LITHIUM ABUNDANCE IN CLUSTER GIANTS: CONSTRAINTS ON MERIDIONAL CIRCULATION TRANSPORT ON THE MAIN SEQUENCE

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ABSTRACT

The observed lithium abundances in giants are used to constrain meridional circulation transport on the main sequence. It is shown that meridional circulation, operating over the main-sequence lifetime, can lead to lithium depletion in the upper radiative envelope \([M(r)/M_\ast \geq 0.1]\) and eventually to extreme Li underabundance in first-ascent giants, following convective dilution on the lower giant branch. It is found that in the mass range \(1.2 M_\odot \leq M_\ast \leq 2.0 M_\odot\), stars having equatorial rotational velocities greater than 30-35 km s\(^{-1}\) on the ZAMS should destroy most of their Li. This value is rather insensitive to the details of nuclear burning and to assumptions made regarding the evolution of the surface rotation rate on the main sequence.

These results are then compared to recent Li abundance determinations in three moderately old clusters, NGC 7789, NGC 752, and M67. They fit reasonably well the data for M67 and NGC 752, but not the large fraction of NGC 7789 giants that show Li abundances in agreement with the standard dilution scenario; i.e., that seem to have suffered very little main-sequence Li depletion. This is surprising, since most factors having been neglected in our model would tend to increase Li depletion, meaning that our calculations give a minimal value for main-sequence depletion. It would be possible to explain the NGC 7789 observations by postulating, ad hoc, a peculiar rotational velocity distribution for these more massive stars while they evolved on the main sequence. It appears more likely that the efficiency of particle transport in the upper envelope has been much smaller than the value corresponding to meridional circulation, as calculated in stellar models whose upper envelopes are assumed to be strictly chemically homogeneous. Various constraints on transport processes in stellar envelopes that can be inferred from our calculations are discussed.

Subject headings: clusters: open — diffusion — stars: abundances — stars: late-type — stars: rotation

I. INTRODUCTION

Because of its extreme fragility to \((p, \alpha)\) reactions, lithium is a key tracer of stellar transport processes. If a star's photosphere has a normal lithium abundance, it implies that very little mixing has occurred between these layers and deeper regions of the envelope. Since lithium can survive only in the outermost mass fraction of a star, its existence also puts constraints on possible mass-loss rates. Proper modeling of the time evolution of its abundance can also yield indirect information regarding other transport processes, such as turbulent diffusion or convective overshoot, that cannot be fully parameterized from first principles. Given the large array of different transport processes that can potentially affect the lithium abundance in stars, it is critical to investigate the effects of each, if a coherent picture of lithium abundance evolution in stars is to be developed.

As the star evolves away from the main sequence, the mass in its superficial convection zone (hereafter SCZ) increases by many orders of magnitude, dredging up matter previously located at higher temperatures. Matter located in the upper regions of the envelope while on the main sequence rapidly gets mixed with matter that has been subjected to partial nuclear processing, including Li burning through \((p, \alpha)\) reactions. The decrease in photospheric Li abundance then gives a measure of Li destruction over that larger mass. Studying the Li abundance in giants can thus become a mean of \(a \text{ posteriori}\) probing hydrodynamical processes occurring deep in the envelope during the main-sequence phase. Furthermore, Li abundance determinations in stars on the giant branch of clusters of different ages allows us to study the mass dependence of these main-sequence hydrodynamical processes.

From the observational standpoint, the Li abundance in evolved stars is a puzzle; if one includes chemically peculiar giants and supergiants, the observed abundance range covers nearly seven orders of magnitude, with values ranging from log \([N(Li)] \approx 5\) on a scale where \(N(H) = 10^2\), two orders of magnitude above the local galactic value, down to log \([N(Li)] \approx -2\), in which case only upper limits are determined (see the recent reviews of Michaud and Charbonneau 1989 and Arnould and Forestini 1988). Furthermore, there appears to be no clear correlation with luminosity, effective temperature, or abundance of other chemical species. If one restricts the sample to chemically "normal" giants and supergiants, then the abundance range still spans over three orders of magnitude, but usually does not exceed the value log \([N(Li)] \approx 1.5\), in agreement with standard evolutionary calculations (discussed in §4 below). There exist a few chemically normal giants that have a Li abundance about equal to that of young main-sequence stars \((\log [N(Li)] \approx 3)\), but these appear to be rather rare (see Wallerstein and Sneden 1982). The low Li abundances observed in normal giants were never considered a major problem, since it is well known that many main-sequence stars destroy Li efficiently, particularly stars less massive than about \(1.2 M_\odot\). Some more massive field or cluster main-sequence stars also show low Li abundances (see Michaud and Charbonneau 1989, and references therein), although in these latter cases it may be that the observed low abundances merely reflect a surface depletion, involving only the outermost layers of the stars. This would be the case if gravitational settling is...
responsible for the observed low Li abundances of these hotter stars (see, e.g., Michaud 1986). In this case, the convective dredge-up occurring at the base of the giant branch would restore the surface abundance to the expected value. It can also be argued that the giants with excessively low Li abundances are not first-ascent giants, but rather stars in more advanced phases of evolution, having thus passed through additional dredge-up events or having suffered nonnegligible mass loss, which can lead to low surface Li abundances.

It has repeatedly been suggested that meridional circulation is responsible for the variations of $^{12}$C/$^{13}$C and $^{13}$C/$^{14}$N ratios observed in giants and supergiants having early main-sequence stars as progenitors (Paczynski 1973; Tomkin, Luck, and Lambert 1976; Luck 1977; Cottrell and Norris 1978). As the star evolves on the main sequence, partial CNO processing can occur outside the hydrogen-burning core, leading to an accumulation of $^{13}$C, and, to a somewhat lesser extent, $^{14}$N near the base of the radiative envelope [at $M(r)/M_* = 0.4$ for a $2 M_\odot$ star; see, e.g., Fig. 2 of Dearborn, Eggleton, and Schramm 1976]. As the star leaves the main sequence and moves toward the giant branch, a deep convective envelope develops and dredges up matter located as deep as $M(r)/M_* \approx 0.3$ (see, e.g., Sweigart and Gross 1978), which then leads, through convective mixing, to altered surface abundances. This way, the photospheric $^{12}$C/$^{13}$C ratio can diminish from the solar value ($\approx 90$) down to values of 20–30. However, it is not possible to obtain ratios as low as 4–10, as are observed in a number of giants (see, e.g., Tomkin, Luck, and Lambert 1976), whether by postulating a deeper dredge-up on the lower giant branch or by allowing for a reasonable spread in initial abundances. The alternate possibility is to allow for some form of mass transport, already operative before the dredge-up event (such as meridional circulation), thus permitting the removal of $^{13}$C from $M(r)/M_* = 0.4$ and its "storage" higher up in the envelope. When dilution eventually occurs as the star approaches the giant branch, a $^{12}$C/$^{13}$C ratio lower than 20 can be obtained, since there is now available above $M(r)/M_* \approx 0.3$ more $^{13}$C (and less $^{12}$C) than expected. Note that this mixing must occur in such a way that the surface region of the envelope remains uncontaminated during the main-sequence phase; otherwise, abnormal CNO abundances would appear on the main sequence, contrary to observations. Detailed calculations of this effect were not generally carried out, in view of the large uncertainties associated with meridional circulation models.

The most detailed calculations to date appear to be those of Sweigart and Mengel (1979); these authors considered the mixing due to meridional circulation on the giant branch, rather than on the main sequence. Further difficulties appear in this case, related to angular momentum redistribution as the star evolves up the giant branch and to the presence of strong gradients of mean molecular weight, which are believed to inhibit greatly meridional circulation. These authors found, through various approximations and hypotheses regarding angular momentum distribution, that CNO abundances characteristic of weak G band stars may be obtained with realistic rotation rates. However, as was rightfully pointed out by Lambert, Dominy, and Sivertsen (1980), the corresponding rate of mixing would lead simultaneously to strong Li depletion. Although, this in in qualitative agreement with the observed $^{12}$C/$^{13}$C vs N(Li) relationship observed in "normal" late-type giants, it leads to a contradiction for weak G band stars, which show low $^{12}$C/$^{13}$C ratios but abnormally high Li abundances (see the discussion in Lambert and Sawyer 1984).

In this paper, we calculate the effect of meridional circulation transport occurring on the main sequence on the Li abundance to be observed once the star becomes a giant. Our motivation for doing so is twofold; first, following the development by Tassoul and Tassoul (1982a; hereafter $T^2$) of a self-consistent model, one now has available meridional circulation solutions with essentially no arbitrary parameters, whose predictions can be tested. Second, recent observations of cluster giants, for which reasonably accurate mass estimates are available, have shown that in the mass range 1–2 $M_\odot$, many first-ascent giants show upper limits to Li abundances which are two orders of magnitude below the value expected from standard models. Our purpose is thus to compute what fraction of lithium initially present in the upper regions of a ZAMS star can be destroyed over the main-sequence lifetime by the combined effects of meridional circulation transport and nuclear burning. No attempts are made to model meridional transport on the subgiant or giant branch, since this would probably require a full time-dependent solution for the meridional circulation velocity field. We thus obtain a lower limit to the expected Li destruction.

Accordingly, § II presents the modifications to be introduced in the meridional circulation model of $T^2$, in order to calculate the circulation velocity field in main-sequence evolutionary models. The calculations of the expected lithium destruction rate are presented in § III for stellar evolutionary models with masses ranging from 1.2 to 2 $M_\odot$ and various rotation rates. The corresponding Li depletion factors are compared to observations in § IV, and the uncertainties associated with the various underlying assumptions made in the calculations are discussed critically in § V. Finally, the implications for stellar hydrodynamics are analyzed in § VI.

II. MERIDIONAL CIRCULATION IN REALISTIC STELLAR MODELS

This section presents a brief outline of the procedure followed by $T^2$ in order to specify the changes made in these calculations when modeling meridional circulation in stellar evolutionary models with SCZs. The evolutionary models are also briefly described.

To first order in $\epsilon$, the components of the meridional circulation velocity field $U$ in the chemically homogeneous part of the radiative zone of a rotating star are given by equations (117) of $T^2$, which can be written as

$$U_r(r, \theta) = \epsilon u(r) P_2(\cos \theta),$$

$$U_\theta(r, \theta) = \epsilon v(r) \frac{dP_2(\cos \theta)}{d\theta},$$

with $\epsilon$, the rotational parameter, given by

$$\epsilon = \frac{v_\phi^2 R_*}{GM_*},$$

where $v_\phi$ is the equatorial rotational velocity. The solution of $T^2$, based on a truncated series expansion in powers of $\epsilon$, will obviously break down for large values of $\epsilon$. It can be easily verified that for a 1.5 $M_\odot$ star, $\epsilon \approx 0.1$ for $v_\phi \approx 140$ km s$^{-1}$, a value well above the rotational velocities to be used below. It is also straightforward to show that the requirement of mass conservation forces the following relation between the functions $u(r)$ and $v(r)$:

$$v(r) = \frac{1}{6\rho r^2} \frac{d}{dr} \left[ \rho r^2 u(r) \right].$$

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Calculating the function $u(r)$ thus completely determines the velocity field in a given stellar model. It is shown by $T^2$ that this function must satisfy the following differential equation:

$$\mathcal{L}^V u - \frac{4\pi G^2 m^3}{3 L \rho r^4} n - 1.5 \left( \frac{2 - m}{r - m} \right) = \frac{8 \pi G p^3}{p} \left[ h' + \frac{1}{r - m} \right],$$

(5)

where $\mathcal{L}^V$ is a sixth-order differential operator, given by equation (76) of $T^2$. A prime indicates a derivative with respect to $r$, $n$ is the effective polytropic index, and $m$ the mass below radius $r$. All other symbols have their usual meanings. The function $h(r)$ corresponds to the gravitational part of the total potential and is obtained by solving Poisson’s equation in the barotropic approximation (see $T^2$, § IV). In the case of negligible viscosity, $\mathcal{L}^V u$ vanishes, and equation (5) becomes a mere algebraic equation, whose solution $(u = u_0)$ was first obtained by Sweet (1950), using a different approach:

$$u_0 = \frac{2 L^4}{G^2 m^3} n - 1.5 \left[ h' + \frac{1}{r - m} \right].$$

(6)

Substitution of this expression, first in equation (4) and then in equations (1) and (2), yields a velocity field that shows some unwanted pathologies near the surface: (a) the radial component $U_r$ does not vanish there; (b) this, in turn, yields an angular component $U_\theta$ that becomes extremely large near the surface, increasing as $\rho'/\rho$. It follows that one cannot properly streamline this surface. Near the boundary of a convective zone ($n = 1.5$), the solutions also break down completely as $u(r) \to \infty$ and $v(r) \to \infty$. $T^2$ proceed to remove these unwanted singularities by retaining viscous dissipation near the boundaries so that it is possible to prescribe the proper boundary conditions at the surface and core-envelope interface. Matching between these conditions and Sweet’s solution is obtained by retaining the viscosity terms in equation (5) and subsequent application of a boundary layer treatment near the two boundaries. They then go on to obtain solutions that remain finite everywhere in the radiative envelope and that satisfy the required boundary conditions (in particular, the fact that the currents must flow along the boundary surfaces).

For purposes of simplicity, $T^2$ calculated their solutions in Cowling models, with the opacity given by Kramers’s formula or simple electron scattering. This is certainly a reasonable approximation, and the differences resulting from the use of Los Alamos opacities are small. However, the Cowling models used by $T^2$ to calculate their solutions have no superficial convection zones. This can again be considered a good approximation in the case of the 3 $M_\odot$ models they were using, since stars of such masses are believed to have extremely shallow SCZs (of the order of $10^{-8}$ to $10^{-10}$ in mass fraction). Stars of spectral type later than about F5 have SCZs whose mass rapidly increases as $T_{\text{eff}}$ decreases. Can the solution of $T^2$ remain valid for these stars? It is certainly possible to assume that a viscous boundary layer also forms at the base of a massive SCZ, in a manner similar to that leading to the formation of such a layer at the core-envelope interface in the more massive models of $T^2$. The boundary layer analysis is then essentially identical to that described in § Va of $T^2$ and will not be repeated in details here. Assuming that the boundary layer is extremely thin, and that all physical quantities [except, of course, the effective polytropic index and the function $u(r)$ itself] remain essentially constant within the layer, one can approximate equation (5) by retaining only derivatives of the highest order in $\mathcal{L}^V$, and approximating $n(r)$ by a truncated Taylor series around $r = r_{SCZ}$. The resulting equation is then expressed in terms of dimensionless variables and solved numerically by $T^2$. The solution they obtain is shown on their Figure 1 and can be used directly after proper rescaling in terms of the boundary layer thickness, given here by

$$\delta = \left( \frac{L}{24 \pi G^2} \left( \frac{\mu^2 \rho (n + 1)^2}{m^3 p^3 n'} \right)^{1/7} \right).$$

(7)

The sign difference between the above expression and equation (91) of $T^2$ comes from the fact that in the situation treated here, $n$ decreases as the SCZ is approached, hence $n' < 0$. The quantity $\mu$ represents the total viscosity, parameterized by $T^2$, as

$$\mu = 10^N \mu_0,$$

(8)

where $\mu_0$ is the usual radiative viscosity coefficient and $N$ is some positive number ($0 \leq N \leq 6$, say). It must be stressed again that, as far as the meridional circulation velocity field is concerned, the solutions in the bulk of the radiative zone do not depend on the choice of $N$, since viscosity is totally negligible there. The solutions are only affected by this choice near the boundaries, since the thickness of the boundary layers does depend on the adopted value of $N$. Even then, the dependence is rather weak, since $\delta$ varies as $\mu_0^{1/7}$.

It has been verified a posteriori by $T^2$ that the boundary layer thickness remains indeed very small at the core-envelope interface. This must remain true at the base of the SCZ, if the working hypotheses behind the boundary layer treatment (neglecting lower order derivatives, variations in nearly all physical quantities within the layer, etc.) are to remain valid. Figure 1 shows the variation of the boundary layer thickness $\delta$, as calculated from equations (7) and (8) in several main-sequence stellar models, for $N = 0$ and $N = 6$. The pressure scale height at the base of the SCZ (dashed line) is also shown. It can be seen that we have indeed $\delta \ll r_{SCZ}$, and that, for all but the warmer models, the layer is much thinner than the scale height.

![Fig. 1.—Thickness of the SCZ boundary layer vs. effective temperature for main-sequence F and G dwarfs. Plot actually shows the dependence of the ratio $x/r_{SCZ}$ on effective temperature, where $x$ is either the boundary layer thickness as calculated from eq. (7) with $N = 0$ or $N = 6$ (solid lines), or the pressure scale height at the base of the SCZ (dashed line). All models used have log $g = 4.3$.](image-url)
Figure 2.—Variation of $u(r)$ with depth in various 1.4 $M_\odot$ models. Solid and dashed lines refer, respectively, to calculation of $u(r)$ in an evolutionary main-sequence model, with and without boundary layers under the SCZ. Dotted line is a scaled Cowling model, taken from $T^2$. Dash-dotted line corresponds to solution at equal density in a Paczyński envelope; this procedure was used in previous work and appears to be a reasonable approximation to the true solution except immediately below the SCZ. All models have $L_s = 3.15 L_\odot$ and $log g = 4.35$. Dotted tick marks on the abscissa correspond to $r = r_{zc} - \delta$ and $r = r_{zc} - 6\delta$. (a) Solution for a large portion of the envelope, (b) base of the SCZ. It can be seen that the solutions, with and without boundary layers, join smoothly in the vicinity of $r_{zc} - 6\delta$, which can then be loosely interpreted as the total boundary layer thickness.

Figure 2 shows the variation of the function $u(r)$ as a function of depth as calculated in three different 1.4 $M_\odot$ models. Figure 2b is an enlargement of the section of Figure 2a located immediately below the SCZ. The dashed line corresponds to the function $u_s(r)$, as defined by equation (6) and calculated in a standard evolutionary model. The function is calculated all the way up to the SCZ without imposing any boundary condition at its base. The divergent behavior of $u_s(r)$ under the SCZ is obvious. The solid line shows the same solution, after application, of a boundary layer treatment under the SCZ, as described above; physically, this corresponds to imposing a no-slip boundary condition at the base of the SCZ and ensuring continuity between the boundary and the function $u(r)$ simply by retaining the viscosity terms in the equations of motion. The dotted line is obtained by directly scaling the $T^2$ solutions for Cowling models of the appropriate mass and luminosity. The dash-dotted line corresponds to the solution obtained by interpolation of the $T^2$ solutions at equal densities between the Cowling model and a Paczyński envelope model, the procedure used in Michaud (1982), Michaud et al. (1983), and Charbonneau and Michaud (1988a, b). The dotted tick marks above the $x$-axis correspond to distances of $\delta$ and $6\delta$, respectively, from the bottom of the SCZ ($\delta$ being given by eq. [7]). Below $6\delta$, the solutions with and without boundary layers coincide to better than 1%. This can be interpreted as the total boundary layer thickness.

Figure 3 shows some meridional circulation streamlines for two of the models described above. The solid streamlines correspond to the evolutionary model with SCZs (solid line in Fig. 2), and the dotted streamlines to the scaled $T^2$ solution in the appropriate Cowling model (dotted line in Fig. 2). It must be emphasized that the solid streamlines do not penetrate the SCZ, but run along its base from the pole toward the equator. A similar behavior is obtained by $T^2$ at the core-envelope interface. The fact that both sets of streamlines do not coincide exactly in the deeper regions of the envelope is due mostly to the somewhat different mass distribution of the two models, which leads to (a) slightly different radial velocity profiles (see Fig. 2), and (b) notably different radial to angular velocity ratios at a given depth, via equation (4) [note also in Fig. 2 the different slopes of the $u_s(r)$ profiles in the deeper parts of the envelope].

The stellar evolution code (Proffitt 1988), with which the models used throughout this paper were calculated, is very similar to evolutionary codes used by other workers (e.g., Kippenhahn, Weigart, and Hofmeister 1967). An initial He abundance of 0.272, a metal abundance of 0.0169, and a ratio of mixing length to pressure scale height of 1.5 were used for all models. The subroutines used for calculating energy generation, nuclear reaction rates, opacities, the atmospheric boundary conditions, the equation of state, and related quantities were taken from VandenBerg (1983a, b). The opacities of Huebner et al. (1977) are used for temperatures above 10,000 K, while those of Alexander (1975) are used for lower temperatures. Nuclear reaction rates are taken from Harris et al. (1983). The scaled solar $T-\tau$ of Krishna Swamy (1966) was used to calculate the pressure at the photosphere. The equation of state subroutine is based on the equation of state of Eggleton, Faulkner, and Flannery (1973).

Abundances of $^1$H, $^3$He, $^4$He, $^{12}$C, $^{13}$C, $^{14}$N, and $^{16}$O were followed at each spatial zone. Composition changes due to mixing and nuclear reactions were solved simultaneously for these isotopes using a numerical differencing scheme similar to...
that suggested by Cloutman and Eoli (1976). The \(^{7}\)Li abundance was also separately estimated at each point in the star. Initial models were scaled from pre-main-sequence models of VandenBerg (1983b) and were allowed to evolve onto the main sequence. Each sequence was started at a time sufficiently in advance of the main sequence that the bulk of the luminosity was still coming from gravitational contraction rather than nuclear burning, and that no significant amount of H had yet been burned to He. Hence, any errors in the initial models due to errors in scaling from a model of different mass or initial composition will have no significant effect by the time the star reached the main sequence, apart from a very small error in the age of the star at the ZAMS (<5%), which is not important for the problem considered here.

III. LITHIUM BURNING VIA TRANSPORT BY MERIDIONAL CIRCULATION

Throughout this paper, atomic diffusion and turbulence are neglected. Lithium depletion is assumed to occur only through the combined effects of meridional circulation and nuclear burning. We also assume that meridional circulation can be neglected. For the purpose of this discussion, it is then possible to divide the star in two regions, the first extending from the center out to the radius, \(r_{Li}\), at which \(T = 2.6 \times 10^6\) K, where Li burns extremely rapidly, and the second containing the remaining upper region of the envelope where Li remains undepleted. Note that this second region contains a small fraction of the stellar mass (about 1.8% in a 1.2 \(M_\odot\) star, down to 0.8% for 2 \(M_\odot\)). When meridional circulation is “turned on,” Li-depleted matter enters the second region near the pole and Li-rich matter leaves the second region near the equator, moving down into the lower region where Li is promptly destroyed. The net effect of this process is thus to decrease the total quantity of Li contained in the second region; i.e., the total quantity of Li which will eventually get diluted when the star leaves the main sequence.

The quantitative time-evolution of the lithium spatial distribution will now be determined. One could write a two-dimensional transport equation for the lithium concentration, with an advection term corresponding to the circulation velocity field \(U\), and a sink term \(S[p(r), T(r)]\) describing nuclear burning; one would then have to solve

\[
\rho(r) \frac{\partial c(r, \theta, t)}{\partial t} = -\nabla \cdot [\rho(r)c(r, \theta, t)U(r, \theta)] + S[p(r), T(r)]\rho(r)c(r, \theta, t),
\]

with appropriate boundary conditions. This linear hyperbolic partial differential equation (PDE) is rather awkward to solve numerically, in view of the rapid spatial variations of the velocity field \(U\) and of the sink term \(S\), for which a proper discretization level can only be achieved with a very fine two-dimensional spatial mesh. It was thus decided to use the kinematic method developed in Charbonneau and Michaud (1988b); the reader is referred to § IV of that paper for details regarding this technique and to § V of the same paper for a discussion of its limitations when other transport processes become important.

At \(t = 0\), the Li abundance profile is assumed discontinuous, the concentration being zero below \(T = 2.6 \times 10^6\) K and equal to the initial value \(c_{Li}^0\) above this position. One may then define an abundance front, which at \(t = 0\) corresponds to a spherical shell of radius \(r_{Li} = r(T = 2.6 \times 10^6\) K). As meridional circulation is turned on, this initially spherical abundance front slowly changes shape. In the equatorial regions, where \(U_r\) is negative, lithium is carried down and burned quasi-instantaneously at \(r_{Li}\); in these regions, the abundance front then remains spherical. The situation is different in the polar region, where Li-depleted matter is pushed upward; the abundance front moves upward and gets slowly deformed as it is advected by meridional circulation. Figure 4 shows the time evolution of this abundance front for a 1.5 \(M_\odot\) star with \(v_\infty = 50\) km s\(^{-1}\). Knowing the shape of the abundance front at a given time allows one to compute angular concentration averages as a function of depth; i.e., horizontally averaged abundance profiles. The time evolution of these profiles is shown in Figure 5 at the same epochs as those for which the shapes of the abundance fronts are plotted in Figure 4. It must be remembered that the transport process

![Fig. 4.—Time evolution of the Li abundance front, in a 1.5 \(M_\odot\) star rotating with \(v_\infty = 50\) km s\(^{-1}\). Labels of the curves from A to J correspond to 0, 0.198, 0.274, 0.284, 0.286, 0.288, 0.292, 0.316, 0.347, and 0.783 (Gyr), respectively.](image)

![Fig. 5.—Time evolution of lithium radial abundance profiles, for circulation-induced burning. Profiles shown are horizontally averaged as described in the text. Calculations were done in a 1.5 \(M_\odot\), log \(g = 4.3\) envelope model, with \(v_\infty = 50\) km s\(^{-1}\). Labelling of the curves is identical to that of Fig. 4. It has been assumed here that the circulation penetrated the SCZ; applying a boundary layer treatment, as described in § II herein, would lead to no depletion in the SCZ, but would leave the profiles essentially unchanged below.](image)
considered here is bidimensional; Figure 5 shows its global effects. Depletion is first felt deep in the envelope. After 0.284 Gyr (see curve D, Fig. 4), Li-depleted matter reaches the SCZ, lithium (the time laps between curves D and G in Figs. 4 and 5 is only 0.008 Gyr; this is to be compared with 0.198 Gyr horizontally averaged abundance deep in the envelope is then between curves A and B, and 0.346 Gyr between I and J.) The remaining Li-depleted matter remains no lithium in the outer portions of the star (above \( \Delta M/M \approx 0.3 \), say).

To a very good approximation (see discussion below), the rate at which the total quantity of Li present in the star decreases is directly proportional to the net downward flux into the Li-burning region. Recalling that the vertical component of the circulation velocity changes sign at \( \theta = \arccos \left( \frac{1}{2} \right) \), it follows that the number of Li nuclei destroyed per second is given by

\[
\phi_{Li} = \frac{2}{m_{p}} \int_{\theta_{min}}^{\theta_{max}} \left. c_{Li}(\rho r^{2}U_{r}) \right|_{\theta = \theta_{i}} \sin \theta d\theta d\phi ,
\]

where \( m_{p} \) is the proton mass and \( U_{r} \) is the vertical component of the meridional circulation velocity field. After substituting equation (1) in the above expression and integrating, one obtains

\[
\phi_{Li} = \frac{2.42}{m_{p}} \int_{\theta_{min}}^{\theta_{max}} \left. c_{Li}[\rho u(r)r^{2}] \right|_{\theta = \theta_{i}} dt.
\]

The total number of Li nuclei destroyed in the main-sequence lifetime (hereafter \( \Delta_{Li} \)) can then be obtained by integrating equation (11) from \( t = 0 \) to \( t = t_{\text{ms}} \). However, some evolutionary effects must be taken into account; as it evolves on the main sequence, the star expands a little. Furthermore, the function \( u(r) \), which defines the meridional circulation solution, also evolves in time. This includes global variations, related to changes in luminosity, and local variations, related to changes in the mass distribution (see eq. [6] above). For now, one can simply write

\[
\Delta_{Li} = \int_{0}^{t_{\text{ms}}} \phi_{Li} dt = \frac{2.42}{m_{p}} \int_{0}^{t_{\text{ms}}} \int_{\theta_{i}}^{\theta_{max}} \left. \frac{v_{r}^{2}R_{*}(t)}{GM_{*}} \left. [\rho u(r,r^{2})] \right|_{\theta = \theta_{i}} dt \right. d\phi .
\]

The true situation is unfortunately not that simple; as the star evolves on the main sequence, the structure of the outer portion of the radiative envelope also changes. This can affect the Li destruction rate in two ways.

The first is related to the slow expansion of the envelope, which can be expected to affect the rotation rate, which in turn affects meridional circulation. The time variation of the equatorial rotational velocity depends on the assumptions made regarding angular momentum redistribution during main-sequence evolution. It can easily be shown that strict local angular momentum conservation (LAMC), i.e., no redistribution whatsoever, implies

\[
\frac{v_{e}(t)}{v_{e0}} = \frac{R_{*0}}{R_{*}(t)},
\]

where \( v_{e0} \) and \( R_{*0} \) are, respectively, defined as the equatorial rotational velocity and stellar radius on the ZAMS. The other extreme possibility is that of full angular momentum redistribution, which enforces solid body rotation (SBR). In this case, the time evolution of \( v_{e} \) is given by

\[
\frac{v_{e}(t)}{v_{e0}} = \left[ \frac{R_{*}(t)}{R_{*0}} \right] \mathcal{J}(t),
\]

where \( \mathcal{J} \) is the moment of inertia of the star, which for a spherically symmetry mass distribution is given by

\[
\mathcal{J}(t) = \int_{0}^{r_{Li}} \rho(r,t)r^{2} dt .
\]

Figure 6a shows the time variations of \( v_{e} \) corresponding to the two (extreme) cases described by equations (13) and (14), for evolutionary main-sequence models of various masses. The curves start on the ZAMS and terminate at the times when the corresponding models start to move up the lower giant branch. It can be seen that assuming SBR rather than LAMC has a larger effect for the more massive models. In both cases, it is further implicitly assumed that the total angular momentum of the star remains constant throughout main-sequence evolution; i.e., that angular momentum loss through stellar wind or any other process is negligible; uncertainties related to the possible breakdown of this hypothesis will be briefly discussed in § V.

Unless otherwise stated, solid body rotation is assumed in all subsequent calculations. Note that since one is dealing here with main-sequence models, whether or not solid body rotation is continuously enforced through angular momentum redistribution is not of critical importance, since the small expansion of the star on the main sequence leads, through

\[
\begin{align*}
(13) & \quad \frac{v_{e}(t)}{v_{e0}} = \frac{R_{*0}}{R_{*}(t)}, \\
(14) & \quad \frac{v_{e}(t)}{v_{e0}} = \left[ \frac{R_{*}(t)}{R_{*0}} \right] \mathcal{J}(t), \\
(15) & \quad \mathcal{J}(t) = \int_{0}^{r_{Li}} \rho(r,t)r^{2} dt .
\end{align*}
\]
equation (13), to a relatively small state of differential rotation. This would not be the case if one considered the evolution of the surface rotation rate on the giant branch (see the discussion in Sweigart and Mengel 1979). Uncertainties introduced by the choice of the time evolution of the equatorial velocity are discussed further below.

The other important evolutionary effect on the Li destruction rate is the slow increase in time of the mass fraction above the point where $T = 2.6 \times 10^6$ K; i.e., the depth at which Li is assumed to burn instantaneously. This is shown in Figure 6b, for the same models as Figure 6a; it affects Li burning in two distinct ways: first, the depth $r_{Li}$ at which the various quantities appearing in the expression for $\phi_{Li}$ are to be evaluated becomes a function of time; second, this slow sinking of the Li-burning front can actually lead to an important decrease in the Li destruction rate. If this front moves down faster than meridional circulation in the equatorial regions, Li destruction effectively ceases. It then is important to calculate, for a given stellar model, the limiting rotation rate, $v_{eL}$, at which this happens. Knowing the speed $v_{BF}$ at which the burning front moves down as well as its position at a given time, it is a simple matter to use equations (1), (4), and (14) and compute $v_{eL}$. Because the star slowly expands as it evolves on the main sequence, one must use the following expression for $v_{BF}$:

$$v_{BF} = \frac{d}{dt} \left( r_{Li} - r_{AMu} \right),$$

where the first term on the right-hand side is, as before, the radius of the shell where $T = 2.6 \times 10^6$ K, while the second one is the radius, at the end of the time step, of the shell where $T$ was equal to $2.6 \times 10^6$ K at the beginning of the time step; this essentially amounts to subtracting the expansion velocity from the burning front velocity. If the mass above $T = 2.6 \times 10^6$ K remains constant within a time step, then both terms on the right hand side of equation (16) are equal, and one obtains $v_{BF} = 0$ even if $r_{Li}$ varies in time. Results for $v_{eL}$ are shown in Figure 7; the irregularities on the curves are due to the relatively small number of models along each evolutionary sequence for which new velocity fields were calculated; the time derivatives appearing on the right-hand side of equation (16), calculated by backward finite differences, are consequently not very accurate. These small fluctuations do not affect the subsequent calculations in any significant way. The $r_{AMu}$ values are of the order of a few tens kilometers per second; the increase of the mass fraction above $T = 2.6 \times 10^6$ K can be expected to be important in modeling the Li abundance evolution, as it introduces a "cutoff" rotational velocity under which Li burning induced by meridional circulation transport becomes inoperative. Furthermore, since $|U|$ is maximum at the equator (see eq. [1]), then in stars having rotational velocities only slightly greater than that shown in Figure 7, Li destruction proceeds at a considerably smaller rate than that shown here, then Li burning effectively ceases. All curves are labeled according to stellar mass, in solar units. For the 1.4 $M_\odot$ model, Be is essentially undepleted up to about $r_r = 30$ km s$^{-1}$, and is completely depleted for rotational velocities greater than 55 km s$^{-1}$. As in the case of Li, there is only a weak time derivative appearing on the right-hand side of equation (16), which leads to

$$\Delta_{Li} = \frac{4\pi_{Li}}{m_p} \int_{v_{BF}}^{v_{eL}} \frac{\rho r^2}{GM_\star} \left( u(r, t) P_2(\cos \theta) - v_{BF}(t) \right) \cdot \rho r^2 \sin \theta d\theta,$$

where $v_{eL}(t)$ is obtained by using as a lower limit for the surface integral the angle $\theta_{eL}$ at which $v_{eL} = U$, (in general, a function of time and of the adopted $v_{eL}$ value) instead of $\theta_{crit}$, and by subtracting $v_{BF}$ from the circulation velocity:

$$\phi_{Li}(t) = \frac{4\pi_{Li}}{m_p} \int_{v_{BF}}^{v_{eL}} \left[ \frac{\rho r^2}{GM_\star} u(r, t) P_2(\cos \theta) - v_{BF}(t) \right] \cdot \rho r^2 \sin \theta d\theta,$$

which reduces to

$$D_{Li}^{\text{am}} = 1 - \Delta_{Li} \left[ 1 - \frac{m(r_{Li})_{ZAMS}}{M_\star} \right] \frac{\Delta m_{Li}}{m_p}, \quad 1 \geq D_{Li}^{\text{am}} \geq 0.$$

Note that in cases for which the rotation rate is such that $U > v_{BF}$ at all times, then the second term in the integrand becomes negligible, $\theta_{eL}(r, t) = \theta_{crit}$, and the above expression reduces to equation (12).

It is then possible, knowing the initial total Li content of the star on the ZAMS, to compute the main-sequence depletion factor (hereafter $D_{Li}^{\text{am}}$). One simply writes

$$D_{Li}^{\text{am}} = 1 - \Delta_{Li} \left[ 1 - \frac{m(r_{Li})_{ZAMS}}{M_\star} \right] \frac{\Delta m_{Li}}{m_p}, \quad 1 \geq D_{Li}^{\text{am}} \geq 0.$$
Fig. 8.—Velocity dependence of the main-sequence Li depletion factors ($D_{\text{Li}}$), as defined by eq. (19) in the text. A value of 1 corresponds to no depletion, while a value of 0 amounts to complete depletion. (a) Results for the four evolutionary sequences considered in the text. If the Li-burning front were to remain at the same depth throughout main-sequence evolution, the curves would have an inverted parabola shape (note how this is nearly true of the 1.2 $M_\odot$ curve). All these curves were calculated assuming that Li burns quasi-instantaneously at $T = 2.6 \times 10^6$ K; (b) For the 1.4 $M_\odot$ sequence, the effect of varying this Li-burning temperature; (c) the effect of assuming local angular momentum conservation (LAMC) rather than solid-body rotation (SBR) for the time-evolution of the equatorial rotational velocity, in the case of the 1.2 and 2.0 $M_\odot$ sequences.

The above results can now be used to calculate the fraction of stars of a given mass that will destroy their Li completely while evolving on the main sequence. This requires knowledge of the distribution of rotational velocities at a given spectral type. It has often been argued (Kraft 1970; Deutsch 1970, and references therein) that the observations are compatible with a Maxwellian distribution of $v \sin i$. However, many authors have now found counterexamples; Conti and Ebbets (1977) found that the O stars data could only be properly fitted using two distinct Maxwellian distributions. Guthrie (1982) has argued that the observed velocity distribution of O and early B stars appears Maxwellian for field stars, but not for cluster stars. Wolff, Edwards, and Preston (1982), upon compiling data from various sources and adding their own, found that the $v \sin i$ distributions at each spectral type, from O to late A, show a net excess of stars at low velocities as compared to a Maxwellian distribution. Note that the magnitude of this low-velocity excess shows a nonmonotonic variation with spectral type; it is as large as 45% for B stars, about 10% at spectral type A0, and somewhere in between for late A stars.

In order to obtain an estimate of the uncertainties associated with this low velocity excess, we have used the following two velocity distributions in our calculations:

$$f_1(jv) = \frac{4}{\sqrt{\pi}} (jv)^3 \exp\left[-(jv)^2\right],$$

$$f_2(jv) = \frac{3}{4} \left(\frac{4}{\sqrt{\pi}} (jv)^3 \exp\left[-(jv)^2\right]\right) + \frac{1}{4} f_3(jv),$$

where

$$f_3(jv) = \begin{cases} (jv_c)^{-1}, & v \leq v_c, \\ 0, & v > v_c, \end{cases}$$

with $j = \langle(v \sin \theta)^2\rangle^{-1/2}$. The first distribution is a standard Maxwellian distribution, while the second is a Maxwellian distribution with an excess of slow rotators, accounting for 25% of the total number of stars, spread uniformly in the interval $0 \leq v \leq v_c$. We chose a value of 40 km s$^{-1}$ for the cutoff velocity $v_c$.

The fraction of stars of a given mass having destroyed, say, 50% or less of their initial Li content is then obtained by integrating these distributions, with a value of $j$ obtained from observations, and using as the upper integration limit the velocity leading to $D_{\text{Li}} = 0.5$, as deduced from Figure 8a. Figure 9a shows, as a function of mass, the fraction of stars having $D_{\text{Li}} \geq 0.5$ and $D_{\text{Li}} \leq 0.1$. The solid and dash-dotted lines refer to Li, the dashed and dotted lines to Be. The solid

Fig. 9.—Fraction of stars containing, at the end of their main-sequence lifetime, a fraction of Li (or Be, as indicated) with respect to their initial content either inferior to 0.1 or superior to 0.5 (see labels on the curves), as a function of mass. Curves labeled MD refer to calculations assuming at each mass a Maxwellian velocity distribution, and those labelled NMD to a Maxwellian distribution with 25% excess below 40 km s$^{-1}$ (eqs. [20] and [21] respectively).

(a) Adopted $(v \sin \theta)$ — profile. Solid line corresponds to the relationship observed in the Hyades. Dashed curve is the $(v \sin \theta) - M_*$ relation obtained by extrapolating at low masses the Kraft curve derived by Kawaler (1987, see text).
and dashed lines are computed using the $f_1$ distribution above, while the dash-dotted and dotted lines are computed with the distribution $f_2$. Comparing the corresponding curves thus gives a measure of the uncertainties associated with these two possible equatorial rotational velocity distributions. Note how the curves for both distributions are very similar at lower masses. The solid line in Figure 9b shows the $v_e - M_\ast$ profile used, which is obtained from the observed $\langle v \sin i \rangle$ -- $T_{\text{eff}}$ relationship in the Hyades, after translation in terms of masses using the $T_{\text{eff}}$'s of our evolutionary sequences at the Hyades age (0.8 Gyr). We have also assumed randomly oriented rotation axes, i.e., $\langle v \sin i \rangle = 2\langle v_e \rangle/n$. The dashed line in Figure 9b corresponds to velocities obtained from the extrapolation to low masses of the observed mass-angular momentum relationship (i.e., the so-called Kraft curve) as recalculated by Kawaler (1987), using evolutionary stellar models to compute the moment of inertia required to obtain angular momenta from (observed) rotation rates. Such a $v_e - M_\ast$ relationship, when coupled to the $f_1$ distribution above, would lead to nearly total Li destruction for all stars in the whole mass range considered here; using instead the $f_2$ distribution leads to total Li destruction in about 75% of stars, again independent of mass.

Whichever distribution is used, the general interpretation of Figures 8 and 9 remains clear: most stars with $M_\ast > 1.3 M_\odot$ should destroy the greater part of their initial Li content while evolving on the main sequence. This simply reflects that stars with $v_e > 35$ km s$^{-1}$ burn all their Li on the main sequence through the combined effect of meridional circulation transport and nuclear burning, and that the observed average rotation rate on the main sequence increases with mass. In the following section, the Li depletion factors calculated above are compared to recent Li abundance determination in cluster giants.

### IV. COMPARISON TO OBSERVATIONS

Once the level of main-sequence depletion has been computed, it is a simple matter to calculate the total depletion expected in giants of a given mass; it is simply the product of the pre-main-sequence depletion, main sequence depletion, and giant branch dilution. These three processes are now briefly discussed separately, after which the total expected Li depletion for the models considered in the preceding section are compared to Li abundances observed in evolved stars. Additional constraints that can be inferred from beryllium abundances and the $^{12}$C/$^{13}$C ratio are also briefly discussed.

#### a) Pre-Main-Sequence Depletion

Pre-main-sequence Li burning occurs on the lower portion of the Hayashi track, as the temperature at the base of the receding convective envelope becomes sufficiently high, in certain cases, to burn lithium through $^7$Li($p$, y)$^8$He. Because of the rapid convective mixing, the (average) Li abundance in the whole convective envelope slowly decreases. As the star approaches the ZAMS, the base of the convective envelope eventually moves up above the depth at which $T = 2.6 \times 10^6$ K, and the surface lithium abundance stops decreasing. Bodenheimer (1965) calculated that no significant pre-main-sequence $^7$Li depletion should occur in stars more massive than about 1.2 $M_\odot$. This result was corroborated by the recent computations of D'Antona and Mazzitelli (1984; see also Proffitt and Michaud 1989) for their standard models (i.e., models without "extra mixing"). Pre-main-sequence depletion is thus expected to be small in all models considered above, except the coolest one, where it should be a factor of 0.98 according to Table 7 of D'Antona and Mazzitelli (1984), while Proffitt and Michaud (1989) obtain destruction factors of 0.9 and 0.7 in their 1.2 $M_\odot$ model with $Z = 0.0169$ and 0.024, respectively (see their discussions as to the origin of the discrepancies).

#### b) Evolution of the Average Envelope Li Abundance in Main-sequence Stars

Consider a cluster of a given age, sufficiently well populated to permit suitable comparison with theoretical evolutionary tracks, and whose global properties (metallicity, etc.) are known; it is then possible to infer, with reasonable accuracy, the mass of the stars populating that cluster's first-ascent giant branch. It can be expected that the first-ascent giants of that cluster have been depleted in lithium by a factor which is a function of their (prior) main-sequence rotation rate (see § III). In what follows, the rotational velocity distribution in the Hyades was used to evaluate the main-sequence depletion factors to be expected in older clusters.

Whether or not the Li depletion is to be observed on the main sequence is unclear. If circulation is free to penetrate the SCZ, then Li underabundances should develop (see § IV of Charbonneau and Michaud 1988b). It is also possible, on the other hand, that this penetration does not occur, perhaps because of the formation of a boundary layer at the base of the SCZ, as described in § II herein. In this latter case, the photospheric Li abundance could remain normal throughout the main-sequence lifetime, even though Li destruction does occur deeper in the envelope.

Since we are concerned here with stars more massive than 1.2 $M_\odot$, the process leading to large Li underabundance in the Sun and G stars needs not be taken into account. As far as global Li depletion is concerned, microscopic diffusion processes can also be neglected since, essentially, they merely redistribute the Li in the outermost fraction of the mass located above the point where $T = 2.6 \times 10^6$ K.

#### c) Post-Main-Sequence Dilution and Total Li Depletion

As the star leaves the main sequence, the outer portions of its envelope expand, following the onset of shell H-burning. The effective temperature decreases, and the base of the SCZ moves deeper, down to the point where it engulfs up to 70% of the total stellar mass. Although the temperature at the base of this growing convective envelope is never high enough to burn Li, the surface abundance decreases, as the leftover lithium gets mixed with increasing quantity of matter previously located at temperatures higher than 2.6 $\times 10^6$ K, and thus Li depleted. Iben (1967a, b) obtained, at the tip of the giant branch, maximum dilution factors of 43, 48, and 60 for stars of 1.25, 1.5, and 2.25 $M_\odot$, respectively. Note, however, that the greater part of the dilution occurs at the very bottom of the giant branch. Since the level of Li dilution is a function of the maximum downward extent of the convective envelope at the base of the giant branch, it can be expected that this will be influenced to some extent by the opacities used, the choice of $\alpha$ (the ratio of mixing length to pressure scale height; Iben used $\alpha = 1$), metallicities, etc. It turns out that the use of updated opacities and higher values of $\alpha$ does not affect greatly the maximum dilution factors, although the more gradual onset of the dilution process in subgiants does not appear to fit the expected pattern very well (Alschuler 1975).
d) Lithium in First-Ascent Giants

It is interesting to compare these expected dilution factors to the Li abundances in cluster giants. Consider first the older cluster M67; its age is about 5 Gyr (VandenBerg 1985), which using this author's evolutionary tracks for $Z = 0.0169$, leads to a mass of $\sim 1.3 \, M_\odot$ for the first-ascent giants of that cluster (see also Fig. 3 of Garcia Lopez, Rebolo, and Beckman 1988). Given a ZAMS velocity distribution similar to that of the Hyades (i.e., the solid line on Fig. 9a), one should expect that those giants had $\langle v \sin i \rangle \approx 50$ km s$^{-1}$ prior to their evolving off the main sequence. Pilachowsky, Saha, and Hobbs (1988) could only determine upper limits in their sample of 17 giants. Garcia Lopez, Rebolo, and Beckman (1988) obtained an abundance of $\log [\text{Li}]/H \approx 0.75$ for the single M67 giant that they observed. Figure 10a shows the expected dilution factor, indicated by a solid line. The dotted and dashed vertical arrows indicate the additional main-sequence depletion corresponding to $v_e = 27$ and 30 km s$^{-1}$, respectively, for a 1.3 $M_\odot$ model. According to Figure 8, all stars with $v_e > 35$ km s$^{-1}$ should have destroyed all their Li, so it is not surprising to find upper limits down to two orders of magnitude below the expected dilution value. Note that if indeed one had $\langle v \sin i \rangle \approx 50$ km s$^{-1}$ for these stars while they were still on the main sequence, then it can be expected, given a Maxwellian velocity distribution, that 88% of these rotated with $v_e > 30$ km s$^{-1}$; this is consistent with the fact that the lithium line is detected in only one giant. Note, however, that according to the results of § III, one is forced to suppose that none of these stars were slow rotators (i.e., $v_e < 20$ km s$^{-1}$). Using a Maxwellian velocity distribution, it can be verified that this latter condition should have been met by 4% of the 1.3 $M_\odot$ stars. If, instead, the non-Maxwellian distribution $f_2$ (see eq. [21]) is used, one finds that the condition would now be met by 15% of the 1.3 $M_\odot$ stars. Given the size of the sample, this cannot be excluded.

The cluster NGC 752 (age $\sim 2.2 \pm 0.3$ Gyr, according to Twarog 1983) does not have a well-populated giant branch, so that it is difficult to distinguish between first-ascent giants and stars in the core He-burning clump. Using again VandenBerg's $Z = 0.0169$ evolutionary tracks yields a value of $1.6 \pm 0.05 \, M_\odot$ for the mass of the first-ascent giants. On the main sequence, a large fraction of giants should then have been fairly rapid rotators ($\langle v \sin i \rangle \approx 90$ km s$^{-1}$ for 1.6 $M_\odot$ Hyades stars), and therefore should have destroyed a substantial fraction of their initial ZAMS Li. Pilachowsky, Saha, and Hobbs (1988) obtained upper limits to the Li abundance in nine giants which they tentatively identify (partly on the basis of the observed low Li abundances) as clump giants. They also obtained Li abundances in two other giants, to which they assign the status of first ascent giant. The observed abundances in these two giants are $\log [\text{Li}]/H \approx 1.2$, a value which, within the observational uncertainties, is in agreement with standard dilution values (see Fig. 10b). The dotted and dashed vertical arrows in Figure 10b indicate the Li depletion to be expected in 1.6 $M_\odot$ stars which rotated on the main sequence with $v_e = 33$ and 29 km s$^{-1}$, respectively. The two giants in which Li is observed must have had $v_e < 30$ km s$^{-1}$ on the main sequence. Assuming random orientation of rotation axes and a Maxwellian distribution of rotational velocities, the latter condition should have been met by only 2% of the 1.6 $M_\odot$ stars in NGC 752 (20% if the non-Maxwellian distribution $f_2$ is used). Thus, the model also appears compatible with the NGC 752 observations.

The younger cluster NGC 7789 (see Fig. 10c) has an age of $1.6 \pm 0.5$ Gyr (Twarog and Tyson 1985). This leads to masses of 1.6–2.0 $M_\odot$ for the first-ascent giants. Its few main-sequence stars located near the turnoff for which both Li abundance and rotational velocity have been measured show a normal $v \sin i$ distribution for their spectral type and a Li abundance about equal to the local Galactic value (Pilachowsky 1986). However, quite surprisingly, about three quarters of the giants observed show Li abundances in agreement with the expected dilution value, implying that these stars must have had $v_e < 30$ km s$^{-1}$ on the main sequence. Since this is unlikely, one must investi-

![Image of predicted total Li depletion in first-ascent giants of various clusters. Solid lines indicate the expected Li abundance variations due to dilution on the subgiant branch. Vertical arrows indicate the effect of main-sequence depletion through meridional circulation transport. (a) M67, mass of giants $\sim 1.3 \, M_\odot$; additional depletion factors corresponding to $v_e = 27$ and 30 km s$^{-1}$ are also shown (dotted and dashed arrows, respectively). (b) NGC 752, mass of giants $\sim 1.6 \, M_\odot$; additional depletion factors for $v_e = 28$ and 33 km s$^{-1}$ (dotted and dashed arrows) are shown. (c) NGC 7789, mass of giants $\sim 1.8 \, M_\odot$; main-sequence depletion for $v_e = 30$ and 35 km s$^{-1}$ (dotted and dashed arrows, respectively) are also indicated. Note that one of the two giants with log $[\text{Li}]/H \approx 2.5$ has tentatively been identified as a weak G-band giant (these often show abnormally strong Li lines; see, e.g., Lambert and Sawyer 1984), while the other is probably not a member of the NGC 7789 cluster (Pilachowski 1986). Triangles indicate upper limits while open circles correspond to measured abundances. The M67 and NGC 752 data are from Pilachowski, Saha, and Hobbs (1988), and from Pilachowski (1986) for NGC 7789.
gate mechanisms that could have reduced the expected transport through meridional circulation. Possible candidates are discussed in § V below. A similar problem is encountered with the few observed Hyades giants (age \( \approx 0.8 \) Gyr, turnoff mass \( \approx 2-2.5 \) \( M_\odot \); see, e.g., Lambert, Dominy, and Sivertsen 1980) in which the Li line has been measured. These giants also show, within observational uncertainties, Li abundances compatible with the standard dilution scenario. One cannot simply avoid the problem by arguing that the Li destruction rates calculated in § III were overestimated by a factor of 10, since in that case the M67 observations are unexplainable. Clearly, as far as the depletion process suggested here is concerned, the observations in NGC 7789 and M67 are in contradiction, unless one is willing to make some arbitrary assumptions regarding the respective main sequence \( r \sin i \) distributions of these two clusters.

The field giant and supergiant data are more difficult to analyze in a quantitative fashion, in view of the large uncertainties associated with mass determination for these objects. Lambert, Dominy, and Sivertsen quote a mean mass of 0.8 \( M_\odot \leq M_\star \leq 1.2 \) \( M_\odot \) for their sample, but the actual range of masses may well go as high as 3 \( M_\odot \), where as the supergiants of Luck (1977) and Luck and Lambert (1982) most probably have masses in the range 5–12 \( M_\odot \). It must simply be noted here that the existence of a large fraction of giants and supergiants with low Li abundances (i.e., log \( [N Li] \) \(< 0.0 \)) can be naturally explained by the transport process described above, without invoking other ad hoc destruction mechanisms. Note, however, that in the less massive of these field giants, the same process responsible for Li depletion in the Sun may have contributed to the total Li depletion.

e) Beryllium Burning through Transport by Meridional Circulation

Additional constraints on main-sequence models can be obtained from beryllium abundances in giants. Meridional circulation transport, as modeled above, is expected to lead to Be destruction in exactly the same way as for Li, with burning occurring at \( T = 3.5 \times 10^6 \) K instead of 2.6 \( \times 10^6 \) K. For a given rotation rate, the main-sequence Be depletion is smaller than that of Li, since the reservoir of Be is larger; although the downward mass flux due to circulation is nearly independent of depth, the mass contained above 3.5 \( \times 10^6 \) K is about 3 times larger than that above 2.6 \( \times 10^6 \) K. However, as can be seen from Figure 9, Be is significantly less depleted than Li only in a relatively restricted stellar mass range, corresponding approximately to the mass of first-ascent giants in M67. Nevertheless, the expected pattern is in agreement, at least qualitatively, with observations of Be in a few Hyades giants that might show somewhat larger Li overdepletion than Be overdepletion (with respect to the expected depletion through dilution), as inferred from upper limits on Be abundances (Boesgaard, Heacox, and Conti 1977). More Be abundance determinations in giants of NGC 7789, NGC 752, M67 would most likely yield important constraints. In particular, many giants in M67 should show a significantly smaller over-depletion of Be than Li, whereas in NGC 752, similar over-depletion of Be and Li are predicted for most giants.

f) The \( ^{12}C/^{13}C \) Ratio in Giants and Supergiants

It would be interesting to calculate the effect of meridional circulation transport on the expected \( ^{12}C/^{13}C \) ratio on the giant branch. Unfortunately, the numerical technique used here is a poor tool to model \( ^{12}C \) transport, since \( ^{12}C \) production occurs in a relatively large portion of the envelope, and therefore cannot be reasonably approximated by a step function. One would need a full solution of the two-dimensional transport equation (9). The various uncertainties related to the development of a mean molecular weight gradient deep in the envelope are also more important, since one has to model meridional transport practically all the way down to \( M(\rho)/M_\star \approx 0.3 \), where nuclear reactions have modified the initial mean molecular weight profile. A rough evaluation of the expected results can nevertheless be made. It can be seen from Figure 8 that, as one increases the rotation rate above about 20 \( \text{km s}^{-1} \), Li depletion rapidly increases, being nearly total at 30 \( \text{km s}^{-1} \). This is only partly due to Li burning occurring instantaneously below a certain depth. It is mostly a consequence of the strong dependence of the circulation velocities on the rotation rate \( (U \propto v^2) \). Therefore, a similar behavior can be expected in the case of \( ^{12}C \) transport; it is already apparent in the approximate calculations of Dearborn, Eggleton, and Schramm (1976; see their table 2). This would then lead to a rather strong correlation between the \( ^{12}C/^{13}C \) ratio and the Li abundance, as observed (see Fig. 5 of Luck 1977; Fig. 9 of Lambert, Dominy, and Sivertsen 1980).

V. UNCERTAINTIES IN THE CALCULATIONS

a) Penetration of the Convection Zone by Meridional Circulation

It has been assumed implicitly in the calculations presented here that meridional circulation could operate unhampered at the boundary between the radiative envelope and the SCZ; i.e., that meridional circulation penetrated the SCZ as if it were a radiative zone. However, one could equally assume that particle transport is negligible across the base of the SCZ by arguing that a boundary layer forms there as described in § II. Obviously, these two possibilities lead to very different photospheric Li abundance evolution on the main sequence. It turns out, however, that the results presented in this paper are not modified substantially if the penetration of the SCZ by circulation is only partial, or even completely absent. If there is no penetration, the Li in the convection zone can at most remain at its original level, which means that the fraction of the ZAMS lithium that can be "stored" in the SCZ is of the order of the ratio \( m(r_{sc})/m(r_{Li}) \). For all models used in the preceding sections, it can be easily verified that this quantity is always very small, about 0.05 for the 1.2 \( M_\odot \) model, down to \( 10^{-5} \) for the 2 \( M_\odot \) model, with \( z = 1.5 \). The uncertainty regarding the behavior of the circulation velocity field under the SCZ therefore, cannot modify the Li depletion rates calculated above by more than 5% in the less massive model. At higher masses, the effect on the global Li destruction is negligible.

b) Evolutionary Effects on Meridional Circulation

The \( T^2 \) solution procedure is strictly valid only in the case of static circulation velocity fields, since the time derivatives have been set to zero in the \( r \) and \( \theta \) components of the Navier-Stokes equations. What is done in § III is essentially to mimic the time evolution of the meridional circulation field by obtaining static solutions in a sequence of quasi-static evolutionary models. Although this is a reasonable approximation on the main sequence, where evolutionary timescales are much longer than thermal time scales, it may no longer be so when the star starts moving toward the giant branch. The same problem

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would be encountered in trying to obtain meridional circulation solutions along pre-main-sequence evolutionary tracks. One would then have to use circulation velocity fields obtained from a full time-dependent solution for the function $u(r)$. To avoid these difficulties, the Li transport by circulation calculated in § III is instantaneously "turned on" on the ZAMS, and "turned off" when hydrogen burning stops in the core.

Some form of circulation is, however, expected in rotating pre-main-sequence models that are close enough to the ZAMS to have developed a large radiative envelope. This evolutionary phase is of very short duration with respect to the main-sequence lifetime, and so transport by meridional circulation is not expected to lead to significant Li depletion. Stopping Li transport completely and instantaneously when core H burning terminates is a more dubious hypothesis; from the evolutionary sequences of VandenBerg (1983), one can easily determine that the time laps between the onset of gravitational contraction off the main sequence and the beginning of the first dredge-up at the bottom of the giant branch is of the order of 10% of the main-sequence lifetime for a 2.0 $M_\odot$ star, but of up to 40% for a 1.2 $M_\odot$ star. Pushing the calculations of § III past $t_{*0}$ would certainly lead to some increase in Li depletion, even when the reduction in the rotation rate due to the expansion of the envelope is taken into account. The effect can be expected to be more pronounced for less massive stars, which take more time (relative to the main-sequence lifetime) to move from the main sequence to the bottom of the giant branch, and may partly explain why the giants of M67 seem to have been so much more efficient than their more massive NGC 7789 counterparts in destroying lithium. This means that our calculations underestimate the total destruction factor to be expected on the giant branch.

c) Suppression of Meridional Circulation by a $\mu$-Gradient

The development of a mean molecular weight gradient caused by nuclear burning is known to literally "choke off" meridional circulation deep in the star (Mestel 1965; Tassoul and Tassoul 1984; see also Huppert and Spiegel 1977). In early main-sequence stars, He settling may play the same role (Moss 1979), although the process is more complicated to calculate properly since meridional circulation slows down gravitational settling, effectively stopping it for sufficiently large rotation rates (of the order of $100 \text{ km s}^{-1}$ for a $3 M_\odot$ star; see Charbonneau and Michaud 1988a). This barrier effect of a $\mu$-gradient due to He settling has not been properly modeled quantitatively, but may lead in these massive stars to the survival of Li in a appreciable fraction of the region where $T < 2.6 \times 10^6 \text{ K}$, and may be partly responsible for the large fraction of giants in NGC 7789 that appear to have suffered no main-sequence Li depletion.

A rough estimate of this effect can be made nevertheless, by using the criteria put forth by Huppert and Spiegel (1977); these authors argue that a $\mu$-gradient cannot seriously impede mixing until $N_r > \Omega$, where $N_r$ is the buoyancy frequency due to the $\mu$ stratification (see §§ II and III of Huppert and Spiegel 1977), and $\Omega$ is the rotation rate. Consider He settling in a 1.6 $M_\odot$ star; a strong $\mu$-gradient develops under the SCZ within a few $10^\text{yr}$ yr. Such a $\mu$-barrier, if the Huppert-Spiegel criteria is used at face value, turns out to be impenetrable (to significant depth) for $v_r < 500 \text{ km s}^{-1}$. This result must be interpreted with caution, since when $v_r > 100 \text{ km s}^{-1}$ meridional circulation greatly inhibits He settling. Furthermore, even if a $\mu$-barrier sufficiently strong to impede meridional mixing develops, a closer look at the time evolution of the He abundance profile in a nonrotating 1.6 $M_\odot$ model (see, e.g., Charbonneau and Michaud 1988a) reveals that this $\mu$-barrier reaches a depth of no more than $\Delta M/M \approx 10^{-4}$ after a few Gyr. Although this is considerably more than the SCZ's mass, it is well below the mass contained above $T = 2.6 \times 10^6 \text{ K}$ ($\Delta M/M \approx 10^{-2}$), so that the $\mu$-barrier could at most protect 1% of the Li initially located above $2.6 \times 10^6 \text{ K}$. Our results would then not be modified substantially by this effect.

On the other hand, Huppert and Spiegel did not include in their calculation the effect of horizontal $\mu$-gradients. These are known to be very efficient in choking meridional circulation in the solar core (see Tassoul and Tassoul 1984). They also develop under the SCZ under the combined effect of He settling and meridional circulation (see Fig. 2 of Charbonneau and Michaud 1988a), and could conceivably inhibit meridional circulation transport over a much larger mass than vertical $\mu$-gradients. There is no quantitative estimate of this effect as yet.

d) Effect of Angular Momentum Loss

Assuming that the observed rotation rates in a cluster are representative of the original rotation rates on the ZAMS actually provides a lower limit to the true-averaged rotation rate, and thus to the time-integrated Li depletion by meridional circulation over the main-sequence lifetime; this is because late-type stars are known to suffer rotational braking, most probably through angular momentum loss via magnetically confined stellar winds (see Kawaler 1988 for a clear discussion). The rotation rate may have been larger in the past and may also be larger deeper in the star than at its surface where it is measured. Uncertainties related to these effects are expected to be small for models of masses greater than about 1.5 $M_\odot$, which appear to suffer little rotational braking, but may become more important for the 1.2 $M_\odot$ model. How much the destruction is underestimated depends on the rate of angular momentum loss, as well as on the efficiency of angular momentum redistribution inside the evolving star. From observations of G and K dwarfs in young clusters, it appears that G and K dwarfs complete their major spin-down phase in a lapse time ranging somewhere between the Pleiades and Hyades ages (see, e.g., Stauffer, Hartmann, and Latham (1987 and references therein), with some stars being already slow rotators at the age of the Pleiades or even less (Balachandran, Lambert, and Stauffer 1988). This is small compared to the main-sequence lifetime of these stars. The spin-down rate of the underlying radiative envelope is, however, also dependent on the level of mechanical coupling between the radiative and convective zones (see, e.g., Kawaler 1988; Pinsonneault et al. 1988). This is another reason why the calculated Li depletion is expected to be a lower limit. For example, using a $(\varv \sin i) - M_*$ relationship corresponding to the extrapolation at low masses of the Kraft curve (see, the dashed lines in Fig. 9b) leads to nearly total Li destruction even for the 1.2 $M_\odot$ sequence; this is obviously an extreme case, but it illustrates the effect on Li abundances of having the stellar interior spinning down on a much longer time scale than the SCZ.

VI. IMPLICATIONS FOR HYDRODYNAMICAL MODELS

We have shown that the dredge-up occurring on the subgiant branch offers a key opportunity to probe, in an a posteriori fashion, the various transport processes that can lead to light-element destruction in regions of the envelope of main-
sequence stars which are inaccessible to direct observation. In this paper, we have focused on one such transport process: transport by meridional circulation. Since strict hydrostatic equilibrium is essentially impossible in a rotating radiative zone, some form of mass transport is bound to develop as a consequence. Meridional circulation is probably the simplest "solution" to the problem, and its potential side effects can be modeled with enough confidence to allow comparison with observations.

The comparison to observations (§ IV) shows that the destruction of Li via meridional circulation transport occurred as expected in M67 and NGC 752. However, in NGC 752, the possible presence of clump giants complicates the comparison to observations (see § IVd). The fact that the observations of NGC 7789 giants are essentially compatible with no main-sequence Li destruction is a surprising result. It provides a strict upper limit on particle transport above the point where \( T = 2.6 \times 10^6 \) K, which contradicts the expected meridional circulation transport and severely constrains any form of turbulent transport. This is surprising since, as discussed in § V, most factors having been neglected in the calculations above lead to underestimating the destruction of lithium.

The only neglected factor that leads to overestimating Li destruction is the effect of a \( \mu \)-gradient in reducing meridional circulation. However, this appears likely to protect at most 1% of the Li mass initially located above the depth where \( T = 2.6 \times 10^6 \) K. While this may still be a possibility, it remains to understand why it would be efficient in reducing Li transport in NGC 7789 and not in the other clusters. It could be related to the age of this cluster and to the higher mass of the observed giant stars.

Part of the explanation may also be found in other transport processes, whose effects have been neglected above. For instance, it may be that while the star evolved on the main sequence, some of the Li initially located below the SCZ was pushed upward and maintained in the SCZ by radiative acceleration (see Lambert and Sawyer 1984; Charbonneau and Michaud 1988b); this effect is only possible in stars with \( T_{\text{eff}} > 7000 \) K (the exact value being a function of metallicity and the chosen value for \( \alpha \), as Li is not supported at the base of the SCZ of cooler stars. However, only a small fraction (\( \lesssim 0.1\% \)) of the Li initially located above the depth at which \( T = 2.6 \times 10^6 \) K can be "saved" this way; to obtain nearly no main-sequence depletion would require all the Li above \( \Delta M / M \approx 10^{-2} \) to be rapidly pushed upward by radiative acceleration, whereas this occurs only above \( \Delta M / M \approx 5 \times 10^{-6} \), for nonrotating models in this mass range (see Fig. 7a of Charbonneau and Michaud 1988b).

The preceding discussion assumes that the NGC 7789 giants had a normal \( \alpha \sin i \) distribution on the main sequence. Given the variation with mass of the excess of slowly rotating stars in clusters (see the discussion at the end of § III), the large Li abundances in NGC 7789 could perhaps be explained, within the framework of the model presented above, by the existence of a much larger number of slowly rotating stars in NGC 7789 than in field stars.

While meridional circulation is the only transport process that can currently be modeled with little arbitrariness from first principles, it is not the only one that is expected to be present in stellar envelopes. Some form of turbulence must also be present; it is actually assumed to exist, via the appearance of a turbulent viscosity coefficient, by \( T^2 \) in arriving at their meridional circulation solution.\(^2\) It would than be interesting to calculate the effect on the Li abundance in giants of turbulent transport operating on the main sequence. More specifically, the various models having been recently put forth to explain the main-sequence Li abundance (Vauclair 1988; Pinsonneault et al. 1988) should be put to this test, for both Li and Be. It may be that Li destruction is due to turbulent mass transport occurring in parallel with angular momentum transport through the stellar envelope. This would provide a simple explanation as to why the more massive giants in NGC 7789, having suffered little angular momentum loss over their main-sequence lifetime, have destroyed little Li, as opposed to the less massive giants of M67, which probably have experienced some angular momentum loss. This has already been suggested in a somewhat different context by Balachandran, Lambert, and Stauffer (1988), who note that in the young cluster \( \alpha \) Per, the rapidly rotating G and K dwarfs show much greater Li abundance than the more slowly rotating ones. The detailed understanding of the abundance anomalies on the main sequence, and their link to abundance anomalies on the giant branch, may lead to a proper understanding of the transport processes operating in stellar interiors. If turbulent transport turns out to play a major role, it will remain necessary to explain why meridional circulation is not as important as calculated above.

Note added in manuscript.—After this paper was written, we received a preprint from Gilroy (1989) containing \(^{13}C/^{12}C\) ratios and Li abundance determinations in giant stars of open clusters with ages ranging from 0.05 to 0.8 Gyr; the corresponding range of masses for their first-ascent giants is from 6 to 2.2 \( M_{\odot} \). Some of these giants show Li abundances that are compatible, within uncertainties, with the standard dilution value; however, in no case is the fraction of "Li-normal" giants as high as that seen in NGC 7789. It would thus appear that high Li abundances are not the rule for clusters younger than 2 Gyr and that NGC 7789 may be truly peculiar in this respect. Nevertheless, Gilroy's sample contains a smaller fraction of strongly Li-depleted giants (\( \log [N(Li)] < 0.0 \)) than predicted by the model presented in this paper.

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\(^2\) It should be noted that the presence of large turbulent viscosity (i.e., momentum transport) does not necessarily imply the existence of correspondingly large particle transport (see the discussion in Tassoul and Tassoul 1988). Along similar lines, note also that Pinsonneault et al. (1988) found, while calculating rotating evolutionary models for the Sun, that a proper reproduction of the Sun's observed characteristics (including light-element abundances) requires that particle transport occurs at a considerably smaller rate than angular momentum transport. Even then, some Li depletion does occur in their models, over a large range of modeling parameters.

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