

CONSTRUCTING A PULSAR TIMING ARRAY

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ABSTRACT

Arrival time data from a spatial array of millisecond pulsars can be used (1) to provide a time standard for long time scales, (2) to detect perturbations of the Earth's orbit, and (3) to search for a cosmic background of gravitational radiation. In this paper we first develop a polynomial time series representation for these three effects that is appropriate for analysis of the present data with its limited degrees of freedom. We then describe a pulsar timing array program that we have established at the National Radio Astronomy Observatory 43 m telescope with observations of PSR 1620–26, PSR 1821–24, and PSR 1937+21. The results presented in this paper cover a 2 yr period beginning in 1987 July. Individual parameters of these objects are compared to previous measurements. The influence of global parameters—clock, Earth location, and effects of gravitational radiation—on our data is discussed in the context of our polynomial model. Improvements in the data-gathering hardware and the inclusion of data from other observatories will lead to a significant increase in the sensitivity of this effort in the near future.

Subject headings: instruments — pulsars

I. INTRODUCTION

Millisecond pulsars act as precision celestial clocks owing to the combined effects of their large rotational energies, 10^{50-52} ergs, low energy loss rates which correspond to high mechanical Q s of 10^{19-20} , and intense microwave beams rigidly anchored in the stars. Precise timing observations of the first millisecond pulsar PSR 1937+21 (Davis *et al.* 1985; Rawley *et al.* 1987; Rawley, Taylor, and Davis 1988) have demonstrated a fractional frequency stability that is comparable to the best terrestrial time standards on time scales near 1 yr. Pulsar timing requires a transformation from the observer's frame on the moving Earth to the inertial reference frame of the pulsar. The precision of the pulsar observations is now near the limit of models of Earth's motion (Hellings 1989a). The existence of a number of millisecond pulsars distributed on the sky leads to the possibility of timing multiple pulsars against each other without ultimate reference to terrestrial clocks and solar system models. A terrestrial clock error is independent of pulsar direction and will have a monopole signature in the arrival time residuals of all pulsars. An Earth position error will produce a dipole signature in the residuals of objects distributed around the sky.

Detweiler (1979) pointed out that pulsars could be used as point masses in a search for passing gravitational waves. Long-wavelength gravitational radiation that is crossing the solar system has quadrupole and higher order angular signatures that are distinct from the low-order multipole terms mentioned above. Romani and Taylor (1983) and Hellings and Downs (1983) analyzed the arrival time data of slow pulsars for the effects of gravitational radiation. Both papers placed limits on the energy density of an isotropic gravitational wave background of $\sim 3 \times 10^{-4} h_{100}^{-2}$ of the closure density per logarithmic frequency interval in the range of 4×10^{-9} to 10^{-7} Hz, where h_{100} is the Hubble constant in units of H_0 per 100 $\text{km s}^{-1} \text{Mpc}^{-1}$.

Davis *et al.* (1985) used the first 2 yr of timing data from PSR 1937+21 to constrain the energy density of gravitational-wave background radiation (GBR). Later work by Rawley *et al.*

(1987) has limited the GBR logarithmic density to $\Omega_{\text{GBR}} \simeq 4 \times 10^{-7} h_{100}^{-2}$ of the closure density of the universe from gravitational waves with periods near 2 yr. This limit places strong constraints on the possibility that cosmic strings were the sources of large-scale structure and the seeds for galaxy formation (Vilenkin 1981; Romani 1988). If timing residuals are dominated by white noise processes, then further improvements in the pulsar timing residual data will lower the upper limit on Ω_{GBR} in proportion to T^{-4} , where T is the time baseline for the pulsar timing observations. Romani (1989) has discussed methods of analyzing pulsar timing array data to place specific limits on the GBR energy density spectrum.

In this paper we discuss in § II the practical issues that arise in conducting a pulsar timing array experiment with existing instruments and the sparse collection of known millisecond pulsars. We present in § III observational results from the first 24 months of our timing program at the NRAO 43 m telescope in Green Bank, West Virginia. In § IV we analyze these data both for individual pulsar parameters and for global effects of clock instability, unmodeled Earth motion, and gravitational wave perturbations. In the conclusion we discuss directions for the future effort with this program.

II. PULSAR TIMING ARRAY CONCEPT

Timing data from an array of pulsars can be used to measure various effects that produce correlations between data from members of the array. The source of these effects arises in imprecision or incompleteness of the model used to analyze the pulsar data. This section discusses detection of these effects for the case of having a small number of pulsars and a small number of degrees of freedom.

Successive topocentric pulse arrival times from a pulsar are linked together by first referencing the individual pulse arrival times to a proper time scale, and then transforming each arrival time event to the solar system barycenter using a solar system model tabulated in an ephemeris. Astrometric parameters of each pulsar in the reference frame of the solar system model are required for this transformation. The arrival times at

infinite frequency are computed using measured values of the electron column density. In the inertial frame of the solar system barycenter the rotational and, if applicable, orbital parameters of each pulsar can be determined. The parameters for each pulsar must be determined with sufficient precision to remove period ambiguities in the comparison between the pulse arrival time model and the observations.

After the individual parameters of each pulsar have been determined, all of the individual parameters and a number of global parameters can be refitted as part of a global solution. We identify the global parameters according to their multipole effect on the timing residuals: fluctuations in the primary terrestrial time standard lead to a monopole effect, ephemeris errors in the transformation to the solar system barycenter produce a dipole effect, and perturbations produced by gravitational radiation from a stochastic background can be distinguished by their approximately quadrupole signature. While this procedure is discussed below, our data at the present time are insufficient to perform a meaningful global solution.

a) Individual Parameters

Our model t_{Ni} for the arrival time of the N^{th} pulse of the i^{th} pulsar consists of a transformation from the observatory space-time event to the corresponding neutron-star emission event (e.g., Backer and Hellings 1986; Hellings 1986). We are mainly interested in the periodic terms in this transformation since constant and secular terms will be absorbed by the rotational parameters of the neutron star. The periodic terms in the spatial transformation are adequately represented by a scalar product between the location of the observatory with respect to the barycenter of our solar system, $\mathbf{r}(t_N)$ and the pulsar's position which is a slow function of time, $\mathbf{n}(T)$. In the case of a binary pulsar an additional scalar product is required between $\mathbf{n}(T)$ and the vector from the binary barycenter to the orbiting pulsar. When the highest precision timing residuals are available, a parallactic term, which needs the pulsar distance D , is required. The periodic terms in the temporal part of the transformation incorporate all relativistic effects, Δt_R . In addition we represent the correction from the observatory's UTC time scale to that of a standard atomic time scale as Δt_c .

An independent source of error in the transformation to the neutron star emission event is introduced by propagation delays caused by the intervening column of thermal electrons. Multifrequency observations are used to determine the dispersion measure, $DM(T)$, which is a slow function of time T , and then to extrapolate the arrival time to infinite frequency. If the interstellar electron turbulence is modeled as a Kolmogorov power spectrum (Armstrong 1984), then we can estimate the amplitude and time scale of propagation delays as a function of dispersion measure, time, and radio frequency (Foster and Cordes 1990). Variations of the integrated electron column density produce delays Δt_{DM} on a time scale τ at a radio frequency ν whose rms amplitude grows roughly as

$$\Delta t_{DM} \simeq 0.05 \mu\text{s} \left(\frac{DM}{\text{pc cm}^{-3}} \right)^{0.5} \left(\frac{\nu}{\text{GHz}} \right)^{-2} \left(\frac{\tau}{\text{months}} \right)^{5/6}. \quad (1)$$

Rawley, Taylor, and Davies, (1988) and Cordes *et al.* (1990) have reported detection of these variations for PSR 1937+21, and Foster, Backer, and Wolszczan (1990) have observed similar effects for PSR 1951+32. The dispersive delay in the interplanetary medium will vary throughout the year. The delay at 1 GHz for an inverse square law model of the electron

density is $0.08 \mu\text{s}$ at an elongation of 30° using a density at $10 R_\odot$ from Counselman and Rankin (1972).

Delay fluctuations produced by variations in refractive bending of radio waves as they propagate through the turbulent interstellar medium are more difficult to model. The refractive time delay introduced by an increase in the geometric path length grows roughly as

$$\Delta t_{\text{geo}} \simeq 0.012 \mu\text{s} \left(\frac{DM}{\text{pc cm}^{-3}} \right)^{0.5} \left(\frac{\nu}{\text{GHz}} \right)^{-3.3} \frac{\tau}{\text{months}}, \quad (2)$$

where τ saturates after a few months for DM s near 50 pc cm^{-3} (e.g., Foster and Cordes 1990). If more than one term is required for the propagation delay, then observations at several radio frequencies are needed. In this case the effects of pulse broadening by diffraction, which vary roughly as $10^{-5} \mu\text{s} DM^{3.0} \nu_{\text{GHz}}^{-4}$, can also be considered.

After these transformations and corrections are applied, the data can be compared to a simple power-law model for the rotation in terms of sequential pulse number N with coefficients for the initial epoch t_0 , the initial period P_0 , and the first two derivatives of the period, $\dot{P}_0$, $\ddot{P}_0$. The total model is then

$$\begin{aligned} t_N = & t_0 + P_0 N + P_0 \dot{P}_0 N^2/2 + P_0^2 \ddot{P}_0 N^3/6 \\ & + \frac{1}{c} \mathbf{r}(t_N) \cdot \mathbf{n}(T) + \frac{1}{2cD} \{r^2(t_N) - [\mathbf{r}(t_N) \cdot \mathbf{n}(T)]^2\} \\ & + \Delta t_R + \Delta t_c + A \cdot DM(T) \nu^{-2} + \Delta t_{\text{geo}}. \end{aligned} \quad (3)$$

Additional terms are required for pulsars in binary systems. If the electromagnetic torque that is responsible for pulsar spin-down dominates the second derivative of the pulsar period, then the second derivative can be related to the first derivative of the pulsar period by the "spin-down" index n : $\ddot{P} = (2 - n)\dot{P}^2/P$. The observed spindown indices of three young pulsars are close to 2.7. The time interval for which the cubic term resulting from electromagnetic torques is significant can be estimated as

$$T_{EM} = 50y P_{-3} X^{0.33} (\dot{P}_{-18})^{-0.67}, \quad (4)$$

where P_{-3} is the pulse period in units of milliseconds, $\dot{P}_{-18}$ is the period derivative in units of $10^{-18} \text{ s s}^{-1}$, and $X \simeq O(10^{-4})$ is the rms timing residual measured in units of the pulse period and reduced by the square root of degrees of freedom. A second contribution to $\ddot{P}$ can arise from Doppler acceleration in a globular cluster (Wolszczan *et al.* 1989).

The short rotation times (ms), gaps of days or even months between observations, and the large transformation to the solar system barycenter (500 s) all lead to the requirement for an iterative solution. Individual pulsar parameters are determined by looking first at short segments of data and then at successively longer time intervals. Residual arrival time measurements without period ambiguities are obtained by subtracting the arrival times based on the parameter model from the measured arrival times. Taylor (1989) presents a succinct review of these procedures for both single and binary pulsars.

b) Global Parameters

The arrival time residuals from an array of millisecond pulsars distributed widely across the sky can be fitted globally to various perturbations with distinct angular and temporal

signatures. In this section we consider a procedure for the analysis of data recorded from a sparse array of pulsars sampled at uniform time intervals over a small number of years. The angular signatures are decomposed into multipole terms. In addition, each perturbation will have unknown temporal variations. We can determine global parameters only if they produce temporal variations that are distinct from those of the individual pulse parameters discussed in the preceding section. We consider expansion of temporal variations as polynomial series which are appropriate both because the leading terms are matched to individual pulsar parameters, and because the remaining terms are matched to the expected slow variations in the global parameters. The polynomial approach provides a minimum degree of parameterization which is appropriate for the low number of degrees of freedom in current epoch averaged data sets.

i) Clock Errors

Comparison of the best Earth atomic time scales indicates that they are dominated by flicker frequency noise for durations of a few months and then a random walk in frequency for durations exceeding a year (e.g., Guinot 1988). The spectrum of these variations is definitely "red," so that an expansion of the clock error as a polynomial series is appropriate:

$$\Delta t_c(T) = c + \dot{c}T + \ddot{c}T^2|_2 + \ddot{c}T^3|_6 + \cdots \quad (5)$$

This clock expansion mimics the terms of the pulsar rotation polynomial presented in the previous section (eq. [3]). The constant c reflects the arbitrary epoch of the atomic time scale with respect to the pulsar rotations. The linear term $\dot{c}$ reflects the arbitrary definition of the second with respect to the pulsar rotation rates at the reference epoch. Note that the second is defined exactly in terms of an atomic transition of the cesium atom transformed to a reference frame rotating uniformly about the Sun. A quadratic term in the Earth atomic time scale $\ddot{c}$, if sufficiently small, will be absorbed into the period derivatives of all pulsars. The requirement is that the clock perturbation must keep the period derivatives of isolated pulsars positive. This requirement corresponds to a fractional frequency stability limit of 2×10^{-10} for durations of 1 yr, which is many orders of magnitude larger than the current clock performance (Guinot 1988). The requirement for globular cluster pulsars is relaxed owing to the presence of unknown accelerations in the cluster core (Wolszczan *et al.* 1989). The cubic term expected from clock performance $\ddot{c}$ is orders of magnitude larger than the effect that is predicted from the spindown behavior of pulsars. We conclude that a correlated temporal cubic term with a monopole angular signature will dominate the pulsar residuals:

$$\Delta t_c(T) = \ddot{c}T^3|_6 \quad (6)$$

This is the minimum parameterization possible for the clock, or monopole term.

In addition to the slow variations discussed above, steering of Earth atomic time scales will lead to abrupt jumps in the reference time standard, and there may be annual terms in the time scales (Guinot 1988). If clock steering corrections cannot be removed from the Earth-based atomic time standard, then the residuals of the most precisely timed pulsar, or a suitably weighted sum of all residuals, must be used as the reference. Effectively one pulsar in the array is used as the clock and absorbs one global degree of freedom per epoch.

ii) Ephemeris Errors

Ephemeris errors can be divided into those with periods less than that of the data set, and secular errors which may or may not have periods longer than the data set. The current level of precision of the inner planet masses suggests relative accuracies of the ephemeris around 1 m (3 ns) over time scales of several years (Standish 1989). This is a considerable improvement over accuracies estimated by Mulholland (1971).

Uncertainties in the outer planetary masses lead to ephemeris errors that grow slowly over time scales which are much shorter than the orbital periods. These uncertainties contribute the largest errors in the planetary ephemerides in current use. Improvements are expected with the next generation of planetary probes. For example, the recent *Voyager* spacecraft encounter with Neptune has led to a mass correction by 0.5% (Hellings 1989b) which will shift the barycenter location by 3600 μ s in future ephemerides. A mass error will lead to an acceleration derivative of the barycenter:

$$\delta \ddot{r} = \frac{2\pi G \delta M_x}{R_x^2 P_x} \quad (7)$$

where δM_x is the error in the planetary mass, R_x is the distance of the planet from the Sun, and P_x is the planetary period. The numerical limit from current uncertainties in the mass of Neptune is about $2.0 \times 10^{-19} \text{ cm s}^{-3}$. The limits from uncertainties in the masses of Jupiter and Saturn are close behind at $0.7 \times 10^{-19} \text{ cm s}^{-3}$ and $1.0 \times 10^{-19} \text{ cm s}^{-3}$.

Periodic errors in Earth's orbit can be divided into range errors and orientation errors. A simple model of the effects on an orbiting body by a remote perturbing mass shows that these errors will have terms with constant and linearly increasing coefficients (Taff 1985). While only the orientation errors will be absorbed by position and proper motion fits, the common period of 1 yr for both range and orientation errors will lead to strong coupling of these effects by the position and proper motion fits. Perturbations from the other planets will have distinct periods that are not matched by any other fit parameter. The extensive timing of PSR 1937+21 at Arecibo (Rawley *et al.* 1987 and Taylor 1989) limit the level of unabsorbed periodic errors to about 300 ns.

Orientation errors have been discussed in reference to the tie between the extragalactic reference frame established by VLBI techniques and that of planetary observations. The absolute accuracy of the planetary inertial reference frame is 0.01 at present with a mean motion of no more than 0.1 per century (Standish 1988). Limits on the acceleration of the frame are not well known, but are probably less than 0.05 century⁻² (Standish 1989). There are still large and unresolved discrepancies between the radio interferometry reference frame and pulsar timing reference frame which can be as large as 0.3 (Fomalont *et al.* 1984; Bartel *et al.* 1985).

We choose to expand slow ephemeris errors in a time polynomial similar to what was done for the clock error:

$$\Delta t_{\delta r}(T) \simeq \frac{1}{c} (\delta r + \delta \dot{r}T + \delta \ddot{r}T^2|_2 + \delta \ddot{r}T^3|_6 + \cdots) \cdot \mathbf{n}(T) \quad (8)$$

Each term in the equation above has a dipole signature for pulsar positions $\mathbf{n}(T)$ around the sky. The first three terms will be absorbed by the pulsar spin parameters with a different amount going to each object. Note that Harrison (1977), and more recently Cowling (1983), have looked at the spatial dis-

tribution of period derivatives to assess whether there is a bias from an unmodeled solar companion. The maximum secular acceleration that will escape detection as a negative apparent period derivative is given by the minimum observed $c\dot{P}/P$ which is $1.5 \times 10^{-8} \text{ cm s}^{-2}$ for PSR 1953+29. These slow effects include the unknown motion of the solar system in the interstellar and galactic potentials. Similarly, proper motions lead to an effective acceleration (Shklovskii 1969) which is also indistinguishable.

What remains after fitting for P , $\dot{P}$, and position is a cubic term if the coefficient dominates the corresponding apparent spindown rate ($\ddot{P}$ term)

$$\Delta t_{\delta r}(T) = \frac{1}{c} \delta \dot{r} \cdot \mathbf{n}(T) T^3|_6. \quad (9)$$

Following the discussion above we can expect to detect significant ephemeris errors only if the measured differential accelerations from planetary mass errors dominate the corresponding T^3 (or N^3) term from electromagnetic torques; i.e.,

$$\delta \ddot{r} > 3 \times 10^{-20} \text{ cm s}^{-3} \frac{\dot{P}^2_{-18}}{P^2_{-3}}. \quad (10)$$

The current ephemeris accuracies are comparable only to the accuracy obtainable from pulsar timing data.

iii) Gravitational Radiation

A gravitational wave train, which has amplitudes $A_+(T - s/c)$ and $A_\times(T - s/c)$ at a distance s along its path would produce Doppler shifts in the apparent rotational rates of pulsars when the wave train passes through the solar system. The Doppler shifts are correlated around the sky with a distinct angular signature (e.g., Detweiler 1979)

$$z(T) = \frac{(\alpha^2 - \beta^2)A_+ + 2\alpha\beta A_\times}{2(1 + \gamma)}, \quad (11)$$

where α , β , and γ are the direction cosines of the pulsar-Earth line with respect to the Earth-centered wave reference frame, and where the wave is traveling along the $+z$ -axis. In polar coordinates (η, ψ) this reduces to

$$z(T) = \frac{1}{2}(1 - \cos \eta)[\cos 2\psi A_+(T - s/c) + \sin 2\psi A_\times(T - s/c)]. \quad (12)$$

The 2ψ variations indicate an angular signature of quadrupole and higher orders which are distinct from the monopole and dipole effects discussed above. Arrival times are affected by the integral of the Doppler shift. The expression for an arbitrary wave direction is obtained by a two-angle rotation. The wave components A_+ , $A_\times$ specify the instantaneous polarization of the radiation.

Gravitational radiation has at least three different sources: (1) monochromatic sources such as a highly relativistic neutron star or black hole binary, (2) broad-band impulse events of very short duration such as a supernova explosion, and (3) a stochastic background left over as a cosmological remnant. We could search for the effects of discrete sources in the pulsar timing data, but we would need to be lucky, since pulsar timing data are sensitive only to a narrow window in the gravitational wave spectrum that is defined by the data sampling frequency and the duration of the measurements. In this paper we are mainly interested in using the pulsar signals as a probe of the

stochastic background of gravitational radiation (GBR), which is analogous to the 2.7 K microwave background radiation (MBR). The signature of the GBR in the pulsar data is not readily transformed into a multipole angular model with temporal polynomial coefficients.

The metric strain tensor, $h_{ij}(\mathbf{x}, t)$, that specifies the GBR in a volume around the solar system and over a time interval surrounding the present can be Fourier-transformed into an angular and frequency spectrum of gravitational waves, $\tilde{h}_{ij}(\mathbf{k}, \omega)$. The true spectrum of h_{ij} is unknown. The GBR is not like the MBR where the statistics are specified by the tenets of thermodynamics. Furthermore, in our probe of the GBR we are observing the moment-to-moment evolution of the metric strain, whereas with MBR measurements one integrates over $\sim 10^{12-16}$ cycles of the characteristic oscillation frequency. The analog of our GBR measurement in the case of the MBR is a measurement at some point in space of the electric field from picosecond to picosecond, rather than a measurement of statistically averaged, and well-defined, quantities like temperature, polarization, and spectrum.

The average GBR logarithmic energy-density spectrum is

$$S_\omega = \frac{1}{8\pi G} \left(\frac{d\tilde{h}_{ij}}{dt} \right)^2 = \frac{\tilde{h}_{ij}^2 \omega^2}{8\pi G}. \quad (13)$$

Assuming that the spectrum S_ω is flat, then typical strain amplitudes, $\tilde{h}_{ij}(\mathbf{k}, \omega)$, will scale as ω^{-1} . The effect of a strain wave at ω on the arrival time residual will grow as ω^{-1} because the Doppler shift will maintain one sign for a longer interval at a smaller ω . The combined effects of amplitude and frequency lead to expected timing residuals that grow with ω^{-2} , or T^2 . Thus we expect that the slowest variations not absorbed by the period polynomial fit will be the first that can be detected.

All Doppler shifts from the GBR metric strain tensor, h_{ij} , that are constant during the observing interval will be absorbed into individual periods. The first derivatives of the Doppler shifts will also be absorbed by the period derivatives. The second derivatives of the Doppler shifts will produce the first independently detectable effects of the GBR, if the effects exceed those resulting from individual pulsar spin-downs. Unfortunately, there is no simple measurement scheme for deriving the minimum parameters of this effect. The amplitude of the cubic phase residuals that remain after fits for clock and ephemeris effects can be used to constrain the level of the GBR by averaging the derived wave amplitudes over all possible wave directions. Romani (1988) discusses the analysis of a pulsar timing array for a full spectrum of waves.

iv) Summary

The three effects discussed in this section have leading terms which are independent from the individual pulsar parameters and are first manifest as cubic polynomials in time. The clock and ephemeris cubic polynomials can be directly fitted in a global solution. The remaining residuals in the period second derivatives can provide a statistical estimate of the likely level of the gravitational-wave background on the longest possible time scales. These statements assume that the amplitude of these effects is larger than the period second derivatives arising from electromagnetic torques or from acceleration-induced torques in the gravitational field of globular clusters. If not, then the next higher polynomial terms, the quartic term, can be considered.

c) Experimental Design

The goal of our experiment is to construct a pulsar timing array using known pulsars and existing equipment. We confined our observations to a single frequency owing to telescope hardware and scheduling constraints. Use of a single frequency makes the results sensitive to arrival time variations that result from changes in the dispersion measure. We selected the highest practical observing frequency to reduce this effect. The dominant dispersion measure variation in PSR 1937+21 is a linear decrease which leads to a period shift in our data (Cordes *et al.* 1990). Dual frequency timing observations are needed to remove the effects of variable dispersion measure delays in order to achieve 1 μ s timing accuracy.

Since the timing parameters for each pulsar are individually fitted using a least-squares process, we can only begin to solve for global parameters after the number of degrees of freedom for the individual pulsars becomes large. A solitary pulsar has a minimum of five free parameters, while binary pulsars have a minimum of 10. Observations over several years, and for binary pulsars several orbits, are required to accurately determine these individual parameters. After this initial start-up time, each additional year of observing adds significantly to the degrees of freedom in any global solution.

At present the stronger millisecond pulsars are confined to a narrow strip between the Galactic center and the Cygnus region. If an *a priori* assumption can be made about the nature of any of the perturbations, then the number of pulsars needed to determine global parameters will be reduced. For example, if only a clock effect (monopole) is relevant, then any two millisecond pulsars can be used. One effectively is timing one pulsar against the other and excluding the time scale of the terrestrial "second." If drifts in the time standard are negligible, then three distributed pulsars are needed to fit for an ephemeris effect (dipole)—an ultimate form of celestial navigation by triangulation.

If the time scale is not negligible, and this is probably the case, then one pulsar is needed to solve for the clock term, and three additional pulsars distributed around the sky are needed to solve for an ephemeris term. Ninety degree separations provide orthogonality. The "clock pulsar" can be spatially coincident with one of the other three. If the leading polynomial terms are cubic as suggested above, then a minimum of four temporal degrees of freedom are required to determine the various coefficients. A fifth pulsar away from the other four would leave an extra degree of freedom upon which an upper limit can be placed on the amplitude of the remaining residuals that could be ascribed to the GBR. The quadrupole signature of gravitational radiation leads to desirable angular spacings of multiples of 45°.

In summary, an effective pulsar timing array requires a minimum of five precisely timeable pulsars widely separated on the sky. The pulsars should be observed at a minimum of two frequencies to remove dispersion measure variations. The upper limit on sensitivity to gravitational radiation grows rapidly with the length of the observations. The sample interval between observational epochs is set by the need to tie down astrometric and any binary parameters for the pulsars. Data from telescopes around the globe can be brought together to increase the angular coverage of the pulsar timing array via the use of a single pulsar as a reference object. *A priori* connection is possible in principle, but insufficient attention has been given to corrections for the signal processing artifacts in different pulsar timing programs.

III. OBSERVATIONS

Following the discovery of the 3.05 millisecond pulsar PSR 1821–24 in the globular cluster M28 (Lyne *et al.* 1987), we began a pulsar timing array program using the National Radio Astronomy Observatory (NRAO) 43 m radio telescope in Green Bank, West Virginia. The initial goal of our experiment was to obtain high-precision timing data from the two fastest and most widely spaced millisecond pulsars, PSR 1821–24 and the 1.56 millisecond pulsar PSR 1937+21 (Backer *et al.* 1982). The 49° spatial separation between these two objects would allow us to place limits on a dipole perturbation. We planned to include new millisecond pulsars in our timing program as they were discovered. In 1988 January we added the 11.08 millisecond object PSR 1620–26 found in the globular cluster M4 (Lyne *et al.* 1988; McKenna and Lyne 1988). The angular separations between PSR 1620–26 and the other two pulsars, PSR 1821–24 and PSR 1937+21, are 27° and 67°, respectively. We attempted to include the 195 millisecond binary pulsar PSR 0655+64 (Taylor and Dewey 1988; Jones and Lyne 1988) in our array, but the signal was extremely weak at our fixed observing frequency of 1330 MHz. The interval between each observing epoch was 2 months, or six epochs per year.

The pulsar arrival time measurements were recorded using the filter bank technique discussed by Taylor and Weisberg (1982). A dual-mode feed and cooled HEMT amplifiers were used to obtain an antenna gain of 0.28 K Jy^{−1} and system noise temperatures at zenith of 20 K for both senses of circular polarization. The RF signal from the telescope was down converted to 30 MHz and passed into two filter-bank spectrometers each having 32 250 kHz channels. The maximum possible bandwidth coverage with this equipment was 8 MHz. The detected outputs from both senses of polarization were summed before being passed to a digitally clocked delay line dedisperser to remove dispersion effects from the interstellar electron plasma.

The dedispersed total power signal was averaged synchronously with the topocentric pulsar period as predicted from earlier observations. The number of samples per pulse period used for each pulsar varied: 512 for PSR 1620–26, 128 for PSR 1821–24, and 64 for PSR 1937+21. The total bandwidth varied for each pulsar as well, 4 MHz for both PSR 1620–26 and PSR 1937+21, and 8 MHz for PSR 1821–24. Because of the limited speed of the dedispersion clock, the actual pulse resolution is less than the sample resolution. The true pulse widths and the pulse widths measured by the dedispersion equipment are given in Table 1. The predicted arrival times for the 32-channel filter bank, which are estimated from the following formula (Backer 1989)

$$\sigma_{\text{rms}}(s) \simeq \frac{(\Delta t)^{3/2} T_k}{(B\tau P)^{1/2} \langle S \rangle G}, \quad (14)$$

are listed in Table 1; in equation (14) Δt is the half-width of the pulse in seconds, P is the pulse period in seconds, T_k is the system temperature on the source, $\langle S \rangle$ is the flux density in janskys, G is the telescope gain in kelvins per jansky, B is the bandwidth in Hz, and τ is the integration time in seconds. Table 1 also shows the arrival time precision theoretically obtainable with optimal sampling, using the NRAO 43 m telescope, given the listed system temperatures, bandwidths, and integration times. The integration times reflect the typical times spent observing each source over a 3 day epoch. The

TABLE 1
TIMING ABILITY OF ARRAY PULSARS USING NRAO 43 m TELESCOPE

Parameter	PSR 1620–26	PSR 1821–24	PSR 1937+21
True pulse width HWHM (μ s)	~ 250	45	25
Observed pulse width HWHM (μ s) using filter bank	270	260	145
System temperature on source (K) at 1330 MHz	38	38	22
Bandwidth (MHz)	4.0	8.0	4.0
Channels (250 kHz)	16	32	16
Assumed average flux density (mJy)	1.6	1.1	8
Integration time (hours)	12	12	6
Predicted timing residuals using actual setup (μ s)	8.6	15.9	1.5
Measure arrival time residuals (μ s)	45.3	21.6	4.5
Predicted timing residuals using true pulse width (μ s) 20 MHz bandwidth and 12 ^h integration times	3.4	0.7	<0.1

estimated arrival time precisions as predicted (see above) are probably accurate to within a factor of 2, with the pulsar flux densities and the effects of the exact pulse shapes being the largest sources of uncertainty.

Continuously averaged profiles were recorded on magnetic tape every 2–5 minutes along with the time as given by the observatory's hydrogen maser clock. The maser was monitored daily and calibrated against Coordinated Universal Time (UTC) as kept at the US Naval Observatory using signals from the Global Positioning System (GPS) of satellites (Weiss and Allan 1987). For each day of observation a correction was applied to remove the offset between the observatory's hydrogen maser and the USNO version of UTC. Time transfer between the USNO standard and the NRAO site is obtained to at least 100 nanosecond accuracy via the GPS satellites.

IV. ANALYSIS

The data presented in this paper were collected at 11 and 12 different epochs over a 24 month period beginning in 1987 July for PSR 1821–24 and PSR 1937+21, respectively. The pulsar PSR 1620–26 was observed for seven epochs over an 18 month period beginning in 1988 January. The last epoch of observation was in 1989 June. Thirty-minute scan averages were used for both PSR 1620–26 and PSR 1821–24, while two-minute averages were sufficient for PSR 1937+21. The scans were cross-correlated with a high signal-to-noise template consisting of the pulse profile averaged over the entire data set to yield individual arrival time offsets. The arrival time at the telescope of one pulse near the middle of the scan integration comes from combining the scan start time with the arrival time offset and then correcting to the midscan of the integration. The topocentric arrival times were corrected to the solar system barycenter using a version of the TEMPO timing package and the PEP740-R planetary ephemeris in a standard manner (e.g., Rawley, Taylor, and Davis 1988). All source positions were measured in the B1950.0 coordinate frame of the planetary ephemeris. All times quoted in this paper are ultimately referenced to UTC as defined by USNO.

a) Individual Parameters

All timing residuals for each pulsar were averaged over each observing epoch to produce a single normal point. The rms of the normal point residuals for PSR 1937+21 reduced to 4.5 μ s while those for PSR 1821–21 reduced to 21.6 μ s. The rms for

PSR 1620–26 is estimated to be 45.3 μ s based on internal errors seen at each epoch. With only seven epochs of data from PSR 1620–26, there are no degrees of freedom for estimation of residuals from a binary fit.

The arrival time residuals from the two pulsars, PSR 1821–24 and PSR 1937+21, are shown in Figure 1 after removing the best-fit model for the pulsar spin-down and position correction. The residuals from PSR 1620–26 are not displayed since there are zero degrees of freedom in the epoch-averaged fit. The observed spin-down parameters for each pulsar are summarized in Table 2. The errors associated with each term are given as 3 times the formal standard deviation to reflect the large covariances that exist between many of the terms.

i) PSR 1937+21

We can compare the position of PSR 1937+21 from our timing data of mean epoch 1988.3 with the published timing positions reported by Davis *et al.* (1985) for mean epoch 1983.9 and by Rawley, Taylor, and Davis (1988) for mean epoch 1986.0. These data indicate a proper motion of the pulsar that agrees in sign (–, –) with the small value published by Rawley. We conclude that our position is consistent with the position and proper motion given by Rawley.

The spin parameters of PSR 1937+21 from our analysis agree with those obtained by Rawley within the uncertainty of

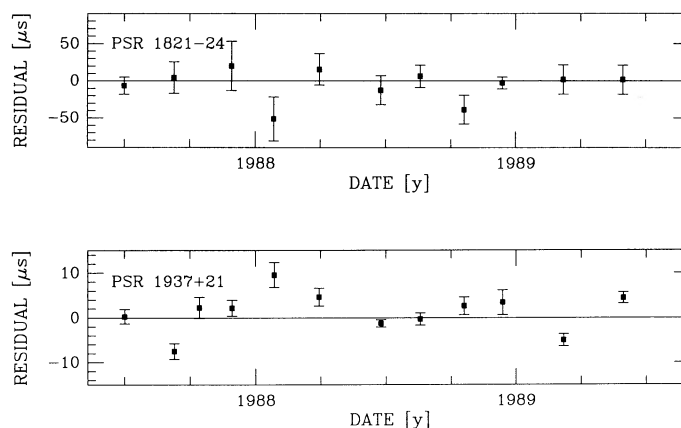


FIG. 1.—Pulsar timing residuals are plotted in microseconds for two pulsars, PSR 1821–24 and PSR 1937+21. The data were collected using the NRAO 43 m telescope at an observing frequency centered on 1330 MHz. Each epoch consists of approximately 12 hr of data averaged over 3 days.

TABLE 2
OBSERVATIONAL DATA FROM PULSARS PSR 1937+21 AND PSR 1821-24^a

Parameter	PSR 1821-24	PSR 1937+21
Right ascension (B1950.0)	18 ^h 21 ^m 27 ^s .3888 (15)	19 ^h 37 ^m 28 ^s .74573 (6)
Declination (B1950.0)	-24°53'50".6 (4)	21°28'01".4587 (2)
Period (s)	0.003 054 314 492 78 (1)	0.001 557 806 464 077 (1)
Period derivative (10 ⁻¹⁵ s s ⁻¹)	0.001 618 7 (3)	0.000 105 092 (7)
Epoch (Julian Ephemeris Date)	2446977.67	2446977.57
Epoch averaged residuals (μs)	21.6	4.5
Number of epochs	11	12
Dispersion measure (assumed) (pc cm ⁻³)	120	71.045

^a All data were collected between the dates of 2,446,977 and 2,447,597 JED and analyzed using the PEP740-R ephemeris.

our data. Recent measurements indicate that the dispersion measure for PSR 1937+21 has decreased by 0.007 pc cm⁻³ between 1984 and 1988 (Cordes *et al.* 1990). Linear and quadratic changes in the dispersion measure will be absorbed by the fitting process and slightly perturb the measured spin parameters. Fluctuations in the dispersion measure will cause arrival time errors. The expected arrival time error at 1330 MHz will have an rms deviation of only ~1.0 μs given the dispersion measure fluctuations seen by Rawley, Taylor, and Davis (1988). This deviation is less than our observed rms residual.

The χ^2 of the post-fit residuals reduced by the 7 degrees of freedom for PSR 1937+21 is 8.6. We examined the rms residual as a function of the number of individual 2 minute scans averaged together for PSR 1937+21 and found that the rms does not decrease with the square root of the number of scans as expected from Gaussian noise. The day-averaged rms values are 2-3 μs larger than the values expected from Gaussian statistics, based on the rms of the 2 minute data. We conclude that the normal point residuals in our data for PSR 1937+21 are the result of an unknown systematic errors; the most likely source of an unaccounted systematic error is with the hardware used to collect these data, or is caused by narrow-band radio frequency interference (RFI) entering our 8 MHz bandpass. Beginning with the last data epoch present in this analysis (1989 June) we upgraded the experimental apparatus to a new generation of dispersion removal hardware that is capable of rejecting narrow-band RFI.

The PSR 1937+21 arrival time data were analyzed with an independent pulsar timing program that uses the JPL DE200 ephemeris (Fairhead 1989). The pre-fit and post-fit residuals agree with the other analysis to within 0.5 μs rms. This is within the errors expected from comparison tests between the use of the two ephemerides after accounting for a rotation and a displacement of the B1950.0 and J2000.0 coordinate frames. Future pulsar timing array data will distinguish between errors in the different ephemerides as discussed in the previous section.

ii) PSR 1821-24

The parameters determined for PSR 1821-24 (Table 2) improve on those published previously (Foster *et al.* 1988). The position that we now derive from the timing data agrees with the VLA position presented by Foster *et al.* within the combined positional uncertainties. However, this timing position for PSR 1821-24 still does not resolve the positional discrepancy of the calibration source 1748-253 that was discussed in our previous paper. Uncertainties in the timing position will decrease as more and better data are obtained. The post-fit

timing residuals for the PSR 1821-24 data give a respectable reduced χ^2 value of 0.8; this is consistent with random Gaussian noise.

The large-period derivative, and therefore young characteristic age and large inferred magnetic field, of PSR 1821-24 is confirmed. The period derivative is large compared to other millisecond pulsars, and is near the maximum expected value for neutron star spinup by the binary accretion process (Alpar *et al.* 1982). However, the large period derivative could be due to the pulsar's interaction with the background gravitational potential in the globular cluster M28. The measured negative period derivative of PSR 2127+11A found in the globular cluster M15 (Wolszczan *et al.* 1989) is most likely the result of acceleration by a gravitational interaction. The inferred acceleration for the M15 pulsar is about an order of magnitude larger than the value predicted using a model of the cluster potential. The large inferred acceleration in M15 may indicate an error in the model rather than chance statistics. If so, then the probability that gravitational acceleration is the principal component of the $\dot{P}$ observed in the M28 pulsar, which was discounted as unlikely by Foster *et al.* (1988), is increased. Timing data shows no indication that the PSR 1821-24 is bound in a weak binary system. A measurement of the period second derivative from the PSR 1821-24 could give an *in situ* probe of the gravitational potential in the globular cluster M28 (Blandford, Romani, and Applegate 1987). Gravitational acceleration of globular cluster pulsars, if detected, will add to the number of parameters required to model their apparent rotations.

iii) PSR 1620-26

The parameters derived for PSR 1620-26 agree with the values published by McKenna and Lyne (1988). At this point we only have seven epochs of data. Our data still have zero degrees of freedom in the fit for 10 parameters over seven epochs; hence we do not have a completely independent timing solution.

b) Global Parameters

With our limited amount of data we can address the influence of global parameters by fitting for cubic phase terms, or $\ddot{P}$, as outlined in § IIb. The fits provide only upper limits on cubic term amplitudes which we take as 3 times the formal error of the fits. The limits obtained for $\ddot{P}$ are less than 6.6×10^{-29} s s⁻² for PSR 1821-24 and less than 1.4×10^{-29} s s⁻² for PSR 1937+21. The limits on $\ddot{P}$ are converted to fractional frequency stabilities, or Allan standard deviation, using equation (6) to place limits on the effect of any clock drift

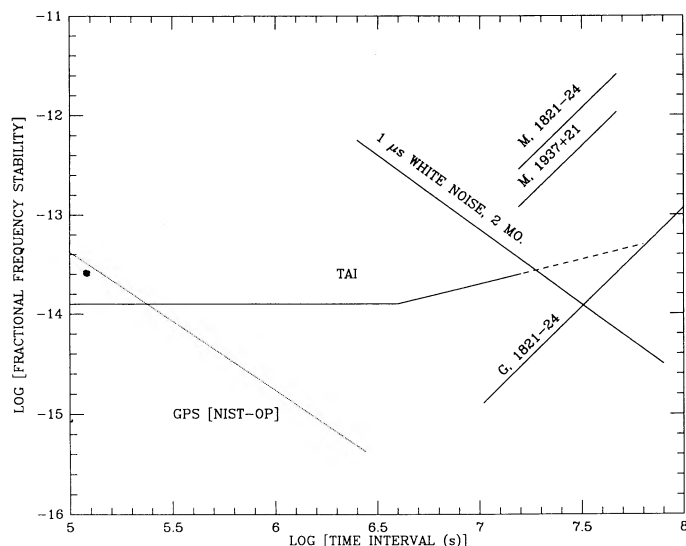


FIG. 2.—Fractional frequency stabilities are plotted for International Atomic Time (TAI) and the transfer of time between the National Institute of Standards and Technology (NIST) and the Observatoire de Paris (OP) via the Global Positioning Satellite system. The horizontal axis is the time interval in seconds. Limits on the stabilities of pulsars PSR 1821–24 and PSR 1937+21 from fits for cubic phase terms are labeled by the letter M and the pulsar name. Also plotted is the stability of oscillator with $1 \mu\text{s}$ white noise which is sampled every 2 months. The estimated limit on the stability of pulsar timing data from PSR 1821–24 owing to gravitational acceleration in the M28 globular cluster is labeled G.

over the time scale of the data set. Figure 2 compares these limits to the stabilities of International Atomic Time, TAI, and of the GPS time-transfer link. Also shown in Figure 2 is the fractional frequency stability of a white noise process with $1 \mu\text{s}$ rms amplitude sampled at 2 month intervals; for white noise the modified Allan standard deviation which averages samples over increasing time intervals to obtain a slope of -1.5 is used. The stability limit from the expected period second derivative for PSR 1821–24 assuming an interaction with the gravitational field of the globular cluster M28 (Blandford, Romani, and Applegate, 1987) is also plotted. On a 1 yr time scale the $\ddot{P}$ term from acceleration in the gravitational potential of globular clusters may begin to be significant at the microsecond level. If the cubic terms from a known acceleration derivative become significant they must be fitted and removed before reaching the background noise level to probe for gravitational radiation.

The amplitudes of acceleration derivatives derived from period second derivatives place a bound on the sensitivity of our pulsar timing array data to dipole perturbations. Figure 3 shows the maximum allowable acceleration derivative in directions along the great circle defined by the plane containing PSR 1937+21, PSR 1821–24, and Earth. In a direction perpendicular to the plane the sensitivity of the two pulsar array vanishes. The characteristic acceleration derivative computed from uncertainties in the mass of Jupiter, Saturn, and Neptune (Standish 1989) are shown (see eq. [10]) in Figure 3. Some reduction in these uncertainties will result from ephemeris development based on the new Neptune mass derived from the *Voyager* spacecraft encounter (Hellings 1989b). Our current data are three orders of magnitude above the expected level of ephemeris uncertainty, but our sensitivity to the ephemeris

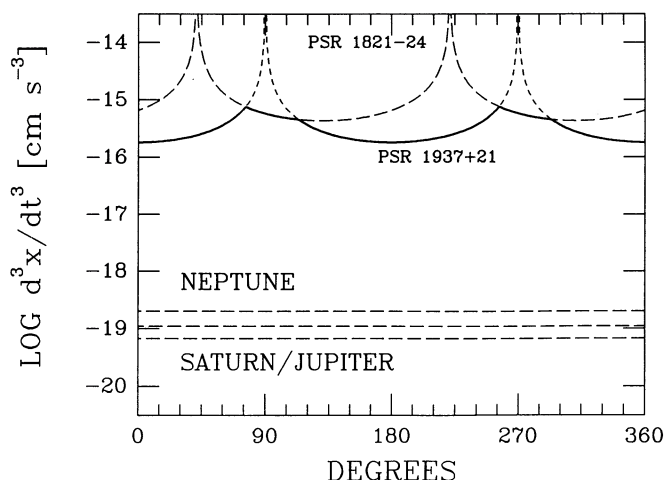


FIG. 3.—Timing data from PSR 1821–24 and PSR 1937+21 are used as a dipole array to place a limit on the amplitude of the acceleration derivative, d^3x/dt^3 . The short dashed line represents the limit from PSR 1937+21, while the long dashed line represents the limit from PSR 1821+24. Together these two objects provide a sensitivity limit set by the dark line. This limit is only valid along a great circle containing the two pulsars and Earth. Perpendicular to this plane the sensitivity drops to zero. Limits on the level of ephemeris uncertainty from errors in planetary masses for Jupiter, Saturn, and Neptune are shown.

perturbations will improve in proportion to the pulsar timing residuals divided by the observation span to the third power.

We can use the rms residual data from PSR 1937+21 alone to place a limit on the energy density of a stochastic background of gravitational waves as a function of wave period (e.g., Hogan and Rees 1984). We determine an upper limit on the energy density due to a stochastic gravitational wave background relative to the closure density of the universe of $\Omega_{\text{GBR}} < 2.0 \times 10^{-4} h_{100}^{-2}$. If Ω_{GBR} is independent of period, then this limit will improve in proportion to the timing residuals squared divided by the observation span to the fourth power. Reliable detection of the angular signature of a gravitational wave (eq. [12]) will require high-precision timing data from multiple pulsars around the sky.

V. CONCLUSIONS

We have presented the status of our effort to establish a pulsar timing array that is capable of fundamental measurements of time, solar system dynamics, and the gravitational-wave background radiation. Methods of analyzing a pulsar timing array data set with limited degrees of freedom have been outlined. Our experiment has provided independent determination of the rotational parameters of two millisecond pulsars. The timing residuals from PSR 1937+21 are an order of magnitude smaller than those of PSR 1821–24. PSR 1937+21 is still the most significant object, in both our experiment and the Arecibo program (Rawley *et al.* 1987; Rawley, Taylor, and Davis, 1988; Taylor 1989), for placing upper limits on the amplitude of the multipole perturbations that affect all pulsar data.

Planned improvements in the pulsar timing equipment will reduce the timing residuals to the $1 \mu\text{s}$ level for each of the three pulsars that are presently included in our experiment: PSR 1620–26, PSR 1821–24, and PSR 1937+21. At this level data from our pulsar timing array experiment and from other experiments will constrain, or even detect, the effects of various

multipole terms affecting the ensemble of objects. This experiment will continue with observing at multiple frequencies to remove the wavelength-dependent propagation delay through the turbulent electron plasma in the interstellar medium. Additional strong pulsars in independent parts of the sky, if found, will improve the sensitivity to the multipole effects discussed in this paper. In particular, the discovery of a millisecond pulsar in the direction of the Galactic anticenter would improve the quality of a pulsar timing array by adding an object to a part of the sky that is currently not covered. Analysis of data from several observatories around the globe will provide the best possible solution for these effects. For example, Arecibo data will always provide the highest precision data for objects in its declination range, while other observatories are required to monitor objects outside that range in the northern and southern hemispheres.

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