

Improved lunar eclipse ephemerides

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New values for two astronomical constants related to eclipses of the Moon for the enlargement of the umbra and the oblateness of the umbra are derived from the analysis of more than 7,000 observations of eighteen lunar eclipses during 1972 to 1989. A suite of micro-computer programs developed for this study has produced data from crater timings, which, when analysed, define the geometric characteristics of the umbra and the relationship with the oblateness of the Earth's atmosphere. These values form part of improved lunar eclipse ephemerides, from which revised primary contact times and lunar eclipse circumstances are predicted. These predictions are compared with observations of the four lunar eclipses of 1988 and 1989.

Introduction

Micro-computer programs have been developed to determine the size and shape of the umbra from a large number of crater-timing observations of eighteen lunar eclipses over the period 1972 to 1989.

The analyses extend the Besselian Elements methodology for the prediction of lunar eclipse circumstances and the determination of the geometric characteristics of the umbra. Computer programs are used to analyse crater timing observations (the time of extinction of lunar craters and other surface features during a lunar eclipse) and of each primary contact made by the Earth's shadow with the limb of the Moon. The computer technique developed by the author is a new methodology which is further described in Appendices A and B.

The data for the study shown in Table 1 and in Figure 2, are from 7488 observations submitted by observers in the southern hemisphere and in Europe, and includes 1085 timings provided by Roger W. Synnott of *Sky and Telescope*. The crater timings were made by the many astronomers listed in Table 2, who took part in the author's crater-timing programme, initially announced in 1984 by the BAA in the *Journal*¹, in IRIS² by the National Association of Planetary

Observers (NAPO), Australia in 1985, and in the News Notes³ of *Sky and Telescope*.

The computed values of umbra enlargement were accepted earlier in the study within a statistical range of 0 to 3% but, finally, limits of 0 to 4% have been used. This range is based on the argument that values outside of these set limits were a direct result of mis-identification of lunar features during the observations.

The umbra oblateness values were determined from a linear regression technique, which included stringent statistical tests of individual observers' timings, and by a less compute-intensive, best-fit-ellipse analysis. Oblateness values were determined only from those data sets with a low error of the mean in the observer's umbra enlargement.

In summary, new values have been determined for two of the constants used in the *Astronomical Almanac* for the calculation of lunar eclipse circumstances and the prediction of primary contact timings. These new values are included in improved lunar eclipse ephemerides and their effect on predicted circumstances are compared with observation reductions in Appendices C to G. Comparisons are made with the observations from two partial eclipses in 1988, on March 3 and August 27, and with the total eclipses of 1989 February 20 and August 17.

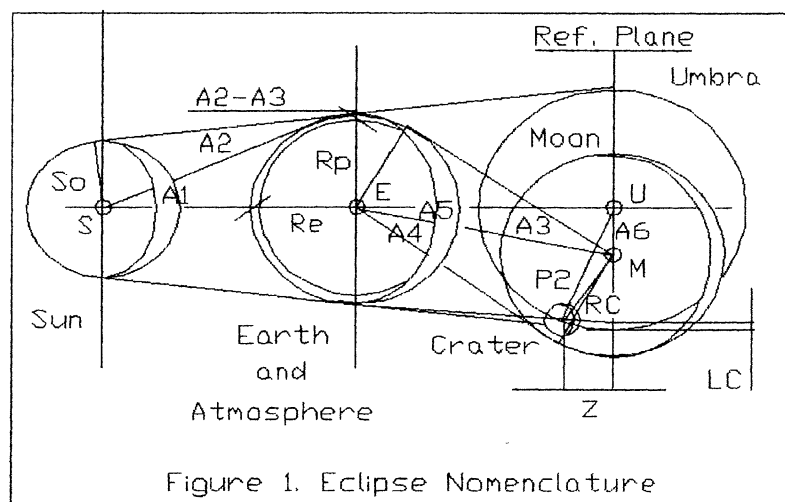


Figure 1. The nomenclature used to determine the geometry of the umbra. Angles are: **A1** the horizontal parallax of the Sun (π_s), **A2** the semi-diameter of the Sun (S_s), **A3** the horizontal parallax of the Moon (π_m), **A4** the semi-diameter of the Moon (S_m), **A5** the declination of the Sun (δ_s), and **A6** the latitude of the crater on the umbra (ψ). Other values are: **RC** the radius of the crater's position on the Moon, **p₂** the theoretical radius of the umbra, **LC** the increase in the apparent radius of the crater due to its closer distance to the Earth **Z**; with the plane of reference through the centre of the Moon-umbra normal to the Sun-Earth line.

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Table 1A. Comparison of crater timing reductions: enlargement of umbra.

Date	Current analyses						Previous analyses ^{1,7,9}					
	Immersions			Emersions			Immersions			Emersions		
	No	%E	m.e.	No	%E	m.e.	No	%E	m.e.	No	%E	m.e.
74 Nov 29	64	1.7410	0.0975	23	2.2021	0.0668	39	1.75	0.07	17	1.83	0.02
78 Mar 24	386	2.1966	0.1371	64	2.1471	0.2289	200	1.86	0.02	38	1.86	0.08
78 Sep 16	689	1.9906	0.1188	57	2.0246	0.1401	468	1.82	0.03	31	1.78	0.09
79 Mar 13	50	2.2303	0.1350	—	—	—	25	1.78	0.11	—	—	—
79 Sep 6	206	1.9591	0.1511	265	1.8620	0.1281	108	1.71	0.05	133	1.66	0.04
81 Jul 17	67	2.2697	0.1922	—	—	—	44	2.03	0.09	8	1.93	0.24
82 Jan 9	359	2.1047	0.1556	1	2.2044	—	155	1.87	0.03	—	—	—
82 Jul 6	58	1.9574	0.2102	289	2.1825	0.1594	42	1.72	0.07	156	1.93	0.04
S&T(1)	478	2.1747	0.1815	133	2.2850	0.1992	538	2.02	—	159	2.24	—
82 Dec 30	45	1.8291	0.1372	88	1.7717	0.1083	6	1.92	0.19	46	1.83	0.07
S&T(2)	273	1.7329	0.1379	57	1.5821	0.1450	298	1.74	—	90	1.74	—
83 Jun 25	98	2.1462	0.1032	58	2.2687	0.1970	22	2.07	0.08	16	2.19	0.09
85 May 4	567	2.1432	0.1125	202	1.9069	0.1197	—	—	—	—	—	—
85 Oct 28	291	2.4873	0.1538	84	1.9198	0.2609	—	—	—	—	—	—
86 Apr 24	683	2.0062	0.0875	425	2.1500	0.1122	—	—	—	—	—	—
86 Oct 17	452	2.2190	0.1378	274	2.1557	0.1126	—	—	—	—	—	—
88 Mar 3	5*	2.8292	0.0676	5*	2.2105	0.0212	—	—	—	—	—	—
88 Aug 27	44	2.2655	0.2176	75	1.9514	0.0995	—	—	—	—	—	—
89 Feb 20	334	2.2425	0.0846	239	2.0021	0.0902	—	—	—	—	—	—
89 Aug 17	357	2.1589	0.1450	83	1.8820	0.2906	—	—	—	—	—	—

* Includes USNO predictions for primary contacts

No is the number of observations statistically accepted
%E is the percent enlargement of the umbra, where 0 < %E < 4 for the current analysis, and 0 < %E < 3 for the previous analyses, with the exception of S&T which is unknown
m.e. is the error of the mean, ie standard deviation/SQR(No)

Table 1B. Comparison of crater timing reductions: oblateness of the umbra.

Date	Current analyses (0 < %E < 4)						Previous analyses ^{1,13} (0 < %E < 3, except S&T)					
	Immersions			Emersions			Immersions			Emersions		
	No	Mean f	s.d.	No	Mean f	s.d.	No	Min f	Max f	No	f	Pole
74 Nov 29	70	266.6	0.01	23	95.7	0.06	—	—	—	—	—	N
78 Mar 24	266	116.2	0.03	24	50.7	0.15	16	201	117	—	—	S
78 Sep 16	532	67.9	0.04	11	83.9	0.01	57	77	22	—	—	N
79 Mar 13	43	254.4	0.01	—	—	—	24	107	29	—	—	N
79 Sep 6	70	161.8	0.07	79	(317.6)	0.04	28	183	62	—	—	S
81 Jul 17	23	93.8	0.01	—	—	—	—	—	—	—	—	N
82 Jan 9	236	133.8	0.03	—	—	—	—	—	—	—	—	S
82 Jul 6	47	(8.9)	0.04	233	30.5	0.02	—	—	—	—	—	W-E
S&T	63	77.5	0.01	11	49.4	0.00	—	—	—	—	—	—
82 Dec 30	38	54.4	0.05	11	111.5	0.07	28	125	121	—	—	N
S&T	84	138.8	0.05	—	—	—	—	—	—	—	—	—
83 Jun 25	92	156.6	0.01	14	52.9	0.09	—	—	—	—	—	S
85 May 4	345	121.8	0.06	102	120.7	0.05	240	54.3	—	1	19	N
85 Oct 28	107	134.3	0.09	38	134.7	0.03	71	42.8	—	—	—	S
86 Apr 24	513	45.9	0.03	164	225.1	0.08	353	21.7	—	208	48.6	S
86 Oct 17	303	84.4	0.03	124	165.5	0.04	—	—	—	—	—	N
88 Mar 3	—	—	—	—	—	—	—	—	—	—	—	N
88 Aug 27	39	(14.2)	0.18	41	143.1	0.02	—	—	—	—	—	S
89 Feb 20	185	180.5	0.05	67	73.1	0.04	—	—	—	—	—	N
89 Aug 17	148	70.0	0.08	13	(9.4)	0.11	—	—	—	—	—	S
Mean	3019	101.36	0.05	944	131.62	0.06	—	—	—	—	—	—

No is the number of observations with their error of the mean < 0.15
Mean f is the weighted reciprocal oblateness, where (doubtful values)
s.d. is the standard deviation
Min f is the minimum oblateness linear regression estimate
Max f is the maximum oblateness linear regression estimate
Pole is the north, south or west-east path of the eclipsed Moon

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Table 2. Participating observers, their sites and eclipses from (1) 74 Nov 29 to (18) 89 Aug 17.

Observer(s)	Observing Location(s) and (Eclipse events)	Observer(s)	Observing Location(s) and (Eclipse events)	Observer(s)	Observing Location(s) and (Eclipse events)
Adcock B. S.	Canberra (2) View Bank, Victoria (12), (13)	Gracias N.	Nazare, Portugal (18)	Moller H. &	Kingsley, Western Australia (13)
Anderson P.	The Gap, Queensland (3), (8), (12), (13), (14), (17)	Grant I.	Traralgon, NSW (5)	Johnson I.	Johannesburg (6)
ASAW	Albury, Wodonga, NSW (17)		Carlton North, Victoria (11)	Munford N.	Napier, New Zealand (9)
Ashdown	Wellington (8)	Grudzie R.	Hobart (5)	Newman J. &	Cootamundra, NSW (8)
M. L. C.		Gutterson G. &	Cootamundra, NSW (8)	Waugh J. W.	
Ashley M.	Canberra (1), (2), (3)	Mowle J.		Newman W. & J.	Cootamundra, NSW (3)
Baroni S.	Milan (12), (14), (18)	Hardy S. & L.	Cook, Australian Capital Territory (5), (7)	Nottage D.	Tailern Bend, South Australia (5)
Barreto M. J. R.	APDA, Brazil (18)		Canberra (3), (11)	Nuland Van J. H.	San Jose, California (5)
Barsby J., Shaw	Johannesburg (4), (6)	Hardy S. &	Canberra (2)	Nurse G.	Pymble, NSW (5)
D. & Hudson B.		O'Keefe R.		Nurse G. & R.	Macquarie University, NSW (11)
Barton W.	Ipswich, UK (14)	Hendman G. &	Wellington (5)	Olsen C. & E.	Kambah, Australian Capital Territory (3)
Bennett G.	Canberra (2), (3)	Kilbey P.		Orchiston W.	Victoria College, Victoria (11)
Bestwick T. &	Cootamundra, NSW (8)	Henshaw C.		Orchiston W.,	Victoria College, Victoria (13)
Harris T.		Herald D. R.		McNabb D. &	
Bezerra U. L.	Bahia, Brazil (6)			Vogrig D.	
Blewett L. & A.	Tumut, NSW (14), (17)	Hers J.		Orchiston W. &	Victoria College, Victoria (12)
Boinck F.	Eindhoven (18)	Holtschke R.	Sedgefield, South Africa (4)	Titcher D.	
Bott P.	Quorn, South Australia (11)	Horton L.	Shrewsbury (18)	Overbee D.	Edenvale, South Africa (6)
Bowden W. & C.	Cootamundra, NSW (7), (8), (13)	Horton L. C. A.	Gawler, South Australia (17)	O'Neill P.	Newtownabbey, UK (12)
Bowden W. &	Cootamundra, NSW (12)	Hughes D.	Wellington (11)	Pagano L.	Cronulla, NSW (3)
Catlin M.			Palmerston North, New Zealand (9)	Parkinson M. L.	Durack, Queensland (8), (12), (13)
Bowron C. F.	Finningley, UK (14)	Hume A.	Canberra (2)		Acacia Ridge, Queensland (5)
Bradbrook A. J.	Malvern, Victoria (12)	Hume A. &	Canberra (1)		Brisbane (3)
Bras J.	Funchal, Madeira (18)	Watson H.		Paynter L.	Radcliffe, UK (12)
Brickell A. A.	Brisbane (5)	Imperiano B. O.	APDA, Brazil (18)	Pearce A.	Woodlands, Western Australia (7), (11), (13)
	Kuaotunu, New Zealand (9), (10)	James A.	Drummoynne, NSW (3), (5), (8), (10)	Pereira C.,	Johannesburg, South Africa (4)
	Whitanga, New Zealand (13)			Spethman K.,	
Brock M.	Murray Bridge, South Australia (11)		Darlington Point, NSW (7)	Richards J. &	
Bulder H.	The Netherlands (14), (18)		Hunters Hill, NSW (11)	Winskill C.	
Carter B.	Cannon Hill, Queensland (5)	Jameson I.	Murray Bridge, South Australia (5)	Perrott M.	Canberra (2)
Castiglioni L.	Milan (18)	Jenniskens C.	Pierlapont, Belgium (18)	Pietschnig M.	Austria (14)
Chorley J. W. A.	Fardon, UK (14)	Jordan, J.	Cootamundra, NSW (7)	Price R. T.	Cootamundra, NSW (2), (8), (11), (13), (14)
Clark M.	Dowerin, Western Australia (5)	Kerr S.	Kogan, Queensland (12), (16)	Price R. T. & E.	Bethanga, Victoria (17)
	Armadale, Western Australia (15), (17)	Kilgour A. &	Nelson, New Zealand (8)	Price R. T. &	Cootamundra, NSW (12)
	Dowerin, Western Australia (3)	Higgs S.		Newman J.	
	Langford, Western Australia (13)	Kinnear G.	Tambellup, Western Australia (9)	Price R. T. &	Cootamundra, NSW (7)
	Rolystone, Western Australia (11)	Kirkland M.	Cootamundra, NSW (7)	Hawkins R.	
	Tambellup, Western Australia (9)	Klekocik A.	Hobart (3), (5)	Price R. T. & R.	Cootamundra, NSW (3)
Cook T.	Canberra (5)	Knight J.	Witbank, South Africa (12)	Reid, H. & J.	Cootamundra, NSW (3)
Cooper I.	Palmerston North, New Zealand (9)	Knight J. & S.	Bernoni, South Africa (14), (18)	Reid H. & M.	Cootamundra, NSW (11), (14)
		Knight S.	Witbank, South Africa (12)	Reid H. & P.	Cootamundra, NSW (7), (8), (12), (13)
Cooper T. P.	Bernoni, South Africa (14), (18)	Kralert P. G.	Perth, Western Australia (7)	Reis P. M.	Rio de Janeiro (8)
	Sasolburg, South Africa (11), (12)	Linthwaite M.	Victoria (12)	Reis P. M. &	Rio de Janeiro (6)
	Vereeniging, South Africa (2), (6)	& Robson K.		Rothchild M.	
Corkery R.	Cootamundra, NSW (8), (10)	Loader B.	Blenheim, New Zealand (10)	Ripero J.	Madrid (11)
Couling G.	Wellington (9)	Lockhart S.	Murray Bridge, South Australia (7)	Roberts N.	Cyprus (12)
Cragg T.	Siding Spring, NSW (3)			Roberts S.	Duffy, Australian Capital Territory (16)
Davis Mrs B.	Tambellup, Western Australia (9)	Loveday N.	Canberra (3)		South Oakleigh, Victoria (13), (14)
De Arruda	APDA, Brazil (18)		Sydney (8)	Robinson P. N.	West Yorkshire, UK (12)
F. S. B.		Low P.	Canberra (5)	Rodriguez D.	Madrid (11)
Delaney P.	Canberra (2)	Lyall A.	Canberra (2), (3)	Rodriguez D. &	Madrid (18)
Demeis S.	Milan (14)	Marciano S.	Melbourne (9), (10)	Ripero J.	
Dieters S.	Altona-Victoria (3)	Marques R.	Parede, Portugal (18)	Rost F.	Kensington, NSW (14)
	Canberra (2)	Marshall G. R.,	Johannesburg, South Africa (4)		Univ. of NSW (10), (11), (12), (13), (15)
Diluca R.	Bologna (14)	Wise P., Zangle		Rothchild P.	Canberra (2)
Dixon G. R.	Donwell, UK (14)	V. & Ferri A.			Kambah, Australian Capital Territory (3)
Dorant P.	Bunbury, Western Australia (11)	Marshall G. R.,	Johannesburg, South Africa (11)	Sanchez R.	Canberra (5)
Drummond J.	RDZ-Gisborne, New Zealand (16)	L. A. & Perez J.		Sarandis J. &	Cootamundra, NSW (14)
		Martin C.	Castle Hill, NSW (8), (10), (11), (12), (13), (14)	Goodwin C.	
Drummond J.	Gisborne, New Zealand (17)			Sarandis J. &	Cootamundra, NSW (13)
& Ledger M.		Martin D. S.	Clayton, Victoria (13)	Blewett A.	
Durham D.	Upper Hutt, New Zealand (10), (11), (12)	McLeavy A.	Kenwick, Western Australia (7)	Searle G.	Canberra (2)
Duthie J. H.	Whakatane, New Zealand (8)	Meagher D.	Melbourne (5)		Concord, NSW (3)
Elso J.	Canberra (2), (3)	Mellor W. W.	Sheffield, UK (14)	Smith C.	Johannesburg (6)
Eynde P.	Antwerp (14)	Metcalf I.	Canberra (2)	Smith S.	Christchurch, New Zealand (8)
van den		Michie D.	Johannesburg, South Africa (6)	Soulsby B. W.	Paradise, South Australia (16), (17)
Felstead M.	Gosnells, Western Australia (7)	Miller D.	Kangaroo Island, South Australia (5), (7), (9), (11), (12), (13), (15)		Canberra (1), (2), (3), (11)
Fernandes J.	Pedrogao Pequeno, Portugal (18)		Welland, South Australia (17)		Mount Stromlo, Australian Capital Territory (7), (8), (10)
Feyth H.	Eindhoven (18)	Moller H.	Cebu City, Philippines (11)		Singapore (12)
Fisher M.	Cootamundra, NSW (7)		Kingsley, Western Australia (11), (16)		
Garcia J.	Oeiras, Portugal (18)	Moller H.,	Kingsley, Western Australia (17)		
	Madrid (14)	Johnson I. &			
Gerritsen A.	Amstelveen, The Netherlands (18)	Pallier O.			
Goncalves R.	Oeiras, Portugal (18)	Moller H.,	Kingsley, Western Australia (14)		
Goodman D.	Wellington (8), (11)	Johnson I. & S.			

(cont.)

Table 2 continued

Observer(s)	Observing Location(s) and (Eclipse events)	Observer(s)	Observing Location(s) and (Eclipse events)	Observer(s)	Observing Location(s) and (Eclipse events)
Soulsby B. W. & A. B.	Weston, Australian Capital Territory (5)	Tabur V.	Rostrevor, South Australia (5)	Weber R.	Vienna (14)
Sowiak P.	Canberra (2)		Adelaide, South Australia (7)	Wheeler D.	Corrimal, NSW (14)
Spencer G.	Canberra (2)	Tregaskis B.	Canberra (2)	Williams I.	Canberra (3)
	Chis olm, Australian Capital Territory (11)		Mount Eliza, Victoria (11), (14)	Wils P.	Niel, Belgium (14)
	Mount Stromlo-Australian Capital Territory (7)	Valls D.	Madrid (11)	Wilson H. N.	Hobart (3), (5)
Steicke D. P.	Murray Bridge, South Australia (5), (7), (11)	Van Den	Belgium (18)		South Hobart, (12)
Stephenson J.	Canberra (2)	Vandenbruaene J.	Burnem, Belgium (18)	Winskill C.	Johannesburg (6)
Sumner B.	Padbury, Western Australia (5)	Vandenbulcke G.	Oostduinkerke, Belgium (18)	Winter G. L.	Rio de Janeiro (8)
	Perth, Western Australia (3), (7)	Vandenbussche	Pierlapont, Belgium (18)	Wolf G. W.	Dunedin, New Zealand (12), (13), (14)
Sumner C.	Perth, Western Australia (7)	B.			Wainuiomata, New Zealand (16)
Sutton T. P.	Sandal, UK (14)	Vollmann W.	Austria (14), (18)	Yvergneaux D.	Wellington, New Zealand (15)
Symonds D.	Murray Bridge, South Australia (7), (11)	Watson N. &	Cootamundra, NSW (13)	Zeipelt A.	Ronse, Belgium (14)
		Gooden D.			Golling, Austria (11), (14)
		Watson N. &	Cootamundra, NSW (12)	Zijl, J.	Benekom, Belgium (18)
		Miles A.			

Constants for change

The first of the two constants studied is the traditional increase by one-fiftieth part (or 2.0%), of the radius of the geocentric shadow of the Earth during a lunar eclipse due to the effect of the upper atmosphere.^{4: p. 257} Extensive data is presented to show that only a small change to this constant is required to agree with the observed mean umbra enlargement of 2.09%.

The other value to be revised in the lunar eclipse ephemeris is the constant 1/297 (or the recent determination 1/298.3)^{4: p. 489} adopted for the oblateness or flattening of the Earth's upper atmosphere. The expected oblateness of the atmosphere is near 1/214 depending upon the geometry of the eclipse as shown by Meeus,⁵ and data presented here show that the Earth's atmospheric oblateness varies from 1.5 to twice the classical value. It is also shown that improved values are found for the apparent radius of the umbra at any position angle during an eclipse when these new observationally determined values replace the constants normally used.

Crater timing results

Umbra enlargement

The enlargement of the umbra has been determined from crater and primary contact observations by the new analytical method described in Appendices A and B. The results are summarised in part in Table 1, which compares the current values with those presented from an earlier analysis¹ which was not fully developed and relied on fewer observations.

The computer-generated values of umbral enlargement for the eighteen eclipses studied were derived from timings within the chosen statistical range, provided by the observers listed in Table 2. Some of these results have been verified by an independent analyst.⁷ Other crater timings supplied by *Sky and Telescope* Associate Editor Roger W. Synnott⁸ for the two eclipses of 1982

on July 5–6 and December 30, have been included and the results compared with the other observations.

Umbra oblateness

The observed oblateness of the umbra has been estimated by a method⁹ which includes statistical linear regression of the computed observed and theoretical umbra radii at the corresponding position angle on the umbra. These results are presented in Figure 3, where

Figure 2. Crater Timing Observations

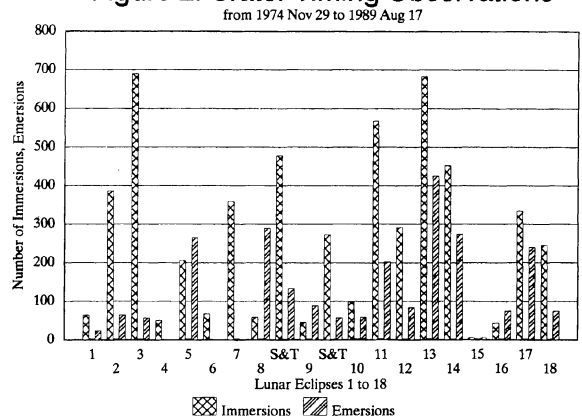
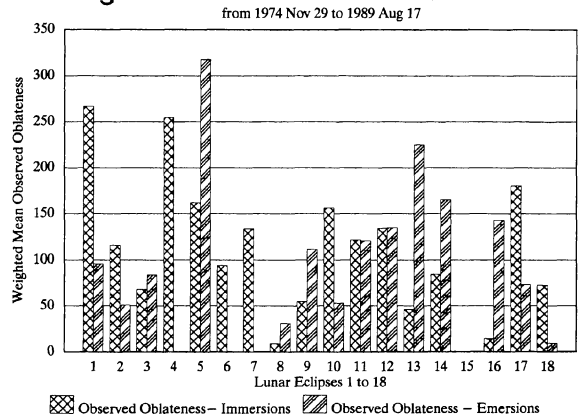


Figure 3. Observed Umbral Oblateness



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the mean umbral oblateness has been determined from 3963 timings made by 75 observers, all of whom exhibited an error of the mean less than $\pm 0.15\%$ in their umbra enlargement data. These reductions have also been verified using an independent analysis based on a best fit to the equation of an ellipse.⁷

The 697 observations from the eclipse of 1982 July 5–6 and the 388 crater timings from 1982 December 30 provided by Roger W. Synnott, have also been analysed for oblateness as requested by *Sky and Telescope*.¹⁰ There were thirteen *S & T* coded observers with an acceptable low error of the mean and their oblateness estimates are included in Table 1. Figure 4 illustrates the departure of the mean observed oblateness from the geometric value for each eclipse.

'Sky and Telescope' observations

The observations from USA provided by Roger W. Synnott were of interest even though most observers reported a small number of mid-feature crater timings. The overall result for percent enlargement of the umbra can be compared but, as Synnott used different acceptance criteria and reduction programs, the *S & T* results are, as expected, not identical with the other reductions presented here.

Figure 3A. Oblateness/FM (Ratio Ks)

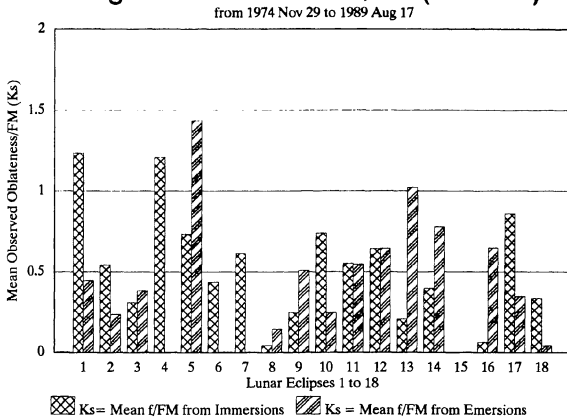
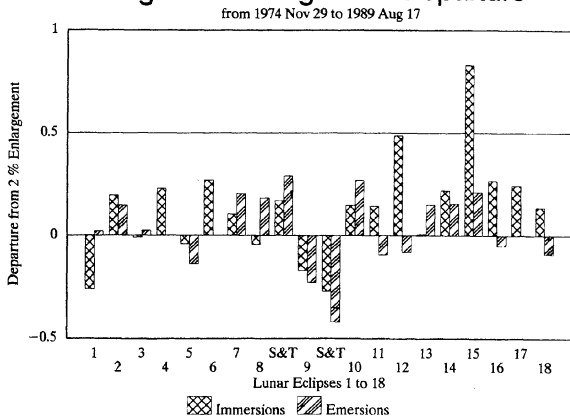


Figure 4. Enlargement Departure



Where there were a low number of timings from each observer, the values of oblateness obtained are perhaps of less significance than the southern hemisphere data sets where a larger number of contacts were timed for each feature (viz. by the observation of both edges of each lunar feature, rather than timing the central peak of craters as adopted by *S & T*). The *S & T* data for 1982 July 5–6 gave a reciprocal umbral oblateness (f) of 78 from five observers who timed 63 immersions and an $f = 49$ for one observer who timed 11 emersions.

Unfortunately, for this eclipse, the southern hemisphere observers' oblateness was unacceptably high for the 47 immersions, but a value of $f = 31$ was found from eight observers who timed 233 emersions, similar to the *S & T* value above. In the *S & T* observations for 1982 December 30, seven observers had an error of the mean for umbral enlargement less than 0.15% and a corresponding $f = 139$ for 84 immersion timings. The southern hemisphere observations of this eclipse produced an immersion value of $f = 55$ for 38 timings and an emersion value of $f = 112$ from 11 timings.

New constants from the observed umbra

Enlargement

The values of enlargement of the umbra within the statistical range of 0 to 4% produced a weighted mean of $2.11 \pm 0.13\%$ from 5149 immersion timings and $2.05 \pm 0.13\%$ from 2339 emersions. The overall mean for the 7488 observations analysed was $2.09 \pm 0.13\%$, a new value which is included as a constant in the improved lunar eclipse ephemeris (referred to here as the ILEE).

Oblateness

The computed values for the oblateness of the umbra are generally much greater than expected when compared with the value adopted for the Earth's solid spheroid and that found by the Meeus geometric considerations.^{5,10} The mean observed umbral oblateness of $1/101.67$ determined from this series of eclipses suggests that the currently adopted value of $1/298.26$ for the Earth and its atmosphere should be increased. This is also considered in the ILEE.

The ILEE

Semi-diameter of the Moon

There have been many improvements made in the methodology for computing lunar eclipse circumstances as a result of changes made in the constants used for prediction. One method uses the ratio ' k ' equal to the radius of the Moon divided by the radius of the Earth for the determination of the Moon's semi-diameter (S_m). The constant ' k ' has varied in value from 0.2724953 in 1924, to 0.2722740 in 1963 and

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0.2725 076 after 1986 for the reasons outlined in the *Explanatory Supplement*⁴: p. 213 and the *Astronomical Almanac*. The ILEE adopts the recently revised value for the Moon's semi-diameter for eclipses as given in the 1988 Almanac (page L4) found from:

$$S_m = \sin^{-1}[0.272\ 5076 * \sin(\pi_m)] \quad (i)$$

which differs from that given for the Moon in the same Almanac (page L6) as:

$$S_m = \sin^{-1}[0.272\ 493 * \sin(\pi_m)]$$

Umbral Radius at Distance of the Moon (ρ_2)

The expression given in the *Explanatory Supplement*⁴: p. 257 for the theoretical radius of the umbra at the distance of the Moon is:

$$\rho_2 = 1.02 * [\pi_1 - S_0 + \pi_0] \quad (ii)$$

where π_1 is $0.998\ 333 * \pi_m$, S_0 is the Sun's semi-diameter, and π_0 is the Sun's horizontal parallax. Each of these values are considered for improvement.

Critical approach by Meeus

A most important critical approach considered for the method developed is that due to Meeus,¹² where allowance is made for the declination of the Sun at the time of eclipse as well as for geometric oblateness. To decide on the appropriate value of oblateness for the determination of the theoretical radius of the umbra (ρ_1) at the position of the Moon, a revised value for the oblateness of the Earth's atmosphere must be considered.

The difficulty here is that the Earth's atmospheric oblateness has been shown to be much greater than the classical value of $1/298.3$ (or $1/298.257$ ¹³: p. 97) using the geometric estimate by Meeus,^{5,11} which is found from the expression:

$$FM = (R_e - R_p) / R_e = \frac{F_e * (\pi_0 + \pi_m) / (\pi_0 + \pi_m) - S_0}{S_0} \quad (iii)$$

$$\text{or,} \quad = F_e * K_m$$

where FM is the geometric theoretical oblateness, R_e the equatorial radius and R_p the polar radius of the Earth's upper atmosphere, and K_m is 1.34 at perigee and 1.43 at apogee. Hence, the reciprocal $F_e = K_m * FM$ at the Earth, with the actual value depending on the eclipse geometry.

Allowing for the Earth's tropospheric altitude differences at the equator and the pole, Ahnert¹¹ has commented that, with a 10-kilometer height differential, the atmospheric reciprocal flattening at the Earth is 204 and, allowing for the Meeus geometric correction, this becomes 147 at the distance of the Moon. This is much less than the mean observed value of $1/102$, which gives an atmospheric height difference of 25 kilometers. The author¹⁴ has also derived an atmospheric height differential of 25 kilometers using a different theory for a reciprocal oblateness of the umbra of 45.[†]

Earth's atmospheric oblateness

Considering the above, the Earth's atmospheric oblateness can be related to the expected value due to Meeus and Ahnert and with an important new relationship with the observed oblateness of the umbra.

Using the Meeus theoretical geometric value based on expression (iii), we can introduce the variable K_m and, with the atmospheric height differential as discussed by Ahnert, we can also introduce a variable K_a . The observed umbra provides a relationship between the Earth's atmospheric oblateness and that observed for the umbra, as follows:

If $K_m = 1.39$ from expression (iii), then $F_e = 1.39 / 298.26 = 1/214.58$, as discussed by Meeus. Allowing for an atmospheric height difference between the equator and the pole of, say, 10 kilometers, then $F_e' = 204$, or $K_a = 214.58/204 = 1.0519$. Then the expected umbral oblateness, $F_o = K_m/204 = 1/146.76$.

Now, if F_o as observed is $1/102$, then the new relationship, $K_s = 146.76/102 = 1.4383$

or, $F_o = 214.58/102 = 2.1 * F_e$, or over twice the value of F_e .

Hence, when using the Earth's solid spheroid oblateness as a constant of $1/298.26$, this needs modification for atmospheric height difference when used for the reduction of crater timings to determine exactly the observed oblateness of the umbra, as follows:

$$F_e = 1/298.26 * K_m * K_a \quad (iv)$$

where K_m is known, but K_a is based on inexact knowledge of the atmospheric height difference and can only be found by reduction of observations for the oblateness of the umbra, which is determined from the ILEE with $F_e = 1/298.26 * K_m$. When the umbra oblateness is determined then $F_e = K_s * F_o$, where $K_s = K_a * K_s$ and is always > 0 .

Theoretical radius of the umbra

Meeus has also developed an expression for the theoretical radius of the umbra which incorporates the Earth's atmospheric oblateness and the declination of the Sun during the eclipse, as follows:

$$\rho_1 = 1 - \cos(\beta) * \cos(\lambda) * \pi_m * \tan(S_0 - \pi_0) - (\cos^2 \delta_0) * (\sin^2 \psi) * F_e \quad (v)$$

where, F_e is the oblateness of the Earth corrected by expression (iv), S_0 is the semi-diameter of the Sun, δ_0 is the declination of the Sun, and β, λ are the libration-corrected selenographic crater co-ordinates. This can be expressed as:

$$\rho_1 = 1 + \alpha \quad (vi)$$

In the ILEE, reduction of all crater timings and primary contacts are compared with the Meeus geometric oblateness and that derived from observations. Computation of the theoretical radii of the umbra includes an allowance for the Sun's declination according to

[†] An initial value of $\frac{1}{45}$ was used but has since been modified; the new observed value is $\frac{1}{102}$.

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Meeus and the initial iteration includes F_e when determining the first value of observed oblateness.

When the value of observed oblateness is included in the crater timing reduction programs of the ILEE, changes in prediction times occur, which are later when preceding opposition and earlier following opposition and are also dependent upon eclipse geometry; this is discussed further in Appendix F.

Equivalent parallax (π_1)

The classical value of π_1 adopts a shadow cone produced from the Earth at mean radius and is based on the parallax of the Moon reduced to the equivalent value at a mid-latitude of 45° , which is a very approximate allowance for the oblateness of the Earth and its atmosphere.

The ILEE adopts a more critical approach and uses the sixth order values of the observed crater's latitude (ψ) with the third order of oblateness (f), to determine the radius at the observer (ρ), which is given in the *Explanatory Supplement*⁴: p. 58, as follows:

$$\begin{aligned} \rho = & 1 - f/2 + 5/16*f^2 + 5/32*f^3 \\ & + [(f/2 - 13/64*f^3)] * \cos(2*\psi) \\ & - [5/16*f^2 + 5/32*f^3] * \cos(4*\psi) \\ & + [13/64*f^3] * \cos(6*\psi), \end{aligned}$$

or, $\rho = 1 + \alpha$ (see expression (vi))

where $f = F_e$, the Earth's atmospheric oblateness as modified. On substitution of a value for F_e of, say, $1/200$, the expression for ρ (or π_1), becomes approximately:

$$\begin{aligned} \pi_1 = & 0.9975\ 0802 + [2.5\text{E}-03 * \cos(2*\psi)] - \\ & [0.7832\text{E}-05 * \cos(4*\psi)] + [2.5\text{E}-08 * \cos(6*\psi)], \end{aligned} \quad (\text{vii})$$

As indicated above, ψ is the computed position angle of the observed crater at the time it is obscured by the umbra. With $\psi = 45^\circ$, π_1 becomes 0.997516. At the equator it is 1.0 exactly and at the pole it is $1 - 1/200 = 0.995$, with these values in terms of the Earth's radius.

Sun's semi-diameter (S_o)

The value adopted for the Sun's semi-diameter in eclipses is $15'59''.63$ without the correction for irradiation of $1''.15$ (or $0.0003\ 19^\circ$). Recent work by Sofia, Fiala et al^{14,15} has indicated that any change in the solar radius is very small. Accordingly, the ILEE uses the ephemerides' values of the Sun's semi-diameter (S_o) but retains an allowance for irradiation also adopted for crater timing analyses by Ashbrook.⁶

Sun's horizontal parallax (π_o)

Since 1896, the value adopted for the solar horizontal parallax in eclipses is $8''.80$, although there is some difficulty with this value when considering the mass ratio of the Sun: Earth-Moon, where a value of $8''.79$ is

more appropriate.⁴: p. 171 Recent determinations are $8''.7984$:p. 490 and $8''.7941\ 48$:¹³:p. 97 the latter is used in the ILEE adopting that given in the appropriate Almanac.

Theoretical umbra radii

ILEE enlargement

The ILEE includes the mean observed umbral enlargement of 1.0209 in place of the traditional value of 1.02. The departures of the observed umbra enlargements from the classical value of 2% are illustrated in Figure 4 for the eighteen lunar eclipses studied.

ILEE oblateness

The ILEE also includes allowance for the mean observed umbral oblateness found from the adjusted oblateness of the Earth. The observed umbra oblateness departure from the theoretical geometric value for each eclipse is illustrated in Figure 5, which also provides an appreciation of the variation in $K_a * K_s = K_s$, shown in Figure 3A.

Improved umbra radius, the ρ_2 expression

The full expression for ρ_2 , the theoretical radius of the umbra combines expressions (i) and (vii) above. Using the terminology illustrated in Figure 1 and the values discussed for %E, F_e , S_o and π_1 , the first expression for the improved theoretical radius becomes:

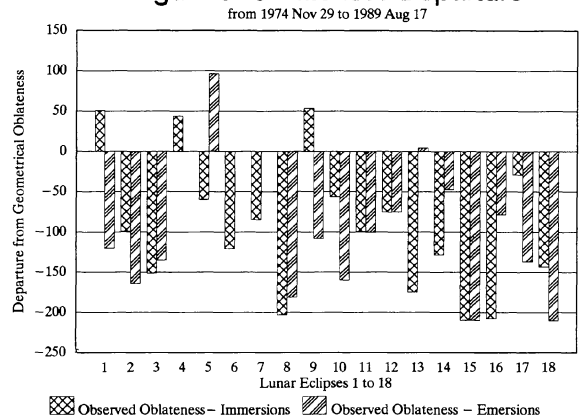
$$\rho_2 = 1.0209 * (\rho * \pi_m + \pi_o - S) \quad (\text{viii})$$

where $\rho = 1 + \alpha$ and $S = S_o + \text{Irradiation} + \text{LC}$.

LC is the umbra increase due to the crater being closer to the Earth than the fundamental reference plane which is at the centre of the Moon (see Fig 1).

$$\alpha = [A + B*\cos(2*\psi) + C*\cos(4*\psi) + D*\cos(6*\psi)*\cos^2(\delta_o)]$$

Figure 5. Oblateness Departure



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and when $F_c = 1/200$, a typical initial value, then as in expression (vii)

$$A = 0.9975\,0802, B = 2.5\text{E-}03, C = 7.832\text{E-}06 \text{ and } D = 2.5\text{E-}08,$$

$$LC = Z \cdot \tan(S_0 + 0.000319 - \pi_0),$$

where $Z = S_0 \cdot (1 - \rho_c^2)^{1/2}$, and ρ_c is the selenographic radius of the crater

or, due to Meeus the expression:

$$\alpha = -\cos^2(\delta_0) \cdot \sin^2(\psi) \cdot F_c.$$

$$\text{Again, } LC = Z \cdot \tan(S_0 + 0.000319 - \pi_0)$$

$$\text{but } Z = S_0 \cdot \cos(\beta) \cdot \cos(\lambda)$$

where β and λ are the libration corrected selenographic crater co-ordinates. It is possible to use either value of α in the ILEE as very close agreement is found with a typical difference in the computed radii of only 6 in 10 000.

Improved penumbra radius, the ρ_1 expression

Similarly, for the penumbral phase of an eclipse the full expression for the theoretical radius of the penumbra ρ_1 at the lunar distance, is:

$$\rho_1 = 1.0209 \cdot [\rho \cdot (\pi_m) + \pi_0 + S] \quad (\text{ix})$$

As described by Meeus¹⁷ the possibility of a penumbral eclipse occurring is very dependent upon the assumption that the penumbra exhibits the same enlargement as the umbra, but a similar change in its shape due to oblateness is not generally considered.

The above expressions are incorporated in the computer program for lunar eclipse circumstances to evaluate if either penumbral or partial eclipses will occur and to study the effect of including the values of observed enlargement and oblateness for an improved understanding of the changes in the geometric properties of the Earth's atmosphere.

Correlation with recent eclipses

The terminally partial eclipse of 1988 March 3

There has been disagreement concerning the 1988 March 3 lunar eclipse in several authoritative works where this event is predicted as either penumbral^{13, 17} or partial.¹⁸ Observation of this eclipse was conducted following computer analysis which had shown that, if the new geometric parameters are applied to the umbra (2.09% enlargement for the size and an oblateness[†] of 1/40 for the shape), then only a penumbral eclipse would be possible.

The ILEE independent analysis indicated that umbral contact would probably not occur and this

[†] The initial value of $\frac{1}{40}$ has since been modified as a mathematical error was found in the prediction program after this event. The new value of oblateness is $\frac{1}{102}$.

prediction was distributed¹⁹ to NAPO members with a request to observe any primary contacts to confirm if this theoretically terminally partial eclipse was penumbral only.

The observations reported did not conclusively verify the ILEE prediction that only a penumbral eclipse would occur; however, the few critical primary contact times made occurred at times nearer to opposition than predicted by the USNO, which gave some confidence in the ILEE.

A photometric record taken by Professor Fred Rost is shown in Figure 6, where the reduction in the relative magnitude of the Moon measured by an integrating light meter, occurred from 16h 10m to 16h 18m, which was around 60% of the USNO predicted time of partiality and agreed well with the ILEE modified predictions. This is discussed further in Appendix F. However, some observers reported primary contacts for this eclipse and these have been included in the overall average value for enlargement.

Partial eclipse of 1988 August 27

A similar prediction was made using the ILEE for time of totality for the partial eclipse of 1988 August 27,²⁰

Figure 6. Magnitude of Lunar Eclipse

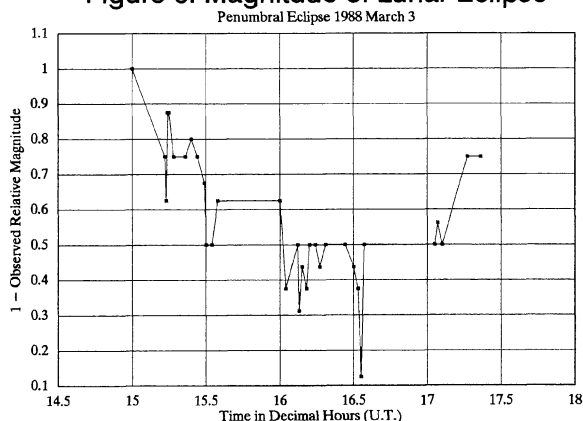
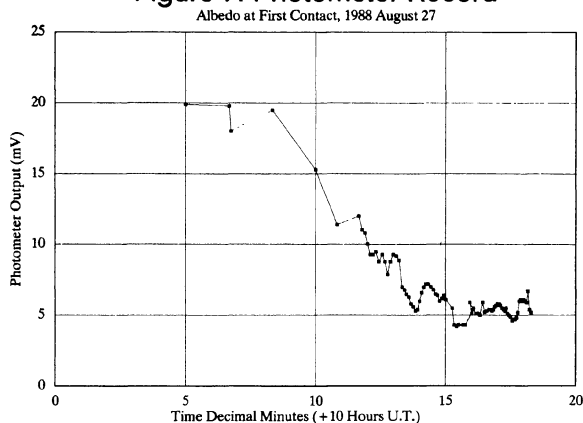


Figure 7. Photometer Record



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where first contact was expected to occur some 3 minutes later than the USNO prediction. This time estimate was found to be too large as shown by the crater timings received and the photometric observations by the author illustrated in Figure 7.

Total eclipse of 1989 February 20

The total lunar eclipse of 1989 February 20 was also observed to verify similar ILEE predictions for later first and second primary contacts. Only eight observers attempted this event as only limited announcements²¹ were made to Australasian astronomers for this eclipse. The results are discussed in Appendix E.

Conclusions

From the observation of eighteen lunar eclipses, the value used as the constant for umbra enlargement of 2% has been found to be very near the correct value, but requires a marginal increase. Considerable evidence has been collated to indicate that the oblateness of the Earth's atmosphere at the time of lunar eclipse has been grossly underestimated.

The analysis of many observations has shown that the astronomical constant for the oblateness of the Earth is too low by a factor of at least two, it should be increased using factors for geometric and tropospheric differences so that an improved relationship with the observed umbral oblateness can be formed.

When these new values for the constants and other improvements are incorporated within the basic theories¹ of Link, Meeus, Ashbrook, Synnott *et al.*, for the reduction of crater timings from the Besselian coordinates methodology, improvement is achieved in the predictions and reduction of lunar eclipse circumstances.

Recommendations

Finally, it is recommended that the values of two astronomical constants used in the prediction of lunar eclipse circumstances should be changed. The allowance for the enlargement of the umbra due to the effect of the upper atmosphere of the Earth should be slightly increased from 2.0 to 2.09%, and the oblateness of the Earth's atmosphere considerably increased by a factor of 2, and that the oblateness relationship found between this adjusted value and that of the observed umbra be recognised. Adoption and use of the ILEE should be encouraged to achieve more accurate prediction of lunar eclipse circumstances and for use in the reduction of the observation of primary contacts and crater timings.

It is anticipated that presentation of this work will influence the IAU Commission 4 (Ephemerides) to accept these new values for the constants and the oblateness relationship established and to recommend

adoption of the Improved Lunar Eclipse Ephemerides for use in the *Astronomical Almanac* in the near future.

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The analyses and observations supplied by Roger W. Synnott are also gratefully acknowledged and the independent analyses of umbral enlargement by W. Nijenhuis from the Netherlands as well as his best-fit-ellipse analysis for oblateness estimates.

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Appendix A

Method of analysis

Introduction

The micro-computer analyses of the crater timings were commenced in 1984 by the author with programs developed in GWBASIC, an extension of Microsoft BASIC, on a 256 Kilobyte RAM micro-computer, the SIRIUS 1 with dual floppy disk drives with the capacity of storing 1.2 Megabytes of data on each double sided formatted 5 $\frac{1}{4}$ inch floppy disk.

The approach and an outline of the strategies developed for the computer programs used, mainly a batch processing program, Reduction of Crater Timings by Date (RCTD.BAS), Crater Timing Oblateness Plot (CTOP.BAS), and the independent analysis Best Fit Ellipse by Date program (BFED.BAS) are given in Appendix B.

Methodology

The statistical analysis adopted considers all timings reported for contact of the umbra with both edges of features observed and timed for each phase of the eclipse, so providing up to four values where a crater is fully observed. The computer program used includes allowances for the diameter and positional change of the crater during each of these four contacts.

Mid-feature and point object timings such as mountains and prominences, are also reduced and corrections are made to all feature selenographic coordinates for lunar librations.

The uncorrected coordinates have been provided by the Arroral Valley Lunar Laser Ranging Station in a *Gazetteer of Named Lunar Features* produced by the Positional Astronomy Section, Division of National Mapping, Canberra which includes latitude and longitude data for 7074 recognised lunar features. The crater diameter data have been taken from several other sources.

The statistical analysis excludes any results of %E outside the limits set, either 0 to 3% based on the standard reported¹ previously by the author, or a more symmetrical range of 0 to 4% as adopted here. These criteria were based on acceptance of results within ± 2 standard deviations (sd). The standard error of the mean is $sd/\sqrt{n}$.

Oblateness estimates

The analysis used for the sensitive estimate of the observed oblateness of the umbra was presented by the author at the 9th National Australian Convention of Amateur Astronomers (NACAA) held in Geelong, Victoria, Australia in 1980. The oblateness, or flattening of the umbra is due to the non-spherical shape of the Earth's troposphere which influences the form of the shadow cast onto the Moon's surface at the time of eclipse.

The umbra shape is determined by using selected observer's results from the crater timing analyses for computed theoretical umbra radii, observed radii and corresponding position angle. Only those observers with a large number of crater timing observations which resulted in a standard error of the mean less than 0.15% have been included in these estimates.

Appendix B

Micro-computer program strategies

Introduction

The micro-computer programs developed by the author are based on earlier work which used several algorithms written with the assistance of David Herald in Canberra, for the

Hewlett Packard programmable calculators HP67 and HP41CV which feature 144 and 600 programming lines, respectively.

This approach was of great assistance in 1978, and improved further in 1981 with the more advanced HP41CV, but both entailed a considerable amount of data key-input, and as a printer was not available, a large amount of hand written reduction output on work forms was necessary.

The crater timing reduction micro-computer program suite was commenced in late 1984 with considerable development since then in my spare time, and includes routines for the altitude-azimuth of the Moon at times of primary contact, lunar eclipse circumstances, crater timing prediction and statistical-printing programs, which were all prepared in Microsoft GWBASIC (a graphics extension to Microsoft MSBASIC).

In addition, many ephemerides and observer data files were prepared, for the latter a self-developed data-base program (INPUTS.BAS) was used to provide correct sequential file format for all observers' timings to expedite reduction.

The two main programs used (RCTD.BAS and CTOP.BAS) are a considerable improvement in application and theory to the logic adopted in the Hewlett Packard versions, with the main areas in the crater timing reduction program (RCTD.BAS), which includes exact expressions for Besselian Elements etc. and compares output with the predictions published in the *USNO Almanac*.

Program strategies

The reduced crater timing programs were developed based on the programming strategy presented below:

1. Input 500 sets of crater data to RAM from CRATER-2.DAT sequential files as:
name, latitude (β), longitude (λ), diameter (or identifier)
2. Read one data set from 18 lunar eclipse ephemerides sequential file(s) as:
date, time reference (ET, DT or UT), solar semi-diameter ($S_\odot$), solar parallax ($\pi_\odot$), zero time, opposition time, mid-time, ΔT , lunar latitudes (β) for 1C to 4C.
3. Read one data set from 18 lunar ephemerides sequential file(s) as:
date, daily polynomial coefficients for δ , α and π_m
4. Read one data set from 18 solar ephemerides sequential file(s) as:
date, δ , α , librations in latitude (β_l), co-longitude ($90 - \lambda_l$) and PA of axis (ψ) of the Moon for four hours centred on mid-time of the eclipse.
5. Read observations from batched observer data file(s) prepared as:
name, date, number of lines of timings, time format (H.MS, H.MM or H.HH), primary contact timings,...; crater name, timings for 1C, 2C, 3C, 4C; and where there are also mid-timings list these alternatively (or separately), as each line must have 4 entries as all data files are read sequentially. The data-base program (INPUTS.BAS) was designed to provide logic options to give this format.
6. Reduce all observations from batched observer file(s) as:
four primary contacts, crater timings for 1C, 2C, 3C, 4C, mid-crater contacts, or mountains, etc. and write data sequentially from buffer to disk.
7. Statistically eliminate and print batched observation reductions:
Use program PRMOD.BAS to find where %E is outside the range set for enlargement and write to disk and print results in a table format.
8. Estimate umbra oblateness from selected observer file(s) by using program CTOP.BAS to statistically evaluate all accepted crater timing results from observers with an error of the mean less than 0.15% and a large number of %E accepted values. Print Meeus geometric reciprocal oblateness, observer name, site, observed and theoretical oblate-

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ness with corresponding probability and tolerances using null hypotheses with t , Z statistical criteria. Where the oblateness observed is statistically accepted, plot linear regression diagrams.

9. Provide reduction results report sheets to each observer as an important feedback using program PRCTRM.BAS, in the form of observer name, number of observations submitted, immersions or emersions, crater, contact type, time, theoretical umbra radius (R_c), observed umbra radius (R_o), difference in radii ($R_c - R_o$), % umbra enlargement and position angle on the Umbra (PSI) and, where appropriate, their oblateness estimates and linear regression diagrams.

Appendix C

Terminally partial eclipse of 1988 March 3

Introduction

The circumstances for this eclipse using a non-oblate umbra were published by the USNO, predicting an umbral phase of only 13 minutes 24 seconds. The ILEE developed by the author, showed that only a penumbral eclipse would be possible when using the new values for the lunar eclipse constants for improved predictions. A NAPO¹⁹ circular was forwarded to participating observers to verify the prediction.

Observations

Several astronomers, including the author, were prevented from observing this eclipse due to very heavy cloud over South Australia and Canberra. However, the reports received are given with analysis of the primary contact observations in a computer generated table.

Conclusions

The normal analysis of these timings gave large umbral enlargement of 2.83% for timed 1C and 2.21% for 4C.

For a partial eclipse to have occurred the enlargement of the umbra would have had to be large, 2.8% at first contact. Due to the extreme difficulty of timing 1C, it is doubtful that this value is reliable.

Appendix D

Partial eclipse of 1988 August 27

Introduction

The circumstances for this eclipse using a non-oblate umbra were published by the USNO, predicting an umbral phase of 1 hour 54 minutes 12 seconds. The ILEE, showed that first contact would occur 3 minutes later than predicted and with a shorter umbral phase. A NAPO¹⁹ circular was forwarded to participating observers to verify this prediction.

Observations

Several astronomers observed this eclipse including the author, and where primary contacts were timed, two obtained a first primary contact later than predicted by the USNO while one did not. However, analysis of the observations submitted is shown in Fig. 2.

The author's observations for first contact were photometric²¹ and Figure 7 illustrates the measured change in integrated albedo during the first primary contact, with its minimum value at 10h 10m 23s and a maximum slope in the brightness curve which occurred at the USNO predicted time

of 10h 07m 30s. The mean values for umbral enlargement were 2.27% for 44 immersions and 1.95% for 75 emersions.

Conclusions

The later times for primary first contact and other crater timings agree sufficiently well with that predicted by the ILEE to support change to the current lunar eclipse ephemerides.

Appendix E

Total lunar eclipse of 1989 February 20

Introduction

This event presented the first opportunity to test the ILEE for prediction of primary contacts during a total lunar eclipse, and in particular for first and second contact where the geometry of this eclipse gave maximum change due to the expected umbral oblateness effects at the respective position angles. It was found that with an umbral enlargement of 2.15% and an initial oblateness of 1/40, first and second contact would occur later than the times given in the *Astronomical Almanac*.

The NAPO publication IRIS was expected to promulgate these later predictions with my *Observers' Guide* for this eclipse but failed to do so due to recent administrative changes. A limited distribution of the *Guide* was made by the author at short notice and in the ASSA Bulletin,²¹ which resulted in ten observers providing crater timings with most reporting the important primary contact timings. The timings provided by these observers with umbral enlargement values within the statistical range of 0 to 4%, gave a total of 560 accepted observations with 189 contributed by Harry Moller and his wife Christine observing from Kingsley and a record 215 timings from Maurice Clark at Armadale; both of these sites were near Perth, Western Australia.

Observed oblateness

The oblateness of the umbra has been revised to a little less than half the 1/40 value given in earlier work by correction to a mathematical error. This revised value has been confirmed from this eclipse by the 185 immersion observations which gave a statistically accepted mean oblateness of 1/181 with an error of the mean of 0.05%, and a greater value of 1/74 $\pm$ 0.04 from 67 emersions.

**Table E1. Primary contact timings (H.mmss),
1989 February 20.**

Source or observer and location	1C	2C	3C	4C
Ephemeris (USNO)	13.4331	14.5541	16.1505	17.2716
Ephemeris (ILEEP.BAS) [†]	13.4359	14.5626	16.1523	17.2772
Soulsby Paradise-SA	13.4319*	14.5610*	16.1505*	—
Miller Welland-SA	13.4350	14.5626	16.1459	17.2710
Horton Gawler-SA	13.4420	14.5505	16.1545	17.2725
Moller Kingsley-WA	13.4202	14.5530	16.1424	17.2647
Price Bethanga-VIC	13.4323	14.5613	16.1527	—
Clark Armadale-WA	13.4311	14.5554	16.1442	17.2836
Blewett Tumut-NSW	—	14.5710	—	17.2809
ASAW Albury-W-VIC	13.4300	—	—	—
Anderson The Gap-QLD	—	—	—	17.2738

[†] Using $f = 100 \times$ Video record — see text.

Improved lunar eclipse ephemerides

Observed enlargement

The overall umbral enlargement for the eclipse was $2.24 \pm 0.08\%$ from 334 immersion crater timings and from 239 emersion timings it was $2.00 \pm 0.09\%$.

Correlation with ILEE

With only a small number of observers participating in this eclipse it was difficult to confirm the ILEE predictions for change in the first two primary contacts, and the contact timings reported indicated inconsistent departure from the USNO predictions as shown in Table E1.

The maximum departures from the USNO predictions were +49s for 1C (Horton) and for 2C + 45s (Miller). Additionally, CCD video records taken by the author during the eclipse gave a good record of 2C occurring about 30s later than the USNO prediction, about two thirds the difference predicted by the ILEE.

Conclusions

The ILEE prediction for later primary contacts prior to totality have not been completely verified during this eclipse. However, an improved value for the umbral oblateness has been found and is supported by this event, as is the general value of 2.10% for enlargement. As the initial predictions for improved primary contacts were made using $\%E = 2.15$ and $f = 1/40$, modification using 2.10% and 1/100 respectively, gave ILEE predicted times of 28s later than the USNO ephemerides for first contact and 45s later for second contact. However, the CCD video records obtained by the author for both of these primary contacts gave first contact 12s earlier and second contact 29s later than the USNO values, that is some 40s and 16s earlier than the ILEE modified predictions.

Total eclipse of 1989 August 17

The total eclipse of 1989 August 17 was not visible in the southern hemisphere. Using the larger oblateness of 1/40, predictions for later 1C (53s) and 2C (1m 27s) in the *Observer's Guides* were forwarded to South America, South Africa and to Europe, and in particular to the Lunar Section of the BAA. Modified ILEE predictions using the current value of 1/100 for umbral oblateness reduced the expected delay in the times of these primary contacts to 26s and 13s respectively. As expected these small differences were difficult to observe and time exactly as primary contacts for purposes of comparison with the USNO ephemerides.

Appendix F describes the modifications to these predictions. The observation of this eclipse confirmed the ILEE predictions for later primary contacts, as described in Appendix G.

Total lunar eclipse of 1990 February 9

The modified ILEE has been applied to produce primary contact timings for this eclipse visible in favourable sites in both hemispheres, and presented another good opportunity to verify these predictions. A preliminary ephemeris gave later contact times for 2, 3 and 4C only, as first contact was near the umbral equator where the effect of oblateness is minimal.

This eclipse was observed for all four primary contacts (and of course for crater timings) in the United Kingdom, Italy, Spain, Belgium, Johannesburg South Africa, Perth Australia, and for 1 to 3C in South Australia, but only for 1 to 2C in the east of Australia. The eclipse was not visible in South America or New Zealand.

A comprehensive *Observer's Guide and Crater Timing Prediction Sheet*, using final predictions from the ILEE, was

provided to corresponding observers at the sites mentioned above.

Appendix F

Improved lunar eclipse ephemerides prediction

Introduction

Even though high accuracy plotting of the computer generated Besselian coordinates, the semi-diameter of the Moon and umbra was used initially, it was decided to improve one of my programs for contact prediction to give greater flexibility by using the Improved Lunar Eclipse Ephemerides or ILEE theory.

The 60CTP.BAS crater timing prediction program was extended to ILEEP.BAS which produces prediction of primary contacts and mid-time contacts for 60 selected craters with allowances included for the observed umbral enlargement of 2.09% and oblateness of 1/102.

The initial value of the umbra, $p_2(1)$ is given in the *Astronomical Almanac* at a position angle of 45° , and this was used to correct the umbral radius p_2 at position angles from the pole to equator using the observed oblateness. The predictions generated were then checked against observation timings for all primary contacts and for some of the 60 craters used in the program.

ILEEP.BAS theory

Based on the linear regression analysis used for the estimates of observed umbral oblateness, it can be shown that the reciprocal oblateness $f = A/2*B$ where A = umbra radius at 45° latitude (p_2) and B is the slope coefficient of best fit linear regression. Hence, at any position angle on the umbra the approximate umbral radius is found from:

$$p_2 * 2*B = p_2(I) * (1 + \cos(2*\psi)/2*f(I))$$

here, $p_2(I) = A * 2*f/(2*f + 1) * 1.209/1.2$ (Fi)

Using this improved value of umbral radius at any position angle ψ , the predicted time of contact can be modified by using the new radius in the quadratic equation solution for t , the time of contact as given in the *Explanatory Supplement*.⁴ p. 257-262 This gives a time of contact of sufficient accuracy to see a change when introducing f values of 102.

Correlation

Running the ILEEP.BAS program for the last three lunar eclipses produced interesting results with extremely good correlation with the high accuracy plots for primary contacts using computer generated Besselian elements, and gave contact times nearer to those observed as shown in the following Table F1.

Observed values

The observed timings for primary contacts are given in some detail in each of the relevant appendices for the four recent

Table F1. Comparison of ILEEP.BAS predictions with USNO (H.mmss).

	88 Mar 3		88 Aug 27		89 Feb 20		89 Aug 17	
	USNO	ILEEP	USNO	ILEEP	USNO	ILEEP	USNO	ILEEP
1C	16.0606	16.0946	10.0724	10.0901	13.4324	13.4359	1.2039	1.2105
2C	—	—	—	—	14.5542	14.5626	2.1955	2.2008
3C	—	—	—	—	16.1500	16.1523	3.5631	3.5641
4C	16.1930	16.1439	12.0141	12.0131	17.2712	17.2722	4.5547	4.5538

eclipses. However, the ILEEP.BAS predictions are more consistent in their departure from the USNO values than that given by high accuracy plotting and will be used for the event of 1990 February 9. The predictions published²² for 1989 August 17 required small modification to the times of primary contact to that shown in Table F1.

Table G1. Comparison of ILEEP.BAS predictions with USNO (H.mmss).

	1C	2C	3C	4C
USNO	1.2039	2.1955	3.5631	4.5547
ILEEP	1.2105	2.2008	3.5641	4.5538
Observed values for this eclipse were as follows:				
<i>n</i>	4	7	4	3
Timings	1.2106	2.1946	3.5614	4.5537

Appendix G

ILEE and the total lunar eclipse of 1989 August 17

Introduction

The observations of this eclipse were made by 18 astronomers in the UK, the Netherlands, Portugal, Belgium, Italy and Spain, as well as in Brazil, South America. The mean values of enlargement were $2.14 \pm 0.15\%$ from 245 immersions, and $1.91 \pm 0.26\%$ from 74 emersions.

It was decided to improve the analysis by determining the value of K_s , the ratio of the observed umbral oblateness to FM; the Meeus theoretical or geometric value, using an initial oblateness for the Earth's atmosphere of $F_e = K_m * FM$, from which values of $K_s = F_o / FM$ were found.

This allowed comparison of the variation in the relationship of the observed umbra to Earth's atmospheric oblateness

as modified by K_m and the derivation of actual K_s values for each eclipse where sufficient data was available from statistically acceptable observed oblateness. The values determined for the new constant K_s are illustrated in Figure 3A.

Comparison of primary contacts

The ILEEP program was used to predict primary contact times which due to the ILEE differ from that made by the USNO. The comparison of these times are shown in Table G1.

A reasonably close correlation was found for two of the primary contacts, first and fourth but the observed mean timings for second and third contact were both earlier than the USNO predictions.

A historic moment for the BAA



Garner Principal King of Arms, Sir Alexander Colin Cole, accompanied by Mr H. E. Paston-Bedingfeld, Rouge Croix Pursuivant of Arms, presents to the President of the British Astronomical Association, the Letters Patent setting out the Grant of Arms to the BAA during the Centenary Meeting on 24 October 1990. The Association was also honoured by the presence of the Earl Marshal, His Grace the Duke of Norfolk, for this ceremony.