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The low-luminosity stellar mass function

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SUMMARY

The stellar mass function for low-mass stars is constrained using the stellar luminosity function and the slope of the mass–luminosity relation. We investigate the range of mass functions for stars with absolute visual magnitude fainter than $M_V \approx +5$ which are consistent with both the local luminosity function and the rather poorly determined mass–absolute visual magnitude relation. Points of inflexion in the mass–luminosity relation exist because of the effects of H^-, H_2 and of other molecules on the opacity and equation of state. The first two of these correspond to absolute magnitudes $M_V \approx +7$ and $M_V \approx +12$, respectively, at which structure is evident in the stellar luminosity function (a flattening and a maximum, respectively). Combining the mass–luminosity relation which shows these inflexion points with a peaked luminosity function, we test smooth mass functions in the mass range $0.9–0.1 \mathscr{M}_\odot$. The smoothest stellar mass function which is consistent with all observational data has the form of a Gaussian in $\log_{10} m$ for mass, m, greater than $0.35 \mathscr{M}_\odot$ and is flat for m less than about $0.35 \mathscr{M}_\odot$. A mass function that continues to increase as an inverse power-law to low masses is not consistent with observations. All mass functions which are consistent with stellar data for stars more massive than $0.1 \mathscr{M}_\odot$ are convergent when extrapolated to zero mass. The total mass included in objects of masses lower than $0.35 \mathscr{M}_\odot$, assuming that the mass function continues smoothly below the minimum mass for hydrogen burning stars, is less than ~ 20 per cent of the total mass in more massive stars.

1 INTRODUCTION

Like the stars themselves, discussions of the absolute number of very low-mass stars have tended to generate more heat than light. In part this is due to the intrinsic importance of very low-mass stars to an understanding of the location and initial mass function of stellar formation: do low-mass stars form in the same places as high-mass stars and what is their possible contribution to the various dark matter problems? And in part it is due to the rather poor observational and theoretical information which has been available until recently. Salpeter (1955) first showed that the initial mass function can be written approximately as a power law of the form $\xi(m)\,dm = 0.013 m^{-\alpha} dm$, with $\alpha = 2.35$, for masses in the range $0.4–10 \mathscr{M}_\odot$. Very little information on lower mass stars was available at that time.

Miller & Scalo (1979) later carried out an extensive study of the stellar mass function, showing that the best available representation is a half Gaussian in $\log_{10} m$ in the mass range $0.1 \mathscr{M}_\odot \le m \le 60 \mathscr{M}_\odot$. The Miller–Scalo mass function flattens at a mass of approximately $0.6 \mathscr{M}_\odot$ so it can be extrapolated to zero mass without divergence. It is important to remember that the stellar mass function derived by Miller & Scalo assumes a distribution of field stars by magnitude (luminosity function) which increases smoothly with increasing magnitude to $M_V \approx +15$, and a mass–luminosity relation which is roughly linear in $\log_{10} m$ for low masses.

The most detailed study of the stellar mass function was made by Scalo (1986). He derived a mass function for low-mass stars that peaks at $m \approx 0.3 \mathscr{M}_\odot$ and decreases rapidly for $m < 0.2 \mathscr{M}_\odot$. Again this conclusion is sensitive to the assumption that the mass–luminosity relation is roughly a power law for low masses.

In all this work the shape of the derived stellar mass function has reflected the shape of the stellar luminosity function. However, it is important to remember that the stellar luminosity function is related to the stellar mass function through the *slope* of the mass–luminosity relation. This crucial point has been stressed before by D'Antona & Mazzitelli (1983). They assumed a theoretical mass–luminosity relation which flattened below about $0.13 \mathscr{M}_\odot$, and obtained a luminosity function with a peak at $M_V \approx 14$ from an underlying monotonically increasing mass function. The luminosity function derived by D'Antona & Mazzitelli is not consistent with recent determinations of the observed luminosity function (see Scalo 1986, for further discussion). Nevertheless, the possibility that the mass–luminosity relation can be the origin of features, such as a change of slope or even of a maximum, in the luminosity function, and hence that there may be no such features in the stellar mass function, deserves further investigation. Such an investigation requires consideration of both the stellar luminosity function and the stellar mass–luminosity relation.

There have been a variety of recent determinations of the stellar luminosity function for low-luminosity field stars near the Sun, following the method of Reid & Gilmore (1982). The most extensive are those by Gilmore, Reid & Hewett (1985) and Hawkins & Bessell (1988). Recently, these have been combined with a new survey by Stobie, Ishida & Peacock (1989). They obtained a luminosity function which agrees well with the other recent determinations. It is now well established that the stellar luminosity function from all these surveys of low-mass field stars in the solar neighbourhood shows a maximum at $M_V \approx 12$ and a subsequent decrease at least as far as $M_V \approx 16.5$. At lower luminosities poor statistics, a shortage of stars with good parallax distances to calibrate the absolute magnitude–colour relations and considerable uncertainties in the bolometric corrections required for comparison with evolutionary tracks, continue to limit confidence in the available results, in spite of their internal consistency. The consistency of recent studies of the solar-neighbourhood luminosity function for stars with $M_V \leq 16.5$ allows us to place new limits on the underlying stellar mass function.

Essentially we are dealing with the following question in this paper: how much of the observed luminosity function might be due to structure in the mass function and how much must be due to structure in the mass–luminosity relation? The form of the low-mass mass–luminosity relation is not well known so we test the following hypothesis: we assume that the maximum in the luminosity function near $M_V \approx 12$ is due to an inflexion in the mass–luminosity relation and is not caused by a real feature in the mass function. To do this we choose a range of mass functions and then calculate the resultant mass–luminosity relation which converts each mass function into a luminosity function, consistent with that derived by Stobie *et al.* (1989). These mass–luminosity relations are then tested for consistency with the available observational constraints on the mass–luminosity relation. The subset of the mass functions which is consistent with the observed luminosity function is identified. We emphasize that this method assumes that all structure, consistent with the rather weak observational constraints on the mass–luminosity relation, is real. This is unlikely to be correct. However, available theoretical mass–luminosity models do indeed show that structure with a similar amplitude and similar location to that required to explain all the observed structure in the stellar luminosity function for stars with $M_V \geq +5$ may be due to structure in the mass–luminosity relation.

2 FROM LUMINOSITY FUNCTION TO MASS FUNCTION

The aim of the present investigation is to determine the maximum range of stellar mass functions which is both consistent with the observed stellar luminosity function and allowed by the observational constraints on the stellar mass–luminosity relation. In practice the luminosity function is determined to better precision than the mass–luminosity relation. Thus we proceed by adopting a smooth (numerical) description of the luminosity function, generate a range of mass functions, and determine which of these mass functions is consistent with both the adopted luminosity function and the observational constraints on the mass-absolute visual magnitude ($m-M_V$) relation. The constraints on the adopted $m-M_V$ relation are the observations of binary stars, which were compiled by Popper (1980). We restrict the discussion to the stellar mass function for stars less massive than $\approx 0.9\mathcal{M}_\odot$ since these stars have not evolved off the main sequence. This avoids the need for a model-dependent and, for the present purposes, irrelevant consideration of the star formation history of the Galaxy.

The luminosity function for low-mass stars which we adopt is shown in Fig. 1. For stars with $M_V > 7$ this is taken from the Stobie *et al.* (1989) compilation and re-analysis of all recent determinations for very low-luminosity stars. For stars with $3 \leq M_V \leq 7$, it is taken from the extensive study of all available determinations by Scalo (1986). The luminosity functions of Stobie *et al.* and of Scalo are in good agreement in their region of overlap.

We require a smooth fit to the luminosity function for the analysis below. The adopted fit is shown in Fig. 1, and is a seven-knot cubic spline fitted to $\psi(M_V)$, where $\psi(M_V)\,dM_V$ is the number of stars per cubic parsec with magnitudes between M_V and $M_V + dM_V$. We emphasize that this representation of the luminosity function has no theoretical basis. We require only that it be a smooth and satisfactory representation of the data. The reduced chi-squared of the fit is 1.011, confirming its adequacy for present purposes.

A further assumption, which is implicit in our analysis, is that the $m-M_V$ relation is single valued. For this to be valid all stars must be isolated, have similar composition and lie on the zero-age main sequence. This last requirement is not likely to be met for stars with $m > 0.9\mathcal{M}_\odot$ where significant evolution across the main sequence can have occurred in the life-time of the Galaxy, and for very low-mass stars, which have long contraction times to the main-sequence, and may still be somewhat above the zero-age main-sequence. Both these effects make a star over-luminous for its true mass, and lead to the assigned mass being too large. At the high-mass end this tends to raise the amplitude of the resultant mass function for the highest mass stars and to flatten the slope slightly. At the low-mass end the effect is to lower the amplitude of the mass function for the lowest masses and again to flatten the slope of the mass function slightly (assuming a decreasing mass function with increasing mass). Both these effects should be small, and have been minimized by our restriction of the mass range of interest to be $0.09\mathcal{M}_\odot \leq m \leq 0.9\mathcal{M}_\odot$.

The effects of an abundance dispersion on the mass–luminosity relation are likely to be minor since the luminosity function data are for field stars within 100 pc of the Sun. The abundance dispersion for such stars is $\sigma_{[Fe/H]} \approx$ 0.2 dex (Gilmore & Wyse 1985). The slope of the absolute magnitude–abundance relation is ≈ 1.16[Fe/H] magnitudes per dex, for [Fe/H] ≥ -0.6, so that any real features in the mass–luminosity relation will tend to be smoothed with a Gaussian of dispersion ≤ 0.23 mag in M_V.

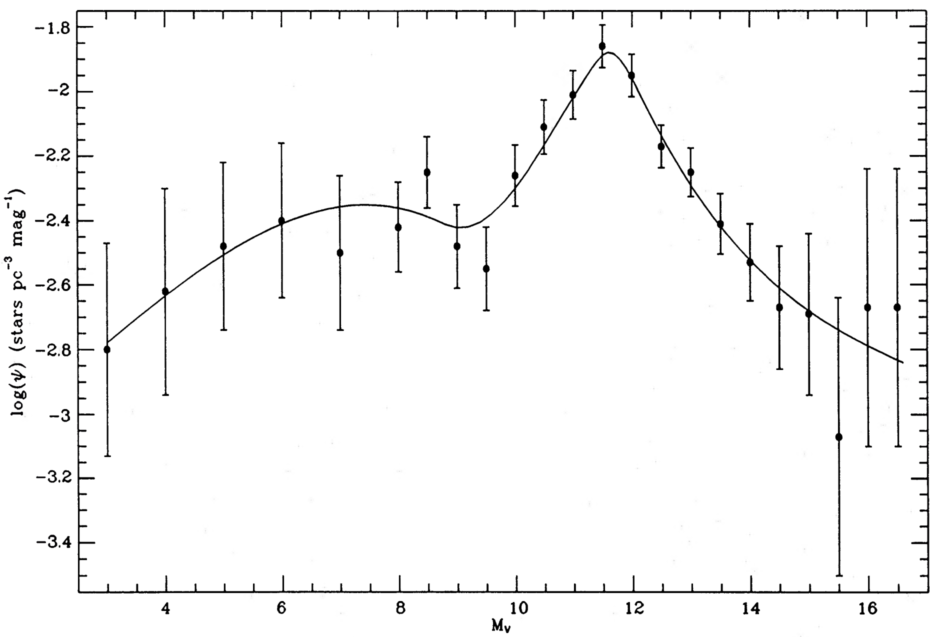

Figure 1. The stellar luminosity function for field stars near the Sun (points) together with a smooth fit (solid line) adopted for the present analysis. The luminosity function data are from Stobie *et al.* (1989) for $M_V \geq +8$, and from Scalo (1986) for more luminous stars.

This will also lead to a small Malmquist-like bias, but will not affect the results of this paper significantly. Other effects which will lead to a real dispersion in the mass–luminosity relation include mass loss, rotation, magnetic fields and binary interaction. None of these is likely to be important for the samples of interest here since the uncertainties are dominated by the small number of points available to calibrate the mass–luminosity relation.

From the definition of the luminosity function we have

$$\psi = \frac{dN}{dM_V} = -\frac{dN}{dm}\frac{dm}{dM_V} = -\xi\frac{dm}{dM_V}, \qquad (1)$$

where N is the number of stars per cubic parsec, m the stellar mass, M_V the absolute magnitude and $\xi(m)\,dm$ is the number of stars per cubic parsec with masses between m and $m+dm$. Equation (1) can be integrated to obtain the relation between mass and absolute visual magnitude. To select suitable boundary conditions we consider a zero-age solar mass model as a fixed point on the m–M_V relation and from a theoretical model (see Section 3.1) we take the visual magnitude of such a star to be $M_V = 5.15$. We also fix the relation so that it passes through a mass $m = 0.09\mathcal{M}_\odot$ when $M_V = 16.5$. This ensures that the mass–luminosity relation is consistent with the theoretical curve discussed below and also ensures that the relationship is consistent with the observational data points for the lowest mass stars which have reasonably precise mass and luminosity determinations available (Popper 1980).

Thus, integrating, we have

$$\int_{16.5}^{M_V} \psi\, dM'_V = -\int_{0.09}^{m} \xi\, dm', \qquad (2)$$

and

$$\int_{16.5}^{5.15} \psi\, dM'_V = -\int_{0.09}^{1} \xi\, dm'. \qquad (3)$$

The second integral has the effect of normalizing the mass function, ξ, so that the number of stars observed between $M_V = 16.5$ and $M_V = 5.15$ is always equal to the number of stars with masses between $0.09\mathcal{M}_\odot$ and $1.0\mathcal{M}_\odot$. To compute the mass–luminosity relation we first apply this normalization and then compare the values of the two integrals in equation (2) over the range of interest. The various choices for the mass functions and the results are discussed in Section 4.

We emphasize that the m–M_V relation computed here from the luminosity function and an assumed mass function has no theoretical basis, but is entirely observational. We also emphasize that both the stellar luminosity function and the observational constraints on the m–M_V relation are based directly on observations in the visual photometric pass band. Thus the derivation of the acceptable range of mass functions in this paper is entirely free of the need to adopt a bolometric correction scale. In the section below we discuss various theoretical models of the m–M_V relation to see if the structure required to convert a smooth mass function into the observed peaked luminosity function is compatible with current theory. This comparison is sensitive to the adopted bolometric corrections. We emphasize this point in consequence of the detailed analysis of the form of the low-mass stellar mass function by Piskunov & Malkov (1987), who considered the effects of uncertainties in available theoretical mass–luminosity relations on mass functions derived by direct multiplication of the observed stellar luminosity function by the slope of a theoretical mass–luminosity relation. They show that uncertainties in the theoretical models themselves, compounded by further uncertainties in the derivation of appropriate effective temperatures and bolometric corrections, are so large that no useful mass function can be derived in this way. It is for this reason that we have followed the inverse procedure here, and allow any m–M_V relation which is not inconsistent with direct observational constraints.

3 THE MASS–LUMINOSITY RELATION

3.1 Theoretical models

As a means of illustrating the level of complexity in the mass–luminosity relation, we have calculated theoretical models of zero-age main-sequence stars using the evolution program written and developed by Eggleton (1971, 1972). The basis for the equation of state used in this code is described by Eggleton, Faulkner & Flannery (1973). In its current (updated) form it allows for ionization of hydrogen and helium, and also for the dissociation of H_2, but no other molecules. A better correction for pressure ionization is also included. Convection is treated by a mixing length theory (Böhm–Vitense 1958) as described by Baker & Temesvary (1966) with α, the ratio of the mixing length to the pressure scaleheight, taken to be 1.5. Unfortunately this is the weakest part of stellar structure theory and cannot be expected to be a good approximation for low-mass stars with deep convective envelopes. For this reason more than any other these theoretical models are not to be trusted quantitatively. Also important in the very outer layers of these stars are the radiative opacities which are taken from Cox & Stewart (1970). Included in these are the effects of H_2, H^-, H_2^+ and He^- along with all the normally ionized species for a population I mixture. Other molecules such as CO and H_2O that may well be important are not included nor is Coulomb pressure.

For lower masses than approximately $0.15\mathscr{M}_\odot$ (which are not important for present purposes) the present set of models is inaccurate, mainly owing to the neglect of other molecules (CO, H_2O, etc.) and the effect of grain formation, and because we do not fit a detailed model atmosphere. Rather, we adopt an Eddington approximation (Woolley & Stibbs 1953) for simplicity. However, the models presented here are sufficiently reliable to illustrate the important point for present purposes, namely that considerable structure exists in the theoretical mass–luminosity relation at just those absolute magnitudes where structure is seen in the luminosity function.

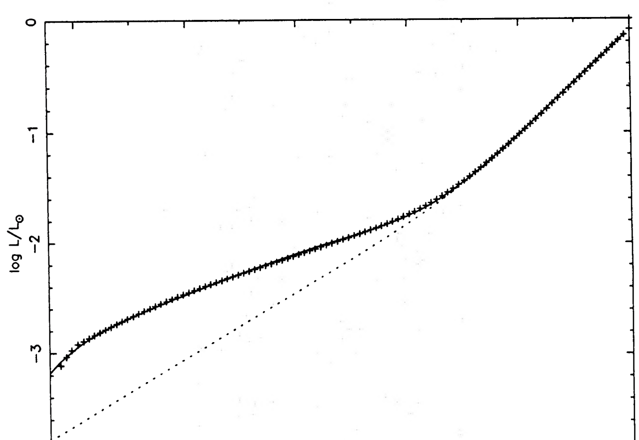

Figure 2. The bolometric luminosity–mass relation for the theoretical models described in the text, both including the effects of molecular hydrogen on the equation of state (solid line) and suppressing its effect (dotted line). The models are indicative rather than definitive, but emphasize the important feature of all available models for present purposes, namely that there is a point of inflexion in the mass–luminosity relationship near $0.3\mathscr{M}_\odot$. This mass corresponds approximately to the luminosity at which a maximum is evident in the luminosity function of Fig. 1.

We have computed 103 models for masses between $1.000\mathscr{M}_\odot$ and $0.093\mathscr{M}_\odot$ equally spaced in $\log_{10}m$ and with a standard population I mixture ($X=0.7$, $Y=0.28$ and $Z=0.02$). Fig. 2 illustrates the bolometric luminosity–mass relation for these models and also shows a fairly good analytic fit given by

$$L=\frac{0.1400m^2+46.81m^9}{2.395\times10^{-10}m^{-9}+1+57.99m^{4.5}}\mathscr{L}_\odot, \quad (4)$$

where the mass is in solar units. The radius can be fitted by

$$R=\frac{m^{1.25}(0.8478m^2+0.1159)}{2.241\times10^{-11}m^{-8}+4.613\times10^{-2}+m^2}\mathscr{R}_\odot, \quad (5)$$

where $\mathscr{L}_\odot$ and $\mathscr{R}_\odot$ are the solar bolometric luminosity and radius, respectively.

Using these and assuming a blackbody relationship we can obtain the effective temperature and thence estimate the bolometric correction by interpolating the tables of Popper (1980) and of Greenstein, Neugebauer & Becklin (1970, for the low temperatures) using a seven-knot spline. From the bolometric luminosity and the bolometric correction we then obtain the absolute visual magnitude, M_V. We expect the errors introduced due to the very uncertain bolometric corrections at the lowest temperatures to be appreciable, making our models quite uncertain for masses below approximately 0.15 $\mathscr{M}_\odot$. We will not discuss the uncertainties in detail, as the theoretical models implemented here are required only to test the plausibility of the existence of real structure in the m–M_V relation. It is sufficient to stress the fact that theoretical models in general are very uncertain in the very low-mass regime.

There is a point of inflexion in the m–M_V relation near $0.33\mathscr{M}_\odot$. This change in the m–M_V relation is a result of the formation of hydrogen molecules on the equation of state in the outer layers (coupled with the interior by convection) of lower mass stars. The physical basis of this result depends on the fact that the mean molecular weight of hydrogen molecules is twice that of hydrogen atoms. Thus the formation of hydrogen molecules means that the pressure is reduced, causing contraction and core heating. The result is higher luminosity than if recombination were prevented. In addition, there is a rapid change in the interior structure of a star near $0.33\mathscr{M}_\odot$, with more massive stars having radiative cores and lower mass stars being completely convective, further enhancing the contraction and increasing the luminosity.

To confirm the origin of this effect we calculated another set of models in which hydrogen recombination was suppressed by reducing the dissociation energy from 4.48 to 1.00 eV, but with unaltered opacities. These models are shown as a dotted line in Fig. 2 and indeed the inflexion has disappeared. We emphasize that this effect is not a new feature of these models, but has been established for many years. Copeland, Jensen & Jørgensen (1970) first discussed the existence of this point of inflexion, and attributed it to the lowering of the adiabatic gradient in the H_2 dissociation zone.

A recent and comprehensive study of models of low-mass stars, by Dorman, Nelson & Chau (1989; their fig. 2), also shows this feature in all the models studied. They also illustrate the limitations of the models described here outside the range of present interest. The agreement is good between the two sets of models for masses above about $0.15\mathcal{M}_\odot$, but at lower masses the more detailed models of Dorman *et al.* are systematically cooler and of lower luminosity than those described here. The cause of this difference is the effect on the opacity and equation of state of the star of the other molecules (H_2O, CO, etc.) and grains. A comparison of the sophisticated low-mass models constructed by Burrows *et al.* (1989) and those of Dorman *et al.* (1989) shows that there remains a substantial uncertainty in the theoretical m–M_V relations. We re-emphasize that these uncertainties in the models do not affect the conclusions of this paper. Both the $m-M_V$ relation and the stellar luminosity function, in units of stars per unit volume per absolute visual magnitude, are based entirely on observations in the visual photometric system and are independent of uncertainties in bolometric corrections and in available stellar models.

3.2 Observational constraints on the mass–luminosity relation

Available observational constraints on the mass–luminosity relation have been compiled by Popper (1980). These data, excluding obviously evolved stars and with Popper's derived observational errors, are shown in Fig. 3, as is a ten-knot spline fit. To fit the spline we utilized Popper's data in the mass range 0.1–$1.6\mathcal{M}_\odot$.

Subsequent to Popper's (1980) review, mass–luminosity data have been derived for several other low-mass stars. These recent data are well reviewed by Liebert & Probst (1987) and are essentially in agreement with the data compiled by Popper. In most cases, however, the formal uncertainties are sufficiently large that these data provide no useful information for present purposes. They are not shown in Fig. 3.

Two exceptions to this deserve comment. The binary Ross 614, shown in Fig. 5 with large open circles, appeared anomalous in fig. 7 of Scalo (1986). The more recent photometric data reported by Liebert & Probst (1987) show the members of Ross 614 to be in good agreement with the data for other stars. The second exception is Wolf 424, for which Heintz (1989) has determined the masses of the equally luminous components ($M_V = 15.04$) to be $0.059\mathcal{M}_\odot$ and $0.051\mathcal{M}_\odot$ with small formal errors – in striking disagreement with data for other stars. These results were obtained at the resolution limit of the equipment used (the orbital semi-major axis has a separation of 0.7 arcsec, and the predominant number of observations are photographic exposures) so that confirmatory observations would be of considerable interest. In the interim we do not include the data for Wolf 424 in the present analysis.

The theoretical models discussed in Section 3.1 are consistent with the observational data, supporting the identification of real structure in the mass–luminosity relation. Points of inflexion in the mass–luminosity relation occur near $M_V = +7$, mass $\approx 0.6\mathcal{M}_\odot$, owing to the H^- opacity, and near $M_V \approx 12$, mass $\approx 0.3\mathcal{M}_\odot$, owing to the effects of molecular hydrogen. Further structure is expected at lower masses where additional molecules and grains become important but where current theoretical models are still uncertain. The observational data shown in Fig. 3 are the constraints on the range of mass–luminosity relations which we utilize to limit the available range of mass functions.

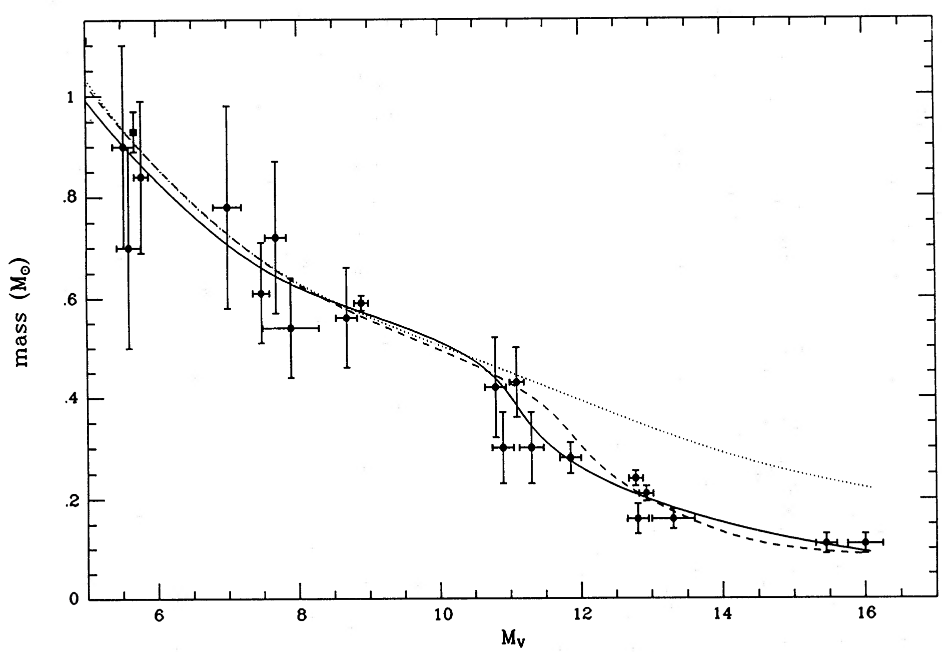

Figure 3. Available observational data to determine the mass–absolute visual magnitude relation, from the compilation by Popper (1980). Also shown are a ten-knot spline fit (solid line), the theoretical model from Fig. 2 including the contribution from molecular hydrogen (dashed line) and the theoretical model from Fig. 2 without molecular hydrogen included (dotted line).

4 STELLAR MASS FUNCTIONS

The identification of points of inflexion in the $m-M_V$ relation forms the basis for our set of constraints on the stellar mass function. Since the $m-M_V$ relation is not required to be (nearly) linear at low masses, we have the freedom to allow features in this relation to be the explanation of features in the observed luminosity function. Thus a maximum in the observed luminosity function need not correspond to a maximum in the stellar mass function. However, since the available theoretical mass–luminosity relations remain uncertain, we proceed by allowing any level of structure in the $m-M_V$ relation which is not inconsistent with the direct observational constraints. We specify a wide range of smooth mass functions, and determine which can be made consistent with the observed luminosity function by adoption of an allowed $m-M_V$ relation.

Using these ideas and a variety of mass functions, we calculate the required mass–luminosity relations and compare them with the data in Fig. 3 using a χ^2 test:

$$\chi^2=\sum_{i=1}^{N}\left[\frac{m_i-m(M_{Vi})}{\delta m_i}\right]^2$$
$$=\nu\chi^2_\nu, \quad (6)$$

where χ^2_ν is the reduced χ^2 and $\nu=N-n-1$ is the number of degrees of freedom, with N being the number of data points constraining the mass–luminosity relation, and n the number of free parameters used in the model. m_i, M_{Vi} correspond to the ith star and δm_i is the error in its mass. We calculate χ^2 over the interval $M_V=5-16$, so that $N=23$ (only 21 points appear as two pairs are the same). We assume that the deviations follow a normal distribution and use the reduced χ^2_ν to identify a 5 per cent two-sigma confidence limit for consistency with the observations. Mass functions that require a greater value of χ^2_ν are rejected as being inconsistent with the observations constraining the mass–luminosity relation.

To compare with the theoretical model we calculate the variance (used here as a probability indicator)

$$s^2=\frac{1}{M-1}\sum_{i=1}^{M}(m_i-m_i')^2, \quad (7)$$

where m_i are mass values from the computed mass–luminosity relation corresponding to m_i' from the theoretical relation, and we evaluate it over the range $M_V=5-16$, with 440 points (M) used to represent the continuous fit to the models.

We consider the following mass functions and in each case record n, the number of free parameters that are varied to minimize χ^2:

(i) Gaussian (GS)

$$\frac{1}{k}\xi_{\rm GS}(m)=\frac{1}{\sigma}\exp\left[-\frac{(m-\mu)^2}{2\sigma^2}\right], \quad (8)$$

where σ and μ are the free parameters and $n=2$;

(ii) Miller-Scalo (MS)

$$\frac{1}{k}\xi_{\rm MS}(m)=\frac{1}{|\log_{10}\sigma|}\exp\left[-\frac{(\log_{10}m-\log_{10}\mu)^2}{2(\log_{10}\sigma)^2}\right], \quad (9)$$

where σ and μ are the free parameters and $n=2$;

(iii) a Miller–Scalo function with a linear extension to low masses from the mass, m^*, at which the slopes are equal (MSLi)

$$\frac{1}{k}\xi_{\rm MSLi}(m)=\begin{cases}\dfrac{1}{|\log_{10}\sigma|}\exp\left[-\dfrac{(\log_{10}m-\log_{10}\mu)^2}{2(\log_{10}\sigma)^2}\right], & \text{if } m\geq m^*\\[2ex] \delta\times m+\dfrac{1}{k}\xi_{\rm MS}(m^*)-\delta\times m^*, & \text{if } m\leq m^*\end{cases} \quad (10)$$

where $\delta=d/dm[(1/k)\xi_{\rm MS}(m^*)]$ with σ, μ and δ the free parameters and $n=3$;

(iv) power-law (Po)

$$\frac{1}{k}\xi_{\rm Po}(m)=m^{-\alpha}, \quad (11)$$

where α is the free parameter and $n=1$. The factor k allows for the normalization of equation (3) in all cases.

By varying the parameters we have minimized χ^2 for the four mass functions listed above (MS, GS, Po and MSLi with $\delta=0$ [MSLiI]). The resulting parameters and the corresponding χ^2 and s^2 values are listed in Table 1. For comparison we also calculate and present in Table 1 the χ^2 for a Salpeter (SL) mass function which is the power law with $\alpha=2.35$.

We have used the 5 per cent confidence limit to place constraints on the low-mass region by considering a range of linear extensions to the Miller–Scalo mass function with various positive and negative slopes, but constraining μ and σ to

Table 1. Mass function properties.

Mass Function	Parameters			χ^2	s^2	k	$\rho(M_\odot pc^{-3})$
	$\mu(M_\odot)$	$\sigma(M_\odot)$	$\delta(M_\odot^{-1})$				$(m\leq 0.35M_\odot)$
MSLiI	0.35	0.54	0.0	18.6	$2.7\cdot10^{-4}$	0.02290	0.0052
MSLiII	0.35	0.54	+6.0	30.6	$7.9\cdot10^{-4}$	0.02449	0.0048
MSLiIII	0.35	0.54	−5.0	30.8	$3.7\cdot10^{-4}$	0.02093	0.0058
MS	0.23	0.42		17.6	$4.7\cdot10^{-4}$	0.03720	0.0078
GS	0.23	0.42		18.4	$2.4\cdot10^{-4}$	0.03701	0.0050
Power Functions	α						
Po	0.70			52.1	$17.0\cdot10^{-4}$	0.03231	0.0063
SL	2.35			1552.0	$587.0\cdot10^{-4}$	0.00301	∞

the values corresponding to a flat extension to low masses. We have $\nu=21$ for which $\chi_\nu^2<1.56$ for a 5 per cent confidence interval. The mass functions with the most extreme allowed linear extrapolations of the Miller–Scalo mass function consistent with the 5 per cent confidence limit, designated here MSLiII and MSLiIII, are included in Table 1. The resulting mass–luminosity relations are illustrated in Fig. 4.

In addition to summarizing the quality of the various numerical descriptions of the mass function adopted here, Table 1 also gives the normalization constant k introduced in equations (8)–(11).

The χ^2 values for mass functions defined by a Gaussian, by the Miller–Scalo function, and by the Miller–Scalo function with a flat extension to low masses all lie within the 60

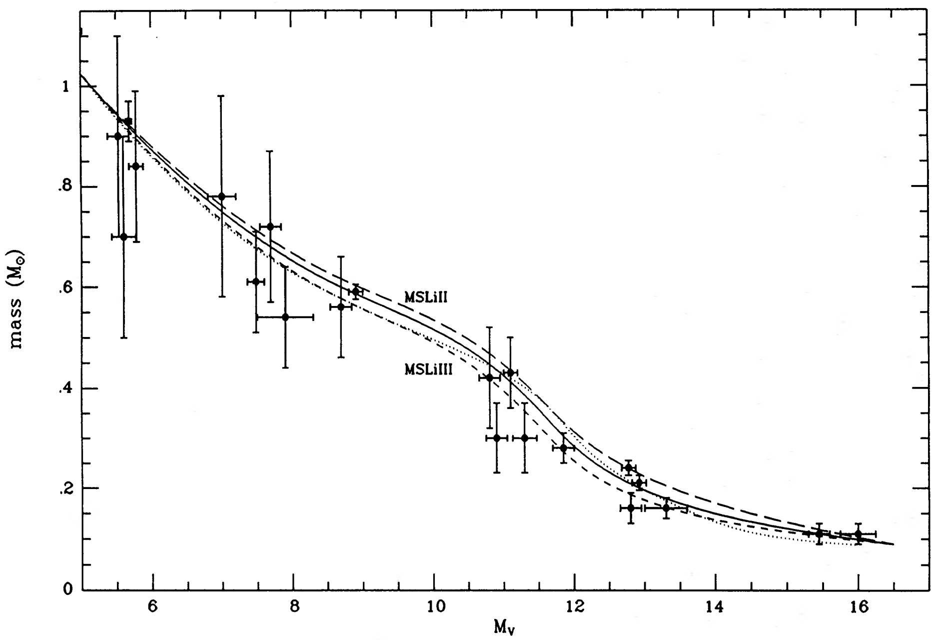

Figure 4. The mass–absolute visual magnitude relations required to transform between the observed luminosity function shown in Fig. 1 and the MS mass function, with three different linear extrapolations to very low masses (equation 10). The three extrapolations are a mass function which is constant below $0.35\mathcal{M}_\odot$ (solid line; MSli I), and the maximally decreasing (long dashes; MSli II) and maximally increasing (short dashes; MSli III) mass function extrapolations allowed by the observational constraints on the mass–luminosity relation. The theoretical model from Fig. 2 is also shown for comparison (doted line). The points show the observational data from Popper (1980).

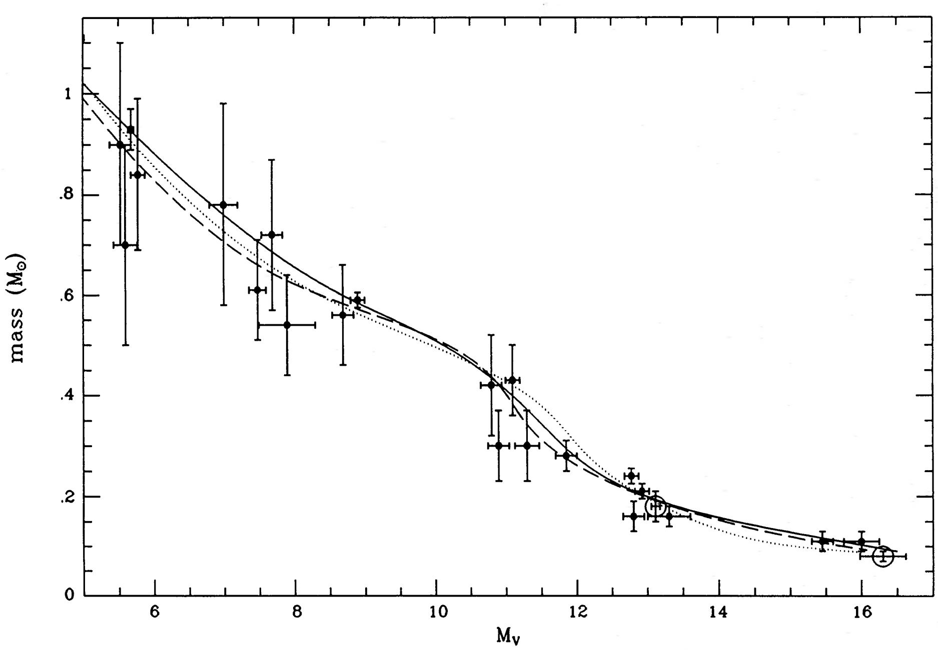

Figure 5. The mass–absolute visual magnitude relation required to transform between the observed luminosity function shown in Fig. 1 and the MS mass function (solid line and equation 9). Also shown are the ten-knot spline fit (dashed line) and the theoretical model curve from Fig. 3 including the contribution from molecular hydrogen (dotted line). The points show the observational data from Popper (1980), apart from the two data points represented by large open circles, which represent Ross 614 A and B from Liebert & Probst (1987).

per cent confidence limits and are so similar that we cannot make a significant distinction between them. A power-law mass function lies well outside the 0.1 per cent confidence level corresponding to $\chi^2=47$, while the Salpeter mass function is completely ruled out. Similar conclusions follow from consideration of the s^2 statistic used to test consistency with the theoretical models. Fig. 5 shows the mass–luminosity relation that results when the Miller–Scalo function is adopted (or similarly the Gaussian function or Miller–Scalo function with a flat extension to low masses) with the numerical best-fit and the theoretical curve from Fig. 3 included for comparison.

The mass–luminosity relations for the two power laws, Po and SL, are shown in Fig. 6. The strikingly poor fit of the two power-law mass functions requires comment. A single power-law is in fact an adequate description of the mass function for masses above $\sim 0.4\mathcal{M}_\odot$, but deviates substantially, by predicting too many stars, at lower masses. Below about $0.4\mathcal{M}_\odot$ the mass function can again be described well by a power law, but this time with a flat slope. Since all our mass–luminosity relations are forced to agree with the observational mass–luminosity relation at both $1.0\mathcal{M}_\odot$ and at $0.09\mathcal{M}_\odot$, and are also forced to provide a correct value for the total number of stars in the luminosity function over the same mass range, by being a solution of equation (3), the power-law models are unable to find an acceptable compromise fit. This simply highlights the inadequacy of a single power law to describe the stellar mass functions over the range of masses from $\sim 0.1\mathcal{M}_\odot$ to $\sim 1.0\mathcal{M}_\odot$.

A recent extension of the luminosity function by Leggett & Hawkins (1988) suggests that it may be increasing at fainter magnitudes than $M_V \approx 18.5$ ($M_R \approx 16.5$). If this result is real it may be an artefact of yet unidentified structure in the m–M_V relation or perhaps represent lower mass stars that are still contracting and have an appreciable accretion luminosity. Alternatively it may indicate an increase in the mass function. It is not possible to extend the analysis of this paper reliably including the Leggett & Hawkins result, since there are no observational mass determinations in the relevant absolute magnitude range while theoretical models and bolometric corrections are very uncertain. We note at this point that even a *flat* luminosity function at magnitudes fainter than $M_V \approx 16$ ($m \approx 0.1\mathcal{M}_\odot$) may translate into a *rising* mass function because the slope of the m–M_V relation appears to approach zero at the hydrogen burning limit.

The range of mass functions which is consistent with both the observational constraints on the m–M_V relation and with the observed stellar luminosity function for field stars near the Sun is shown in Fig. 7. The two power-law functions are included for comparison.

With the parameters in Table 1 it is straightforward to calculate the mass density deduced for the solar neighbourhood by integrating the *extrapolated* mass functions to $m=0$ from $m=0.10\ \mathcal{M}_\odot$. All the mass functions shown to be adequate in this study are convergent, and the resulting mass densities are given in the final column of Table 1. The Salpeter (1955) mass function can be used to calculate the mass in stars more massive than $0.35\mathcal{M}_\odot$, which is $0.05\mathcal{M}_\odot$ pc^{-3}. Roughly one-half of all the mass in the stellar mass function is contained in stars with masses near and below $1\mathcal{M}_\odot$. Therefore comparing the integral of the Salpeter (1955) and the MS mass function over the range 0.35–$1.3\mathcal{M}_\odot$ is of interest. The Salpeter mass function contains $0.020\mathcal{M}_\odot$ pc^{-3} in this range, while the MS function contains $0.025\ \mathcal{M}_\odot$ pc^{-3}. As expected, these values are similar since the Salpeter mass function is similar to that of the MS mass function over the range which contains most of the stellar mass.

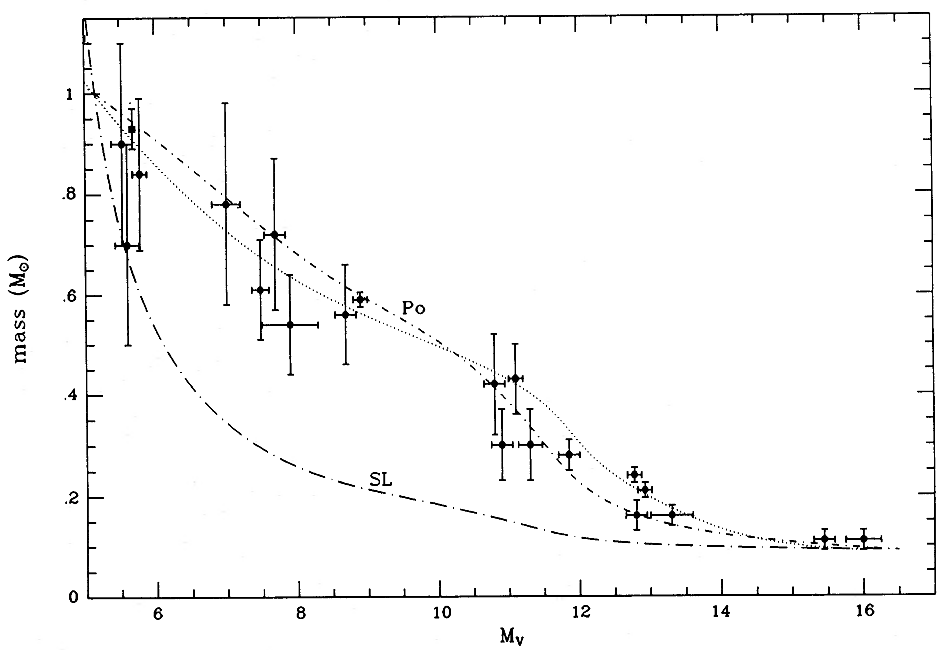

Figure 6. The mass–absolute visual magnitude relations required to transform between the observed luminosity function shown in Fig. 1 and two power-law mass functions, forced to satisfy the boundary conditions discussed in Section 2. The SL function is the power law with Salpeter's (1955) index (Section 4), while the curve labelled Po is a power law with index -0.70. Also shown is the theoretical model curve from Fig. 3 including the contribution from molecular hydrogen (dotted line). The points show the observational data from Popper (1980).

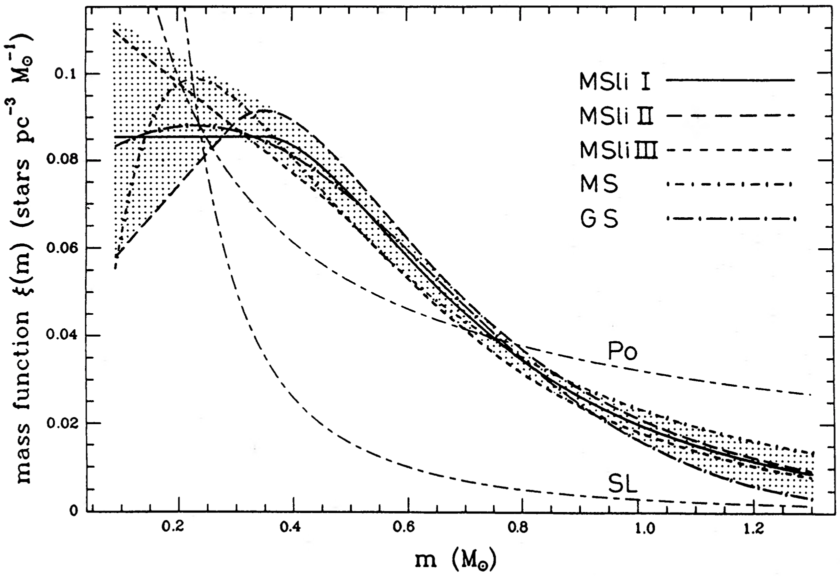

Figure 7. All the mass functions considered in this work. The shaded area represents the range of mass functions which are consistent at the 5 per cent level with the observed stellar luminosity function and with the observational constraints on the m–M_V relation.

We emphasize that all the mass functions consistent with both the observed luminosity function and the mass–luminosity relation are convergent when *extrapolated* to low masses. The total amount of mass contained in stars of mass less than $0.35\mathcal{M}_\odot$ assuming a linear extrapolation of the stellar mass function to zero mass from $0.10\mathcal{M}_\odot$ is less than about 20 per cent of the mass contained in the stellar mass function for higher mass stars. Over the range from $\sim 0.1\mathcal{M}_\odot$ to $\sim 1.0\mathcal{M}_\odot$ the mass function for field stars near the Sun is smooth. The several features evident in the stellar luminosity function over this range are all consistent with being due to points of inflexion in the mass–luminosity relation.

5 CONCLUSIONS

The stellar m–M_V relation contains points of inflexion where the state of the stellar matter changes and where the dominant opacity source in the stellar atmosphere changes rapidly. These changes in the slope of the mass–luminosity relation mean that a smooth mass function will correspond to a stellar luminosity function which shows corresponding structure, for stars of sufficiently low mass not to have evolved significantly in a Hubble time.

An example of these changing opacity sources is the flattening of the m–M_V relation near mass $\approx 0.7\mathcal{M}_\odot$, $M_V \approx 7$, due to the increasing importance of H^- as an opacity source. This leads to a flattening of the observed stellar luminosity function above $M_V \approx 7$, a feature which has sometimes been called the 'Wielen Dip'.

The formation of molecular hydrogen changes the stellar equation of state, and thereby affects the m–M_V relation at fainter absolute visual magnitudes. Available models for very low-mass stars are inadequate to provide a reliable prediction of the detailed form of the mass–luminosity relation, or of the bolometric corrections. Nevertheless, indicative theoretical models show that molecular hydrogen produces a point of inflexion in the mass–luminosity relation near a mass $m = 0.35\mathcal{M}_\odot$, $M_V \approx 12$. This absolute visual magnitude is similar to that at which a maximum is seen in the stellar luminosity function.

The above two examples suggest the possibility that *all* the observed structure in the stellar luminosity function is due to equation of state and opacity-generated structure in the mass–luminosity relation, and that the stellar mass function is smooth.

We have examined this hypothesis by determining the allowed range of stellar mass functions which can be made consistent with the observed stellar luminosity function, and at the same time be consistent with the observational constraints on the stellar mass–luminosity relation. We confirm that the observed maximum in the stellar luminosity function near $M_V = +12$ is consistent with being an artefact of a point of inflexion in the mass–luminosity relation, caused by the effect of H_2 on stellar structure. The underlying stellar mass function is consistent with being smooth and monotonically rising through the region which shows the maximum in the stellar luminosity function, and need not have a maximum near $0.35\mathcal{M}_\odot$. We emphasize that a smooth stellar mass function is not *required* by available data any more than is a mass function with structure. However, the important result here is that a smooth mass function is *consistent* with all available data and that one with structure is not required. Below $\approx 0.1\mathcal{M}_\odot$ the mass function remains unknown, as the m–M_V relation is unconstrained by data for such low masses.

The allowed range of mass functions, summarized in Fig. 7, allows us to calculate the total mass in objects below the minimum mass for hydrogen burning stars included in any *hypothetical* linear extrapolation of the stellar mass function. *Assuming* a linear extrapolation, the total mass in stars and substellar mass objects below $0.35\mathcal{M}_\odot$ is $\leq 0.008\mathcal{M}_\odot$ pc^{-3}, while that in more massive stars is $\approx 0.05\mathcal{M}_\odot$ pc^{-3}. Thus a dynamically insignificant mass is present in any allowed linear extrapolation of the stellar mass function to very low-mass objects and there remains no robust evidence to support intrinsic structure in the stellar initial mass function. This result has considerable implications for galactic evolution models invoking bimodal star formation processes and identification of dark matter.

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