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GEOMETRIC PROPERTIES OF SOLAR FLARES AND THEIR ENERGETICS

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ГЕОМЕТРИЧЕСКИБ СВОЙСТВА СОЛНЕЧНЫХ ВСПЫШЕК И ИХ ЭНЕРГЕТИКА

На основе статистического анализа 18 параметров для 404 вспышек было установлено, что энергетически значимые вспышки (I-я популяция в настоящей работе) приобретают большие размеры вспышечной аркады (x 14, x 11, x 12) а также площади и объемы побочных вспышечных фокусов (x 18, x 15). Самые низкие H-альфа петли вспышечной аркады (x 13) имеют сходные размеры как для импульсного ($N_{II} = 330$), так и для динамического ($N_I = 74$) типов вспышек. Мощность (x 6) и время жизни (x 8, x 9) вспышек H-альфа отчетливо больше для энергетически значимых вспышек. Средние значения переменных ($x\,1 - x\,18$) для вспышек импульсного (II) динамического (I) типов даны в табл. 1. Взаимные связи переменных ($x\,1 - x\,18$) анализировались методами PCA (анализ основных компонентов) и FA (факторный анализ). Результаты иллюстрируются частью корреляционной матрицы (рис. 2 и табл. 2) и содержанием факторов F_1 — F_4 в отдельных переменных (рис. 3 и 4).

The mutual relationship of 18 geometrical and energy variables of 404 flares has been considered. It was found that 1) Approximatelly 60% of the total variance of the system ($x\,1 - x\,18$) could be explained by 3—4 F_j factors. 2) The lowest H-alpha loops of the flare arcade (x 13) have comparable mean dimensions for both studied flare groups, i.e. for "major flares" ($N_I = 74$) as well as for impulsive flares ($N_{II} = 330$). During the maximum H-alpha phase, the height of the lowest flare arcade loops is about 7000 km. 3) Energetically significant "major flares" acquire larger lengths of the flare ribbons (see $\bar{x}$ 11 in Tab. 1), as well as the ranges and volumes of the supplemental flare structures ($\bar{x}$ 15 and $\bar{x}$ 18) than impulsive flares. 4) Commonly held views about simple relationships between H-alpha or SXR flare importance and the flare duration are in error. A relationship was found between H-alpha duration and the comprehensive flare index (CI = x 5).

Key words: solar flare — impulsive flare — gradual (extended, dynamical, explosive) flare

1. Introduction

For all essential flares observed in the course of the International Geophysical Year (IGY, from July 1957 to December 1958), the Meudon IGY heliographic maps had enabled measurements of the geometric characteristics of the H-alpha flares. The significance of the individual measured variables (x 1 to x 18) is shown in Section 2 of this paper. All flares indicated on the Meudon heliographic maps were distributed into two energetically different groups:

I — "Major flares" conforming to the conditions defined by Dodson and Hedeman (1971), i.e. those flares, which satisfied any one of the following 5 criteria: SID and H-alpha flare importances larger than degree 3, the flux on 10 cm larger than 500 × × 10^{-22} W m^{-2} Hz^{-1}, type II burst and type IV radio emissions with durations of over 10 minutes. Our sample accounts for 74 such "major flares". According to recent flare classification rules (De Jager 1988, Sturrock 1987, Tanaka 1987), these "major flares" were perhaps, in their overwhelming majority, of the gradual flare type. Coincidence between "major" and gradual flares is called into question, when the flare displayed only an isolated type II burst (it is a "major" but perhaps not a gradual flare.)

II – The flares observed in the course of the IGY, which failed to satisfy the conditions for "major flares". This group numbered 330 flares. In accordance with the current flare classification, these flares mainly belong to the impulsive flare type.

The question of the suitability of processing comparatively obsolete observation materials, comprising geometric flare properties as obtained in the course of the IGY, might justifiable arise. As a matter of fact, the IGY maps represent unique homogeneous sets of H-alpha flares which enable the geometric dimensions of a large number of flares to be measured. International flare research programmes, organized at a later date, published no flare maps.

The purpose of this paper is

a) the mutual comparison of the same variable x_i in two, energetically different populations of flares (groups I and II, see Tab. 1 and Fig. 1),

b) the determination of the mutual relationship between the measured variables ($x\,1 - x\,18$).

2. Observational Material

The variables ($x\,6 - x\,18$) were measured for all 404 flares indicated on the IGY Meudon Solar Activity Maps. The variables ($x\,1 - x\,5$) were taken from the Report UAG-14 (Dodson and Hedeman 1971). The significance of the variables ($x\,1 - x\,18$) is as follows:

$x\,1$ – SID (Sudden Ionospheric Disturbance). The $x\,1$ scale ranges from 0 to 3.

$x\,2$ – The magnitude of the flux on 10 cm wavelength. The value of $x\,2$ is characteristic of the decadic logarithm of the F_{10} flux, measured in units of 10^{-22} W m^{-2} Hz^{-1}.

$x\,3$ – Dynamic spectrum of the flare. ($x\,3 = 1$ for type II burst, 2 for radio continuum, 3 for type IV with duration exceeding 10 minutes).

$x\,4$ – 200 MHz flux (F_{200}), expressed in the same unit as $x\,2$.

$x\,5$ – Comprehensive flare index (CI). Its value was defined by Dodson and Hedeman (1971) as $x\,5 = x\,1 + x\,2 + x\,3 + x\,4 + x\,6$.

$x\,6$ – H-alpha flare importance (old scale ranges from 0 for subflares to 3 for the largest flares).

$x\,7$ – Duration (in minutes) of the increasing phase of the H-alpha flare, expressed in decadic logarithms.

$x\,8$ – Duration of the decay phase of the H-alpha flare, expressed similarly to $x\,7$.

$x\,9$ – Total flare duration, i.e. $x\,9 = x\,7 + x\,8$.

$x\,10$ – Number of flare ribbons.

$x\,11$ – Length of flare ribbon (L), expressed in heliographic degree, i.e. 1° (at latitude 15°) = $= 11{\cdot}7 \times 10^3$ km.

$x\,12$ – Outer radius of the flare arcade (r_1), i.e. half the distance between the outer edges of the mutual parallel flare ribbons, expressed in heliographic degrees. The value of $x\,12$ is related to the flare phase, in which it has its maximum area, in the H-alpha line.

$x\,13$ – Inner radius of the flare arcade (r_2) expressed similarly to $x\,12$; this is half the distance between the inner edges of two mutual parallel flare ribbons. $x\,13$ usually increases with the developing flare. Our value of $x\,13$ represents the lowest height of the flare arcade loops as measured at the time of the H-alpha maximum.

$x\,14$ – Volume of the flare arcade $V = \frac{1}{2}\pi(r_1 - r_2)^2 L$, expressed in decadic logarithms.

$x\,15$ – Volume of the supplemental flare structures, expressed similarly to $x\,14$.

$x\,16$ – Association of the flare with filament activity. Filaments are recorded in the IGY Solar Activity Maps only once per 24 hours, hence this value only has tentative significance.

$x\,17$ – The sunspot area covered by flare ribbons, at the time of maximum flare development. $x\,17$ is expressed in units of heliographic degrees squared.

$x\,18$ – The maximum distance of the supplemental flare structures from the centre of the flare arcade. $x\,18$ is measured in heliographic degrees.

3. The Frequency Distribution Functions of the Variables ($x\,1$—$x\,18$)

The frequency distribution functions of the variables ($x\,1 - x\,18$) are shown in Figs 1a–1c. The mutual comparison of the mean values $\bar{x}_i$ and distribution functions of the same variable x_i in two, energetically different flare populations (I and II) are summarized in Tab. 1, which has been arranged as follows:

2nd and 3rd columns – Mean value of the variable $\bar{x}_i$(I) and its standard deviations s(I), calculated for "major flares" (population I).

4th and 5th columns – Mean value of the variable $\bar{x}_i$(II) and its standard deviation s(II) for impulsive flares (population II).

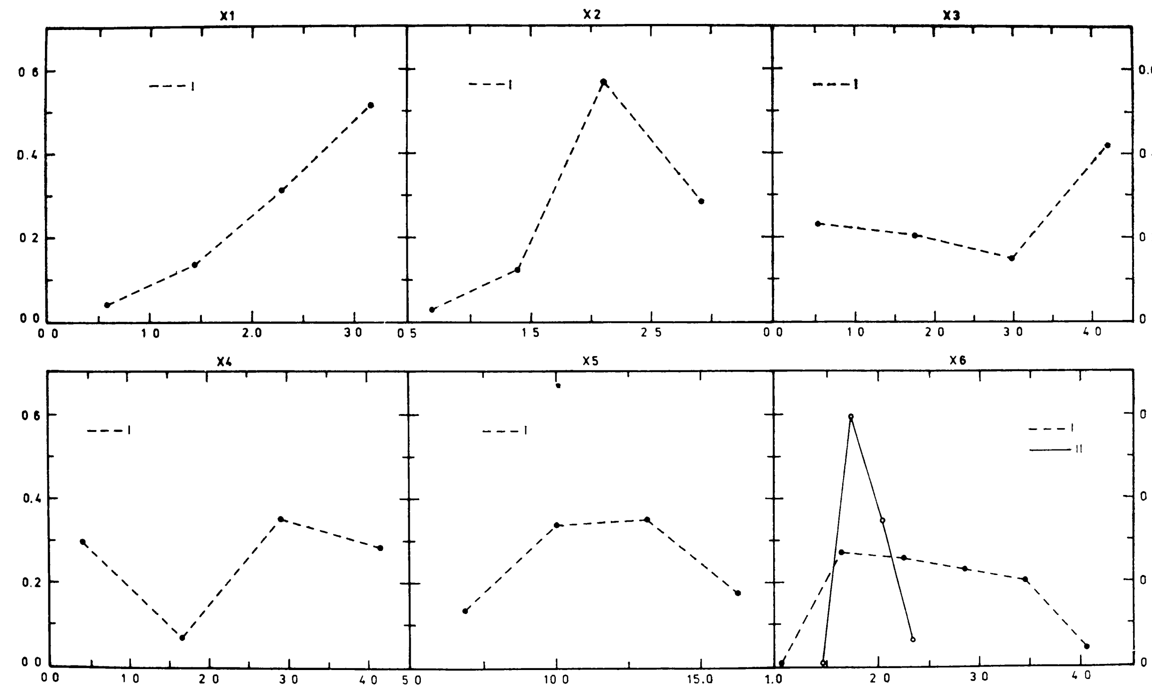

Fig. 1a. The frequency distribution functions of the (x 1—x 6) variables. The x-axis represents the x_i variable, the y-axis its percentual representation within the given flare population. The gradual "major" flares (Population I, $N_I = 74$) are shown by dashed lines, the impulsive flares (Population II, $N_{II} = 330$) by full lines. The x 6 variable (H-alpha flare importance) has different distribution functions for both compared flare populations.

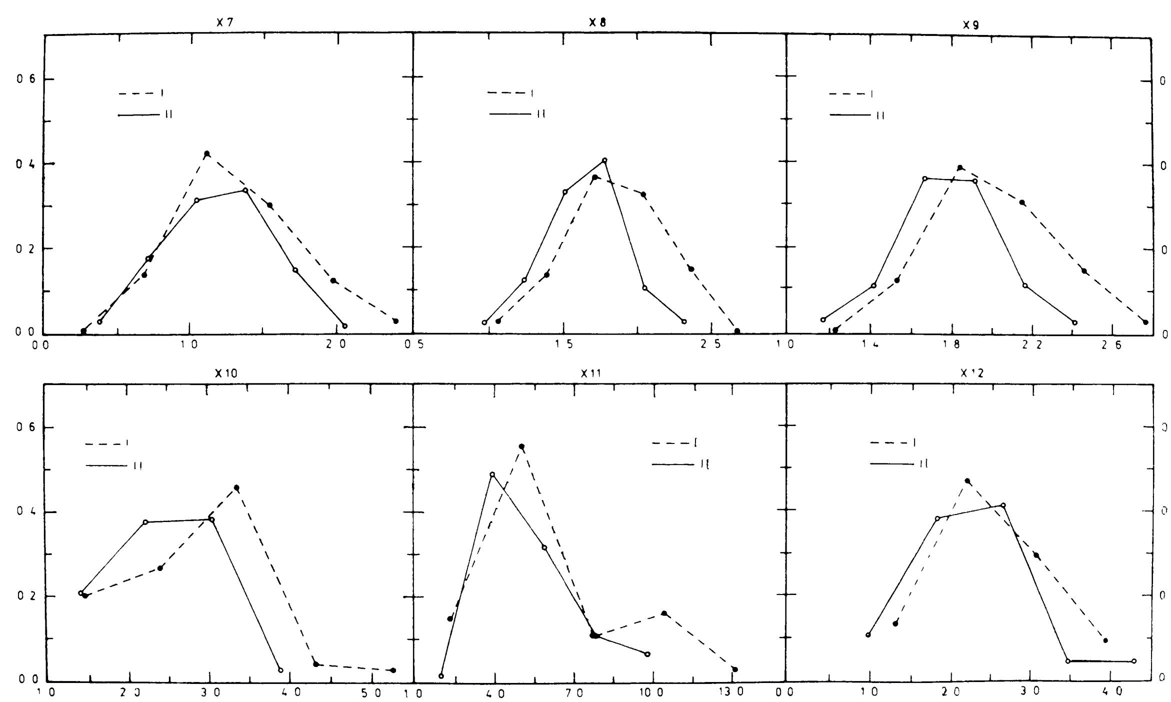

Fig. 1b. The frequency distribution functions of the (x 7—x 12) variables. Largest differences for both flare types were observed for the duration of the decay phase (x 8), total flare duration time (x 9), and the length of the H-alpha flare ribbons (x 11). The number of flare ribbons (x 10) in both flare populations has a mean value exceeding 2·0.

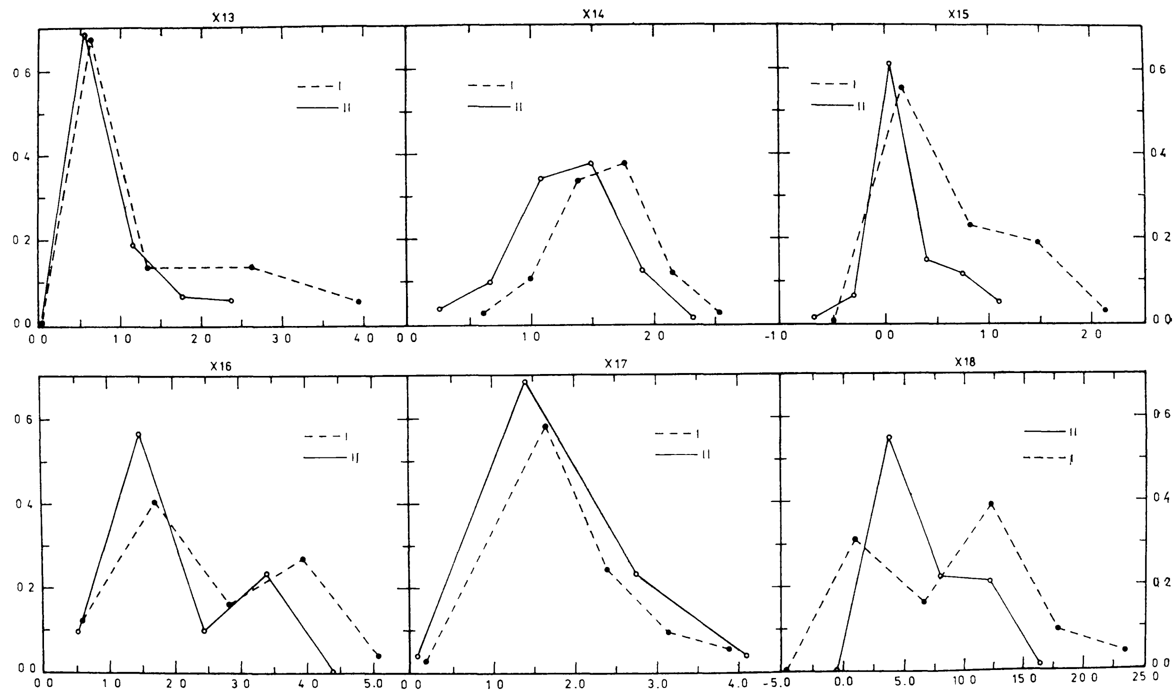

Fig. 1c. The frequency distribution functions of the (x 13—x 18) variables. The flare populations differ in the arcade volume (x 14), volume of the supplemental flare structures (x 15) and the flare action radius (x 18 — the distance of the supplemental flare structures from the centre of the flare arcade). At the time of the maximum flare development, both the impulsive and the "major" (gradual) flares display their lowest arcade loops (x 13) approximately at the same altitude of 7 000 km, i.e. 0·6°.

Table 1.
Distribution functions of x_i in the 1st (major) and 2nd (impulsive) flare populations

x_i	$\bar{x}_i$(I)	s(I)	$\bar{x}_i$(II)	s(II)	u	F	λ
x 6	2·241	0·605	1·730	0·303	**7·05**	**3·99**	**3·21**
x 7	1·113	0·431	1·047	0·336	1·25	**1·64**	0·48
x 8	1·712	0·326	1·514	0·270	**4·87**	1·46	**2·69**
x 9	1·838	0·310	1·664	0·249	**4·51**	1·55	**2·97**
x 10	2·419	0·951	2·236	0·824	1·53	1·33	1·63
x 11	5·054	2·713	3·932	1·963	**3·37**	**1·91**	0·91
x 12	2·182	0·878	1·810	0·831	**3·33**	1·12	1·13
x 13	0·628	0·704	0·553	0·609	0·85	1·36	0·76
x 14	1·381	0·384	1·079	0·413	**6·03**	1·16	**2·20**
x 15	0·159	0·659	−0·161	0·469	**3·95**	**1·97**	1·60
x 16	1·703	1·119	1·476	0·959	1·62	1·36	**1·70**
x 17	1·655	0·744	1·407	0·673	**2·64**	1·22	**1·68**
x 18	6·554	5·645	3·673	4·217	**4·14**	**1·79**	**2·05**

u — statistical test, $u_{0.01}$ = 2·58
F — Snedocor distribution, $F_{0.01}$ = 1·56
λ — Smirnov-Kolmogorov statistics, lambda$_{0.01}$ = 1·63
— The deviations of u, F, lambda exceeding the significance level 0·01 are bold faced
$\bar{x}_i$— mean value x_i; s— standard deviation, s^2 — variance

6th, 7th and 8th columns – Statistical tests u, F, lambda of the distribution functions, characterizing the same variable x_i of two mutually compared flare populations. The values u, F, lambda, which exceed the confidence interval for alpha = 0·01 are boldfaced.

3.1 *Statistical Test*: *u*, *F and Lambda*

Comparison of the frequency distribution functions of x_i in the first ($N_{\rm I}$ = 74 gradual flares) and second ($N_{\rm II}$ = 330 impulsive flares) populations:

a) The statistical u-test verifies the assumption that the frequency distribution functions of x_i of both the compared flare groups are such that their mean values are equal, i.e. $\bar{x}_i(\mathrm{I}) = \bar{x}_i(\mathrm{II})$. The deviation parameter u is defined by the equation

$$u = \frac{|\bar{x}(\mathrm{I}) - \bar{x}(\mathrm{II})|}{s_u} \quad (1)$$

where

$$s_u = \left(\frac{s_{\mathrm{I}}^2}{N_{\mathrm{I}}} + \frac{s_{\mathrm{II}}^2}{N_{\mathrm{II}}}\right)^{1/2}$$

s is the standard deviation. It is assumed that u has a normal distribution N; $u \in N(0, 1)$ and for alpha = 0·01, $u_{0.01} = 2{\cdot}58$.

b) Snedocor's F-statistics verify the assumption that the variances of s^2 in the 1st and 2nd flare populations are equal; F is defined by the equation

$$F = \frac{s_{\mathrm{I}}^2}{s_{\mathrm{II}}^2} \quad (2)$$

For alpha = 0·01 its value $F_{0.01} = 1{\cdot}56$.

c) The Smirnov-Kolmogorov lambda test varifies the assumption that the distribution function of the variable x_i in the 1st and 2nd flare populations follows identical patterns. Parameter lambda is defined by the equation

$$\lambda = D(N)^{1/2} \quad (3)$$

where

$$D = \sup_x |S_{\mathrm{I}}(x) - S_{\mathrm{II}}(x)|$$

$$N = \frac{N_{\mathrm{I}} N_{\mathrm{II}}}{N_{\mathrm{I}} + N_{\mathrm{II}}}$$

$S_{\mathrm{I}}(x)$ and $S_{\mathrm{II}}(x)$ are cumulative distributive functions. For alpha 0·01 the lambda value is 1·63.

3.2 *Results of the Comparison of the Frequency Distribution Functions for the "Major" (I) and Impulsive (II) Flares (Figs 1a–1c, Tab. 1)*

a) Properties concordant for both flare types (x 10– number of flare ribbons; x 13 – inner radius of the flare arcade)

In this section we adress the question: How does the occurrence of two parallel ribbon flares vary with the flare types?

According to all three statistical tests, distribution functions for the variables x 10 (number of flare ribbons) and x 13 (inner radius of the flare arcade) follow the same patterns for "major" and impulsive flares. This means that the occurrence of two-ribbon flare morphology is very frequent in the whole flare population. This basic flare morphology does not respond to energetic differences of impulsive and gradual flares (Kučera and Antalová 1988).

The analysis of variable x 13 (the height of the lowest loops of the flare arcade) shows a similar case. At the time of the maximum flare development, both the impulsive and the "major" (gradual) flares display their lowest arcade loops (x 13) approximately at the altitude of 7·0 thousand km (0·6 heliographic degree). The lowest part of flare arcade, therefore, has the same radius for both kinds of flares (Kučera and Antalová 1988).

b) Considerable different distribution of variables x 6 and x 18 (x 6 – H-alpha flare importance; x 18 – the range of flare action)

The variables x 6 (H-alpha flare importance) and x 18 (the range of flare action) follow different patterns in the 1st and 2nd flare populations, as indicated by all three statistical tests (Tab. 1).

The mean H-alpha importance is $\bar{x}\ 6(\mathrm{I}) = 2{\cdot}2 \pm 0{\cdot}1$ for the gradual "major" flares, while for the impulsive flares $\bar{x}\ 6\ (\mathrm{II}) = 1{\cdot}7 \pm 0{\cdot}1$ (see Fig. 1a).

Similarly, the supplemental flare structures are located on a larger area in "major flares" as compared to impulsive flares. The mean distance of the supplemental flare structures from the main flare arcade is $\bar{x}\ 18(\mathrm{I}) = 6{\cdot}5° = 76 \times 10^3$ km for the "major flares", while for the impulsive flares $\bar{x}\ 18(\mathrm{II}) = 3{\cdot}7° = 43 \times \times 10^3$ km. The range of flare action (x 18) is considerable larger for the "major flares" than for the impulsive flares.

c) Different distribution of x 11 and x 15 in the u- and F-tests (x 11-the flare ribbon length; x 15 – the volume of the supplemental flare structures, Fig. 1b)

As can be seen in Section 3.2a, the flare morphology and inner radius of the flare arcade play little part in the energy differentiation of flares. Unlike to 3.2a, the good symptoms of gradual ("major") flares are variables x 11 – the length of flare ribbons and x 15 – the volume of the supplemental flare structures.

The mean flare ribbon length ($\bar{x}$ 11) for the "major" flares is approximately 60 000 km and for the impulsive flares 46 000 km.

Similarly, the volumes of the supplemental flare structures ($\bar{x}$ 15) have been found to differ by 2 orders of magnitude.

Table 2

Three maximum values of the correlation coefficient between variables ($x\,1$—$x\,18$), chosen from the correlation matrix

x_i	r_1	x_i	r_2	x_i	r_3	x_i	r_1	x_i	r_2	x_i	r_3
"major flares" — 1st population						impulsive flares — 2nd population					
$x\,1$ — SID (Sudden Ionospheric Disturbance)											
$x\,5$	0·60	$x\,2$	0·53	$x\,17$	0·31	—	—	—	—	—	—
$x\,2$ — F_{10} (The flux on 10 cm wavelength)											
$x\,5$	0·62	$x\,1$	0·53	$x\,17$	0·40	—	—	—	—	—	—
$x\,3$ — Dynamic spectrum											
$x\,5$	0·73	$x\,4$	0·53	$x\,6$	0·44	—	—	—	—	—	—
$x\,4$ — F_{200} (200 MHz flux)											
$x\,5$	0·71	$x\,3$	0·53	$x\,2$	0·26	—	—	—	—	—	—
$x\,5$ — CI (Comprehensive flare index)											
$x\,3$	0·73	$x\,4$	0·71	$x\,2$	0·62	—	—	—	—	—	—
$x\,6$ — Importance H-alpha											
$x\,9$	0·62	$x\,8$	0·59	$x\,11$	0·55	$x\,15$	0·39	$x\,18$	0·37	$x\,14$	0·35
$x\,7$ — the duration of the flare increasing phase											
$x\,9$	0·71	$x\,6$	0·49	$x\,8$	0·47	$x\,9$	0·70	$x\,8$	0·43	$x\,6$	0·33
$x\,8$ — the duration of the flare decay phase											
$x\,9$	0·94	$x\,6$	0·59	$x\,7$	0·47	$x\,9$	0·93	$x\,7$	0·43	$x\,6$	0·26
$x\,9$ — the duration of the whole flare ($x\,7 + x\,8$)											
$x\,8$	0·94	$x\,7$	0·71	$x\,6$	0·62	$x\,8$	0·93	$x\,7$	0·70	$x\,6$	0·32
$x\,10$ — number of flare ribbons											
$x\,15$	0·69	$x\,18$	0·61	$x\,13$	0·55	$x\,15$	0·63	$x\,18$	0·63	$x\,12$	0·39
$x\,11$ — the length of the flare ribbons											
$x\,6$	0·55	$x\,14$	0·45	$x\,7$	0·42	$x\,10$	−0·28	$x\,6$	0·27	$x\,12$	−0·25
$x\,12$ — the outer radius of the flare arcade											
$x\,14$	0·74	$x\,13$	0·55	$x\,16$	0·29	$x\,14$	0·76	$x\,13$	0·67	$x\,10$	0·39
$x\,13$ — the inner radius of the flare arcade											
$x\,10$	0·55	$x\,12$	0·55	$x\,14$	0·32	$x\,12$	0·67	$x\,14$	0·40	$x\,10$	0·35
$x\,14$ — the arcade volume											
$x\,12$	0·74	$x\,11$	0·45	$x\,13$	0·32	$x\,12$	0·76	$x\,13$	0·40	$x\,6$	0·35
$x\,15$ — the volume of the supplemental flare structures											
$x\,18$	0·70	$x\,10$	0·69	$x\,6$	0·51	$x\,18$	0·67	$x\,10$	0·63	$x\,6$	0·39
$x\,16$ — association with filament											
$x\,14$	0·36	$x\,6$	0·33	$x\,11$	0·30	$x\,13$	0·13	$x\,15$	0·13	$x\,18$	0·10
$x\,17$ — the area of the covered sunspots											
$x\,2$	0·40	$x\,5$	0·31	$x\,1$	0·31	$x\,10$	0·38	$x\,18$	0·21	$x\,15$	0·19
$x\,18$ — the range of flare action											
$x\,15$	0·70	$x\,10$	0·61	$x\,6$	0·49	$x\,15$	0·67	$x\,10$	0·63	$x\,6$	0·37

The variables $x\,6$, $x\,11$, $x\,13$, $x\,14$, $x\,16$ and $x\,17$ have, for the 1st ("major") and the 2nd (impulsive) populations, different representation of the mutually correlated variables.

d) Different distribution of x 8, x 9, x 14 and x 17 in the u- and lambda tests

Considerable differences have been found in the distributions of the total durations of flares (x 9). The mean value $\bar{x}$ 9(I) for the "major flares" is 69 minutes and $\bar{x}$ 9 (II) for impulsive flares 46 minutes. Similarly, the duration of the flare decay phase ($\bar{x}$ 8), the total volume of the main flare arcade ($\bar{x}$ 14) and the mean value of sunspot area covered by flare ribbons ($\bar{x}$ 17) also showed different patterns.

e) One positive statistical test (x 7, x 12, x 16)

Three variables had only one positive test when the 1st and 2nd flare populations were compared. This means that these variables have approximately the same dimensions for both the studied kinds of flares. These are:

x 7 – duration of the flare increasing phase
x 12 – outer radius of the flare arcade
x 16 – association with active region filament (its proximity).

4. Mutual Dependences between Variables (x 1—x 18)

Mutual dependences between the 18 flare characteristics were studied using the Principal Component Analysis – PCA and Factor Analysis – FA methods (Jakimiec and Bartkowiak 1986).

A correlation matrix for 18 studied variables was calculated. Table 2 shows the three maximum values of the correlation coefficient between variables (x 1 – x 18). On the lefthand side of Tab. 2 are the correlation coefficients for the "major flares", on the right-hand side, for the impulsive flares. The variables x 6, x 11, x 13, x 14, x 16 and x 17 have, for the 1st (major) and 2nd (impulsive) populations, different representation of the mutually correlated variables. Below, only the most interesting relationships of the correlation matrix are discussed.

Comprehensive index (x 5)

– The values of the flare comprehensive index – CI mostly correlate with the dynamic flare radio spectrum (x 3 – 0·73), also with the flare flux on 200 MHz (x 4 – 0·71) and the flare flux on the 10 cm wavelength (x 2 – 0·62).

H-alpha flare importance (x 6)

– The H-alpha flare importance of the "major flares" correlates with the total flare duration (x 9 – 0·62), as well as, with the duration of its decay phase (x 8 – 0·59) and with the length L of the flare arcade (x 11 – 0·55).
– In the case of the impulsive flares, there is weak correlation with the volume of the supplemental flare structures (x 15 – 0·39), the range of flare action (x 18 – 0·37) and the volume of the flare arcade (x 14 – 0·35).

SID (x 1) and F_{10} flux (x 2)

– An interesting correlation was found between the SID value (0·31), the flare F_{10} flux (0·40) and the sunspot area, covered with ribbons during the flare (x 17).

4.1 *The Dimensionality of the Set of Variables* (x 1 – x 18)

The PCA and FA methods (Jakimiec and Bartkowiak 1986) to determine the dimensionality of our set of variables (x 1 – x 18) were used. With the PCA method, it was found that approximately 60% of the total variance of the system (x 1 – x 18) could be explained by $3-4F_j$ factors. Each further factor can only explain less than 5% of the set variance (see Fig. 2).

Assume that our variables (x 1 – x 18) are a linear function of four F_j variables (designated as factors) with j = 1, 2, 3, 4. The following equation can then be written:

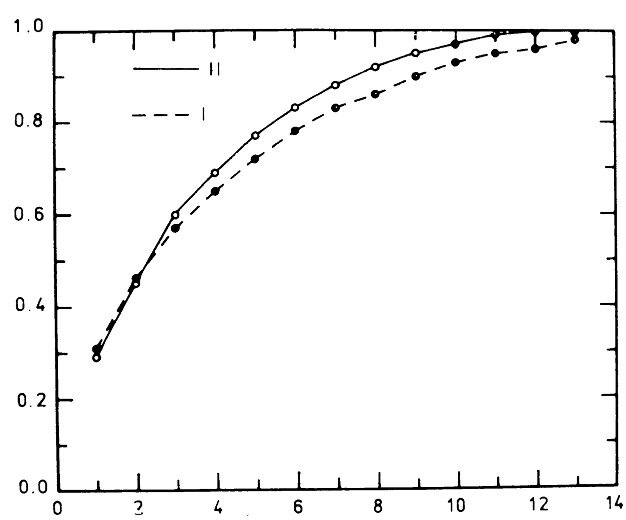

Fig. 2. The part of the covariant matrix illustrating the fact that 60% of the complete variability of the variables system (x 1 – x 18) can be described by F_j (j = 1, 2, 3, 4). Each further factor (F_5...) describes only approximately 5% of the total variability of the (x 1 – x 18) system. The definition of F_j is given in the text of the paper. The x-axis represente F_j, the y-axis the percentual complete variability of the system (x 1 – x 18).

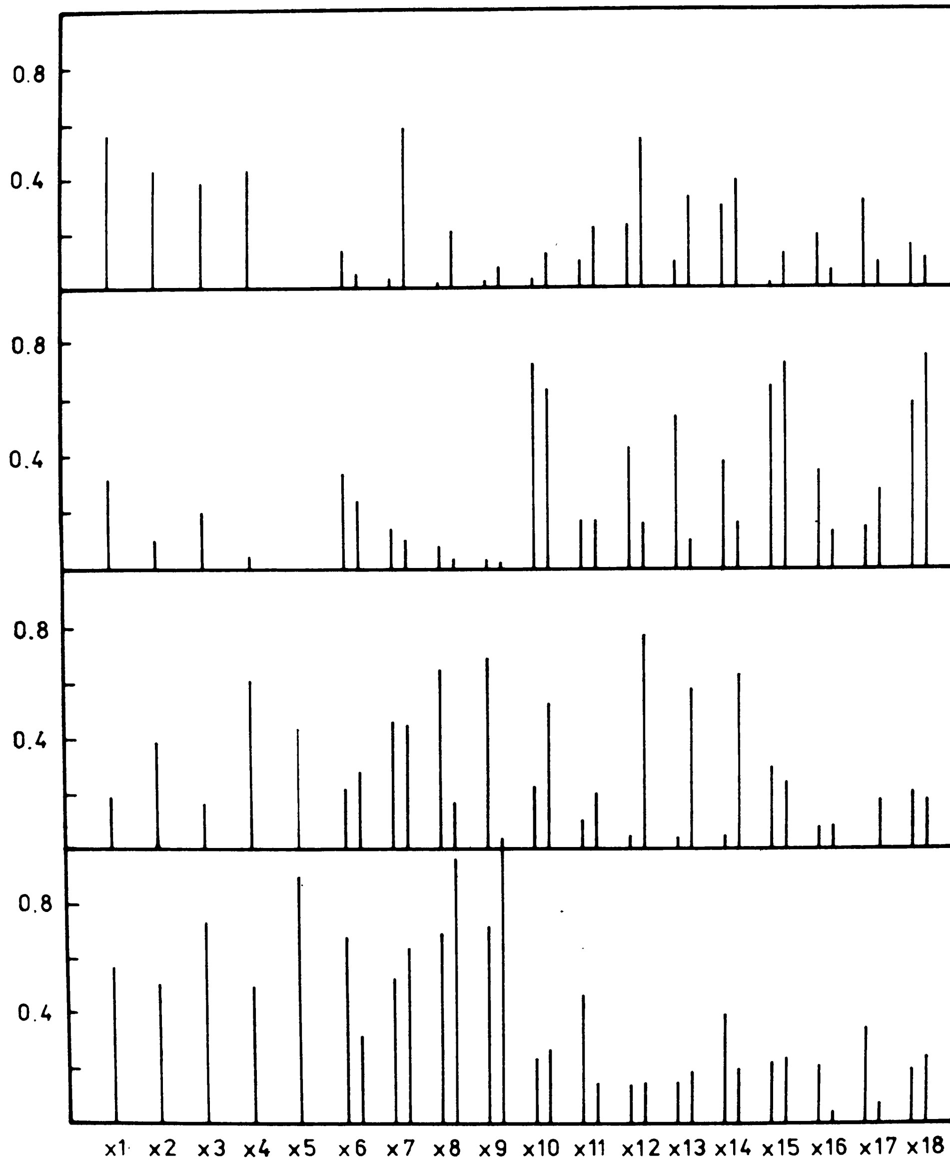

Fig. 3. Graphical record of the FA (Factor analysis) of our variables system (x 1—x 18). Each line shows the factor dependence on variables (x 1—x 18), with the bottom line showing the most significant F_1 factor, the top line relates to F_4. The representations of the F_1—F_4 factors in the individual x_i variables are indicated by vertical segments; for example, x 5 shows only a dependence on F_1 and F_2.

$$x_i = l_{1i}F_1 + l_{2i}F_2 + l_{3i}F_3 + l_{4i}F_4 + v_iU_i \quad \text{for } i = 1, 2, \ldots, 18 \tag{4}$$

The last term in right-hand side of the Equation (4), i.e. v_iU_i, contains the specific variable U_i and the specific variance v_i^2 as shown in Fig. 4.

The factor loadings l_{ji} of the individual F_j factors in the given x_i variable are shown in Fig. 3, in the vertical direction.

For example, the volume of the supplemental flare ribbons (x 15) for both flare populations mostly depends on the F_3 factor (the third line from the bottom in the Fig. 3), while the dependence of x 15 on F_1 (the first line from the bottom in the Fig. 3), the F_2 and F_4 factors were marginal.

4.2 *Discussion of the Factor Analysis*

The first factor F_1 (bottom line in Fig. 3), depends on the flare emissions (x 1 – x 6) as well as on the flare duration (x 7—x 9). The remaining variables are not represented in F_1, with exception of x 11 (length of the flare ribbons) and x 14 (volume of the main flare arcade) for the 1st flare population. F_1 describes 30% of the (x 1—x 18) set variance, as illustrated in Fig. 2.

In the first "major flare" population, the second factor, F_2, depends on the $x\,4$ (F_{200}), $x\,5$ (CI) and ($x\,7 - x\,9$) variables, and in the second population on the ($x\,10 - x\,14$) variables. F_2 describes 15% of the system variance.

The third factor, F_3 (the 3rd line from bottom in Fig. 3), probably relates to the definitions of the variables $x\,10$ (number of flare ribbons), $x\,15$ (volume of the supplemental flare structures) and $x\,18$ (activity radius of flare).

In the 1st population of "major flares", the fourth factor, F_4, depends on the $x\,1 - x\,4$ (radio and SXR emissions) variables and on the $x\,14$ (volume of flare arcade). For the 2nd population of the impulsive flares on the $x\,7$ (duration of the increase flare phase), $x\,12$ (outer flare arcade radius) and $x\,14$ (volume of flare arcade) variables. F_4 is illustrated by the top line of Fig. 3.

The specific variance v_i^2 is shown in Fig. 4. The geometric characteristics ($x\,11 - x\,15$) and the $x\,16$, $x\,17$ (the sunspot area covered by flare ribbons) variables maintain high values of the specific variance for both flare populations.

The $x\,7$, $x\,12$ variables had different specific variances for the 1st and 2nd flare populations. In both cases, a large variance is observed for the "major flares", as well as, for $x\,7$ (duration of the increasing flare phase) and for $x\,12$ (outer radius of the flare arcade). A large variance for $x\,6$ (H-alpha flare importance) is observed for the population of the impulsive flares (see Fig. 4).

5. Results

The PCA and FA analyses of the set of variables ($x\,1 - x\,18$) for two flare populations (I – "major", $N_I = 74$ flares; II – impulsive, $N_{II} = 330$ flares) resulted in the following conclusions:

5.1 *The Dimensionality of the Set* ($x\,1 - x\,18$)

A comparatively large interrelation exists between the variables ($x\,1 - x\,18$). 60% of the total variance of the set ($x\,1 - x\,18$) can be described by the factors $F_1 - F_4$. Each further factor F_j (5, ...), described approximately only 5% of the total set variance ($x\,1 - x\,18$). The representation of the variables in the factor structure is shown in Fig. 3; for the F_1 factor, see the bottom line. The $F_2 - F_4$ factors are shown in the subsequent higher lines. The information on the system ($x\,1 - x\,18$) dimensionality is supplemented by the specific variance value v_i^2 (Fig. 4).

The high variances of the geometric arcade dimensions ($x\,11$, $x\,12$ and $x\,13$) indicate large variabilities of the structures of the actual flares.

5.2 *Comprehensive Flare Index (CI) and Duration of "Major Flares"*

Dodson and Hedeman (1971) defined the comprehensive index of "major flares" as a function of the radio, H-alpha and SXR flare emissions ($x\,1$ to $x\,6$). The Factor Analyses of the set or variables

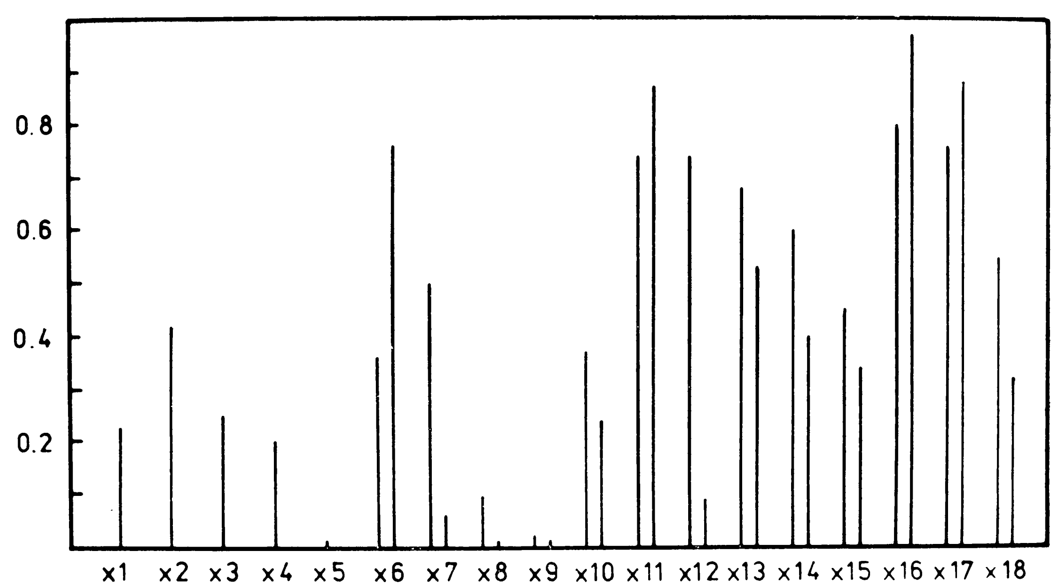

Fig. 4. Specific variance v_i^2 of the variable system ($x\,1 - x\,18$). The smallest variance was observed in $x\,5$ (CI) and ($x\,8$, $x\,9$) — the flare duration. The different variances in $x\,7$ (the duration of the increasing flare phase) and the high values of the specific variances of the geometrical characteristics ($x\,11 - x\,18$) for both flare populations are observed.

($x\,1-x\,18$) indicate that CI also depends on the duration of the "major flares" ($x\,7-x\,9$). This results from the fact that variable $x\,5$ (CI) is a function only of the F_1 and F_2 factors for the "major flares" (see the bottom line and 2nd line from bottom in Fig. 3). CI is not a function of F_3 and F_4, and its variance (Fig. 4) is also very low. As was shown in Section 4.2, the F_1 and F_2 factors depend mainly on the $x\,1-x\,4$, $x\,6$ and $x\,7-x\,9$ variables. The first two dependences are identical with the CI definition, the latter is new. This result coincides with the distribution of flares into gradual and impulsive types (De Jager 1987, Dennis 1988, Sturrock 1987, Tanaka 1987, Tsuneta 1987, Bai 1986) as well as with the finding that the duration of the H-alpha flares is related to their radio type IV emission (Antalová 1986, 1988).

5.3 *Identical Flare Characteristics for Gradual and Impulsive Flares*

Practically the same x_i curves in both compared flare populations, regardless of their energetics, were found for the following variables:

- duration of the increasing flare phase ($x\,7$), with a mean value of 11 – 13 minutes (in H-alpha)
- the number of flare ribbons ($x\,10$), with a mean value of 2·2 – 2·4, i.e. more than 2 ribbons
- the inner radius of the flare arcade ($x\,13$) – mean value 7000 km during the flare maximum

5.4 *Different Flare Characteristics for Gradual and Impulsive Flares*

The most significant differences observed, indicating various flare conditions mainly during the post-maximum phase, were related to:

- the total flare duration in H-alpha ($x\,9$) and the duration of the post-maximum phase ($x\,8$): mean value $\bar{x}\,9$(I) for the "major flares" 69 minutes and 46 minutes for the impulsive flares
- the length of the flare arcade ($x\,11$), with a mean value of 5·1° heliographic degrees for the "major" and 3·9° for the impulsive flares
- the H-alpha flare importance ($x\,6$) – see Tab. 1 and Fig. 1a
- the maximum volume of the flare arcade ($x\,14$), with a mean value 2-times larger (24 cubic degrees) for the "major flares" as compared to the impulsive flares
- the volume of the supplemental flare structures ($x\,15$), with a mean value of 1·4 cubic heliographic degrees for the "major" and 0·7 cubic degrees for the impulsive flares
- flare action radius ($x\,18$), measured from the centre of the main arcade. For the "major flares" $x\,18$ has a mean value of 76×10^3 km and $49 \times \times 10^3$ km for the impulsive flares.

5.5 *Relations between H-alpha Flare Importance and Flare Volumes*

Due to the definition of the H-alpha flare importance ($x\,6$) a strong relation between $x\,6$ and the flare dimensions is naturally expected.

For the impulsive flares the expectations were really fullfiled, i.e. the H-alpha importance ($x\,6$) depends on the volume of the flare arcade ($x\,14$) and on the volume of the supplemental flare structures ($x\,15$), but the coefficients of correlation are too small (0·39 – 0·35).

For the "major flares" correlation (0·55) was found between H-alpha importance ($x\,6$) and the length of the flare arcade ($x\,11$) – see Tab. 2. Practically the same (0·51) correlation was found between the volume of the supplemental flare structures ($x\,15$) and H-alpha importance ($x\,6$). It is interesting that the SXR and radio emissions ($x\,1-x\,5$) of the "major flares" are reflected to a smaller extent in the volume of the supplemental flare structures ($x\,15$) than H-alpha flare importance ($x\,6$ – 0·39).

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