GIUSEPPE CALANDRELLI AND HIS FIRST APPLICATIONS OF GRAVITATIONAL THEORY TO THE COMETARY ATMOSPHERES

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To Giuseppe Calandrelli, founder and director of the Observatory of Collegio Romano from 1787 to 1824, we owe the first calculations concerning the cometary masses. He get the idea from the apparition of the great comet of 1807 to carry out some studies of their constitution.

It is noted that Newton, accepted by the most part of astronomers, assumed the formation of cometary tails caused by an apparent repulsion, that is the formation was caused by the rising of cometic vapours, mixed with ether, in a kind of solar atmosphere. Newton stated that the tail formation was similar to the smoke coming out of a chimney.

Calandrelli instead was a supporter of the repulsive theory mentioned first by Kepler and accepted afterwards also by Euler and Laplace. Just to contradict Newton and Boscovich's theory, Calandrelli made geometric demonstrations to define the tail's inclination, the escape velocity from the nucleus of the effluxies and so on. According to Calandrelli the theory of apparent repulsion would be possible only when the comet was without mass; but for him this was not possible because the comet, before forming the tail, retains a spheric atmosphere around its own nucleus. Referring to Euler's concept he is of the opinion that the comets possess a solid nucleus and that the missing observation of the phases is due to the strong refraction received from the Sun's rays in crossing the atmosphere of the comet.

Calandrelli had at his disposition at that time instruments that were very modest, in fact he carried out the observations with a simple equatorial of Dollond with 17 inches of F.L. and capable of only 70 enlargements (1). The observations were ef= fected from 28 September until the first days of December 1807. For the calculations Calandrelli chose the position of the co= met in the evening of 30 September. In fact that evening the tail of the comet was perpendicular to the declination circle. This permitted the calculation of the angle of deviation of the axis of the tail, in comparison to the vector radius, easier that the method of Cheseaux. Calandrelli, after having found the angle of deviation of 6° 49' 6", determined the length of the tail; he observed that the smokiness that goes to form the tail moves along a line passing by the Sun; such smokiness lea= ves the surface of the comet, but in different successive points of its orbit in such a way that the convexity of the tail will be turned towards the zone of the orbit already described.

On the base of such considerations Calandrelli calculates the length of the tail equal to 4826912 Roman miles and the speed of the comet in that part of the orbit equal 157177 feet per second (2). The 30th September the comet was near the ascending node, that is 220° R.A. and 1° 44' D.

To contest the theory of Boscovich, Calandrelli carried out many different calculations and demonstrations, one of wich we illustrate below.

Let MD be a tract of the orbit (fig. 1), SC the vector radius and CN the direction of the tail. We have $\widehat{BCN}$ = 6° 49' 6" the comet travels over the tract CI in 4h 22' 31". On the basis of the Boscovich theory the vapours should rise due to the ef= fect of the specific gravity, and for this reason when the comet is in I the smokiness goes towards N (Boscovich in fact does not admit, differently from Newton, that there can be motion of translation along the orbit due to the resistance of the Sun's atmosphere). Consequently, if the comet is in C, the end part of the tail is in N. To cover this distance the vapours take 4h 22' 31"; but even admitting that they rise with the full acce= leration of the Sun's gravity, calculated constant all along its course, the vapours would cover only the distance of 1075 miles against the 4905852 miles of the distance IN.

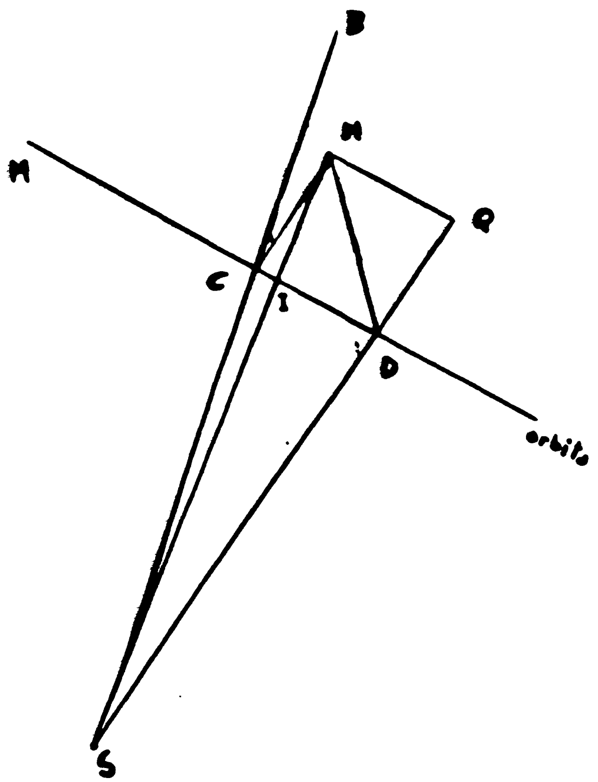

fig. 1

Calandrelli therefore also contests Newton's theory which, as we have already said, had admitted not only the rising of the vapours but also the movement of such along the direction DC. In such a case, when the co= met is in D, the vapours would be pushed towards C, that is in the direction DN. When the comet has reached C, the smoke farthest away will find itself in N. To arrive at N they will take 2d 21h 21' 57", that is the time taken by the comet to pass from point D to point C. In such time the components due to the specific gravity difference is the stretch DQ = CN, that is 4826912 Roman miles. Admitting as before an elevation caused by an entire acceleration of the Sun gravity, in that spare of time the vapours would cover 278435 miles, instead of the entire length of DQ. (3).

As it is noted, the comets, not caused perturbations in the planetary movements, because of their very light mass, do not consent to the calculation of their own masseswith the classic method of the celestial mechanic. Laplace had calculated a superior limit for the mass of the comet Lexell that on the return in 1770 had notably come near to the Earth. The comet, according to Burchardt, had passed very near to Jupiter, inside the orbit of the 4th satellite (4). In such return of the Lexell comet there was no perturbation observed in the revolution of the Earth or in the system of Jupiter. After such consideration Laplace had deducteda superior limit of the mass equal to 1/5000 of the mass of the Earth.

Calandrelli again disapproves a demonstration of Boscovich which was to show the inexistence of the Sun's repulsive strength. Boscovich had observed in the comet of 1744 the convexity of the tail facing that part of the orbit still to be described, while a push of the Sun's rays would have provoked a convexity facing the opposite side, that is towards the orbit already described. Calandrelli objects that if, in addition to the rising of the vapours, we admit a motion of translation we can fund the convexity of the tail of the comet of 1744.

This objection of Calandrelli is needed to ascertain that the resistence of the etherial fluid cannot cancel the movement of translation of the "little globes" which make up the atmosphereof the comet because these gravitate around the nucleus. This explains also the presence of an enormous atmosphere around the cometary nucleus which, due to the small mass of the nucleus, in the upper zones will feel the attraction very little. If there= fore the resistence of the etherial fluid, in which the comet are immersed, annuls the movementof the upper zones, this would re= main behind forming a tail in the direction of the tangent of the orbit. Before the formation of the tail, that is when the comet is far from the Sun more than 1 AU (5), it can be observed a spheric atmosphere that accompanies the nucleus and which belongs to the comet.

Calandrelli says that the nucleus has always around it a sphere of attraction and everything that finds itself in this sphere, which means ether together with vapour of the atmosphere, all gravits towards the centre of the comet. Even if the vapours arri= ve at their maximum rarity they will always remain around the nucleus (6).

Calandrelli then calculates the limit of the sphere of attraction of the comet.

Let be: M = mass of the Sun; m = mass of the comet;
h = radius of the Sun; G = acceleration of gravity at the Sun's surface; k = m/ M

At the point P (fig. 2) the attraction of solar gravity is:

$$\frac{G\,h^2}{(a - x)^2}$$

At the distance h we should have the following gravity of the comet: $G\,\frac{m}{M}$, that is Gk.

At the distance x we have: $G\,k\,\frac{h^2}{x^2}$

fig. 2

Calandrelli besides considers, referring to the theory of tide for as the point P will be very near to the comet, that the gravity of the Sun at P will owe to be diminished of the attra= ction of the comet towards the Sun, that is: $G \frac{h^2}{a^2}$ (7).

Definitively the equilibrium will be given by the equation:

$$\frac{G h^2}{(a - x)^2} - \frac{G h^2}{a^2} = G k \frac{h^2}{x^2} \qquad (1)$$

Imposed a = 1 and developed:

$$x^4 - 2 x^3 + k x^2 - 2 k x + k = 0$$

In consideration that k and x are extremely little in compari= son with M and a, Calandrelli neglects the terms where is their product and neglects the 4th power of x. He obtain definitively:

$$x = \sqrt[3]{\frac{k}{2}}$$

Calandrelli applies this formula to calculate the mass of the comet of 1807 and considers that the limit of the sphere of at= traction coincides with the limit of the atmosphere which enve= lops the nucleus. He obtains, observing araund the nucleus a cometic atmosphere with a radius which subtends an arc of 5'30", a cometic mass equal to $14 \cdot 10^{-3}$ terrestrial masses.

Applying the same proceding to the comets of the years 1759 and 1811 he obtains respectively $3 \cdot 10^{-5}$ and 0,109 terrestrial masses. The result is no exact but we mast consider the absolu= te will and uncertainty of the x, that is of the limit of the sphere of attraction of the comet.

The simple formula of Calandrelli will be taken again by Roche only in the year 1854. Roche takes in consideration the free surface (8) and considers that the cometary atmosphere is into this one. He takes as the limit of the sphere of attra=

ction the apparent nucleus of the comet, and he obtains an inferior limit of the value of the cometary mass. On the basis of these considerations Roche obtains:

comet	mass		
1807	$38 \cdot 10^{-9}$	terrestrial	masses
1759	$93 \cdot 10^{-12}$	"	"
1811	$3 \cdot 10^{-7}$	"	"

References

1) Osservazioni e riflessioni sopra la cometa del settembre anno 1807, in Opuscoli Astronomici, v. II (1808), p. 19.
2) In Rome, at that time, was usual to use the Parigian foot equal to 0.32484 metres.
3) Osservazioni e riflessioni..., cit., p. 34.
4) Late the research of Le verrier will contradict this result of Burchardt; from the calculations of Le Verrier the di-stance of Jupiter will be equal to 3.5 rays of the orbit of the 4th satellite.
5) It is noted that Laplace had calculate that an eventual solar atmosphere could not extend itself besides the 9/10 of the mean distance Sun-Mercury.
6) Osservazioni e riflessioni..., cit., pp. 45-46.
7) Some years before Regner had obtained the uncomplete equation:

$$\frac{G\,h^2}{(a-x)^2} = G\,k\,\frac{h^2}{x^2}$$