

3. ERROR ALLOCATION AND ACCURACY ASSESSMENT

This chapter deals with the related concepts of error allocation, performance verification and accuracy assessment, as viewed by ESA and the industrial contractor. Error allocation is the process whereby specifications are placed on the specific satellite or data reduction (sub-)systems, so that the total system will give the required accuracy. Performance verification entails measuring or modelling individual components and their performance. Given estimates of the components' performance, it is then possible to reverse the process and make an assessment of the accuracy that the whole system will yield. These considerations were particularly important during the satellite's detailed design phase.

3.1. Introduction

The unprecedented precision of the measurements to be performed by the Hipparcos satellite necessitates a particularly careful control of all potential error sources. These include the statistical properties of the signals from the stars, limitations imposed by the manufacture of the instrumentation, the thermal and structural stability of the satellite, the performance of the attitude control and measurement subsystems, and the capabilities of the data reduction.

The procedures adopted to ensure that the overall main-mission accuracy conforms to the system requirements are based on an allocation of errors from system level down to equipment level, in accordance with a well-defined plan. Budgets have been established for the contributions of different effects to particular errors, and the payload test programme was designed to determine, as far as possible, the actual performance. The measured payload performance has then been used as input for the generation of simulated data, which have been reduced to provide the best indication of the expected mission accuracy.

These procedures have been in use throughout the design of the hardware, up to the final pre-launch reviews. In addition, pending the final data reduction, calibration measurements made during commissioning will be used as inputs to the accuracy estimation software, to provide the first assessments of the actual in-orbit performance.

3.2. Error Allocation

The Hipparcos project has introduced many innovations to the concept of scientific satellite development. Not the least of these is the responsibility of the satellite manufacturer for the final mission performance, subject to clearly specified assumptions about the data-reduction process. This responsibility can only function in an environment of close contact with the scientists, coordinated and monitored by ESA, which has become a characteristic of the mission. The multiple-step approach to data reduction has greatly facilitated this partition of responsibilities.

Principles of Error Allocation

The object of the error allocation process was to convert the scientific performance requirements into a set of specifications usable by the satellite designer. Two major obstacles complicated this process: (a) the final outcome of the Hipparcos mission stems not only from the satellite performance, but also from the performance of the data reduction carried out on the ground by the scientific consortia; and (b) there is no unique solution to the error allocation process, and a choice must be made among the many possible sets of specifications that eventually lead to satisfactory mission performance.

The first obstacle (a), which is specific to the Hipparcos programme, required the satellite designer to have a thorough understanding of the data-reduction scheme. As illustrated in Figure 3.1, there are two major stages in the information flow, from the emission of light by the star, to the establishment of its astrometric properties: (i) the 'signal propagation stage', where the light emitted by the star is collected and modulated by the optical subsystem, sensed and sampled by the detection subsystem, formatted by the on-board computer, and transmitted to the ground by the telecommunication subsystem; and (ii) the 'data-reduction stage', where the raw data sent to the ground by the satellite throughout the mission are processed by the data reduction consortia, and combined into a consistent set of astrometric data. The error allocation reflects both of these stages, even though the data-reduction stage is not under the control of the satellite designer.

The second obstacle (b), namely the selection of a set of specifications among many possible candidates, is a problem of a very general nature, which must be faced in order to break a high-level specification into a number of lower-level specifications. The respective merits of the various candidate breakdowns must be weighed against achievability in terms of feasibility, planning, and costs. Although there was neither an ideal selection to be made, nor a straightforward way to attain it, it was possible to converge towards a solution by following general working steps:

- (i) all the terms that contribute to the final performance were identified;
- (ii) the expected characteristics and potential for improvement of each contributor were evaluated;
- (iii) the separate impact of each contributor on the final performance was assessed.

Once these preparatory steps had been completed, if the final performance obtained from the individual expected characteristics of the various contributors was out of specification, effort was concentrated on improving those contributors having either a large potential for improvement, or a strong influence on the final performance. This approach led, after a few iterations, to a set of specifications on the various contributors that fulfils the final performance requirements.

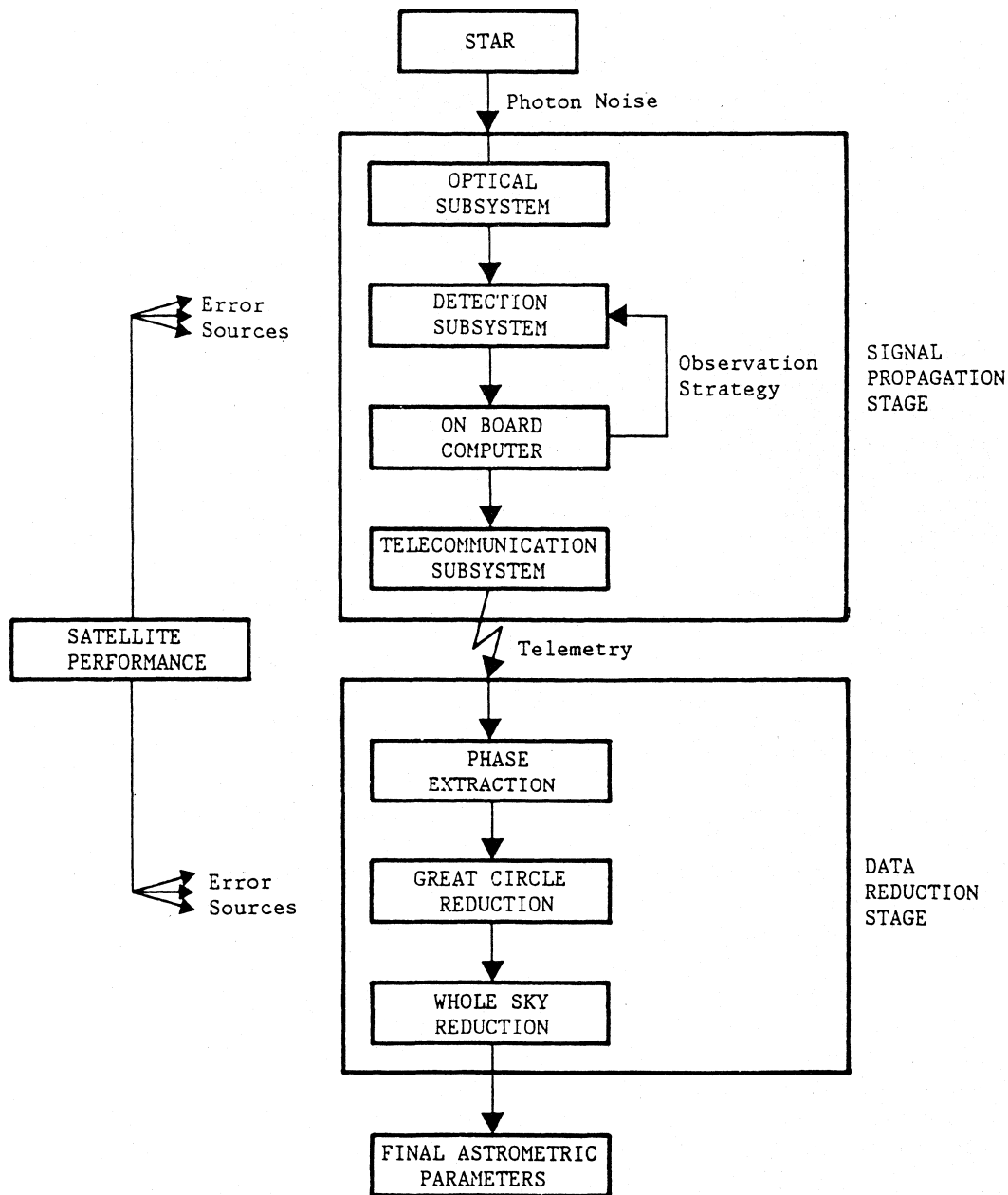


Figure 3.1. The error propagation flow.

The Data-Reduction Scheme

Unlike most scientific missions, the data from individual measurements by Hipparcos cannot be reduced individually, but have value only in their relationship to all the other measurements made during a period of at least six months. However, the volume of such data precludes its simultaneous reduction in one activity. Therefore, a reduction scheme (described in detail in the accompanying volume describing the Hipparcos Data Reductions), encompassing several successive steps, was adopted. As depicted in Figure 3.2, these steps are:

- (i) determination of the relative positions of the star images in the field of view, at a given instant of time, by analysis of the modulated photoelectron counts transmitted through the telemetry. This gives the longitudinal field coordinates, relative to the undetermined satellite attitude;
- (ii) positioning of all the stars observed over a set of several successive scans by the satellite relative to each other along a reference great circle, giving the so-called 'star abscissae' relative to an undetermined zero point;
- (iii) the whole-sky reduction for the so-called 'primary reference stars', a subset of the 120 000 programme stars. This step provides the zero-point corrections required to complement step 2;
- (iv) back-substitution of the abscissa zero points for each individual star, yielding the five astrometric parameters for all programme stars.

For a correct understanding of the limitations and capabilities of the data-reduction process, a close relationship had to be established between ESA, the satellite prime contractor, and the data-reduction consortia. This was achieved through teams of scientific consultants set up by ESA and by the contractor.

The Error-Allocation Process

The error-allocation tree is made up of two parts. The first part of the allocation tree is an 'upside-down' version of the data reduction stage and leads from the specified astrometric parameter rms errors (the final outcome of the mission), via the star abscissa rms errors (determined by the great-circle reductions), to the longitudinal field coordinate rms errors (determined at observation frame level by the phase extraction part of the data reductions).

The required star abscissa rms error at the level of the great circles, σ_α , is derived from the astrometric rms error, σ_a , by means of the equation:

$$\sigma_\alpha = \frac{\sigma_a}{I_i F} \quad [3.1]$$

where F is a degradation factor induced by any mission observation 'dead time', and I_i are the 'coefficients of improvement' for the five astrometric parameters, which result from the combination of all the great circles during which a star is observed throughout the mission.

The coefficients of improvement corresponding to the five different astrometric parameters, a_i , are obtained from a software package that generates directly the covariance matrix associated with the values of a_i (the actual coefficients of improvement are also dependent on the assumed mission duration, as well as on the ecliptic latitude of the star: see Volume III, Chapter 18). For the purpose of error allocation, the most penalising coefficient of improvement, which turns out to be that corresponding to proper motion in longitude, I_2 , is considered.

Going from the star-abscissa rms error, σ_α , to the field-coordinate rms error, σ_η , is much more delicate, because the error propagation mechanism, through the great-circle reduction, varies according to the nature of the error source. An identification of the

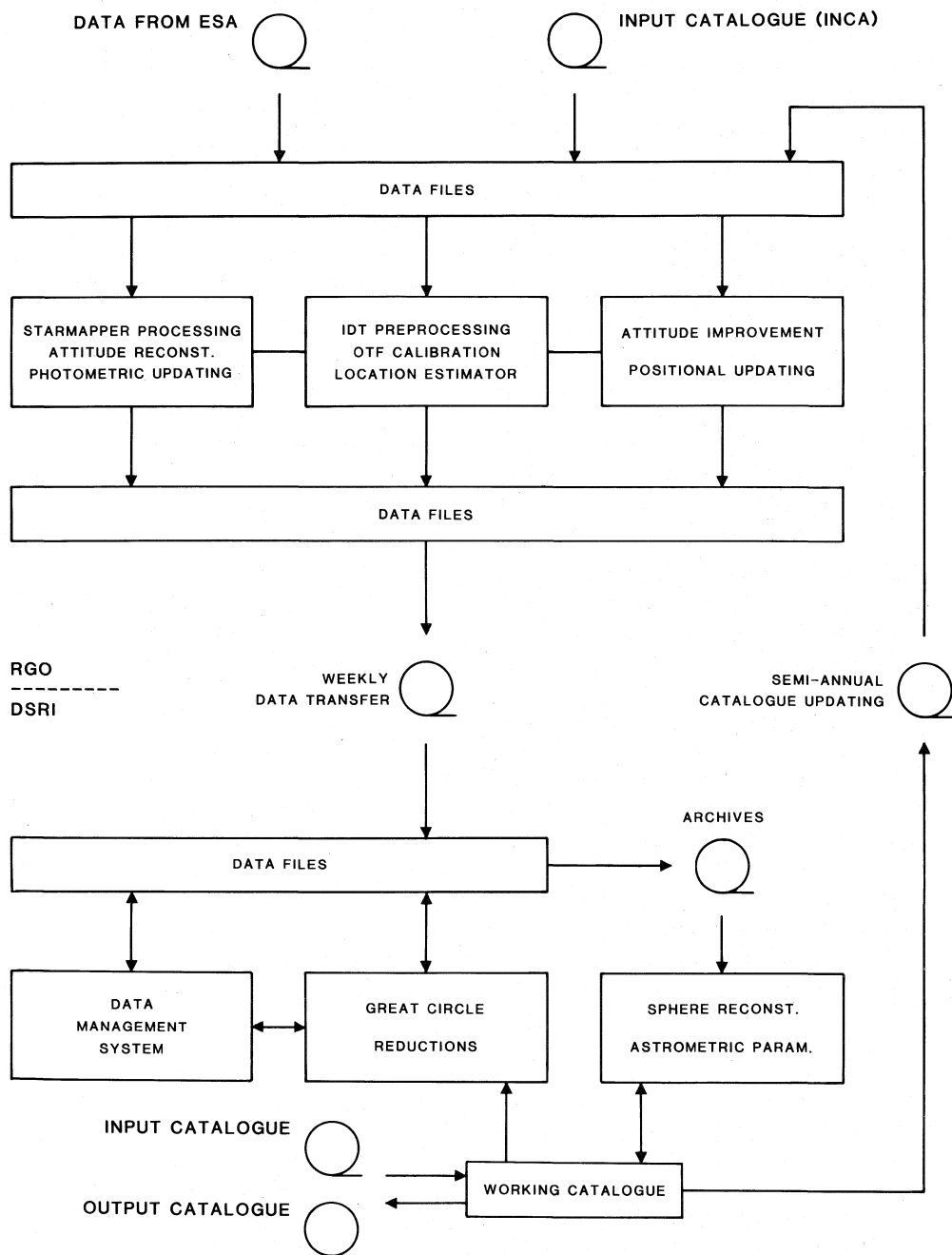


Figure 3.2. An overview of the data-reduction scheme (as adopted by the NDAC Data Reduction Consortium). Further details of the data reductions are given in Volume III.

various types of propagation mechanism is thus required at this point, and a preliminary allocation at star abscissa level is to be made accordingly. Simulation software, referred to as the 'great-circle reduction' software, which represents rather faithfully the actual great-circle reduction step, was developed. However, the results it gives are too detailed to be used directly as an error-allocation tool. Instead, an empirical, analytical formula derived from physical considerations, which could quite satisfactorily fit the great-circle reduction simulation results for all uncorrelated error sources, was established.

The second part of the allocation tree is the reverse process to the signal propagation stage, and leads from the allocated field coordinate rms errors to the equipment specifications. The implementation of the great-circle reduction formula and the allocation of the field coordinate rms error, σ_η , between the various error sources, require a knowledge of the dependence of σ_η on the star colour, magnitude, and observing time. This is given by a second simulation program, called the 'Image Location Stage' software, which simulates the image dissector tube modulated photoelectron counts, including all known error sources, and which also simulates the phase-extraction process.

The Image Location Stage software was used to find where there is cross-coupling between the various error sources and demonstrated that their effects can be split into four terms that can be independently added at the variance level, which simplifies the field-coordinate error allocation process.

The error allocation may be illustrated by considering the photon noise error, the most important of the four types identified by the Image Location Stage software:

- (i) an allocation is first made between the inherent photon-noise error, characterised by a theoretical Cramér-Rao bound, and the various degradation factors that apply to it, reflecting imperfections of the payload;
- (ii) the Cramér-Rao bound itself is a function of the instrument detection efficiency and modulation factors, and an error allocation between those terms is made accordingly;
- (iii) the payload detection efficiency is finally split between the various relevant optical and electronic equipments, yielding spectral transmittance specifications for these elements.

In practice, the error allocation process may not be as 'clean' a backward process as indicated above—it is rather a backward-and-forward process involving a trial-and-error approach. Nevertheless, one is strongly dependent on the correctness and flexibility of the error-allocation tools, in order to converge rapidly towards a satisfactory set of specifications.

3.3. Performance Verification Approach

The satellite performance is assessed on the basis of a combination of analyses, simulation software, test measurements, and calibrations, both on-ground and in-flight. Clearly, as the project life progressed, test and calibration results have played an increasing role in the performance assessment, which has thereby become more and more realistic. In parallel, the assessment of the mission accuracy has been regularly repeated, taking account of the updated satellite performance.

Main Satellite Performance Estimates

The payload performance contributes to the final mission performance, mostly through the photon-noise error, as characterised by the Cramér-Rao bound, and through the degradation factors applied to it at the level of the field coordinate. The Cramér-Rao

bound is determined by the photometric performance of the payload, while the main degradation factors arise from the geometrical characteristics of the telescope and the main grid.

The overall error allocation process eventually allocated a value of 7 milli-arcsec for the Cramér-Rao bound, corresponding to a 1 s observation of a star of $B = 9$ mag and $B - V = 0.5$ mag. The currently predicted performance is based on payload measurements, made during the on-ground photometric calibration (see Chapter 25), and lies in the range 6.5–6.9 milli-arcsec (depending on the detection chain—prime or redundant) at the start of the mission.

The most critical geometrical characteristics are the stability of the basic angle, the stability of the optical distortion, and the instrumental chromaticity. Current estimates are:

- basic-angle stability: 0.83 milli-arcsec (peak-to-peak) over 2.8 hours;
- distortion stability: 0.7 milli-arcsec (rms) over 24 hours with peaks of 4 milli-arcsec at eclipse transients;
- chromaticity: 0.6 milli-arcsec mean value over both fields of view and over the mission lifetime;
- 0.8 milli-arcsec rms variation over both fields of view and over the mission lifetime;
- 0.7 milli-arcsec mean difference between the two fields of view.

In practice, the above requirements correspond to:

- (i) relative displacements or rotations between the optical elements of the order of a few microns or tenths of arcsec;
- (ii) polishing quality of the mirrors of the order of $\lambda/60$ rms (at $\lambda = 500$ nm);
- (iii) short-term temperature variations of the order of a few tenths of a degree.

It is also important that the main grid pattern be written to such a high quality that small-scale defects do not exceed a few nm, while the medium-scale defects must be calibrated on ground, to an accuracy of better than 34 nm rms. Calibration of the flight grid has demonstrated that the electron-beam pattern generators, used to write and calibrate the grid, do allow these requirements to be met.

The above effects are barely observable directly, and very sophisticated verification methods, based on a combination of analyses, simulations, and tests, have to be considered. Their determination, as well as that of the more fundamental, but less demanding, photometric characteristics of the payload, are addressed in some detail in subsequent chapters, covering payload testing and in-orbit calibration.

3.4. Accuracy Assessment

Starting from the satellite performance on the one hand, and from applicable working assumptions on the other, an estimate of the mission accuracy is derived, using a combination of simulation software and analytical formulae.

The approach adopted for the accuracy analysis was driven by the number and complexity of the error sources to be considered. It was known at a very early stage that the final mission accuracy is dependent upon many system parameters and error sources, but it was unclear how. There was a need to understand what effect they had on the final science data, so that they could be optimised, and so that the final design could be demonstrated to meet the mission accuracy requirements.

It was decided that a complete analytical treatment would be unmanageable, and that the interpretation of the results obtained would prove very difficult. A Monte-Carlo simulation was deemed the best approach. The advantages are numerous:

- simplifying assumptions can be used, where appropriate, without dramatic effects on the resulting accuracy;
- reliable software is easier to produce, there being fewer mathematical computations involved;
- the effects of changing system parameters and data-reduction algorithms can easily be studied;
- individual error sources can be studied in isolation from one another and the coupling of error sources can be studied;
- new error sources can be added without the need to alter large amounts of software.

A particular disadvantage is that, if a parameter change produces a small effect, this may be difficult to assess. For example, to measure a 5 per cent change in the variance to an accuracy of 10 per cent means the variances must be obtained to an accuracy of about 0.35 per cent in each of two runs. This, in turn, requires about 160 000 Monte-Carlo simulations, which can lead to considerable computer overheads. In order to construct an analytical model for an error source, such accuracy is often necessary. The problems were kept manageable by choosing parameters that made individual simulations run quickly, and by making effects larger and hence easier to measure by increasing the size of the corresponding error source.

3.5. The Image Location Stage Program

The second of the two simulation programs mentioned in Section 3.2 is used to assess the accuracy of the individual measurements of the position of a star on the grid, the 'Image Location Stage' of the data reduction. A Monte-Carlo approach is used to generate many observation frames, and the number of stars in each frame, their magnitudes, colours and positions on the grid can be fixed or can be randomly generated from realistic probability density functions. Over 30 individual error sources can be included in the simulation. The simulated image dissector tube data generated for each star in each frame is then reduced and converted to positional estimates of the stars in view. Various phase-estimation procedures are implemented, such as least-squares and maximum-likelihood. The output generated is the positional error covariance matrices for the different frame configurations simulated.

Input Parameters

The inputs for the Image Location Stage program are grouped into two categories: those parameters defining the star and background distribution, and the satellite and error models; and those parameters defining the nature of the simulation to perform.

These two types of input data are held in separate data files, so that it is very easy for the user of the software to create inputs for several different simulation runs using the same sky, satellite and error models. Each of the categories of data are now described in more detail.

Sky Model

Observations of the astronomical sky are modelled by the following parameters:

- the number of programme stars designated for observation in the whole sky;
- the probability density function for the B magnitude of the programme stars;
- the probability density function for the colour index, $B - V$, of programme stars, for each B magnitude;
- the probability of a star being binary;
- the probability density functions for the B magnitude differences and angular separations of binary pairs;
- the number of non-programme stars, together with their B magnitude probability density function, down to $B = 15$ mag.

As an alternative, a direct catalogue interface has been constructed. This can be used to perform simulations using actual strips of sky.

Satellite Performance

The satellite configuration is defined by a large number of detailed parameters grouped into the following categories:

- sampling time hierarchy (sampling time, interlacing time, frame time);
- scanning law;
- telescope optical characteristics;
- detector efficiency;
- parameters used by the on-board software for star observing strategy and sample data compression.

The parameters define the basic form of the modulated image dissector tube signal used in the simulation, before error sources are included, which is given by:

$$I(t) = I_0[1 + M_1 \cos(2\pi\omega t + \theta) + M_2 \cos(4\pi\omega t + 2\theta)] \quad [3.2]$$

where $I(t)$ is the instantaneous count rate at time t , I_0 is the mean count rate for the observed star, M_1 is the first modulation coefficient for the observed star, M_2 is the second modulation coefficient, t is the time relative to mid-frame time, ω is the modulation frequency of the image dissector tube signal in Hz, and θ is the phase of the signal.

Error Models

There are numerous error sources that are modelled and can be included in any of the simulations:

- (i) photon noise: the simulated sample count is the output of a Poisson random number generator with an input mean of the expected photon count, i.e. the integral of $I(t)$ over the sample period;
- (ii) background effects: sky background, as well as earth, moon, and sun straylight and detector dark current are modelled by adding a constant count rate, I_b , to the modulated signal for the star. A more serious error source accounted for in the software is the presence of modulated signals from bright stars also in the combined field. This 'veiling glare', resulting from the extended response of the instantaneous field of view profile, can be caused by programme stars or by non-programme stars;
- (iii) detector effects: the sensitivity of the image dissector tube falls as the star image moves away from the the centre of the instantaneous field of view. The combination of the star's motion during the instantaneous field of view repositioning period of $\frac{1}{150}$ s, piloting errors due to inaccurate calibration, instability of the calibration and *a priori* star position uncertainty, are all modelled by the software, and the resulting change in sensitivity is used to modulate the value of I_0 in equation [3.2]. Further, a facility

is provided to alter the shape of the instantaneous field of view sensitivity function, depending on where it is pointing on the grid. Inhomogeneities in the photocathode of the image dissector tube are modelled by changing the sensitivity of the instantaneous field of view from one repositioning period to the next. The change is defined by a Gaussian distributed random variable, whose mean and standard deviation can be made position dependent. Thus, both small- and large-scale inhomogeneities can be modelled. Finally, the non-linearity of the detector due to saturation effects is modelled with a simple saturation law;

(iv) optical effects: the modulation transfer function, defined by M_1 and M_2 in equation [3.2], is already star-colour dependent. Large-scale effects can be modelled by making it position dependent and a third harmonic term can be added. Chromatic aberrations are modelled by adding a colour-dependent phase shift to a star's phase. Large-scale geometric distortions are modelled by adding a position-dependent n -th degree polynomial error to each star's estimated grid position. This polynomial is assumed to define the resultant distortion after calibration has been performed. Separate polynomials can be defined to make the distortion field of view and colour dependent;

(v) grid effects: locally, small-scale grid defects are modelled by generating Gaussian-distributed random errors in the grid slit spacing and positions, thus introducing extra frequency and phase errors into the image dissector tube signal. Medium-scale effects, caused by the method of manufacture of the grid, are modelled by generating position-dependent phase shifts from a sawtooth function. These phase shifts can be left uncorrected, or can be the residual errors after calibration results have been applied;

(vi) attitude errors: attitude jitter is a very important error source to consider, because it causes correlated position errors between the stars in view. It is modelled in the simulation by generating a time-dependent phase shift, with power spectral density defined by adding in parallel the output from a set of recursive digital filters, each corresponding to a different normal mode of the spacecraft structure. These filters are driven by a sequence of impulses, with some random variation, to simulate gas jet actuations. The constant acceleration introduced by perturbing torques can also be included in the simulation;

(vii) other errors: also included in the simulation are quantisation errors, due to data compression and subsequent de-compression of the samples; the possible loss of a sample when repositioning the instantaneous field of view to another star; and the effects of jitter in the on-board clock.

Signal Processing

The position of the star on the grid at a fiducial time, t , such as the time at the middle of the observation frame, is defined by the slit number of the star, N_G , together with the phase of the signal:

$$G = (N_G + \frac{\theta}{2\pi}) \quad [3.3]$$

Thus, the star's along-scan position on the grid is calculated by estimating the phase, θ , of the signal and combining it with the relevant slit number, obtained from *a priori* star positional information. The Image Location Stage program allows three different phase estimation techniques to be used: a simple Fourier estimator, a least-squares estimator and a maximum-likelihood estimator. Each of these will produce estimates of the star's phase, both from the first harmonic and from the second harmonic, of the image dissector tube signal. These two estimates are combined by computing their weighted average, and this is used as the best estimate of the signal phase.

Results of the Image Location Stage Investigations

The Image Location Stage program has been used extensively to investigate the nature of the error sources described above. All individual error sources have been considered and the couplings between them were checked when there was any reason to suspect that there might be a coupling. Also, extensive cross checking was used to verify that there were no unexpected couplings. The main conclusions of those investigations are described below.

Photon Noise

The variance of the position measurement errors due to the presence of photon noise is found to be inversely proportional to both the intensity of the star and to the amount of observing time allocated to the star. In other words, it only depends on the total number of detected photons. It does not matter how the observing time is arranged—the results are identical if the time is consecutive or interlaced into shorter discrete observations, so long as the total time remains constant. In the simple case of only photon noise being present in the signal, the Cramér-Rao statistical limit for the best possible performance of the phase estimate can be analytically evaluated as:

$$\sigma_{\text{CR}}^2 = \frac{s^2}{2\pi T I_0 I_{\text{CR}}(M_1, M_2)} \quad [3.4]$$

where T is the observing time (s), I_0 is the signal intensity (counts s^{-1}), s is the grid spacing (milli-arcsec), and $I_{\text{CR}}(M_1, M_2)$ is given by:

$$I_{\text{CR}}(M_1, M_2) = \pi \left[8 - \frac{M_1^2}{M_2} + \frac{(M_1^2 - 8M_2(1 - M_2))^{0.5}}{M_2} \times \right. \\ \left. [M_1^2 - 4M_2(1 + M_2) - 4M_2((1 + M_2)^2 - M_1^2)^{0.5}]^{0.5} \right] \quad [3.5]$$

It is found that the measurement variances produced in the Image Location Stage program by the maximum-likelihood estimator agree extremely well with this limit, provided that a minimum of about 100 photons are detected.

Several of the error sources have a direct multiplicative effect on the variance obtained in the photon noise case. These are: sample loss; third harmonic present in the signal; instantaneous field of view sensitivity changes; and small- and large-scale photocathode inhomogeneities.

Background Effects

Without photon noise, the constant extra background light does not contribute to the measurement variance. With photon noise, it causes an approximate increase in the variance which is inversely proportional to the intensity of the star squared, and to the observation time. Analytically, the effect of adding the extra constant signal, I_b , to the signal defined in equation [3.2] is to increase the signal intensity whilst reducing the amount of signal modulation in the following way:

$$\begin{aligned} I_0^* &= I_0 + I_b \\ M_1^* &= \frac{M_1 I_0}{(I_0 + I_b)} \\ M_2^* &= \frac{M_2 I_0}{(I_0 + I_b)} \end{aligned} \quad [3.6]$$

where the * indicates the modified values. Thus, it is possible to evaluate the Cramér-Rao bound using the modified signal parameters in exactly the same way as was done in

the previous section. If the background is small, the result follows by Taylor expansion, and has been verified by simulation. Typical predicted background count rates (in Hz) are given in Table 3.1, both for the main and star mapper detectors.

Time-Dependent Errors

It is found that two of the error sources cause an additive increase in the measurement variance that is inversely proportional to the observation time, but independent of star intensity. These are instantaneous field of view sensitivity changes and grid defects.

Note that the instantaneous field of view effects cause both an extra multiplicative factor on the photon noise variance and this extra additive variance.

Time-Independent Errors

The final type of Image Location Stage error is that which produces an extra additive variance, independent of both observation time and star intensity. These are large-scale geometric distortions, chromatic errors, and attitude jitter.

The saturation law and quantisation effects cause error variances that are time- and intensity-dependent in a complex manner. Fortunately, they are so small that they can be ignored.

Veiling Glare

One error source, veiling glare, is more difficult to treat. It is caused by a bright star falling in the extended wings of the image dissector tube's instantaneous field of view. This leads to an additive modulated term in the signal, and hence to estimated grid coordinate errors. To a certain extent, its effects can be minimised by choosing the Input Catalogue to avoid faint stars with bright stars close by. A further problem is that bright stars from the other field of view can cause the veiling glare. In this case, the measurement will be rejected, adding to the mission dead time, but propagating no further in the data reduction. The error is highly non-normal in distribution. A separate estimate of the residual errors contributed by less bright stars that are not rejected by the above approach was made and is accounted for, explicitly, in the final accuracy assessment (see Section 3.7.)

Summary of the Results of the Image Location Stage

The results of the Image Location Stage simulation are summarised into four types of error, which can then be accommodated in the great-circle reduction simulation:

- errors whose variances are inversely proportional to star intensity and observation time (photon noise type);
- errors whose variances are inversely proportional to star intensity squared and observation time (background type);
- errors whose variances are inversely proportional to observation time (grid type);
- errors whose variances are constant (distortion type).

In summary, the error at frame level can be written as:

$$\sigma^2 = \left(\frac{A}{I^2} + \frac{B}{I} + C \right) \frac{1}{T} + D \quad [3.7]$$

where A , B , C and D are star colour-dependent constants, I is proportional to the stellar flux, and T is the observation time. There are only three sources of error that do not satisfy this formula and they are negligible or, in the case of veiling glare, treated separately.

Table 3.1. Expected background count rates (in Hz) for both the main image dissector tube detector (IDT) and the two star-mapper channels (B_T and V_T).

Contributor	IDT	B_T	V_T
Dark counts	6	191	240
Sky background	35	860	1267
Sun straylight	35	860	1267
Light leakage	7	172	253
Cerenkov effect	8	1200	1200
Total	91	3283	4227

Table 3.2. Hipparcos model star distribution.

B (mag)	Number of stars	$B - V = -0.25$ (per cent)	$B - V = 0.5$ (per cent)	$B - V = 1.25$ (per cent)	Observing time per star (s)
<6	3000	59	28	13	270
6-7	5400	48	35	17	310
7-8	14800	37	46	17	390
8-9	40800	32	51	17	530
9-10	16000	21	52	27	730
10-11	12000	10	49	41	990
11-12	6000	4	38	58	1330
12-13	2000	5	41	54	1780

Table 3.3. Specified and actual (budget) ‘dead times’.

Contributor	Fractional dead time (per cent)	
	Specification	Budget
Earth and moon occultations	6.0	5.1
Thruster actuations	1.0	0.3
Emergency situations	0.5	0.5
Data loss due to routine operations*	2.5	0.2
Veiling glare effect	2.0	0.0
Margin	2.0	–
Total for satellite	14.0	6.1
Ground segment losses	4.0	4.0
Total	18.0	10.1

* Calibrations, antenna switching, and station keeping

3.6. Great-Circle Reduction Program

The second stage of data reduction is the computation of star abscissae around a strip of sky—the great-circle reduction stage. The simulation proceeds by generating a fairly realistic distribution of stars around the strip of sky to be scanned. The summary results of the Image Location Stage program are then normally used to generate typical individual measurement errors on the stars in view for each frame of the simulation. The individual measurements are then reduced using one of several implemented algorithms to yield abscissae estimates around the great circle. These estimates can then be compared with the true catalogue positions to indicate the achieved accuracy.

The satellite scans the whole sky by the combination of two motions: a rotation about an axis normal to the plane of the two viewing directions at a rate of $168.75 \text{ arcsec s}^{-1}$, while the spin axis itself revolves, inclined at 43° to the sun line, at a rate of 6.4 revolutions per year (see Section 2.3). Both of these motions are simulated in the generation of abscissae.

Catalogue Generation and Frame Simulation

The great-circle reduction simulation starts by first generating the subset of the catalogue of stars in the strip of sky that is to be scanned. A fixed axis in space, close to the satellite spin-axis, is chosen to define a 'reference great circle'. Each star's position is defined relative to this reference by two angles: an abscissa and an ordinate. Thus, for each star in the strip of sky, the following is generated:

- position of star relative to reference great circle;
- *a priori* errors in the position;
- magnitude and colour of the star.

These data are usually generated randomly, from the probability density functions defined for the Image Location Stage program, but facilities do exist for the program to use fixed distributions of any of the above data or for the program to read from an existing catalogue.

The simulation proceeds by generating the positions of the two fields of view for each frame. This requires the two attitude motions (rotation and spin-axis precession) to be simulated. The stars in view for each frame are extracted from the catalogue and the star observing strategy (as used on-board) is used to allocate the observation times to the different stars. The frame simulation can be done in one of two ways:

- (1) 'slow' mode: the Image Location Stage frame simulation can be used—image dissector tube data can be generated and reduced to give the position estimates;
- (2) 'fast' mode: a set of grid positions can be produced by generating typical frame errors and adding them to the true star positions. The errors used are generated with variances derived from Image Location Stage simulations.

In addition to the error sources that are applicable at the frame level, several other effects can be considered at the great-circle reduction stage. These include:

- (1) basic angle variation: the basic angle variations are assumed to be periodic around one scan. This is because the satellite is almost geostationary and the dominant sources of basic angle variations will be thermally induced. A number of such terms with different frequency, amplitude, and phase can be added together to provide a resultant variation as a function of time;
- (2) interruptions (resulting in 'dead time'): groups of frames can be considered 'lost' due to occultations of the earth or moon, or due to gas-jet actuations;

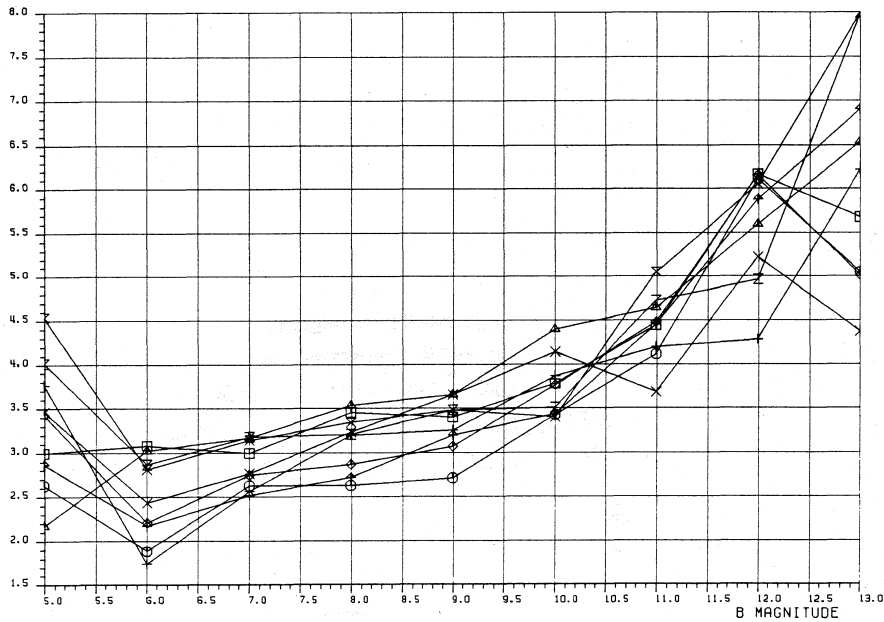


Figure 3.3. The great-circle reduction results (photon noise).

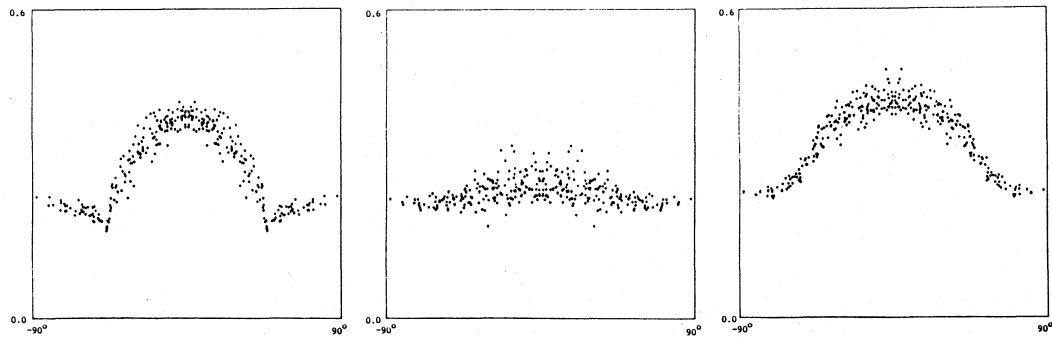


Figure 3.4. The form of the coefficients of improvement in (a) longitude ($\Delta\lambda \cos\beta$), (b) latitude ($\Delta\beta$) and (c) parallax, as a function of ecliptic latitude, based on early simulations by the data-reduction consortia (Lindgren, private communication). The simulations assume a mission lifetime of 2.5 years, $\xi = 43^\circ$, $K = 6.4$, and $R = 12$ revolutions per day for the scanning law (see Chapter 2), and a reference great-circle 'set' duration of 12 hours. The ordinates give estimates of the numerical factors which must be applied to the great circle abscissa errors to derive the final accuracies for each of the astrometric parameters.

(3) spin axis motion: this causes errors in the estimation of the abscissae on the reference great circle due to imperfect determination of the satellite attitude.

Data Reduction

The estimates of the star abscissae around the reference great circle can be combined with the along-scan attitude errors in the following way:

$$\alpha_i - \omega_k = \eta'_{ik} + \epsilon_{ik} \quad [3.8]$$

where α_i is the true abscissa for observed star i , ω_k is the along-scan attitude error for frame k , η'_{ik} is the estimated abscissa of star i based on the measurement in frame k , and ϵ_{ik} is the error in the measurement of star i in frame k .

This form of the observation equations lends itself to a least-squares estimation of the unknowns $\{\alpha_i\}$ and $\{\omega_k\}$. The following equations for the estimates of the star abscissae can be derived from the least-squares equations:

$$\tilde{\alpha}_i = \frac{1}{n_i} \sum_{k=1}^M \left[p_{ik} + (\eta''_{ik} + \frac{1}{p_k} \sum_{j=1}^N p_{jk} \tilde{\alpha}_j) \right] \quad [3.9]$$

where: $\tilde{\alpha}_i$ is the estimated abscissa for observed star i , p_{ik} is +1 if star i occurs in frame k , and 0 otherwise, $p_k = \sum_{i=1}^N p_{ik}$, where N is the total number of stars in the reduction, $n_i = \sum_{k=1}^M p_{ik}$ where M is the total number of frames in the reduction, and $\eta''_{ik} = \eta'_{ik} - \frac{1}{p_k} \sum_{j=1}^N p_{jk} \eta'_{jk}$, i.e. the estimate of the abscissa of star i in frame k less the average star abscissa in that frame.

Equation [3.9] can easily be generalised into a weighted least-squares form. Three different sets of equations can be set up by the great circle reduction program, corresponding to perfect knowledge of the attitude, the simple least-squares solution, and the weighted least-squares solution. These equations are solved using either an iterative algorithm or using a direct sparse matrix inversion algorithm.

Results of the Great-Circle Reduction Investigations

The great-circle reduction program has been used for various investigations and studies into the error sources applicable at the great-circle stage of the data reduction, and into the propagation of the errors from the Image Location Stage. As an illustration of the results, the case of photon noise as the only error source is shown.

Figure 3.3 shows the results, after reduction by least-squares, of nine different great-circle scans on the same strip of sky. The results are plotted as rms abscissae errors against B magnitude. The smaller the number of stars, as is the case for the very bright and very faint stars, the greater the spread of the variances. The general form of the graphs show that there is a constant part to the variances (due, in fact, to the attitude uncertainties) and a magnitude-dependent part, due to the photon noise itself.

The results are found to be well-fitted by the 'rigidity' formula:

$$\sigma_\alpha^2 = \frac{(\sigma_\eta^2 + k < \sigma_\eta^2 >)}{n} \quad [3.10]$$

where n is the average number of observations per star in a great circle, $< \sigma_\eta^2 >$ is the longitudinal field-coordinate harmonic mean variance, and k is a parameter calibrated by the great-circle reduction simulation.

The first term of the right-hand side of the above equation corresponds to the measurement-error variance of the star under consideration, while the second term characterises the geometrical 'rigidity' of the great-circle system of equations, corresponding to errors in the attitude reconstruction.

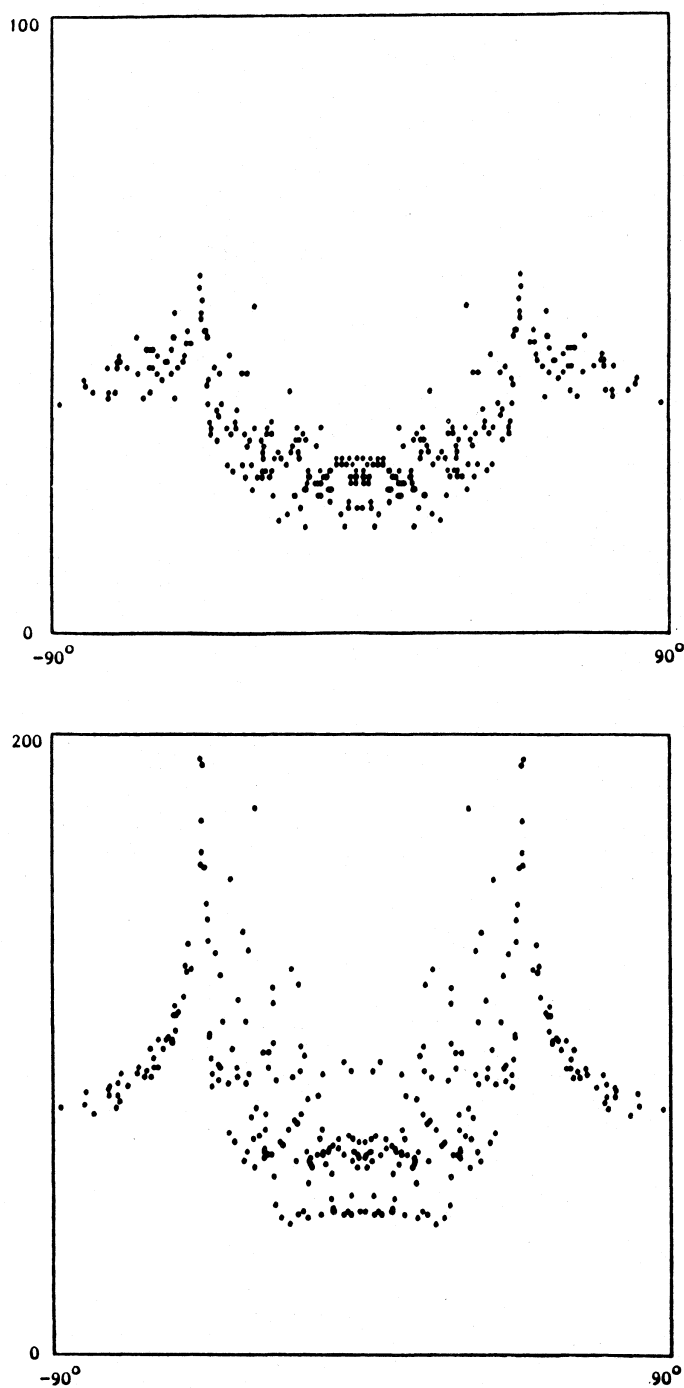


Figure 3.5. The number of (a) great-circle 'sets', and (b) great-circle scans across a star as a function of ecliptic latitude, based on early simulations by the data-reduction consortia (Lindgren, private communication).

Whole-Sky Reduction

The abscissa errors can be transformed directly to yield the final accuracy on the five astrometric parameters, using equation [3.1] to correct for the coefficients of improvement, and the dead time. The coefficients of improvement are computed by a separate software package that assumes that the abscissa errors from different great circles are uncorrelated. This should be the case, at least for errors caused by photon noise and attitude uncertainties. Some residual correlated terms may remain, due, for example, to basic angle variations, but these are expected to be small. If the errors are uncorrelated, the computation of the expected coefficients of improvement is essentially a geometric exercise. For each astrometric parameter, the average number of contributing observations is known from the scanning law, together with an appropriate weight for each, depending on the abscissa variance, plus an error in placing the origin of the great circle, and a projection factor. Thus, the variance on the parameter can be computed and the coefficient of improvement derived. This coefficient is, in general, position-dependent over the celestial sphere, but the scanning law minimises such variations, and the worst-case value has been chosen for the accuracy assessment. The only system parameters that can affect the final accuracy at this stage are related to the scanning law.

Figure 3.4(a-c) illustrate the form of the coefficients of improvement in longitude, latitude and parallax respectively, as a function of ecliptic latitude, based on early simulations by the data reduction consortia. Figure 3.5 illustrates the number of great-circle 'sets' and the number of great-circle scans across a star as a function of ecliptic latitude, also based on early simulations by the data-reduction consortia.

3.7. Summary of the Expected Mission Accuracy

Main Mission

The final mission accuracies are derived from the field coordinate error variance σ_η^2 for different star magnitudes and colours, corresponding to the allocated observation times per star shown in Table 3.2. The harmonic mean variance $\langle \sigma_\eta^2 \rangle$ is calculated on the basis of the model distribution of stars, also shown in Table 3.2, to be 50 milli-arcsec². The terms to be added to the abscissa error variance are:

- (i) 0.7 milli-arcsec² ($B = 9$ mag) or 31 milli-arcsec² ($B = 12$ mag) for the effect of parasitic stars;
- (ii) 0.1 milli-arcsec² for basic-angle variations.

The dead time factor F is 1.05, corresponding to a dead time of 10 per cent (see Table 3.3). The final astrometric errors are then derived, using the improvement coefficients, and are summarised in Table 3.4. The latter shows that in all cases the performance is better than the corresponding specification.

Predicted astrometric accuracies, based on expected instrumental performances, and using a more rigorous propagation of simulated data through the actual data-reduction chain, are given in Volume III, Chapter 18, as a function of B and b .

The Tycho Experiment

The required star-mapper performance is specified in terms of the astrometric and photometric accuracies of the measurements made during a single transit of a star image across the complete star-mapper slit system (transverse and inclined slits). Thus, the only element of data reduction to be taken into account is the performance of the

Table 3.4. Astrometric performance assessment for the main mission (values are given in milli-arcsec).

<i>B</i> (mag)	<i>B</i> − <i>V</i> (mag)	spec	σ_{a_1}	σ_{a_2}	σ_{a_3}	σ_{a_4}	σ_{a_5}
9	−0.25	2.6	1.4	1.9	1.1	1.6	1.7
	0.50	2.0	1.3	1.8	1.0	1.5	1.6
	1.25	2.6	1.3	1.7	1.0	1.4	1.6
12	−0.25	5.2	2.8	4.0	2.3	3.3	3.6
	0.50	4.0	2.4	3.4	2.0	2.8	3.1
	1.25	5.2	2.1	2.9	1.7	2.4	2.6

Key:
 σ_{a_1} = error on ecliptic longitude component of position
 σ_{a_2} = error on ecliptic longitude component of proper motion
 σ_{a_3} = error on ecliptic latitude component of position
 σ_{a_4} = error on ecliptic latitude component of proper motion
 σ_{a_5} = error on parallax
(Improvement coefficients used were: 0.209, 0.292, 0.170, 0.241 and 0.262, respectively)

Table 3.5. Astrometric and photometric performance assessment for the Tycho experiment.

$(B + V)/2$ (mag)	<i>B</i> − <i>V</i> (mag)	Astrometry		Photometry		
		spec (arcsec)	σ_a (arcsec)	spec (mag)	σ_{B_T} (mag)	σ_{V_T} (mag)
9.65	0.0	0.15	0.07	0.375	0.12	0.15
	0.7	0.10	0.08	0.250	0.17	0.14
	1.5	0.15	0.08	0.375	0.25	0.12
10.65	0.0	0.30	0.16	0.750	0.25	0.34
	0.7	0.20	0.19	0.500	0.37	0.30
	1.5	0.30	0.17	0.750	0.57	0.24

filter used to extract the phase of the signal. The accuracy assessment, following lines similar to, but simpler than those of, the main mission, leads to the results presented in Table 3.5. Note that the combinations of stellar magnitude and colour to which the specifications apply differ from those for the main mission, to take account of the different spectral response(s) of the Tycho experiment and the different colour distribution of the stars to be measured.

The accuracy assessment is a complex and lengthy process to carry out completely. Therefore, it has been repeated only at six-month intervals during the Hipparcos project's development. The successive issues of the accuracy assessment have a multiple purpose:

- (i) to ensure a regular monitoring of the achievable final mission performance;
- (ii) to validate the set of specifications generated by the satellite prime contractor at the beginning of the project;
- (iii) to produce a contractually-applicable estimate of mission performance at well-identified milestones in the Hipparcos project, starting with the system Critical Design Review and finishing six months after the beginning of in-orbit operations.

In between two successive issues, the evolution in mission accuracy has been estimated, when necessary, by means of sensitivity analyses.