of the lunar disk. The white light from the penumbral portions has an average brightness of roughly one third times that of the full moon over 80 per cent of the lunar disk. Hence the red light is being swamped by a nearby source that is 130000 times brighter. This is like trying to spot an 8 watt red light bulb next to a megawatt searchlight.

The invisibility of umbral colouration near the outer umbral contacts is amply confirmed by observations, including those by Davis (1982), Mobberley (1986) and my own observations of the 1988 August 27 eclipse. Hence, I conclude that any reddish colouration of the 3 April AD 33 eclipse must be unrelated to the eclipse itself.

However, there is another way the moon can appear 'blood coloured'. We have all seen this when the moon is low on the horizon. The atmosphere acts as a selective filter and will primarily transmit red light. This filtering property can be quantified as a function of wavelength (Allen 1973). When the moon is on the horizon (with the equivalent of 40 airmasses of extinction), the density of the atmospheric filter will be 5·5, 4·1, 3·5, 3·0 and 2·2 at wavelengths of 4500, 5000, 5500, 6000 and 6500 Å, respectively. This is effectively the equivalent of a neutral density 3 filter plus a Wratten 106 filter (Kodak 1981). The Wratten filter is described as 'amber'. Hence, the 3 April AD 33 eclipsed moon would have appeared amber when it was rising, exactly like any other rising moon.

CONCLUSIONS

I have re-examined the starting dates of the Jewish lunar months around the time of the crucifixion by means of an improved algorithm for predicting lunar visibility. These dates can be of use in helping identify the exact date of Christ's death.

The eclipsed moon on 3 April AD 33, which has been widely associated with the crucifixion, is found to have been invisible at moonrise and indeed the eclipse is likely not to have even been noticeable at any time from Jerusalem. In addition, any reddish colouration caused by the eclipse could not have been visible. However, the rising moon (in fact, any rising moon) would have been amber coloured because of atmospheric absorption.

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