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GINGA observations of Abell 2218: implications for H_0

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SUMMARY

GINGA observations of the cluster of galaxies Abell 2218 show that its spectrum is well fitted by an isothermal model with temperature $6.7^{+0.5}_{-0.4}$ keV. Abell 2218 is the cluster in whose direction the Sunyaev–Zel'dovich effect has been searched for most intensively. Assuming a value of -0.69 ± 0.10 mK for the central SZ decrement (Birkinshaw), and calculating the radial gas density distribution by deprojecting the X-ray surface brightness, we derive a value for the Hubble constant, for a uniform cluster gas, $H_0 = 24^{+13}_{-10}$ km s^{-1} Mpc^{-1}. Possible reasons for the discrepancy between this value and the currently accepted higher values derived with other techniques are explored.

1 INTRODUCTION

1.1 The Sunyaev–Zel'dovich effect

Sunyaev & Zel'dovich (1972) demonstrated that the microwave background radiation, in passing through the hot gas in a cluster of galaxies, would be Compton-scattered to higher energies. This leads to a decrement in the observed brightness of the microwave radiation in the direction of the cluster at wavelength longward of the Wien peak (~ 1.4 mm), and an increment at shorter wavelengths.

The mean fractional frequency change of a microwave photon in one scattering is

$$\frac{d\nu}{\nu} = \frac{4kT_e}{m_e c^2}$$

and, in the long wavelength limit, the fractional change in brightness of the microwave background is

$$\frac{dI}{I} = 2\tau_e \frac{kT_e}{m_e c^2},$$

where τ_e = optical depth through the cluster to electron scattering
$= \sigma_T \int n_e \, dl$, and n_e = electron density.

It has been shown by a number of workers (e.g. Silk & White 1978; Gunn 1978) that if the microwave decrement measurement is combined with an X-ray determination of the cluster gas temperature and density distribution, a value for the Hubble constant, H_0, can be derived directly. This is, in principle, a very important derivation as it is free from any evolutionary biases, is independent of the usual stepping-stone arguments and is applicable at much higher redshifts than classical methods.

1.2 Abell 2218

Attempts to measure the microwave decrement have concentrated on the richest known clusters, where the SZ effect is expected to be greatest, and the best-observed cluster is Abell 2218. This cluster was classified by Abell (1958) as richness class 4 and distance class 6. Its redshift is 0.171 (Struble & Rood 1987). Although categorized by Leir & van den Bergh (1978) as Bautz-Morgan class II, it is dominated by a large cD galaxy at

$\alpha(1950.0) = 16\ 35\ 37$, $\delta(1950.0) = 66\ 18\ 44$

(Bruzual & Spinrad, private communication). From *Einstein* observations Boynton *et al.* (1982) find that the projected X-ray surface brightness distribution is smooth, centrally condensed and circularly symmetric. It is therefore a reasonably straightforward cluster for which to model the gas distribution.

Abell 2218, although quite a luminous cluster, is a rather distant, and therefore faint, X-ray source. No previous X-ray experiments have been able to obtain an accurate measurement of its spectrum and hence it has been impossible to make any useful determination of H_0.

GINGA is the first satellite to have the capability to determine the X-ray spectrum, and hence measure the X-ray temperature, and details of such an observation are described in Section 2. In Section 3 we use our measurement of the cluster temperature to determine H_0 on the assumption of a non-clumped isothermal gas. The determination of H_0 is, however, still limited by uncertainties in the measurement of the microwave decrement and in modelling the distribution of density and temperature in the cluster gas. We discuss how allowing extreme values of these parameters could change our derived value of H_0.

2 THE OBSERVATIONS

The Japanese/UK satellite *GINGA* carries a Large Area proportional Counter (LAC) which is sensitive to X-rays in the energy range 1.5–3.0 keV (Turner *et al.* 1988). The observations of Abell 2218 were performed on 1987 November 12–13 with background data being obtained from a blank field observation made one day previously. The background subtraction method used involves modelling the LAC non-X-ray count rate as a function of several background monitors (Hayashida *et al.* 1989). Once a background model has been established then the background for the source observation can be calculated and subtracted to leave the source data.

The solar angle of the LAC during the A2218 observation was less than 90°. In order to ensure that there was no contamination of the data by sunlight scattered from the collimator, only data collected when the satellite was in the Earth's shadow were used in the analysis. We have also excluded data from periods with the highest background. The total resultant on-source exposure time was 13 100 s and the measured count rate in the 2–10 keV band (top layer) was 3.2 ± 0.2 count s^{-1}.

3 DISCUSSION

3.1 The gas temperature

The source count spectrum in the 2–10 keV band was fitted to a thermal bremsstrahlung model (Raymond & Smith 1979) using a chi-squared minimization technique. The LAC is not sensitive to low values of the absorbing column. We accordingly fixed the absorbing column at the line-of-sight value out of our own Galaxy in the direction of Abell 2218 of 3×10^{20} cm^{-2} (Stark *et al.*, private communication), consistent with the upper limit determined with n_H as a free parameter in the fit. Abundances of heavy elements other than iron, which was a free parameter in the fit, were fixed at 0.5 times the solar value, typical for nearby clusters. The best fit value for the bremsstrahlung temperature is $6.72^{+0.5}_{-0.4}$ keV, where the errors quoted are the 1 σ errors for one parameter of interest. We give the 'emitted', rather than 'observed' values, i.e. we correct for the cluster redshift. The abundance of iron determined from the fit was $0.2^{+0.2}_{-0.2}$ of the solar value, consistent with that measured for other nearby clusters. Fig. 1 shows the best fit count spectrum together with the residuals from the fit and the inferred photon spectrum seen at the detector. A summary of the derived parameters is given in Table 1.

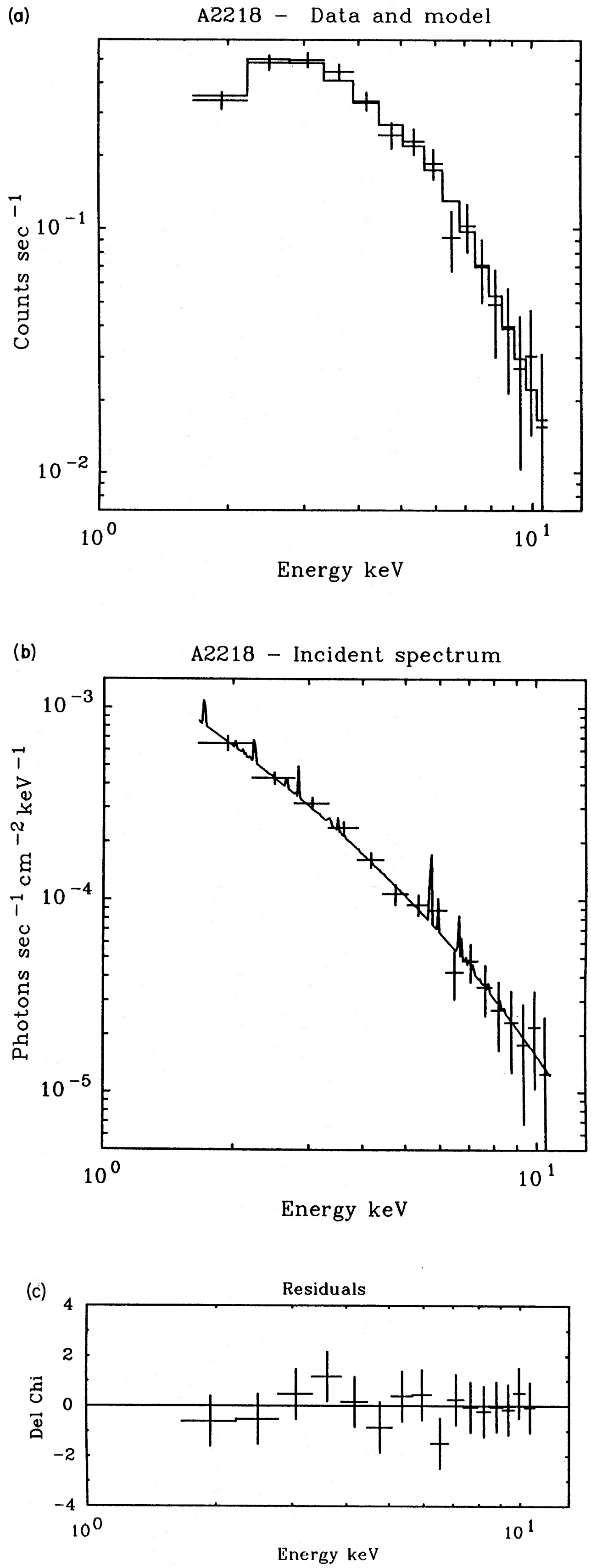

Figure 1. The spectrum of A2218. (a) The *GINGA* count spectrum (data points) and best fit model spectrum (histogram). (b) Data – model residuals. (c) Input model spectrum and inferred source spectrum.

Table 1. Summary of derived spectral parameters for A2218.

Model 'Independent'	
f_x (2-10 keV)	$6.8 \pm 0.5 \times 10^{-12}$ erg cm^{-2} s^{-1}
A_{Fe}	0.2 ± 0.2
Isothermal Model	
kT	$6.7^{+0.5}_{-0.4}$ keV
Polytropic Model	
kT_0	$7.4^{+3.6}_{-1.4}$ keV
γ	$1.02^{+0.7}_{-0.2}$
β	$0.56^{+0.4}_{-0.2}$

The observed count rate corresponds to a 2–10 keV flux of 6.8×10^{-12} erg cm^{-2} s^{-1}, and predicts a flux in the *Einstein* HRI band (0.5–3.0 keV) of $6.2 \pm 0.4 \times 10^{-12}$ erg cm^{-2} s^{-1} in good agreement with that given by Boynton *et al.* (1982), of $5.4 \pm 0.4 \times 10^{-2}$ erg cm^{-2} s^{-1}.

The only previously published measurement of the gas temperature in Abell 2218 was by Perrenod & Henry (1981) who derived a temperature of $10.8^{+\infty}_{-4.3}$ keV from *Einstein* IPC data. This is a spatially integrated measurement ($T_{\rm ave}$) and so is directly comparable to ours. The limited pass-band of the IPC precludes the accurate measurement of temperatures greater than a few keV. Within this limitation Perrenold & Henry's result is consistent with our measurement.

We note that the temperature and luminosity of A2218 derived from the *GINGA* data are entirely consistent with the luminosity-temperature correlations found for nearby clusters with *HEAO-1*, (Mushotzky 1988), and with the recent improved study using *EXOSAT* data (Edge & Stewart, in preparation).

An alternative method of deriving the central gas temperature is to fit the X-ray surface brightness distribution to that expected from a polytropic gas distribution. This results in a measurement of the galaxy to gas scaleheight, β, the polytropic index, γ, and the core radius, θ. The central temperature, T_c, is then derived from the galaxy velocity dispersion, σ_v, using the relationship

$$\beta = \frac{m\sigma_v^\alpha}{kT_c},$$

where it is usual to take $\alpha = 2$. As pointed out by Boynton *et al.*, the parameters β, γ and θ are highly correlated if only the X-ray surface brightness is fitted, and so they introduce an additional constraint on θ by simultaneously fitting the galaxy distribution. Their resultant best fit values are $\theta = 58$ arcsec, $\beta = 0.5 \pm 0.1$ and $\gamma = 1.0 \pm 0.1$. Taking $\sigma_v = 1400 \pm 200$ km s^{-1} they conclude that $kT_c = 24^{+8}_{-12}$ keV.

However, there is strong evidence that the gas temperature does not scale as σ_v^2. Both Mushotzky (1988) and Edge & Stewart (in preparation) find that $\alpha \sim 1.5$. Our measurement of $T_{\rm ave}$ for Abell 2218 is entirely consistent with this relationship. We do not discuss here the possible reasons why $\alpha \sim 1.5$ (e.g. non-isotropic galaxy motions or incomplete sampling of the velocity field) rather than 2, as expected from simple physics, but conclude that gas temperatures scaled directly from galaxy velocity dispersions are prone to error.

We therefore conclude that our measurement of $T_{\rm ave}$ for A2218 is entirely reasonable.

3.2 The microwave decrement

Determination of the microwave decrement is an observationally very difficult task as the expected signal is so small. Problem areas include contributions from discrete radio sources, variations in sidelobe contribution when the telescope is nodded off-source, slow gradients in the background across the cluster. These difficulties, and the ways in which they can be overcome, are described in the papers listed in Table 2. We do not repeat them here.

In order to allow comparison between observations made with different instruments it is most convenient to work in terms of the fractional change in intensity, $dI/I|_0$, along a line-of-sight through the cluster centre as this is then independent of instrumental geometry (e.g. see Radford *et al.* 1986). The determination of $dI/I|_0$ from the observed decrement does require a correction for beam dilution of the central SZ effect which is in itself dependent on the structure of the X-ray emitting gas (Rephaeli 1987). Radford *et al.* assume an isothermal model with $\beta = 0.5$ in their correction and our measurements show that this is indeed the best description of the X-ray data for Abell 2218. We conclude that this correction is unlikely to introduce any error greater than about 5 per cent.

In Table 2 we give the various measurements of the central decrement of Abell 2218, published in the years 1979–84, as tabulated by Radford *et al.* (1986) in their table 4. With the exception of the result of Lake & Partridge (1980), all observations are consistent with a detection of a decrement. However, the quoted errors on individual measurements are a factor of 2–3 lower than the pooled rms uncertainty (Radford *et al.* 1986), indicating the presence of unknown systematic errors.

In addition Birkinshaw (1986) has published the results of extensive observations of Abell 2218 at 20 GHz with a 1.8 arcmin beam at OVRO in the years 1983–86. From these observations he derives a mean central decrement, dT, of -0.69 ± 0.1 mK. As, in the Rayleigh–Jeans limit,

$$\frac{dT}{T} = \frac{dI}{I}$$

Table 2. Microwave decrement observations of Abell 2218.

Group	Frequency (GHz)	$\frac{dI}{I}\vert_0 \times 10^{-4}$
Lasenby and Davies 1983	5.0	$+2.13 \pm 6.66$
Birkinshaw, Gull and Northover 1981	10.6	-9.37 ± 1.87
Schallwich 1982	10.7	-7.73 ± 1.59
Birkinshaw and Gull 1984	10.7	-3.05 ± 1.53
Birkinshaw, Gull and Hardebeck 1984	20.3	-1.93 ± 0.55
Perrenod and Lada 1979	31.4	-7.69 ± 3.55
Lake and Partridge 1980	31.4	$+7.85 \pm 3.83$
1981 Crawford Hill (Radford *et al.*1986)	100-110	$+4.0 \pm 3.1$
1983 Kitt Peak (Radford *et al.*1986)	89.6	$+2.0 \pm 5.2$

then, for a microwave background blackbody temperature of 2.7 K,

$$\left.\frac{dI}{I}\right|_0 = -2.6 \pm 0.4 \times 10^{-4}.$$

This measurement has the lowest associated error yet published and is in agreement with the previous lowest error measurements listed above and we use this value of dT henceforth. (We note that, as the observing frequency changes, the exact relationship between $dI/I|_0$ and dT/T changes and that the parameter we actually need to know to calculate H_0 is dT.)

3.3 The gas distribution

The form of the gas density distribution introduces a large uncertainty into our derivation of H_0. We have attempted to avoid such model dependence by deriving the gas density numerically, in a model-independent way. The method used is the deprojection method described by Stewart *et al.* (1984). In this 'onion-peeling' method the projected surface brightness of the outer annulus of the *Einstein* Observatory HRI image of the cluster is fitted by a particular volume emission measure, thus providing a constraint on n_e and T_e. As the emission measure depends not on n_e^2, but only on $T_e^{-0.5}$, the density is very well constrained, but the temperature is not. To constrain the temperature we assume that the gravitational potential of the cluster can be represented by a truncated King model which gives an emission-measure weighted average temperature in agreement with the derived *GINGA* value. Having derived values of n_e (and T_e) for the outer annulus, values can be successively derived for the inner annuli. This method is only applicable to spherically symmetric clusters such as Abell 2218.

We list, in Table 3, the deprojected values of the electron density as a function of distance from the cluster centre in 30 arcsec bins. Outside the radius where photon noise limits the accuracy of the density distribution we extrapolate the density profile analytically out to 10 arcmin radius assuming the density follows the distribution given by the standard King model, normalized to the measured values at inner radii.

We note that the X-ray surface brightness, S_x, is given by

$$S_x \propto \int \frac{n_e^2 T_e^{-0.5}}{D_L^2}\, dV$$

and so, for fixed T_e and S_x, $n_e^2 \propto H_0$. We list values for $H_0 = 50$ km s^{-1} Mpc^{-1} and scale these values appropriately during later use.

Table 3. Radial distribution of electron density ($H_0 = 50$) in 30 arcsec bins.

Bin Number	Radius in kpc	Density (m^{-3})
1	148.8	$4.65 \pm 0.39 \times 10^3$
2	297.5	$2.42 \pm 0.25 \times 10^3$
3	446.3	$1.54 \pm 0.25 \times 10^3$
4	595.1	$0.92 \pm 0.35 \times 10^3$
5	743.9	$1.06 \pm 0.18 \times 10^3$
6	892.6	$0.43 \pm 0.20 \times 10^3$

Having calculated the density distribution we can immediately calculate the expected decrement, if we know the temperature distribution. We do this as outlined below.

In Section 1.1 we saw that

$$\frac{dI}{I} = 2\sigma_T \frac{k}{m_e c^2} \int n_e T_e\, dl$$

and as

$$dl = \theta \frac{D_L}{(1+z)^2},$$

where dl is the distance corresponding to an angular distance, θ, of 30 arcsec, we can write the microwave decrement

$$dT = 2T\sigma_T \frac{k}{m_e c^2} \frac{\theta}{(1+z)^2} \frac{cz}{H_0} \sum n_e T_e,$$

where $\Sigma n_e T_e$ is the sum of the electron density and temperature product in each 30 arcsec bin, corresponding to the chosen value of H_0. (N.B. Each radial bin is included twice in the summation.)

For the isothermal case ($T_e = 6.7$ keV) we compare our predicted values with the measured value and deduce that

$$H_0 = 24^{+13}_{-10} \text{ km s}^{-1} \text{ Mpc}^{-1},$$

where we have added the errors in temperature, density and microwave decrement in quadrature to derive the errors in H_0.

This value is substantially different from that derived by Boynton *et al.* of 54 km s^{-1} Mpc^{-1}, who assumed a value for T_e of 24 keV and a value for the microwave decrement three times larger than that used here.

4 POSSIBLE SYSTEMATIC ERRORS

There is no *prima facie* reason for the value of H_0 derived above to be in error. However, the optical and radio evidence in favour of a much larger value of H_0 (85 km s^{-1} Mpc^{-1}) is substantial (e.g. see Tully 1988) and so, before rejecting the currently accepted larger value, we consider possible unknown systematic errors in the X-ray and microwave measurements, and any unjustified assumptions in our model for the intracluster gas.

4.1 Gas temperature

As we noted above the X-ray temperature is determined to a high statistical precision and is entirely consistent both with previous measurements for this cluster, and with measurements of other nearby clusters of similar richness.

The cluster is, however, a faint X-ray source and a significant uncertainty in estimating the contribution in the LAC field of view of the diffuse X-ray background can be caused by the fundamental 'fluctuation noise'. The 1 σ fluctuation level is of the order of 5 per cent of the diffuse background intensity, or ~20 per cent of the intensity of A2218 itself. The diffuse X-ray background has a temperature (kT ~40 keV) significantly different from that of A2218. Any error in the subtraction of this component could therefore lead to an erroneous determination of the cluster gas temperature.

To investigate the magnitude of this effect we have artificially added 10 per cent of the average diffuse X-ray background spectrum to the spectrum of A2218 and then re-fit the spectrum in the manner described above. This leads to a temperature estimate of $8.7^{+0.6}_{-0.5}$ keV. Conservatively, we use the upper confidence bound to this estimate and derive a value of 33 km s^{-1} Mpc^{-1} for H_0. This spectral fit, however, results in a significantly worse reduced χ^2 and also overpredicts the flux seen in the HRI band by some 50 per cent.

4.2 Gas structure

4.2.1 Non-isothermality

In Section 3 we assumed that the gas is isothermal ($\gamma = 1$). We also fit the *GINGA* spectrum for the case of a polytropic gas distribution for which the temperature at any point scales with the deprojected density as

$$\frac{T}{T_c} = \left(\frac{n}{n_c}\right)^{\gamma - 1},$$

where T_c is the central gas temperature, and we used the standard formalism to describe the density distribution.

In fact the best fit polytropic model (Table 1) was almost isothermal ($\gamma = 1.02$) with a value for $\beta = 0.56$, in remarkable agreement with the best fit values determined from the surface brightness distribution by Boynton *et al.* (1982). At the 90 per cent level, however, an adiabatic gas distribution with $\gamma = 1.67$ is not ruled out by the *GINGA* data. The best fit central temperature for an adiabatic model depends on the value of β and for $\beta = 0.5$ was 10.1 keV. For $\beta = 1.0$ this rises to 12.6 keV.

For both values the derived values of H_0 are similar and are not in fact very different from the isothermal case, with H_0 being

$$H_0 = 23^{+13}_{-9} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

4.2.2 Non-uniform density distribution

We have assumed throughout that the gas on small scales is uniformly distributed. Any non-uniformity in the gas distribution, however, could in principle affect our determination of H_0.

We have estimated the effect of clumping on our determination of H_0 by considering a two-phase medium in pressure equilibrium (gas on scales below the HRI resolution limit would reach pressure equilibrium in a short time and the smooth surface brightness distribution in the HRI image sets strong constraints on clumping at larger scale sizes). A two-phase medium is quite an extreme case and would have an effect similar to a Gaussian spread of temperatures with the standard deviation equal to approximately half the difference in temperature between the two phases. We have allowed the filling factor of the cooler component to vary between 0.1 and 1.0, and the temperature ratio to vary between 1 and 3. In the most extreme case we underestimate H_0 by ~ 40 per cent.

However, such an extreme case requires a temperature ratio of 3 (and a filling factor of 0.2), which is ruled out by our measurement of the X-ray spectrum which is best fit by an isothermal gas. We note that, in fact, when a second component was included in the model fit to the spectrum, no significant improvement in the fit was found and the two components had similar derived temperatures. From our analysis of the X-ray spectrum we conclude that the upper limit to the temperature ratio is 2. For the most extreme clumping this only results in an underestimate of about 20 per cent for H_0.

If the temperature ratio were very large because the X-ray emitting gas was in pressure equilibrium with a 'cool' phase which was undetectable either with *GINGA* or *EINSTEIN*, then H_0 could be underestimated by factors of a few. The 'cool' phase must have a temperature of less than $\sim 10^6$ K and would be unobservable, except possibly through extended emission lines of high-ionization species. However, for H_0 to be 85 km s^{-1} Mpc^{-1} we require the filling factor of the X-ray emitting gas to be ~ 0.1. As the total mass of X-ray emitting gas would be ~ 3–5 per cent of the virial mass of the cluster, the mass of the much denser cool gas would be several times the virial mass of the cluster inferred from the galaxy velocity dispersion. While such a gas would provide a solution to the 'missing-mass' problem, this seems an unlikely scenario, particularly as the cooling time of the cool gas would be much less than a Hubble time. The EUV and soft X-ray telescopes on *ROSAT* will be able to rule out this possibility.

The alternative scenario, that the X-ray gas is in equilibrium with a 'hot' phase with $T > 25$ keV is also unlikely as this gas would be unbound at large radii in the cluster.

We conclude that clumping of the gas cannot significantly alter the derived value of H_0.

5 CONCLUSIONS

On the assumption of a non-clumped gas we have derived a value for H_0 of 24^{+13}_{-10} km s^{-1} Mpc^{-1} for $\gamma = 1$ and 23^{+13}_{-9} km s^{-1} Mpc^{-1} for $\gamma = \frac{5}{3}$. (The error in H_0 is based on our best estimate of the errors of the contributing quantities; we do not include the unlikely systematic error in the X-ray temperature determination or the possible effect of clumping, both of which we have already described individually.) Such a low value of H_0 has interesting implications. We note, in particular, that the mass of X-ray emitting gas in clusters which, for $H_0 = 50$ km s^{-1} Mpc^{-1}, has been found to be ~ 15 per cent of the mass required to bind clusters (e.g. Stewart *et al.* 1984) would have been significantly underestimated and would, in fact, be almost sufficient to remove the problem of the 'missing mass' in clusters.

However, we note that, by using VLBI observations of expanding supernovae combined with optical spectroscopic expansion velocity measurements, Bartel (1988) has measured distances to galaxies as far as the Virgo cluster and hence determined that $H_0 = 65^{+25}_{-20}$ km s^{-1} Mpc^{-1}. This is in entire agreement with Tully's (1988) recent paper where he argues that all optical data leads to a value $H_0 = 85 \pm 15$ km s^{-1} Mpc^{-1}, but that infall towards the Virgo cluster leads to lower values of H_0 being measured locally. For a distant object such as Abell 2218 we would therefore expect to measure $H_0 = 85$ km s^{-1} Mpc^{-1}. The evidence in favour of this value, and certainly of a value in excess of 50 km s^{-1} Mpc^{-1}, is quite substantial and so we are inclined to suspect our derived value.

Are there any further sources of measurement error in our determination? In the absence of exotic clumping scenarios or large systematic X-ray temperature determination errors, we are only left with the microwave decrement observations, which we therefore conclude must still include unknown systematic errors. $H_0 \propto (dI/I)^{-2}$ and so, if the error resides entirely in the decrement observation, and the gas is uniform, we require $dI/I \sim 0.14 \times 10^{-3}$ to derive a value $H_0 = 85$ km s^{-1} Mpc^{-1}. This is somewhat smaller than the observers would like to accept, but is not impossible. There is reasonable agreement between the work of Birkinshaw (1986) and Uson (1986) in the determination of the SZ effect for the clusters 0016+16 and Abell 665, but Uson finds, however, that Birkinshaw's quoted errors are underestimated by a factor of about 2.5 for those clusters. If this is also the case for Abell 2218 then we can reconcile the X-ray and SZ observations with the currently accepted higher value of H_0.

We therefore conclude that still further observations of the SZ effect, preferably in the increment regime where the problem of contaminating radio sources is much less, are required before this technique can be used as a cosmological probe. We note that, apart from the possibilities of clumping, the fractional error, both statistical and systematic, in H_0 contributed by X-ray measurements is now smaller than that in the SZ observations, and encourage the radio observers to proceed with their difficult work! We suggest that the optimum strategy will be to observe clusters with a high X-ray luminosity (and hence temperature). The all-sky catalogue which will be provided by the *ROSAT* XRT will be crucial in identifying candidates.

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