THE EXPANDING UNIVERSE AND THE EXPANDING SOLAR SYSTEM

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Sommario

E' presentato un collegamento teorico tra la cosmogonia dell'uni= verso e quella particolare relativa al Sistema Solare.
La dimostrazione analitica, per questa connessione, é derivata dal la relatività proiettiva. Come conseguenza dinamica,di queste con siderazioni teoriche, si ottiene una variazione secolare di G nel tempo in buon accordo con quelle ritrovate, da altri teorici e spe rimentatori, seguendo vie differenti.

Summary

A theoretical connection between Solar System and whole Universe cosmogony is here presented and proved. This is obtained by means of an implementation of the model of the Projective Relativity. As an implication of this study, a time variation law of universal gravitation constant G is found.

1 GENERAL

Although a noticeable amount of work has been done in the field of the investigation on the Solar System origin and also in the field of researches on the origin and evolution of the whole universe, there is no theoretical linkage which connects the two separate fields of researches.
In the textbook "The Origin if the Solar System" (Dermott 1978)we

read the sentence: "The old problem of how the world was created is to-day divided in two problems. One is the 'big creation' or how the universe as a whole has originated and developed, which we have discussed to some extent. The other is the 'small creation' or how in a small part of a small part of the universe the Solar System originated. We shall devote the rest of this lecture exclusively to this restricted problem" (H.Alfvén).
Let us note that, while in the first problem the conceptual frame of classical physics is applied, in the second one the conceptual frame belongs to relativistic physics. This kind of separation, in scientific work, has been criticized by the science philosophers; in fact, according to Popper's epistemology the perfect division of labour in research would soon stop the scientific progress. J.Agassi in its textbook "Science in Flux" claims (Agassi, 1975): "Investigators may wish to study the universe as a whole, without even bothering to ask how their partial pictures integrate with man's picture of the universe as a whole". In fact nowadays we can read, in any good textbook of cosmology (see for instance Cavalleri 1987), some sentence like this: "While the whole universe is expanding, any single galaxy, as well the Solar System, absolutely does not participate to this expansion. In other words the Solar System, during its evolutionary history, was never in expansion".
The aim of this work is twofold. At first to check the truth of the foregoing sentence, then to establish, accepting Agassi's challenge, a connection between two fields of researches.
The starting point lies on the knowledge of the following experimental facts. The most probable age of the Earth is about 4.7×10^9 yrs, while that of whole universe is about 15×10^9 yrs. Then there is no big difference between the time intervals. Therefore the task is to look for the possible beforesaid theoretical connection.

2 ANALYTICAL DEVELOPMENT OF THE PROBLEM BY MEANS OF PROJECTIVE RELATIVITY

Coming back to our problem, it should be useful a kind of relativity to be regarded as a physics for "great distance phenomena", both in space and time.
More than thirty years ago it was asserted (Fantappié, 1954) that special relativity derives, as a limiting case, from a new theory

in which the Minkowski space-time, of null curvature, is substituted with the De-Sitter space-time, with constant curvature. Thus we obtain a new "Projective Relativity" which perfects the special relativity and may be studied with the methods of group theory. Furthermore it is maintained the connection with the special relativity to which it reduces when r, universe radius, approaches infinity (Arcidiacono, 1976).

The Projective Relativity can be developed utilizing a flat representation of the De-Sitter space-time. Among the infinite possible representations, the simplest one is that called "Beltrami Geodesics", in which the geodesics of V_4 space-time correspond to straight lines. As a consequence the group of motions in itself of De-Sitter universe V_4 is represented by Fantappié-group, i.e. by the projective of P_4 space-time which changes in itself the Cayley-Klein absolute of equation

$$x^2 + y^2 + z^2 - c^2 t^2 + r^2 = 0 \tag{1}$$

[with t_o = universe age and $r = ct_o$]

here c = light velocity. The Minkowski space-time is replaced with the projective space-time P_4, called Castelnuovo space-time, from the name of the mathematician who introduced it, without giving any physical meaning (Castelnuovo, 1930).

Application of Projective Relativity have been made in astrophysical cosmology and also in understanding elementary particle processes (Gürsey 1964, Prasad 1966).

We have seen that, in Castelnuovo space-time, the group of motions in itself of De-Sitter universe is represented by the Fantappié group. The calculation of the Fantappié group transformations has been made, with whole generality, in the past (Arcidiacono, 1956). Instead of rewriting these complete transformations let us study un more reduced and suitable form for our scope. At first we introduce coordinates x_k [with k = 1, 2, 3, 4] provided by following formulas

$$x_1 = x \qquad x_3 = z$$
$$x_2 = y \qquad x_4 = ict \qquad i = \sqrt{-1}$$

and then we put them into Eq. (1) obtaining

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + r^2 = 0. \tag{2}$$

Then we introduce the projective homogeneous coordinate $\bar{x}_\alpha$

$$x_\alpha = r \frac{\bar{x}_\alpha}{x_5} \qquad (3)$$

$$\left[\text{with} \quad \alpha = 1, 2, 3, 4, 5 \right]$$

and Eq. (2) shall become as follows

$$\bar{x}_1^2 + \bar{x}_2^2 + \bar{x}_3^2 + \bar{x}_4^2 + \bar{x}_5^2 = 0 \ . \qquad (4)$$

The transformations of the Fantappiè group change in itself the a bsolute (4). We limit ourserves to the "simple rotation" of the following type. Let us consider an observer O_s (who is in rest with respect to the reference frame) and the particular transformation consists in considering this observer in two different instants of time. Then we shall have a "simple rotation" ($\bar{x}_4$, $\bar{x}_5$) provided by the particular transformations.

$$\bar{x}_1' = \bar{x}_1 \qquad \bar{x}_2' = \bar{x}_2 \qquad \bar{x}_3' = \bar{x}_3$$

$$\bar{x}_4' = \bar{x}_4 \cos t_4 - \bar{x}_5 \sin t_4 \qquad \bar{x}_5' = \bar{x}_4 \sin t_4 + \bar{x}_5 \cos t_4 \qquad (5)$$

Putting then $\tan t_4 = ic \frac{To}{r} = \gamma i$, it follows that

$$\cos t_4 = \frac{1}{\sqrt{1 - \gamma^2}} \qquad \sin t_4 = \frac{1}{\sqrt{1 - \gamma^2}},$$

To being the time interval between the beforesaid instants. Eqs.(5) will become

$$\bar{x}_1' = \bar{x}_1 \qquad \bar{x}_2' = \bar{x}_2 \qquad \bar{x}_3' = \bar{x}_3$$

$$\bar{x}_4' = \frac{\bar{x}_4 - i\gamma x_5}{\sqrt{1 - \gamma^2}} \qquad \bar{x}_5' = \frac{\bar{x}_5 + i\gamma x_4}{\sqrt{1 - \gamma^2}} \qquad (6)$$

If we substitute the following formulas, on the basic of foregoing ones

$$x = x_1 = r\,\frac{\bar{x}_1}{\bar{x}_5} \qquad y = x_2 = r\,\frac{\bar{x}_2}{\bar{x}_5}$$

$$z = x_3 = r\,\frac{\bar{x}_3}{\bar{x}_5} \qquad ict = x_4 = r\,\frac{\bar{x}_4}{\bar{x}_5}$$

the final goal concerning the considered transformation is (Arcidiacono, 1956)

$$x' = x\,\frac{\sqrt{1-\gamma^2}}{1-\gamma\frac{t}{t_o}} \qquad y' = y\,\frac{\sqrt{1-\gamma^2}}{1-\gamma\frac{t}{t_o}}$$

$$z' = z\,\frac{\sqrt{1-\gamma^2}}{1-\gamma\frac{t}{t_o}} \qquad t' = \frac{t - T_o}{1-\gamma\frac{t}{t_o}} \, . \tag{7}$$

Observer O_s is to-day in rest with respect to the reference frame, having the coordinates x, y, z, o (present epoch $t = o$).

The same observer, at epoch $t' = -T_o$ (also in rest with respect to the reference frame) had the coordinates

$$x' = x\sqrt{1-\gamma^2} \qquad y' = y\sqrt{1-\gamma^2}$$

$$z' = z\sqrt{1-\gamma^2} \qquad t' = -T_o \, .$$

The to-day distance between O_s and origin of the reference frame is $d_a = \sqrt{x^2 + y^2 + z^2}$; while at the epoch t' was :

$$d' = \sqrt{x^2 + y^2 + z^2} \quad \sqrt{1 - \gamma^2} = d_a \sqrt{1 - \gamma^2} \qquad (8)$$

Let us apply the previous results to our problem. Suppose that observer O_S was, at the epoch $t' = -T_O$, i.e. at the first epoch of the Solar System formation, on the limiting surface of the primeval nebula (assumed spherical for the sake of simplicity) while the origin of the reference frame was put in the mass center of nebula (practically coincident with the protosun center). The radius of the beforesaid surface was d', while the distance of the observer from the origin is to-day (at the epoch t = o) d_a.
Since $\gamma = To/t_{\Theta}$ is less than 1 , it follows that $d' < d_a$.
The significan radius observed by O_S , at the initial epoch of the Solar System, was smaller than that observed to-day, as the radius within which the all major objects of the Solar System are now contained.

3 DYNAMICAL IMPLICATIONS OF THE RESULTS OBTAINED IN ITEM 2

The consequences of this space-time deformation, in connection with possible modification of classical dynamics, are now discussed. At first we remember that the bodies masses, which are present within the Solar System during its evolutionary history, shall be affected by a time variation, following the Projective Relativity laws, which shall be now described. Owing the Fantappié group transformation, on the bases also of the fundamental assumption (item 2) of taking into account the whole phenomenon time duration only, the mass m' of any body, at epoch $t' = T_O$, and the to-day mass ma are connected by the following relationship.

$$m' = m_a (1 - \gamma^2) \qquad (9)$$

Let us consider a mass, for instance a unit mass, orbiting around the protosun, at the epoch $t' = -T_O$, at any average distance d' and the same orbiting mass to-day at a distance d_a .
In other words there is the motion of a body (of unit mass) with respect to its primary body; hence we consider the motion evolution during a long time interval. Here is the two-body problem which is worked, in classical celestial mechanics, by means of the fundamental mechanical laws of motion together with Newton's gra-

vitation law. This problem is rigorously solvable because the two integrals of the differential equations of motion are sufficient to yield the complete motion description. Let us study now this problem taking into account Eqs.(8) and (9).

It is convenient to remember the theory of adiabatic invariants. It has been proved (Levi-Civita, 1927; 1928) that, in two body problem, there is a quantity $\bar{W}$, which maintains invariable during the time, provided by the following formula

$$\frac{\bar{W}}{2\pi} = \sqrt{G M a} - c_o \tag{10}$$

Here $\bar{W}$ is called adiabatic invariant,

$M = m_1 + m_2 =$ sum of two bodies masses

(m_1 = primary mass ; m_2 = unit mass)

G = Newton gravitational constat,
a = semimajor axis of m_2 orbit in the motion respect to m_1 ,
c_o = angular momentum of the unit mass orbiting around m_1 = constant (the central field of force is supposed in the theory)

Levi-Civita has showed that $\bar{W}$ is constant not only in the normal case, i.e. when M , G and a are rigorously constant during the time. The time variation law may be any, but it shall be completely independent of any local property of motion; in other words the time variation shall be an adiabatic one. In such conditions it is proved that the product

$$G M a$$

remains constant, together with W and c_o , during our problem, taking into account Eqs.(8) and (9). It follows that

$$G' M' a' = G_a M_a a_a \tag{11}$$

when the primes denote the quantities at the epoch $t' = -T_o$ and the subscripts a denote the same to-day quantities. Then we have

$$G' M_a (1 - \gamma^2) \; a_a (1 - \gamma^2)^{1/2} = G_a M_a a_a$$

from which it follows

$$G_a = G' \; (1 - \gamma^2)^{3/2} \tag{12}$$

Eq.(12) contains a very important result : the gravitational constant G is varied very slowly, during the time interval T_o, according to (12).

This is the dynamical implication following from the Projective Relativity model. On the basis of these data :

$$t_o = 15 \times 10^9 \text{ yrs} \qquad T_o = 6 \times 10^9 \text{ yrs}$$

it is possible to calculate $\frac{\Delta \dot{G}}{G}$, which is -9×10^{-11} parts per year.

4 CONCLUSIVE REMARKS

During the evolutionary history of the Solar System, owing to the whole universe evolution, the spatial scenario of the Solar System formation was certainly smaller than the present one. It follows that the single, partial and subsequant phases of Solar System formation, are surely correctly-analyzed by means of classic physics. In fact G variation, provided by Eq.(12), is important from a conceptual point of view but quantitatively very small. However it is important to consider the geometrical correction, of the beforesaid scenario, owing to Eq.(8).

Assuming $t_o = 15 \times 10^9$ yrs and $T_o = 6 \times 10^9$ yrs we shall have

$$d' = d_a \sqrt{1 - \gamma^2} = 0,92 \; d_a$$

Since Pluto's orbit radius is about 40 A.U., the matter which formed the whole Solar System was mainly contained within a sphere having a radius equal to 36.8. A.U.

As regards the beforesaid calculation of $|\Delta \dot{G}/G|$, we can point out that experimental researches have provided values as 8×10^{-11} yr^{-1} (Van Flandern 1975) of the same order of that of other investigators (Shapiro et alii, 1977).
Very recent results (Damour et alii, 1988) would provide smaller values as 2×10^{-11} yr^{-1}.

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