

THE STRUCTURE OF THE BETA PICTORIS CIRCUMSTELLAR DISK FROM COMBINED *IRAS* AND CORONAGRAPHIC OBSERVATIONS

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ABSTRACT

We have constructed a grid of models of the particulate disk around β Pic, in which the same grains orbiting the star give rise to both the directly observable scattered starlight and the far-IR thermal radiation measured by *IRAS*. The spatial distribution of particles in the disk is determined from the 1986 coronagraph images published by Paresce and Burrows in 1987 and used together with *IRAS* measurements to find a distribution of matter in both the directly observable and occulted disk regions. We derive an upper limit for the tilt between the disk ecliptic and the line of sight of 14° , corresponding to an infinitesimally thin disk. Such an extreme tilt is possible only if disk particles have a more isotropic scattering phase function than Lambert's law of diffusive reflection. As long as the tilt is less than approximately 10° , the intrinsic structure of the disk differs only slightly from the exactly edge-on case. Most models presented in this paper have a constant disk opening angle, out to at least 250 AU from the star, while previous attempts have assumed constant disk thickness. Assuming a nearly edge-on viewing geometry of the disk, half of the total effective scattering area lies within the sector $i < 7^\circ$, where i is the angle between a given element and the ecliptic plane of the system. Thus, although the apparent surface brightness in scattered light falls with radius as $r^{-3.6}$, the normal optical thickness of the disk decreases only as $r^{-1.6}$ to $r^{-1.9}$. This result resolves discrepancies found in previous work.

Maximum entropy inversion of the far-IR spectrum places the maximum normal optical thickness of the disk of $\tau \approx 0.007$ at radii where the grain temperature is $T = 95\text{--}115$ K, and the half-maximum density at $T \approx 160$ K. This gives rise to an inner clearing zone with a half-clearing radius 5–15 AU depending on grain emissivity law. Available information on the near-IR extent of the disk image favors a grain emissivity falling off linearly with wavelength in the far-IR rather than a constant emissivity used in some recent models of the disk. Together with observed neutral scattering of visible and near-IR light, this indicates grain radii between $1\ \mu\text{m}$ and $20\ \mu\text{m}$, approximately. The model-dependent geometric albedo of grains in the visible is determined to be in the interval 0.66–0.76, consistent with a model-independent lower bound of 0.5. Plausible grain composition includes either fine-grained feldspars or olivines, or, alternatively, water and/or ammonia frost as a matrix with a small amount of dark (carbonaceous?) contaminants. If the grains are icy, the central disk gap may represent a sublimation zone.

Subject headings: infrared: sources — stars: circumstellar shells — stars: individual (β Pic)

1. INTRODUCTION

In common with a small number of main-sequence stars (most notably Vega, Fomalhaut, and ϵ Eri), β Pic shows a substantial infrared (IR) excess in the *IRAS* passbands (*Infrared Astronomical Satellite*; Aumann *et al.* 1984 and Aumann 1985). The *IRAS* excesses have been interpreted as being due to thermal radiation from dust typically at about 100 K orbiting the stars. The presence of this solid circumstellar material is now commonly interpreted in the context of planet formation (e.g., Gillett 1986), and its study is expected to contribute greatly to our understanding of the course and the relative frequency of the process of planetary system formation.

The β Pic System is well suited for such studies as it is exceptional in several respects (Hobbs 1986). Of all the Vega-like stars it has the largest overall IR excess seen by *IRAS*, exhibits the only substantial excess at $12\ \mu\text{m}$ (Gillett 1986), and its nearly edge-on circumstellar disk is the only one imaged directly to date, first in the *I*-band by Smith and Terrile (1984),

and recently in broad-band *B*, *V*, *R*, *I*, *I_c* multicolor photometry by Paresce and Burrows (1987), and by B. Smith and R. Terrile (R. Terrile, private communication).

Previous work on the number density of emitting and scattering material as a function of orbital radius provided widely different results. If the number density of grains is proportional to the gas density, one can expect a power-law index of -1 (Vidal-Madjar *et al.* 1986). The same power index seems to follow from Gillett's (1986) fit to the *IRAS* spectrum. On the other hand, the radial dependence of the midplane surface brightness of the disk is consistent with a much steeper profile of index -3.1 (Smith and Terrile 1984). This power law dependence has been challenged by Buitrago and Mediavilla (1986) on the basis that it leads to unacceptable empirical scattering phase functions. These problems are probably due to the poor quality of existing data that does not allow a reliable determination of both the radial distribution and the scattering properties of disk material simultaneously by the standard methods used in zodiacal light studies which require the knowledge of the derivative of image brightness over angular separation. It is thus very important to reexamine more accurately the available disk photometry as well as to investigate the possibility of a variable disk thickness which has been up to

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now assumed constant, to see whether the discrepancy is true or apparent.

Another difficulty is connected with the estimation of the size of particles contributing most of the effective disk area and, hence, determining disk's appearance. We shall define in this paper "small" particles as being small relative to I -band photons, i.e. less than $1\ \mu\text{m}$ in effective size, "large" particles as those with optical properties approximately constant in the wavelength interval up to at least $100\ \mu\text{m}$ which requires, for realistic composition, grain radii larger than $20\text{--}30\ \mu\text{m}$ and, finally, "midsize" particles as those in between. Note that geometrically large particles with small-scale roughness (e.g., fluffy particles) may not be "large" in our definition.

The original argument for midsize or large β Pic disk particles (Smith and Terrile 1984) relied partly on the assumed Poynting-Robertson effect and partly on the apparent neutral extinction of direct β Pic visible flux. The star appears 0.3 mag fainter than a typical ZAMS star, and 0.8 mag fainter than an evolved star of spectral type A5 V, with no reddening seen in the UV or visible spectra (Kondo and Bruhweiler 1985). However, the reality of this effect has been questioned by Diner and Appleby (1986) on the basis of uncertainties in the parallax. Disk models presented in this paper suggest that extinction of direct starlight by the disk may, in fact, be unobservable.

A much stronger argument for the β Pic grains being larger than $1\ \mu\text{m}$, and hence much larger than typical interstellar grains with radii $a < 0.1\ \mu\text{m}$, is provided by multicolor photometry of the disk. Paresce and Burrows (1987) have shown that within observational uncertainties no color differences exist between the disk and β Pic itself between B and I_c bandpasses. Recent results of Smith and Terrile (R. Terrile, private communication) confirm neutral scattering down to B and indicate a downturn in reflectivity in the U -band. They also measure 10% (20% in the innermost observed regions) polarization of scattered light. Mie theory calculations show that silicate grains with radius smaller than $a \approx 0.75\ \mu\text{m}$ show strong color effects in scattered light between U and I bandpasses (for water ice spheres, the critical radius is $a \approx 1.3\ \mu\text{m}$). Therefore, the only possibility for small particles not to show color effects is the presence of a particle size distribution of submicron grains such that particles of each radius contribute equally to the integrated area. However, the polarization in systems in which this situation is thought to occur is significantly higher than observed (50%–70% in VY CMa; e.g., Jura 1975). More importantly, submicron grains lie below the radiative blow-out size limit for the β Pic system (Gillett 1986; Artymowicz 1988), such that they are removed from the disk practically instantly.

On the other hand, Aumann (1985) and Gillett (1986) find large-particle models to fit the *IRAS* fluxes poorly and advocate midsize particles. Gillett (1986) additionally noted that large blackbody particles give disks 10 times smaller than the size suggested by *IRAS* data. We shall consider this issue in detail in § IV. To properly address the crucial question of particle size, we have constructed a grid of models corresponding to both midsize and large grains.

Another discrepancy appears in the literature concerning the possible existence and extent of a central clearing zone in the inner part of the disk which cannot be presently seen directly. Smith and Terrile adopting large grains deduce a sizable (radius 30 AU) inner clearing from the assumed extinction of β Pic equal to 0.5 mag. Diner and Appleby (1986) assume a constant IR emissivity of particles, i.e., large grains, but they attribute no statistical significance to the neutral extinction

and add one free parameter to the functional form of the radial density of grains as compared with the truncated power law of Smith and Terrile. With these assumptions and taking *IRAS* data into account, they obtain a much smoother distribution of disk material with substantial opacity as close to the star as 5 AU. Least-square parametric methods may be inappropriate to this problem as the correct functional form of the spatial distribution of grains around the star is unknown.

Finally, all the previous models assumed that the disk is viewed strictly edge-on. Clearly, the sensitivity of the results to this assumption and an upper bound on the tilt angle need to be carefully evaluated.

The problems just described stimulated us to examine afresh the observational inputs and to construct models satisfying both the disk photometry inferred from the recent observations by Paresce and Burrows (1987) and *IRAS* infrared measurements. Our goal is to investigate the range of model parameters (both geometrical ones and those related to grain optical properties) for which self-consistent models of the disk exist. We assume throughout the present paper that the same disk particles give rise to both scattered and thermally reradiated starlight, and that there are no other sources of radiation in the disk (e.g., gas). While fitting the *IRAS* spectrum we invert the flux integral using maximum entropy techniques and thus avoid limiting the number of free parameters in the unknown distribution of matter while ensuring its smoothness.

In § II, we reanalyze the coronagraphic images of the β Pic disk and derive constraints on the spatial distribution of disk matter. In § III, we review the basic model assumptions and equations, along with the methods for solving them. Then we proceed with the rederivation of *IRAS* excess fluxes, and construct a range of models with the maximum entropy technique. Section IV deals with the results of modeling and the discussion of the edge-on disk models, effects of the finite tilt, the comparison with other recent models of the system, and finally the composition of grains. In § V, we summarize the main results.

II. GEOMETRY OF THE BETA PICTORIS DISK FROM CORONAGRAPH IMAGES

Direct imaging of the β Pic disk in the visible and near-infrared regions of the spectrum shows a highly flattened circumstellar structure, suggestive of a disk seen nearly edge-on. The disk can be traced from $6''$ up to roughly $25''$ from the star in the NE and the SW directions, according to Smith and Terrile (1984). This corresponds to a radial extent of about 400 AU, with an assumed distance to β Pic of 16.4 pc (e.g., Kondo and Bruhweiler 1985). Recently they were able to trace the disk out to more than 1000 AU (Smith and Terrile 1987). They also note that, at a distance of 300 AU, the disk image has a thickness of about 50 AU. Gradie *et al.* (1987) note that the disk may become thicker with increasing distance, in accordance with an earlier suggestion by Smith and Terrile (1984). Unfortunately, all the existing models of the disk assumed, for simplicity, a vanishing tilt angle and constant intrinsic thickness of the disk. These simplifications may ignore the role of possibly different radial dependences of disk's surface density and its midplane (ecliptic) spatial density, which may give rise to the different power-law indices of the radial grain density deduced from IR data versus those deduced from existing coronagraphic models.

In order to put much tighter bounds on the spatial distribu-

tion of disk material, we reanalyzed the broad-band coronagraph observations of Parescé and Burrows (1987, hereafter PB). The field of view is smaller than that of Smith and Terrile's original images: the disk can be traced from slightly less than $5''$ to about $13''.5$ but with improved sampling especially important in the direction perpendicular to the midplane of the disk. For a detailed description of the instrument and the basic reduction technique, the interested reader should consult PB. Here, we only note that the disk images of PB are difference images, i.e., result from subtraction from the object image of an optimally overlaid and scaled control star image. This procedure is expected to eliminate the wings of the seeing disk and most of the instrumental diffraction effects and yield a photometrically accurate image of the disk.

We found it necessary to modify somewhat the reduction process used in PB to compensate for variations in the seeing disk profile between β Pic and the control star due to changing atmospheric conditions. The objective was to improve the subtraction close to the mask (which was irrelevant to the results given in PB on relative magnitudes in different bands). The method used was to subtract a radially dependent multiple $f(r)$ of the control star image from the original so that the difference image was flat perpendicular to the disk. In PB, a constant ratio of the images was assumed. Two remarks on this procedure should be made here. First, the function $f(r)$ deviates by no more than 20% from and for large r tends to $f = 1.1$, the constant ratio utilized in PB. Second, a centrally symmetric mask does not give rise to any spurious nonsymmetrical features in the result and so does not interfere with the position angle or shape of the disk, or produce spurious disk-like features.

We used results of exposures in the I_c band centered on $7900 \text{ \AA}$. Images taken with different filters lead to similar pictures, and because the overall normalization follows closely the broad-band photometry of the star itself, the choice of this particular spectral band is arbitrary. The isophotes of the disk,

spaced in half magnitude per unit area intervals, are shown in Figure 1. We have rotated the image of bilinear interpolation so that the disk axis is vertical, and clipped it to cover an area of 400 by $200 \times 0''.07$ pixels.

The midplane brightness expressed in counts/pixel for a 30 minute exposure is presented in Figure 2 (by the disk midplane we denote here the maximum brightness line on the image, not necessarily the disk ecliptic). The upper solid line corresponds to the SW part of the disk, the lower one to the NE part. Overplotted are entries from Table 1 of PB, applying to the NE extension of the disk. Very good agreement is found between all the PB points and our lower curve, except for the $6''$ point, where our radially dependent correction produces a 30% higher value. The data in Figure 2 are fitted well with a $r^{-3.6}$ dependence shown as a dashed line in Figure 2. The departure from a power law for the innermost observed radii may be caused by registration effects near the occulting finger, rather than representing a real feature. It is safe to assume that the power law holds down to $6''$ from the star with approximately 20% accuracy.

Comparison of the results shown in Figure 1 with the original I_c band false-color image of PB obtained from the same raw data shows no apparent conical void around the disk axis. Instead, the isophotes appear to be discontinuous only when occulted by the coronagraph finger (*right top to left bottom stripe*) or inside the circular central region where the CCD analog to digital conversion was saturated. The isophotes are most inaccurate near regions of high count rate (due to registration errors) and low count rate (due to background errors). The basic image characteristics including the convex shape of isophotes and the ratio of vertical to radial extent of about 0.3 are insensitive to these errors and will be used below to estimate the spatial distribution of scattering matter within the disk. We shall construct several theoretical images corresponding to prescribed density distributions within the disk and compare them directly with Figure 1.

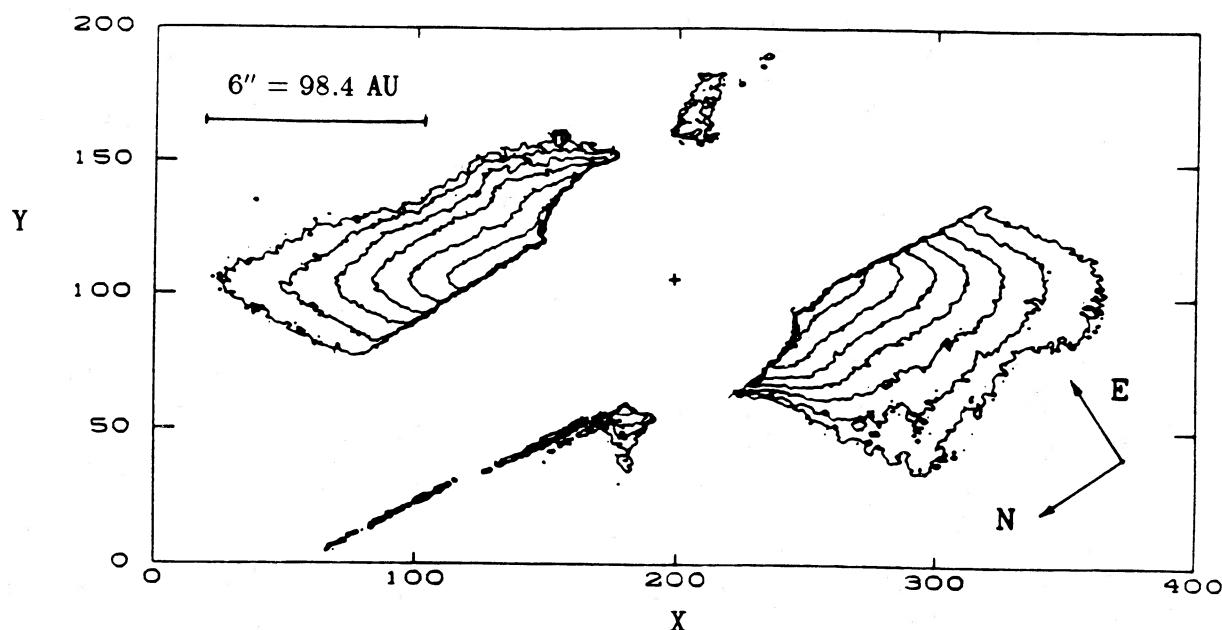


FIG. 1.—Isophotes of the β Pic disk image in the I_c band centered at $7900 \text{ \AA}$. Both coordinates are given in pixel numbers (pixel's size = $0''.0702$). Isophote spacing corresponds to 0.5 mag brightness difference.

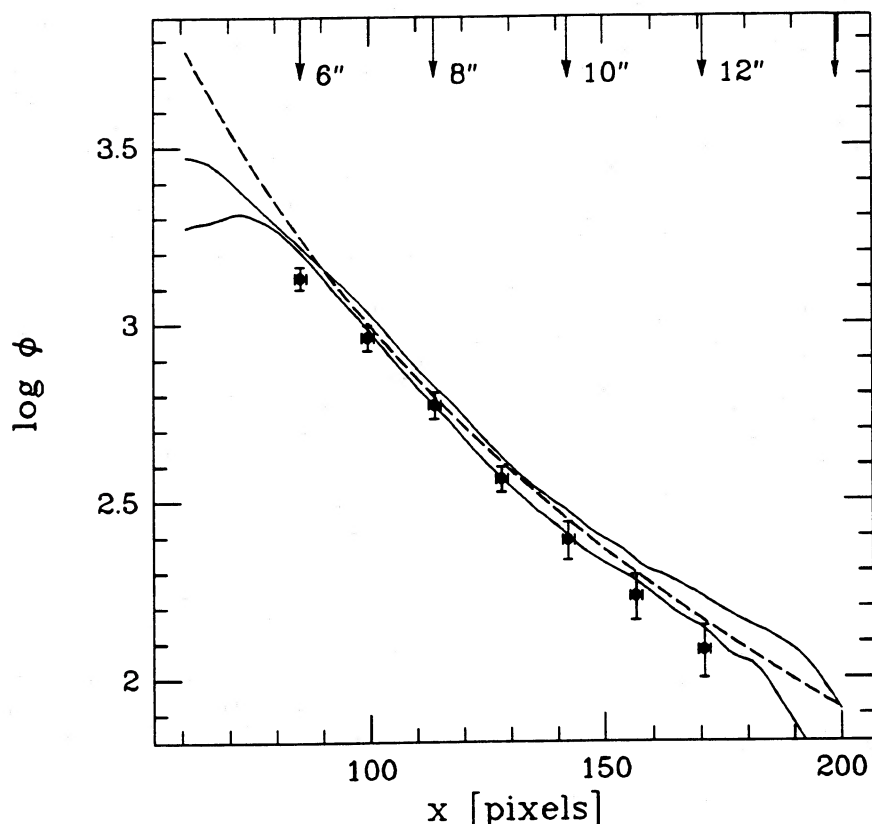


FIG. 2.—Logarithm of count rates per pixel per 30 minutes of exposure along the midplane of the I_c image (Fig. 1), as a function of distance from image center. Corresponding angular distances are indicated on the upper axis. The upper and lower solid lines represent the SW and the NE extension of the disk. Points referring to the NE extension are from BP. The dashed line is the theoretical fit to the data, $\phi = 1790 \times r^{-3.6}$.

To do this, we assume that the density distribution function is axisymmetric and separable in the cylindrical disk coordinates r and z according to:

$$A_s = A_{6S} R(r) Z(z), \quad (1)$$

where A_s denotes the scattering coefficient or scattering cross section per unit volume, i.e., the total projected area of grains cm^3 . If $dn(r, z, a)$ is the number density of spherical grains with radii between a and $a + da$ scattering light with efficiency Q_{sca} , we can express A_s as $A_s(r, z) = Q_{\text{sca}} \pi \langle a^2 \rangle n(r, z)$, where $n(r, z)$ is the total number density of grains, and $\langle a^2 \rangle = n^{-1} \int a^2 dn$. In equation (1), A_{6S} is equal to $A_s(r = 6D, z = 0)$, or the scattering coefficient at the fiducial distance of $6D \approx 100$ AU ($D = 16.4$ AU) corresponding to the angular separation of $6''$. $R(r)$ and $Z(z)$ are the nondimensional radial and vertical profiles of A_s . In the following, we will often express r and z in units of the fiducial distance $6D$ and denote the new nondimensional variables as $r_6 = r/(6D)$, and $z_6 = z/(6D)$. In the regions of the disk further out than about $6''$ from the star, which we refer to as the outer disk, the coronagraphic observations suggest the following functional form for $R(r_6)$

$$R(r_6) = r_6^\alpha, \quad 1 < r_6 < 5 \quad (2)$$

where α is a negative constant to be determined from observations. The upper limit for r_6 follows from an assumed outer boundary at 500 AU. We have to set the finite upper limit on r_6 because the values of α we find imply an infinite mass of the untruncated disk (it is obvious that the actual behavior of disk thickness and α are modified at large radii so as to remove this

formal problem). In any case, the specification of the outer boundary is without consequence for all the following calculations, except the disk mass estimations.

We restrict our attention to two specific functional forms for the vertical profile, $Z(z_6)$, i.e., a Gaussian and an exponential fall-off of scattering efficiency with height. These two choices are representative of distributions that have flat maxima and no extended wings in the first case, and sharp central peaks with broad wings in the second case. In collisionally dominated disks with Maxwellian velocity distribution, the vertical profile is Gaussian. We consider, however, both possibilities to estimate the sensitivity of results to the shape of the vertical profile. We also assume a power-law scaling of the vertical scale height ζ with the radius. The vertical profile is then

$$Z(z_6) = \exp [-(|z_6|/\zeta)^\gamma], \quad (3)$$

where

$$\zeta = \zeta_6 r_6^\beta, \quad (4)$$

and ζ_6 is equal to $\zeta(r_6 = 1)$. The vertical structure parameter γ is equal to 1 or 2 for exponential or Gaussian profiles, respectively.

With any specific choice of γ , our scattering distribution has, thus, four free parameters: A_{6S} , ζ_6 , α , and β . In addition, the conversion of a cylindrically symmetric disk model into the observed projected brightness distribution requires the specification of the disk tilt angle, or inclination with respect to the line of sight, i_{tilt} .

To calculate the theoretical coronagraph image correspond-

ing to a given set of model parameters, we set the lower limit of r_6 in equation (2) to a small value. (The possible disk gap affects only the central, presently unobservable regions of the image occulted by the finger. We defer the evaluation of the maximum extent of the clearing until the derivation of the radial profile of the gap in § III.) Assuming that the disk is optically thin in any direction (we verify this assumption in § IVa), with L_v denoting the β Pic luminosity in any of the spectra bands B, V, R, I , the flux entering a unit volume having disk coordinates (r, z) is $L_v[4\pi(r^2 + z^2)]^{-1}$. This flux is scattered according to a phase function $f(\theta)$ depending on the angle θ between the direction from β Pic to the given point and the line of sight, and it is normalized in such a way that its average over all directions equals one. We define the orthogonal coordinate system (x, y, l) with the center in β Pic, oriented such that the x -axis corresponds to the disk midplane on the coronagraph image, the y -axis is the disk z -axis projected onto the sky, and the l -axis is the line of sight. Then, the observed flux per 1 arcsec² of projected disk area is:

$$I(x, y) = \frac{D^2}{4\pi R^2} \int \frac{L_v A_S(x, y, l) f(\theta) dl}{4\pi(r^2 + z^2)} \\ = \frac{\Phi_v D^2}{4\pi} \int \frac{A_S(x, y, l) f(\theta) dl}{r^2 + z^2}, \quad (5)$$

where R is the β Pic–Earth distance, and Φ_v is the known direct flux of β Pic at Earth. The formulae for r, z , and θ as functions of x, y , and l follow from the appropriate coordinate transformation

$$\begin{cases} r = [(l \cos i_{\text{tilt}} + y \sin i_{\text{tilt}})^2 + x^2]^{1/2} \\ z = y \cos i_{\text{tilt}} - l \sin i_{\text{tilt}} \\ \theta = \cos^{-1} [-l/(x^2 + y^2 + l^2)^{1/2}]. \end{cases} \quad (6)$$

To obtain a theoretical image from equations (1)–(6), we have to specify the phase function $f(\theta)$ and the method of blurring the image to account for observational seeing effects. There are many possible choices for $f(\theta)$, $f = 1$ being the simplest one. As the most realistic choice, we modeled $f(\theta)$ after the empirical phase function of the zodiacal light (Leinert *et al.* 1976) originating from midsize particles (10–100 μm) with rough surfaces (Schiffer and Thielheim 1981) and showing essentially no color effects, just like the β Pic disk. The chosen phase function is flat with a small enhancement in the back-scattering direction and a larger one reaching, in our model's geometry, a factor of 2 or 3 in the forward scattering direction. This degree of asymmetry may be expected of both midsize and large grains with a surface which is rough on a scale length exceeding the wavelength of the illuminating light. We checked numerically that Lambert's law of diffuse reflection yields similar final results (unless the tilt angle is extreme, as discussed in § IIb). We account for seeing by convolving the image given by equation (5) with a Gaussian having a 1" full width at half-maximum. Finally, we mask the central circular region of the image to facilitate the comparison with the actually observed parts of the disk seen in Figure 1.

For a given value of i_{tilt} and γ , we estimate the best-fit values of the remaining parameters by a trial-and-error search in parameter space allowing us to find theoretical images that are acceptable according to the following four criteria. First, we require as exact similarity as practical of the obliqueness of the theoretical and observed isophotes, judged by their distance

from the x -axis at 6" divided by the distance at which they intersect the x -axis. Second, we require the midplane brightness of the seeing-blurred theoretical images to reproduce the power index and the normalization of our fit to the coronagraph data given in Figure 2 (*dashed curve*). The first requirement fixes ζ_6 ; the second one fixes α and A_{6S} simultaneously.

The third criterion concerns the radial dependence of the width of the disk image and, hence, constrains the range of the β parameter. We define the equivalent width $w(x)$ of the measured disk image at a projected distance x as

$$\phi(x, 0)w(x) = \int \phi(x, y)dy, \quad (7)$$

where $\phi(x, y)$ is the image surface brightness. Note that only $w(r)$ is directly observable and, because of the projection effect, never directly gives the intrinsic vertical profile $Z(z)$. The third criterion is satisfied when the theoretical equivalent thickness functions are consistent with those measured in a given image. The fourth criterion requires rejecting models that produce curvature of isophotes near the disk midplane and their slope far from the occulting finger differing from the observed ones. We applied this criterion with caution since it is related to features that may change from one coronagraphic image to another. We have found it possible to construct both edge-on and tilted disk models satisfying the above criteria reasonably well and describe, for clarity, these two cases separately in the following paragraphs.

a) Strictly Edge-on Models

A series of images satisfying the first two criteria obtained for different disk thickness parameters β and vertical structure indices $\gamma = 1$ and 2 is given in Figures 3 and 4. All the model parameters used in constructing a given image are overplotted on these figures. Both seeing-free and seeing-blurred images are presented to allow an easy qualitative extrapolation to seeing disk diameters different from 1". In particular, the unblurred images may become helpful in interpretation of extra-atmospheric observations possible with the Hubble Space Telescope, for example.

We compare the observed (*dashed lines*) and theoretical (*solid lines*) equivalent widths as a function of x in Figures 5 and 6. Although there exists a significant discrepancy between the curves corresponding to the NE (*lower dashed line*) and the SW disk extension (*upper dashed curve*) in the particular image analyzed here, we do not see any evidence for a systematic brightness asymmetry of the disk (either up-down or left-right) in a larger set of images. The allowed region between the dashed lines corresponds roughly to the statistical spread of $w(x)$ seen in a number of β Pic images. It is readily seen in Figures 5 and 6 that the isophote obliqueness criterion used to construct a unique model for each pair of γ and β always leads to reasonable equivalent image widths in the vicinity of 6" projected separation. However, the slope of $w(x)$ over the entire x range depends strongly on the parameter β . We conclude from Figure 5 that the only models with Gaussian vertical profiles consistent with the observational $w(x)$ profiles are those with $\beta = 1$ –1.5. Figure 6 shows that models with exponential vertical profiles all have an equivalent width which is somewhat too small at the smallest observable radii, so that smaller values of β are preferred. On the other hand, at large radii, models with $\beta < 1$ are excluded (the $\beta = 1$ model equivalent thickness curve classifies it as marginally acceptable; we

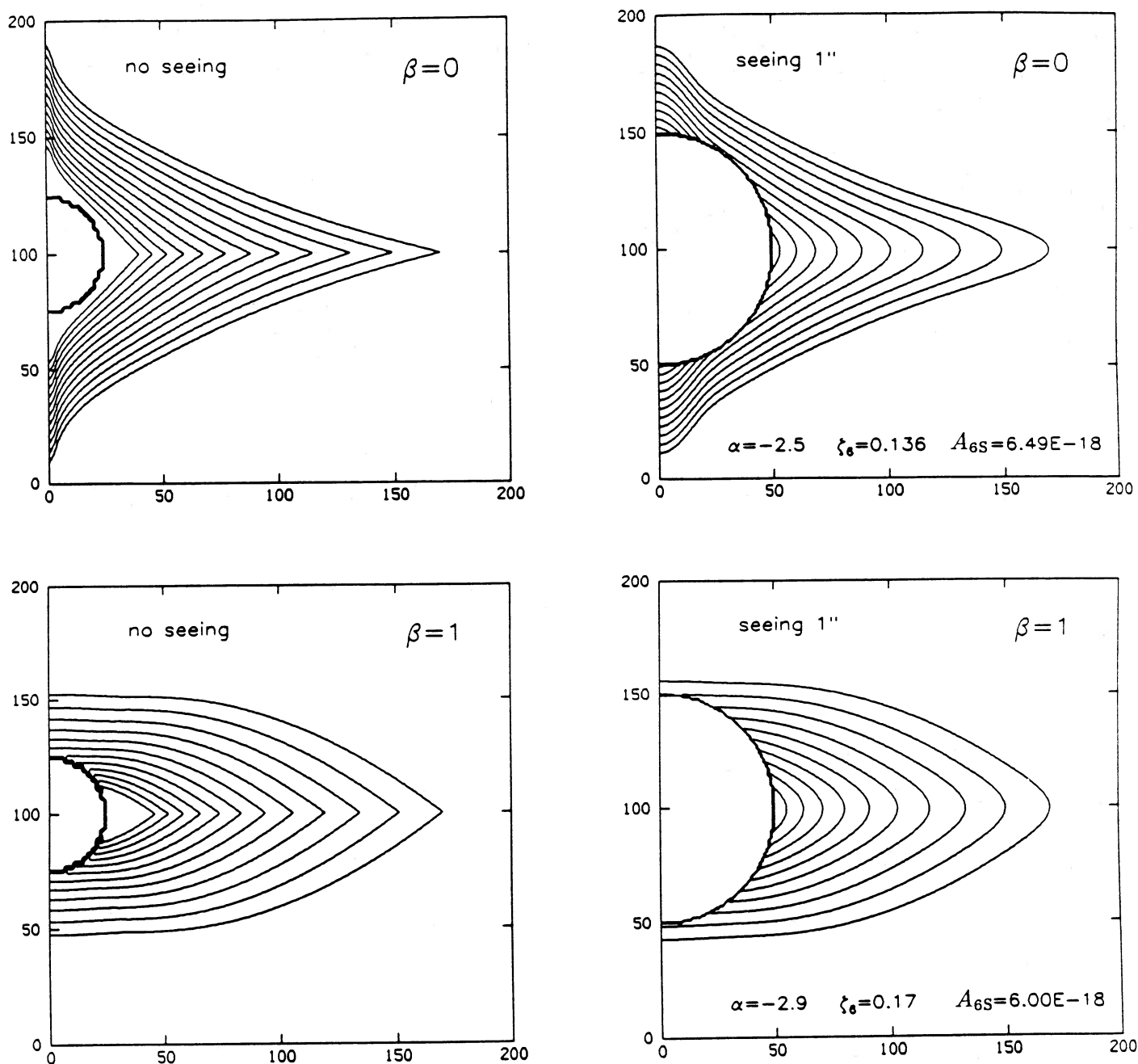


FIG. 3.—Theoretical images of a strictly edge-on β Pic disk, with and without simulated seeing effect, for the models having Gaussian shape ($\gamma = 2$) of the vertical profile of the scattering coefficient. All the model parameters are indicated in the plots. The coordinates are as in Fig. 1. Central circles are masked to simulate the occulting mask/finger of the coronagraph.

retain this model for the sake of comparison with $\gamma = 2$ models, as indicated earlier). Consequently we adopt $\beta = 1.5$ and 1 as compatible with the coronagraphic data for both $\gamma = 1$ and $\gamma = 2$.

Comparison of Figure 3 of PB and Figures 1 and 3 of this paper leads to the conclusion that $\gamma = 2$, $\beta > 1.5$ models should be rejected according to the fourth criterion because of their very small curvature of isophotes near the disk midplane. The same is true of any $\gamma > 2$ distributions. The $\gamma = 1$ models generally produce isophote patterns matching the observed ones better, and so we refrain from rejecting any $\gamma = 1$ model

on the basis of this additional criterion. We have found that $\gamma < 1$ would result in an unacceptable shape for isophotes near the disk midplane. We note here that all the results for edge-on disks proved insensitive to the choice of the scattering phase function in the sense that a Rayleigh scattering function, for example, gives the same $w(x)$ as the zodiacal light phase function within 4%.

In summary, the comparison of midplane brightness, equivalent widths, and isophote patterns of a set of theoretical images with the coronagraph images eliminates all but a limited range of untilted disk models. The first three entries of

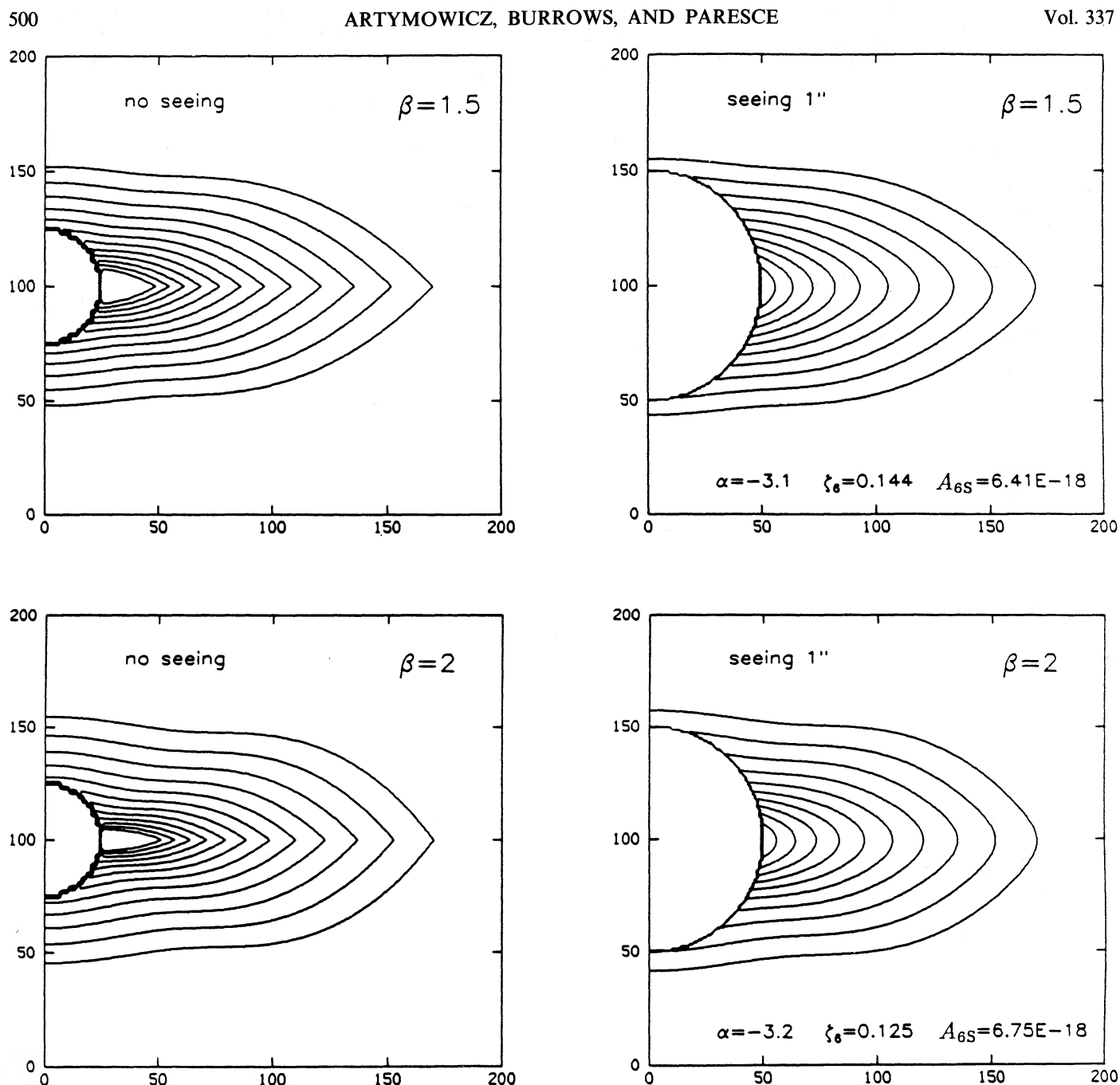


FIG. 3.—Continued

Table 1 give the characteristics of three boundary cases, designated I, II, and III. They form a triangle in the $\gamma - \beta$ plane that contains models best fitting Figure 1. Note that the power-law index of the scattering coefficient is between $\alpha = -2.7$ and $\alpha = -2.9$, in reasonable agreement with Smith and Terrile's value of $\alpha = -3.1$. More importantly, the total amount of scattering area varies with distance as $r^{\alpha+\beta}$, i.e., only with the -1.7 th (model I), -1.9 th (model II), or -1.6 th (model III) power of radius. The full width at half-maximum of the vertical profiles at 100 AU form the star, given in Table 1 as F_6 , is between 25 AU and 38 AU. Denoting by i the absolute value of the inclination of particle position with respect to the ecliptic of the disk (not to be confused with the inclination of particle's

orbit), and average inclination $\langle i \rangle$ is defined by the requirement that one-half of the disk integrated area is contributed by particles having $i < \langle i \rangle$. We find $\langle i \rangle = 6^\circ 7 \pm 0^\circ 3$ in models I and II, and $\langle i \rangle$ growing slowly with distance in model III.

b) Finite Tilt Angle

Using the same four-criteria method, we constructed a series of models with the Gaussian vertical profile and finite i_{tilt} . The first criterion requires that i_{tilt} be less than or equal to 14° , the value at which all the apparent thickness of an infinitely thin ($\zeta_6 = 0$) disk is due to the tilt. We present a series of four models covering the relevant interval of i_{tilt} in Table 1 (models I5–I14). It is readily seen that the thickness parameter ζ_6 , and

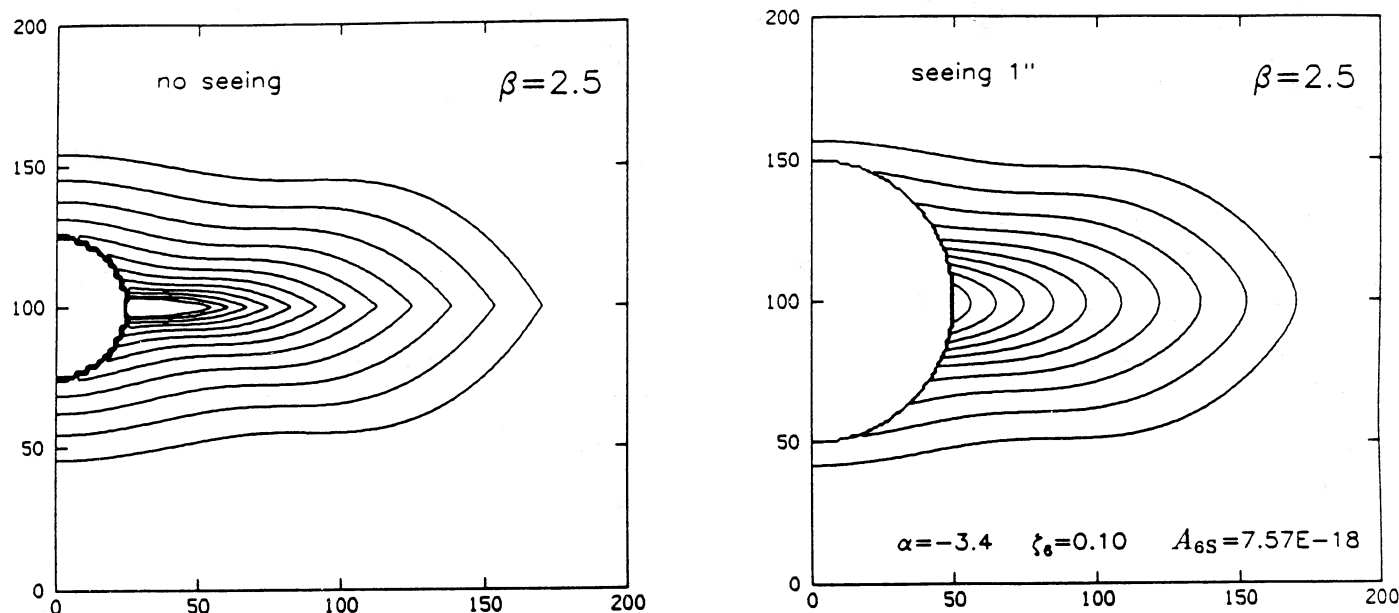


FIG. 3.—Continued

hence also the disk opening angle $\langle i \rangle$, are decreasing with i_{tilt} in a highly nonlinear manner. The scattering efficiency per unit volume in the ecliptic of the disk grows so as to keep the normal to the disk optical thickness roughly constant (decreasing with the growing tilt by less than 15%). Consequently, the structure of the disk shows appreciable departure from the edge-on case only when $i_{\text{tilt}} \gtrsim 10^\circ$. However, an asymmetry in the disk image due to the different efficiency of back-scattering as compared with forward scattering of light by the grains grows roughly proportional to i_{tilt} . Quantitatively, we define the asymmetry S_6 as the average over reliably determined isophotes of the fraction $s = |y_1 - y_2|/(y_1 + y_2)$, where y_1 and y_2 is the absolute value of the y -coordinate of the isophote at the x -coordinate corresponding to a separation of 6'' from the star, above and below the x -axis, respectively. The theoretical values of S_6 given in Table 1 suggest that the asymmetry of the disk image at 6'' from the star should remain small for the whole range of tilt angles, provided the forward-backward asymmetry of the scattering phase function is similar to that of the empirical zodiacal light phase function.

As indicated in § IIa, there is no systematic asymmetry

between the SW and the NE parts of the disk, but a single image, e.g., our Figure 1, can occasionally exhibit as large an asymmetry measure as $S_6 = 0.08 \pm 0.05$, so that we were not able to put tighter bounds on the tilt angle from the asymmetry measurements. The right-hand side panels of Figure 7 present two theoretical images for 10° and 13.5° tilt angle, calculated with a zodiacal light phase function. Two analogous models, but calculated with the phase function corresponding to Lambert's diffusive reflection law, are given in the left-hand side panels of Figure 7. While the former two images are marginally consistent with data in Figure 1, the latter two can obviously be rejected (quantitatively, $S_6 = 0.24$ for the upper-left panel, and $S_6 = 0.4$ for the lower-left panel). Note, also, that the maximum brightness lines (*dashed line* on Fig. 7) on the two sides of the disk would not be parallel in the case of this large asymmetry, an effect not seen in existing disk images. We conclude that tilt angles $i_{\text{tilt}} > 10^\circ$ are possible only if the particles' scattering phase function is more isotropic than Lambert's law [the asymmetry factor of the phase function $g = \int f(\theta) \cos \theta d(\sin \theta) / \int f(\theta) d(\sin \theta)$ should satisfy the relation $|g| < 0.3$]. In other words a very thin disk (like, say, Saturn's rings) with dominant large bodies, to which Lambert's law is applicable, can be ruled out.

In a summary, data in Table 1 predict that a slight tilt of the disk plane with respect to the line of sight does not imply significant changes in the intrinsic model parameters as compared with the strictly edge-on models, a fact that lends some credibility to the assumptions made in the previous models of β Pic.

III. MODELING OF IR FLUX

We relate visible and near-IR coronagraphic images and infrared photometry with the key assumption that the scattering coefficient A_s defined and obtained in the previous paragraph is proportional to the effective area that radiates thermally in every volume element of the disk. We assume the same particles (with fixed optical properties throughout the disk) give rise to both the scattered and thermally reradiated

TABLE 1

PARAMETERS^a OF THE MODELS SATISFYING CORONAGRAPH DATA

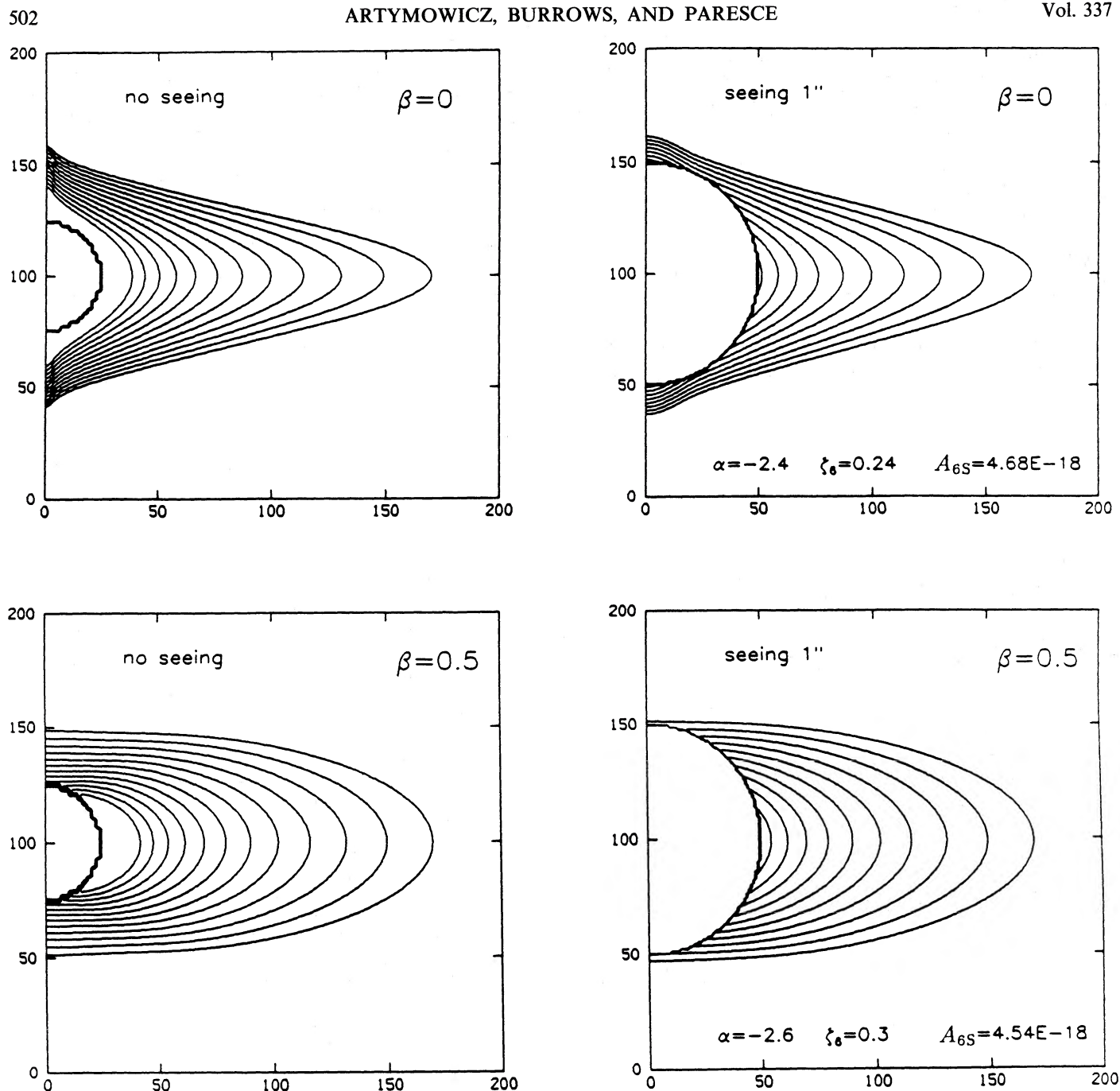
Model	i_{tilt}	α	β	γ	ζ_6	A_{6s}^b (10^{-18} cm^{-1})	F_6^c (AU)	S_6^d
I	0°	-2.7	1	2	0.265	4.66	37.7	0
II	0	-2.9	1	1	0.170	6.00	29.1	0
III	0	-3.1	1.5	1	0.144	6.41	24.6	0
I5	5	-2.7	1	2	0.255	4.85	36.3	0.038
II0	10	-2.75	1	2	0.195	5.83	27.7	0.063
II3	12.5	-2.80	1	2	0.120	8.92	17.1	0.076
II4	13.5	-2.83	1	2	0.025	42.5	3.6	0.088

^a α , β , γ , and ζ_6 are the parameters determining the intrinsic disk geometry defined in eqs. (2)–(4).

^b The scattering coefficient at 6'' from the star, defined in eq. (1).

^c The FWHM of the vertical profile $Z(z)$ at a distance of 98.4 AU (6'') from the star.

^d The theoretical image asymmetry defined in the text.

FIG. 4.—The same as Fig. 3, but for the exponential ($\gamma = 1$) profiles

light. In other words, we shall consider here only a single component particulate disk.

We shall model both large and midsize grains, having absorption and scattering efficiency factors Q_{abs} and Q_{sca} constant throughout the visible and near-IR wavelength interval. In this spectral region let A_r denote the absorption cross section per unit disk volume, and A_s the scattering cross section, as before. The sum $A = A_s + A_r$ is the total projected area of grains that can be imagined to devote Q_{sca} of their face-on area to scattering and Q_{abs} of the area to absorption. The extinction efficiency can be taken as $Q_{\text{sca}} + Q_{\text{abs}} = 1$

because diffraction effects are restricted to almost exactly forward scattering which is presently not observable. Coronagraphic data give $A_s = Q_{\text{sca}} A$, and the *IRAS* data give $A_r = Q_{\text{abs}} A$. These relations evaluated at the (arbitrary) fiducial distance of 6" from β Pic will thus yield both the optical properties (Q_{abs} , Q_{sca}) and the projected area A of the disk.

Consider now the far-IR optical properties of grains. Let $\epsilon(\lambda)$ be the ratio of the far-IR to the visible emissivity of a grain. While large particles have by definition $\epsilon(\lambda) = 1$, midsize grains have an emissivity whose precise form is a function of both size distribution, surface properties, and composition. We adopt,

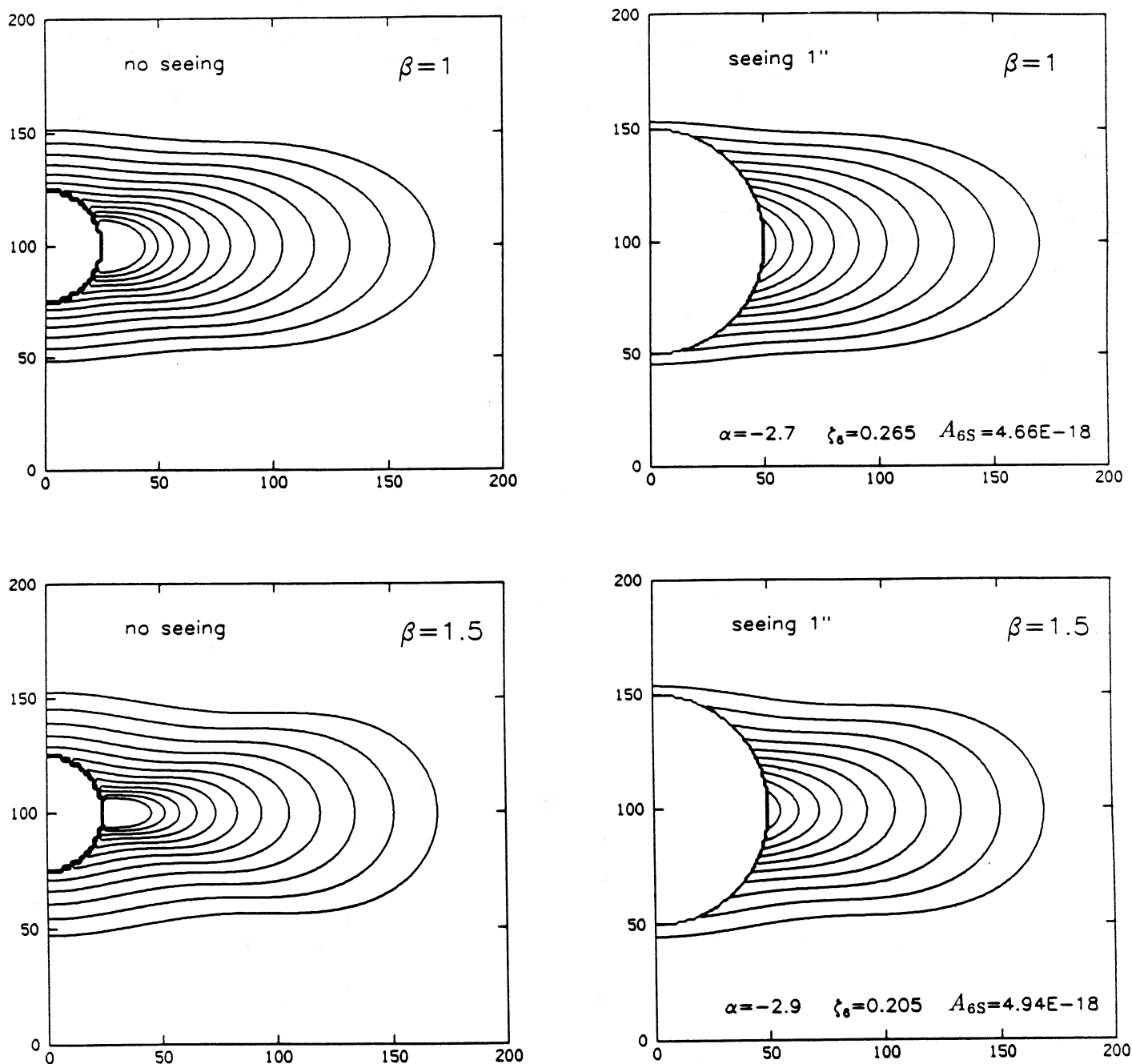


FIG. 4.—Continued

following Gillett (1986), the simple form of $\epsilon(\lambda)$ for midsize grains:

$$\epsilon(\lambda) = \begin{cases} 1 & \text{for } \lambda \lesssim 1 \mu\text{m} \\ \lambda_1/\lambda & \text{for } \lambda \gtrsim 10 \mu\text{m} \end{cases} \quad (8)$$

where λ_1 is a free parameter, and a smooth joining of the two wavelength regions is assumed (but has negligible effect on the results). The above emissivity offers a rough but reasonable first approximation to the silicate grain emissivity (Draine and Lee 1984), in which case λ_1 is of order of the grain size, and Q_{abs} is of order one. It can also reproduce the main features of the midsize (1–10 μm) ice grain emissivity, provided Q_{abs} is con-

siderably less than unity to simulate large albedo in visible light. (Some models were calculated with “real” silicate and ice emissivities, without dramatic effects on results.)

We assume, as before, that the disk is optically thin, i.e., the starlight reaching any particle is not substantially attenuated on its path from the star. We shall see later how this condition is met in our results. The equation for the far-IR flux excess originating from spherical particles is, then,

$$\Phi(\nu) = \epsilon(\nu) R^{-2} \int A_r B_r(T) dV, \quad (9)$$

where $\Phi(\nu)$ is the flux per unit frequency at the frequency ν . The

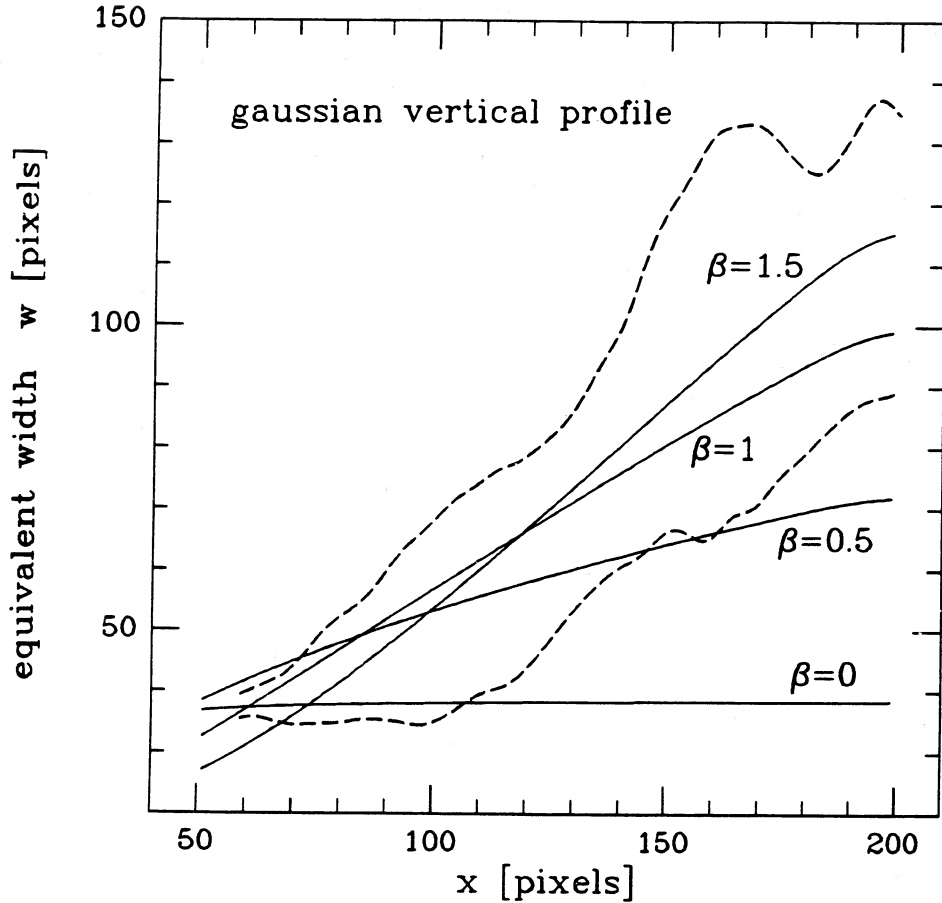


FIG. 5.—Equivalent widths as a function of radial distance from the star for I_c band image from Fig. 1 (dashed lines) and the set of theoretical models with Gaussian vertical profiles (for model parameters, see Fig. 3).

integral is over the disk volume, T is the temperature of the grains, B_ν is the Planck function, and R is the β Pic–Earth distance. It is convenient to replace A_r in equation (9) by the normal (vertical) optical thickness of the IR-radiating area $\tau_r(r) = \int A_r(r, z) dz$. The new form of equation (9) is thus

$$\Phi(\nu) = 2\pi\epsilon(\nu)R^{-2} \int \tau_r(r)B_\nu(T)r dr. \quad (10)$$

In order to calculate T in any disk volume, we assume that the particles are in thermal equilibrium, i.e., they reradiate the total absorbed flux. We are ignoring energy losses by sublimation, or gains by absorption of cosmic rays, as well as possible local inhomogeneities of temperature within the grain. The equation of thermal equilibrium for particles with equal visible and IR emissivity takes the simple form

$$qT^4 = L(16\pi\sigma)^{-1}r^{-2}, \quad (11)$$

where

$$q = \langle \epsilon \rangle_T \langle \epsilon \rangle_{T_\beta}^{-1}. \quad (12)$$

Here, σ is the Stefan-Boltzmann constant, L is the luminosity of β Pic ($L = 6.5 L_\odot$), T_β is the effective temperature of the star ($T_\beta = 8000$ K), r is the radial disk coordinate, and the angle brackets stand for Planck-average with temperature indicated as subscript. For large grains, we obtain $q = 1$ and $T \propto r^{-1/2}$, whereas for midsize grains $q \propto T$ and hence $T \propto r^{-2/5}$.

Taking into account the proportionality of A_s and A_r , and

using equations (3) and (4) and the definition of τ_r , it is straightforward to find $A_r(r, z)$ for every set of parameters γ , ζ_6 , and β , once the normal optical thickness τ_r is determined. An equally spaced radial grid gives an unnecessary spatial resolution in the outer disk and poor resolution in inner regions where the integration kernel in (10) is changing quickly. Instead of using an inconvenient unequally spaced grid, we change the independent variable from r to the grain temperature T and multiply the kernel for numerical convenience by $T^{-\kappa}$, where $\kappa = 3$ for large grains and $\kappa = 4$ for midsize grains. Thus, we have

$$\Phi(\nu) = \epsilon(\nu)R^{-2} \int y(T)B_\nu(T)T^{-\kappa} dT. \quad (13)$$

This equation has the advantage of being more general than either equation (9) or (10). It is independent of the assumptions of thermal equilibrium of grains and low optical thickness of the disk, because $y(T)T^{-\kappa}$ yields the projected area of those blackbody grains which have temperature between T and $T + 1$ K, in any geometrical configuration. The conversion of $y(T)$ to $\tau_r(r)$ by the help of equations (9), (10), and (11) gives $r\tau_r(r) \propto y[T(r)]$.

It is not *a priori* evident that, in the low-temperature range, the solution of equation (13) can have the power-law form $y(T) \sim T^{-2-(\kappa+1)(\alpha+\beta)/2}$, which is equivalent to the scaling $\tau_r \sim r^{\alpha+\beta}$ in the outer disk ($r_6 \gtrsim 1$), required by the coronagraph data combined with the assumption of a single component disk. The fact that *IRAS* bandpasses extend only to

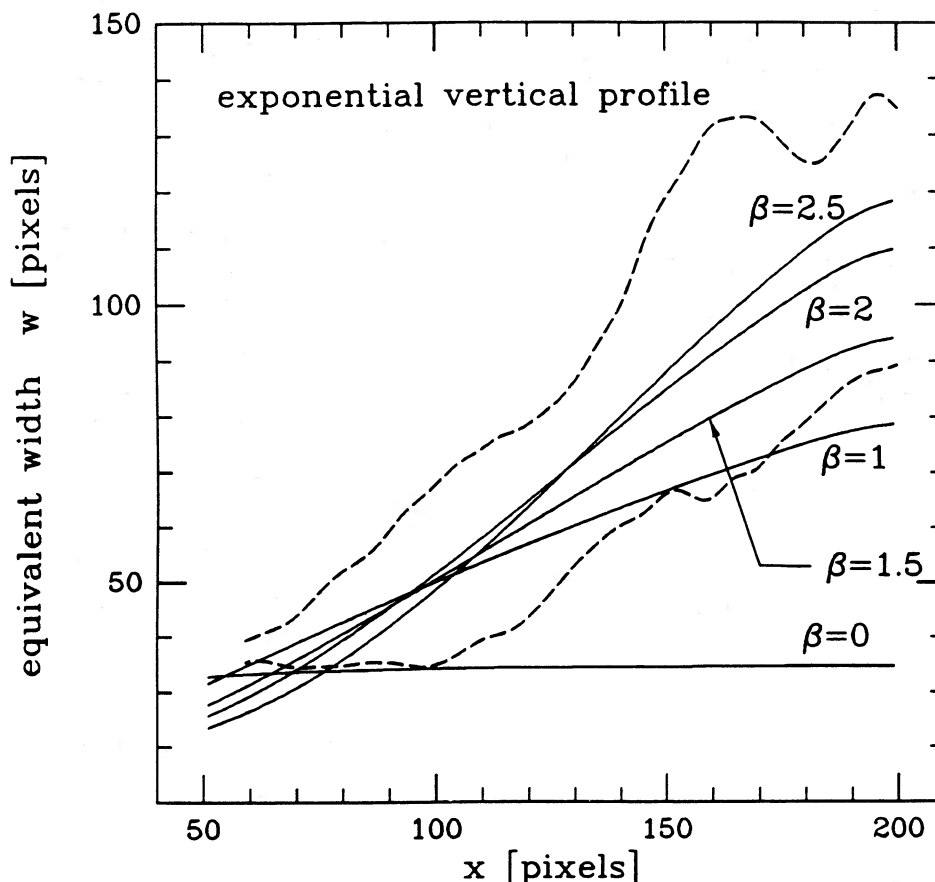


FIG. 6.—The same as Fig. 5, but for the set of models with exponential vertical profiles (model parameters are presented in Fig. 4)

about $100 \mu\text{m}$ implies that $y(T)$ is essentially undetermined for T lower than 30–50 K. Material at less than 30 to 50 K does not contribute significantly to the observed flux, which originates mainly from the hotter disk region with $T = 60$ –120 K. Both in large and midsize particle models considered, this hotter disk region is located inside the occulted part of the coronagraph image. It is therefore reasonable to impose a power-law constraint on the low-temperature tail of the $y(T)$ distribution to obtain a solution that simultaneously satisfies the coronagraph and *IRAS* data.

Before describing the method of solution of equation (13) and the resultant $y(T)$ and $\tau_*(r)$, we briefly present here for completeness our rederivation of *IRAS* excess fluxes. We adopt the total fluxes at wavelengths 12, 25, 60, and $100 \mu\text{m}$ given by Gillett (1986). He evaluates the stellar flux using the *L* band ($3.4 \mu\text{m}$) magnitude and assuming an $L - [12] = 0$ mag difference. We repeated the stellar flux calculations using *M* band ($4.8 \mu\text{m}$) observations of Engels reported by Koornneef (1983), assuming β Pic has a blackbody spectrum with $T = 8000$ K beyond the *L* band. The partition of total *IRAS*-observed flux into stellar and disk contributions is presented in Table 2. The errors of the extrapolated stellar flux result from the 4.7% uncertainty of absolute calibration of the photometric system at the *M* band (Campins *et al.* 1985), and the assumed 0.01 mag error in the β Pic magnitude of $M = 3.51$. The results are practically identical with those of Gillett (1986), except for the $12 \mu\text{m}$ point, where his excess is equal to 1.64 ± 0.1 Jy, and ours 1.59 ± 0.17 Jy.

There are many methods of inverting the integral in equation (13) in such a way that $\tau_*(r)$ has, in the outer disk, the power-law dependence required by the coronagraphic data. We used two: regularized least-squares and maximum entropy (ME). In the first one, finding N unknown $y(T)$ values requires the solution of N regularized normal equations (for review, see Titterton 1985). Practical constraints on N cause the resolution of the method and its speed to be inferior to that of ME. Furthermore, the incorporation of the prior knowledge (e.g., our power-law constraint) into the process of solution is easier in ME. Because we find both methods to give comparable results, we concentrate exclusively on ME in the following.

In ME, the goodness of the fit to the *IRAS* excesses, measured by χ^2 , is a predefined quantity. The temperature distribution should be as smooth as possible, where smoothness is

TABLE 2
STELLAR PHOTOSPHERIC VERSUS DISK CONTRIBUTION TO IR FLUX DENSITY OF β PICTORIS

CONTRIBUTION	BAND (μm)			
	[12]	[25]	[60]	[100]
Total ^{a,b}	2.8 ± 0.1	10.4 ± 0.5	18.8 ± 0.9	11.2 ± 1.0
Star ^b	1.21 ± 0.07	0.29 ± 0.02	0.05 ± 0.0	0.02 ± 0.0
Disk ^b	1.59 ± 0.17	10.1 ± 0.5	18.8 ± 0.9	11.2 ± 1.0

^a *IRAS* measurements taken from Gillett 1986.

^b Flux densities and uncertainties in Jy.

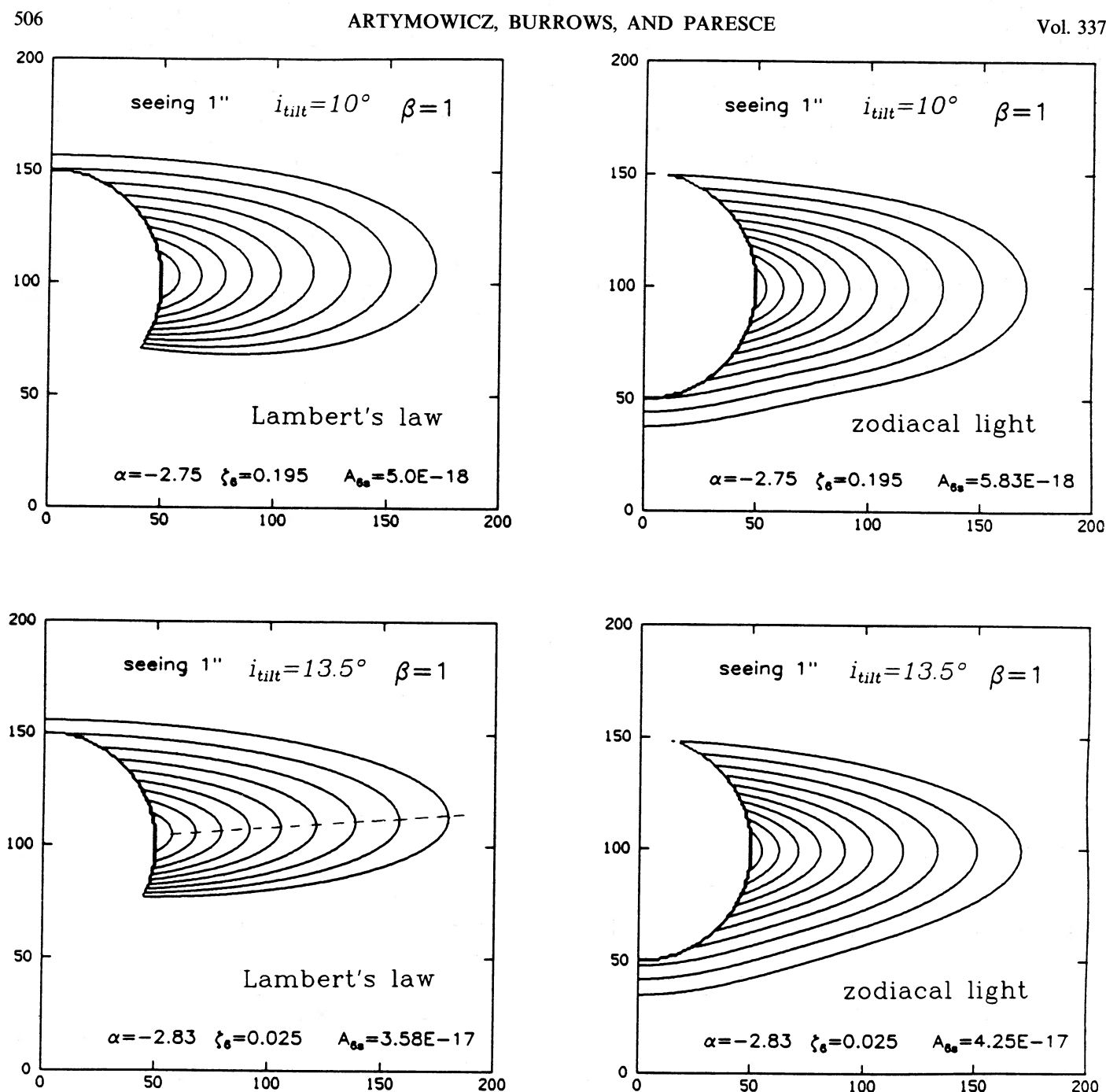


FIG. 7.—Theoretical images of an inclined β Pic disk. The upper panels are for tilt angles $i_{\text{tilt}} = 10^\circ$ (model I10), the lower ones for $i_{\text{tilt}} = 12.5^\circ$ (model I13). All the symbols have the same meaning as in Fig. 3. In the left-hand side panels the Lambert's law of diffuse reflection was used as a scattering phase function. In the right-hand side panels the empiric phase function of the zodiacal light was adopted.

defined by the entropy functional relative to a suitable default or prior image. ME has been applied with success to many image reconstruction problems (for reviews, see Skilling 1984; Narayan and Nityananda 1986). The default image used is a constant joined smoothly to the coronagraphic power law in the region $T \lesssim 45$ K. We used John Skilling's FORTRAN subroutine package implementing the "Cambridge Maximum Entropy Algorithm" (Skilling 1986). When $12 \mu\text{m}$ is the shortest infrared bandpass to be fitted, there exists a divergence in $y(T)$ for large T in excess of 500 K. Addition of the measured zero flux density at $4.8 \mu\text{m}$ with an uncertainty of 0.15 Jy

removes this problem. The value of χ^2 was set to 1.42 per degree of freedom; it will be less than that 75% of the time in any series of experiments with 5 degrees of freedom.

A total of eight ME solutions (four different $\alpha + \beta$ values corresponding to five tilt angles times two choices of κ) were obtained. We discuss the large particle models first ($\kappa = 3$).

Three sets of curves corresponding to ME solutions of equation (13), consistent with the coronagraphic models I–III are plotted in Figure 8. (We retain the symbols I, II, and III for these large-particle models, but it should be noted that, for each set of photometric parameters from Table 1, a model with

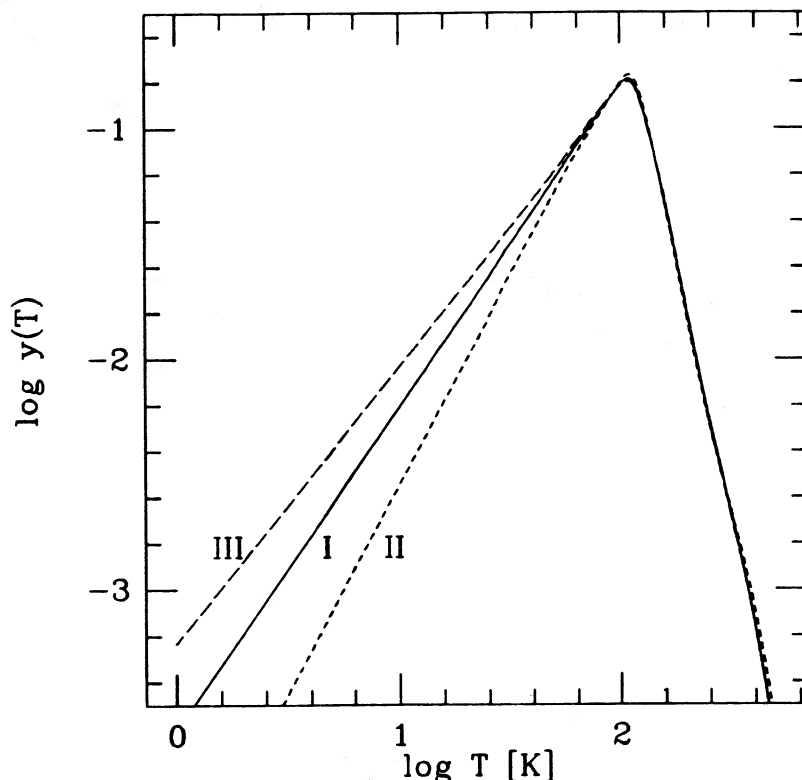


FIG. 8.—Solutions of eq. (13) (units arbitrary) obtained from the maximum entropy fit to the excess IR fluxes, constrained in the region $T \lesssim 45$ K to have the power-law dependences $y(T) \sim T^{1.4}$, $T^{1.9}$, and $T^{1.6}$ required by models I, II, and III, respectively.

midsize particles can also be constructed.) The different slopes imposed in the low-temperature region, as expected, do not lead to appreciable differences in the rest of the profiles, e.g., the maxima roughly coincide. The fit to the *IRAS* points obtained with models I–III is presented in Figure 9. From this figure one can clearly identify the need for additional flux measurements at wavelengths greater than $100 \mu\text{m}$ in order to better model the distribution of cooler, more distant disk material. The remaining photometric models, I5–I14, give results similar to those of model I.

In the case of midsize grains ($\kappa = 4$), the overall shapes of $y(T)$ are similar to those in Figure 8, except for the shift in the location of the maximum toward lower temperatures for the most part due to a different value of κ , but also for the following physical reasons. It can be seen from equation (13) that, for any arbitrary grain composition, the integral problem to be solved is always the same except that the spectrum $\Phi(\nu)$ should be replaced by $\Phi(\nu)/\epsilon(\nu)$. In realistic cases, ϵ increases with ν , and the modified spectrum is shifted toward the lower frequencies, requiring a lower characteristic temperature of the disk as well as a larger spatial scale. The consequence of this is that the large grain models have $T \approx 115$ K at the radius of the maximum $\tau(r)$, while midsize grain models have $T \approx 95$ K at such a radius. However, at the “half-clearing” radius (half the maximum τ) the temperature is $T \approx 160$ K for both grain emissivity prescriptions. Note that, even without the shift in the characteristic temperature, midsize grain models have a larger spatial scale whenever $q < 1$ (see eqs. [11] and [12]). Although the second effect is weaker, it depends on the value of the parameter λ_1 in equation (8) and hence allows us to find the lower bound on λ_1 by requiring that the central gap in the disk

(no grains at high temperatures) be small enough that its presence produces at $r_6 = 1$ less than 20% deviation from the precise power law describing the midplane surface brightness of the photometric model (cf. § II). We discuss these results in the following paragraph.

IV. PROPERTIES OF THE BETA PICTORIS DISK

In this section we describe the results of matching the effective scattering area per unit volume obtained from the coronagraph data with the effective absorbing area per unit volume deduced from the *IRAS* data, in order to calculate grain reflectivity and the structure of the disk. It was shown in § IIa that the photometric models are quite insensitive to the tilt angle i_{tilt} as long as $i_{\text{tilt}} \lesssim 10^\circ$. The same is true for the inversion of the IR flux equation. Therefore, we present first the edge-on models ($i_{\text{tilt}} = 0^\circ$) and then briefly discuss the effects of a finite tilt.

a) Edge-on Models

We begin with the large grain models I–III. The known $y(T)$ distributions yield, with the help of equations (9)–(11), the normal optical thickness due to the absorbing grain area $\tau_a(r)$. The total normal optical thickness of the disk is equal to $\tau(r) = \tau_a(r) + \tau_{6S}(r)$, where τ_{6S} is the normal optical thickness due to the scattering grain area ($\tau_{6S} \propto \tau_a$), normalized with the help of the τ_{6S} values given in Table 1. The absorption efficiency of the grains is equal to $Q_{\text{abs}} = \tau_a(r)/\tau(r)$. Figure 10 presents $\tau_a(r)$ (lowest set of curves) and $\tau(r)$ (upper set of curves) in models I–III for distances less than 120 AU. At larger distances, the optical thickness is decreasing as $r^{\alpha+\beta}$, i.e., as $r^{-1.7}$ in the Gaussian model, and as $r^{-1.9}$ and $r^{-1.6}$ in the exponential models II and III. The overall difference between the three

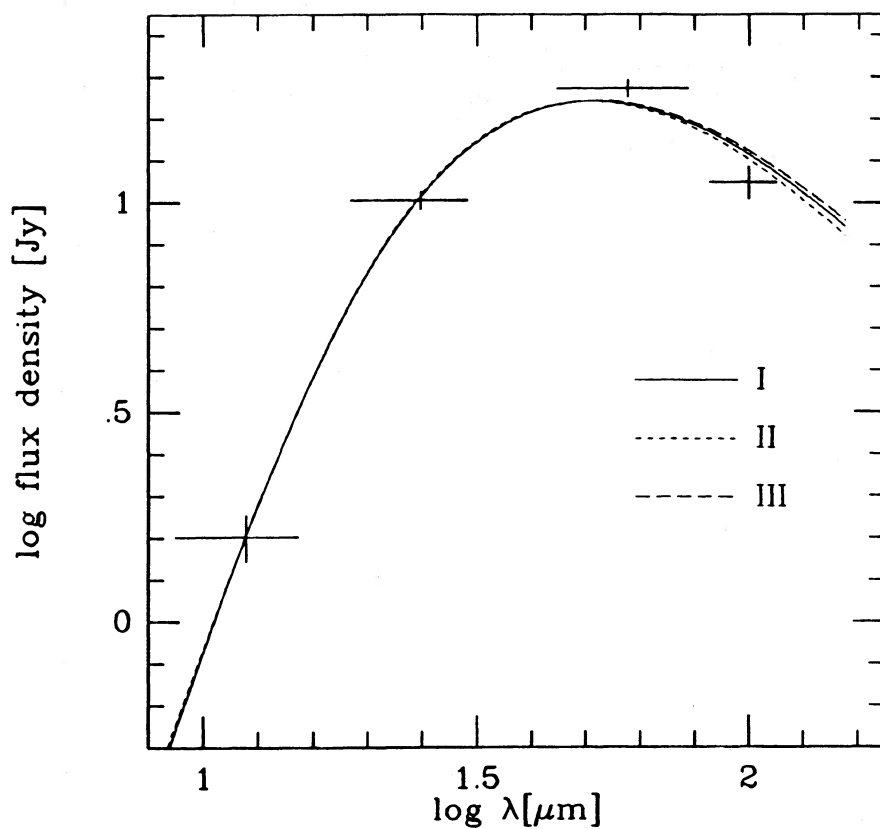


FIG. 9.—Theoretical spectrum in models I–III, obtained from the β Pic excess fluxes (crosses; data from Table 2) by the maximum entropy technique. χ^2 per degree of freedom is equal to 1.42 in each model.

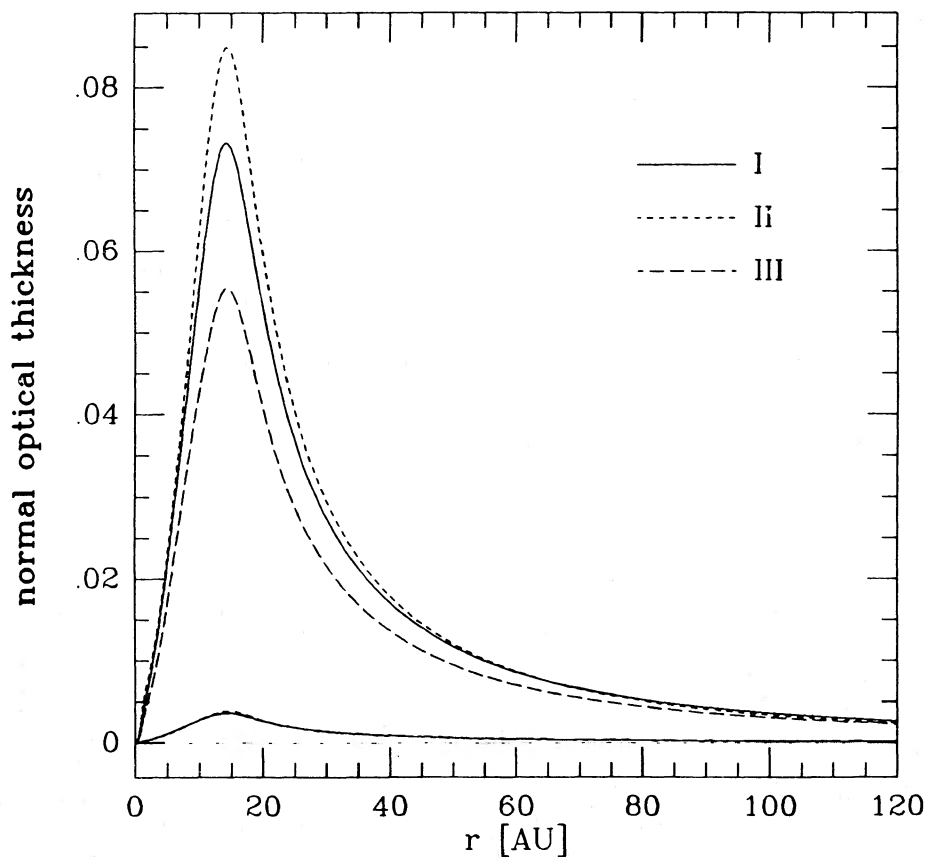


FIG. 10.—Radial dependence of disk's normal optical thickness in large-grain models I–III. Three upper curves represent the total optical thickness $\tau(r)$, the lower ones $\tau_r = Q_{\text{abs}} \tau$, in each model. Only the inner disk regions are presented: at $r \gtrsim 100$ AU each model has $\tau \propto r^{\alpha+\beta}$, where α and β are model parameters given in Table 1.

models is 20% or less, i.e., the normal optical thickness is essentially model-independent. In all three models, the optical thickness obeys the power law down to about 25 AU, peaks at about 15 AU (corresponding to $T \simeq 115$ K), and falls monotonically to zero towards the star.

Table 3 presents τ_6 , i.e., the predicted normal optical thickness at 100 AU, along with the absorption efficiency of grains Q_{abs} . The remarkable feature of all the large-particle models is the large ratio of the scattering to absorbing area of order 15–20, implying very low absorption efficiencies $Q_{\text{abs}} \approx 0.05$ (we discuss this result in § IVd). Table 3 contains also the estimates of the total projected area of particles in the disk A_{tot} and the minimum mass of the disk out to 500 AU from the star, denoted as M_{min} . The total projected area of spherical particles is given by $2\pi \int \tau(r)r dr$, and their mass is numerically equal to their projected area times $4/3 \langle a^3 \rangle / \langle a^2 \rangle$ (using the approximate density of water ice of 1 g cm^{-3}), where the angle brackets denote averaging over the size distribution. For a plausible power-law distribution of the form $dn(a) = a^{-3.5} da$ resulting from collisional break-up (see, e.g., Dohnanyi 1969), $\langle a^3 \rangle / \langle a^2 \rangle$ may become very large (most mass in large bodies). For a single dominant radius of grains a , the mass-to-area ratio is just $4a/3$. The single-size ensemble with $a \approx 1 \mu\text{m}$ yields the minimum mass, which turns out to be comparable to one lunar mass.

Next, we have to check whether the assumption of the transparency of the disk is satisfied. We denote the optical depth calculated from β Pic outward to 500 AU (or infinity, with essentially the same result) in the midplane of the disk by $\tau_{\parallel}$. The midplane extinction cross section per unit radius $d\tau_{\parallel}(r)/dr = A_S(r, z=0)/Q_{\text{sea}}$ is unambiguously defined in the outer disk, but not in the inner disk, where the equivalent thickness of the disk is uncertain (midplane extinction is inversely proportional to the thickness). We checked whether the thickness power law $\zeta \sim r^{\beta}$, given by equation (4), continued into the inner disk regions results in plausible $\tau_{\parallel}$. Model I with the smooth Gaussian vertical profile is characterized by the smallest $\tau_{\parallel}$ (see Table 3), corresponding to 0.22 mag neutral extinction of β Pic light for an exactly edge-on geometry. Model II would show 0.42 mag extinction, and model III (which, in addition to a midplane concentration of matter due to $\gamma = 1$, has $\beta = 1.5$ and, thus, a thickness 3 times smaller in maximum density region than in the $\beta = 1$ case) would have an extinction of 1.06 mag. We see that the extrapolation of the observed thickness behavior from $r_6 \approx 1$ down to $r_6 \approx 0.2$ results in an uncomfortably large extinction only for model III.

In order to be acceptably transparent in this case, the disk at $r \approx 20$ AU must be 1.5–2 times thicker than about 2.5 AU predicted by equation (4). While the midplane extinction of 0.2 mag to 0.4 mag is still consistent with the assumption of disk transparency, it may be sufficient to explain the disputed 0.3–0.8 mag underluminosity of β Pic.

Finally, for the sake of comparison with *IRAS* data in § IVc, we computed theoretical extra-atmospheric images in the $60 \mu\text{m}$ band. Table 3 gives the full width half-maximum angular extents along (X) and perpendicular (Y) to the x -axis (i.e., the maximum brightness line) of the image, expressed in arcseconds, in each model.

Let us now turn to the midsize grain models. A one-parameter family of models can be constructed by varying λ_1 within the interval $\lambda_{1\text{min}} < \lambda_1 < \lambda_{1\text{max}}$. Both limits have been found by trial and error for each set of α , β , and i_{tilt} . The lower limit corresponds to a model with the maximum spatial scale set according to the criterion that no gap be seen in coronagraph image, whereas the upper one corresponds to a $10 \mu\text{m}$ emissivity of grains approaching unity, i.e., $Q_{\text{abs}} \lambda_1 / (10 \mu\text{m}) \rightarrow 1$. We find that $\lambda_{1\text{min}} = 6.5 \mu\text{m}$ and $\lambda_{1\text{max}} \approx 60 \mu\text{m}$. Calculations of all the disk characteristics and the absorption efficiency of the grains proceed in the same way as for the large-grain models. Four models named M7, M15, M30, and M60 (cf. Table 3) calculated with λ_1 equal to $6.5 \mu\text{m}$, $15 \mu\text{m}$, $30 \mu\text{m}$, and $60 \mu\text{m}$, respectively, represent the midsize grain models based on the photometric model I ($\gamma = 2$). We plot the normal optical thickness in these models in Figure 11. The data in Table 3 and Figure 11 demonstrate how the changes in λ_1 involve the spatial rescaling of a model without changing the normalization of τ_r . The value of the product $Q_{\text{abs}} \tau_{\parallel}$ remains constant as well, whenever $\beta = 1$. The radius of maximum τ is given by $r_m = 8'31(\lambda_1/1 \mu\text{m})^{-1/2}$. As a result, the normalization of the total optical thickness, τ , and the total midplane extinction, $\tau_{\parallel}$, increases with decreasing λ_1 as Q_{abs}^{-1} . The rescaling changes the absorption efficiency between $Q_{\text{abs}} = 0.15$ in model M60, and $Q_{\text{abs}} = 0.34$ in model M7, and decreases (with respect to model I) the midplane extinction of the starlight to 0.03–0.07 mag, values much too low for the explanation of a possible underluminosity of β Pic.

b) Tilt Effects

Inspection of Table 1 tells us that the geometrical thickness of the disk decreases rapidly once $i_{\text{tilt}} > 10^\circ$, but that τ_{65} decreases slowly. Therefore, we expect the midplane scattering coefficient and the midplane extinction to increase rapidly and,

TABLE 3
BETA PICTORIS DISK CHARACTERISTICS IN THE EDGE-ON MODELS^a

Model	λ_1 (μm)	τ_6^b (10^{-3})	Q_{abs}	A_{tot}^c (10^{30} cm^2)	M_{min}^c (10^{26} g)	$\tau_{\parallel}$ (mag)	r_m	X	Y
I	...	3.42	0.055	0.75	1.0	0.22	0'91	1'96	0'52
II	...	3.17	0.045	0.68	0.9	0.39	0.91	2.14	0.36
III	...	2.91	0.064	0.66	0.9	0.98	0.91	2.10	0.26
M7	6.5	4.92	0.344	1.34	1.8	0.032	3.26	4.86	0.70
M15	15	4.58	0.295	0.92	1.2	0.036	2.15	3.58	0.62
M30	30	4.22	0.235	0.71	0.9	0.046	1.52	2.81	0.56
M60	60	3.82	0.153	0.67	0.9	0.070	1.07	2.20	0.52

^a Large grain models are based on parameter values from Table 1. Midsize grain models M7–M60 have the photometric parameters of model I, and are parameterized by λ_1 (see eq. [8]).

^b The total normal optical thickness of the disk at $6''$ from the star.

^c Inside the orbital radius of 500 AU.

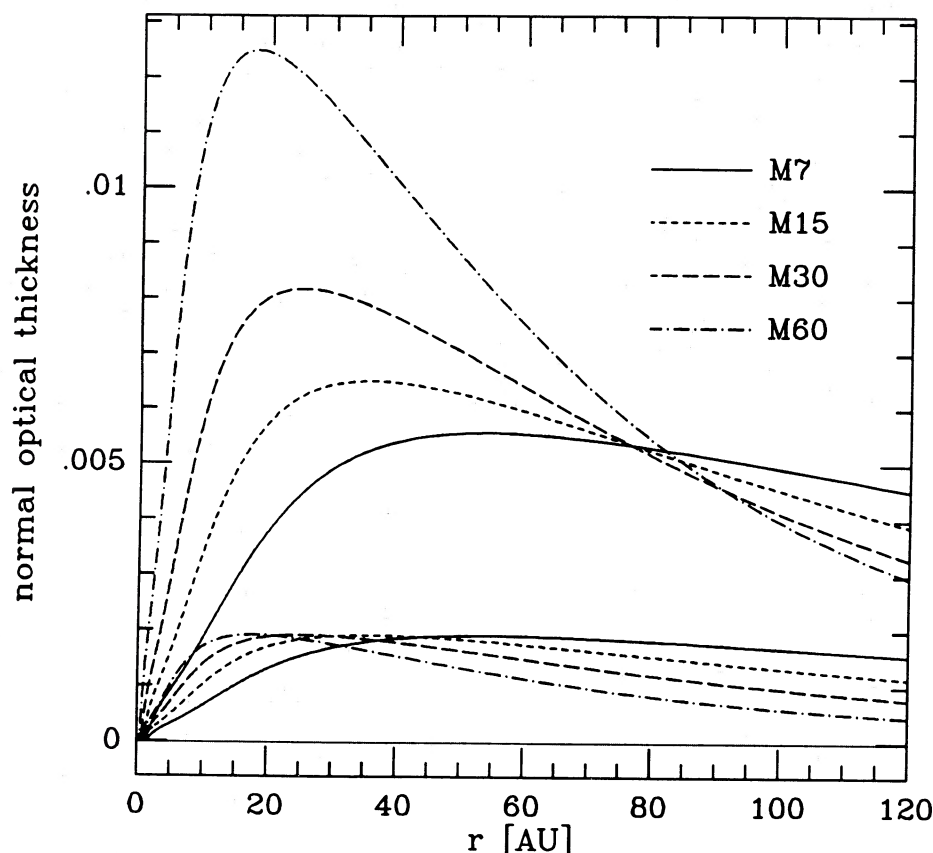


FIG. 11.—Same as Fig. 10, but for midsize grain models M7–M60

in fact, diverge for $i_{\text{tilt}} \rightarrow 14^\circ$, while the absorption efficiency stays roughly constant. We verified this prediction by constructing a grid of models for all the combinations of optical properties and inclinations considered thus far. Indeed, in tilted models, Q_{abs} increases relative to its value in Table 3 by no more than 15%, i.e., is practically tilt-independent. All the model parameters change by less than 10% for $i_{\text{tilt}} < 10^\circ$. Even a larger tilt does not lead to catastrophic changes in the structure of the disk. Tilting a given model from 0° to 12.5° produces twice the initial midplane extinction and so an optically thin disk both in large and midsize grain models. The FWHM (full width at half-maximum) measures of the $60 \mu\text{m}$ image grow by some 20% due to the decreasing projection effect. An extreme tilt of 13.5° involves a tenfold increase in the midplane extinction and renders our large-particle models (but not those with midsize grains) inapplicable. The extent of the $60 \mu\text{m}$ band image is increased by a factor of 1.8 in this case, so that in no model and for no position angle is the FWHM at $60 \mu\text{m}$ greater than $8''.6$.

Although the midplane extinction increases with i_{tilt} , the extinction of β Pic light toward Earth decreases with the growing tilt. For $i_{\text{tilt}} = 5^\circ$, it falls to 0.95; for $i_{\text{tilt}} = 10^\circ$, to 0.53; and finally for $i_{\text{tilt}} > 12.5^\circ$, to less than 0.07 its maximum value. Consequently, only the combination of constant emissivity particles with less than 10° tilt of the disk can produce an appreciable neutral extinction of the direct β Pic light.

c) Comparison with Other Work

In the light of our results, some of the discrepancies between the previous work on β Pic find a reasonable explanation.

Consider first the radial distribution of matter in the visible disk. Smith and Terrile (1984), Diner and Appleby (1986), and apparently Gillett (1986) assumed a constant disk thickness and a strictly edge-on disk position. Our photometric models are consistent with intrinsic thickness of the disk increasing linearly with radius. Assuming $A_S \sim r^{-3.1}$, as in the original photometry by Smith and Terrile, thickness effects scale the normal optical thickness index down to -2.1 to -1.6 (in our models -1.9 to -1.6). These numbers are much closer to the -1 advocated by Gillett. Since he fitted *IRAS* data with the radial distribution of grains in the form of a power law truncated at both inner and outer disk boundaries, this oversimplification may account for the remaining discrepancy. We obtained a good agreement with Smith and Terrile's slope for the matter distribution (-3.1 versus our $\alpha = -2.7$ to -3.1), and a slightly worse one with their index of the midplane brightness (-4.3 versus our -3.6 , both for *I* band images). This may be due partly to the slight flattening of the profile by atmospheric seeing, and partly because the power indices may correspond to different disk regions. Indeed, coronagraphic images obtained recently by two of us (F. P. and C. B.) show a possible steepening of the radial brightness profile from an index roughly equal to -3 at projected distances of 75 to 100 AU from the star, to an index roughly -4 at distances larger than about 150 AU.

As to the *IRAS* spectrum inversion, we feel that an approach involving the nonparametric inversion of the excess flux (eqs. [10] or [13]) is superior to the previous few-parameter models, especially to those which assume a unique density power law to be valid throughout the disk. We estimate that even the

more flexible parametrization of Diner and Appleby results in a χ^2 misfit greater than $3N$, which can be considered a poor fit.

Our large-particle models with zero tilt predict the maximum normal optical thickness of 0.07 ± 0.025 at the orbital radius of 15 AU, a distance corresponding to grain temperature $T \simeq 115$ K. Closer to the star, the normal optical thickness of the disk decreases almost linearly with decreasing radius. The position of the peak compares favorably with model "c" of Diner and Appleby (cf. their Fig. 4), and our $\tau_r(15 \text{ AU}) = 3.7 \times 10^{-3}$ is not much larger than their estimate of 1.5×10^{-3} . The radius of half the peak density occurs at about 3 AU in their model c, and 7.5 AU in our models. Both estimates for the effective radius of "half-clearing" are much less than the 30 AU obtained by Smith and Terrile (1984) from the assumed 0.5 mag midplane extinction. We are thus able to support Diner and Appleby's reservations concerning the existence of a sizable inner clearing region in large particle models of the β Pic disk, as even in midsize grain models the half-clearing occurs at less than about 15 AU.

While the existing coronagraphic and combined coronagraph-*IRAS* models assumed thus far large grains, Gillett (1986) working with *IRAS* data alone concluded that large grains give a poor fit to the IR spectrum. We cannot support this claim and, again, stress that this discrepancy may result from the parametric approach to spectrum inversion. Gillett's second reason for rejecting large particles is that the theoretical size of the 60 μm band image turns out to be about $2''$ in this case, or an order of magnitude difference with respect to the deconvolution of *IRAS* scans that give a FWHM $< 14''$ from the slow in-scan in the N-S direction on the sky, and $22'' \pm 6''$ from the cross-scans at 340° position angle giving, thus, a measure of the disk size at 70° position angle (F. Gillett, private communication). In order to fit the $22''$ size, he needed $\lambda_1 = 3.7 \mu\text{m}$ in his counterpart to our equation 8. The first problem that arises is that $\lambda_1 < 6.5 \mu\text{m}$ in untilted models (and $\lambda_1 < 7.7 \mu\text{m}$ in maximally tilted ones) is incompatible with the coronagraphic observations because of the gap size problem. We find that $\lambda_1 = 0.3 \mu\text{m}$ (rather than the $3.7 \mu\text{m}$ suggested by Gillett 1986) would be needed to obtain $X = 22''$, but the size of the disk would, then, be enormous, the maximum $\tau(r)$ being located at $16''$ or, equivalently, 260 AU from the star. This would contradict the coronagraphic observations unless we abandon the basic assumption of a single-component disk. The problem is further exacerbated by the fact that, on the coronagraph images, the disk is oriented along the 30° position angle, so that the *IRAS* cross-scan gives the information on the extent of the IR image along a line inclined by 40° to our x -axis. Our models predict a FWHM along the line inclined by an angle δ to the x -axis roughly equal to $Y/\sin \delta$ provided $\delta > 20^\circ$. Using the data in Table 3, we predict an in-scan FWHM of $1''$ to $1.4''$ (in agreement with the *IRAS* upper limits) and a cross-scan FWHM of only $0.8''$ to $0.1''$. Even in the absence of the projection effect and the rather extreme case $i_{\text{tilt}} = 13.5^\circ$, none of the models yield either cross-scan or in-scan sizes in excess of $4.4''$. Note also that the disk cannot be expected to be resolved along the line inclined by $\delta = 40^\circ$ to its midplane, while it is not resolved for $\delta = 30^\circ$. It is important to note that the deconvolution of the cross-scans is very difficult and the nonzero value it gives indicates merely a marginal spatial resolution on the 2σ level (D. Backman, private communication). An independent IR size measurement is clearly needed.

After the present work has been completed, D. Backman kindly supplied us with a preliminary account of his ground-

based multiaperture observations of β Pic in N and Q ($10.1 \mu\text{m}$ and $20 \mu\text{m}$) bandpasses. The system shows nonzero flux coming from an annulus between $4''$ and $8''$ apertures. We estimate that the FWHM of the disk seen in these bands cannot be much smaller than $2''$ to produce such an effect. This would favor our models M7-M15, and possibly M30, but seems to exclude model M60, as well as all the large particle models because of their FWHM equal to or less than $1''$ in N and Q bands.

d) Grain Properties

If confirmed, the near-IR extent of the β Pic image restricts the acceptable models of grain emissivity to midsize grains, i.e., inefficient far-IR emitters, with visible absorption efficiency factor between 0.24 and 0.34. Before we discuss the range of possible grain compositions consistent with this and other constraints, we briefly describe a simple consistency check showing that the deduced high reflectivity of the β Pic grains is a result independent of model assumptions. Specifically, if the grains located in the coronagraphically observable disk alone were absorbing as much incoming radiation as they scatter ($Q_{\text{abs}} = 0.5$, or $\tau_{\text{GS}} = \tau_r$), their thermal radiation would equal or exceed the observed fluxes in one or more of the *IRAS* bands. We show this by choosing a set of extremely conservative assumptions as to the density of grains. First, we assume that there are no grains located within 60 AU of the star (current observations probe the disk down to $r \approx 70$ AU, but a gap this big would cause near the coronagraph mask a flattening of the steep brightness power law due to seeing to an extent which has not been observed). Second, we assume $\tau_{\text{GS}} = 3 \times 10^{-3} r_6^{-\eta}$, where η ranges from -1.7 to -3 (the normalization at 100 AU is lower than in original model by Smith and Terrile 1984). We considered large, midsize, and "real silicate" grains of radius $0.75 \mu\text{m}$ to $40 \mu\text{m}$, where "real" refers to Mie theory calculations with dielectric functions of "astronomical silicate" of Draine and Lee (1984). Each type of grain has a different $T(r)$ dependence, and hence produces a different spectrum: small silicate grains give an excessive $25 \mu\text{m}$ band flux, large silicate and midsize grains an excessive $60 \mu\text{m}$ band flux, and large grains give a cool disk producing too much $100 \mu\text{m}$ and $60 \mu\text{m}$ radiation. As a result, an albedo of 0.5 is a lower, model-independent limit. A realistic disk with a smaller gap, consisting of, say, dark carbonaceous material absorbing 90% of the stellar flux would produce an order of magnitude violation of most *IRAS*-measured fluxes.

What is the composition of the grains? On the basis of the downturn in reflectivity in the violet end of the optical reflection spectrum, and a 10%–20% polarization of disk light, it has been proposed that the grains consist of dark, carbonaceous material common in solar system bodies such as meteorites, asteroids and the surface of the comet Halley nucleus (R. Terrile 1988, private communication). As we have just seen, low albedo (typically below 0.1 in the bodies mentioned) is inconsistent with *IRAS* observations. There are at least two ways to resolve the problem, each implying a different composition of grains.

One is to assume the nonvolatile nature of the grains and to look for the material or a combination of materials that is both reflective enough and has the observed spectrophotometric properties. Best candidates with these characteristics are feldspars (plagioclase, orthoclase; see Wagner, Hapke, and Wells 1987 and Crown and Pieters 1987) and olivines (forsterite; cf. Wagner, Hapke, and Wells 1987). The fine-grained samples

(<75 μm grains) have reflectivities from 55% to 80%, and featureless spectra turning down in ultraviolet. Should future (preferably narrow-band) spectrophotometry reveal the exact position of UV absorption edges not corresponding to any known highly reflective mineral, the presence of a small amount of contaminating (carbon-rich?) material may then become necessary.

There are several indications that the nonvolatile grains must not be much larger than a few microns. First, reflectivity is dependent on grain size. Figure 2 of Crown and Pieters (1987) illustrates how a gradual change in plagioclase feldspar grain size from 200–500 μm to about 25 μm increases the visible reflectivity from approximately 0.35 to 0.65 and smooths any spectral features (for other examples, see Yon and Pieters 1987). Second, for any composition and for a radius larger than about 20 μm , grains become effective IR emitters acting as large particles, producing possibly too small a disk extent in the near-IR. The third indication is connected with the observed polarization of disk light. We find with the help of Mie theory applied to “astronomical silicates” (Draine and Lee 1984), and the known spatial distribution of particles, that grains with radii $a = 1.25 \mu\text{m}$ to $a = 1.7 \mu\text{m}$ reproduce the observed polarization well. While slightly smaller grains can exhibit negative sign of polarization, the larger ones ($a > 4 \mu\text{m}$) would produce, in Fresnel reflection limit, a 60%–70% polarization (notice that no depolarization due to multiple scattering is present).

A different interpretation involves a predominantly volatile composition of grains—a “dirty” ice model. The plausibility of icy, rather than refractory, composition for the solid material in disks around main-sequence stars has been demonstrated by Weissman (1984). Indeed, the “astronomical ices” of H_2O , NH_3 , CH_4 , and clathrate hydrates, if sufficiently fine-grained and pure, scatter optical radiation with 90% to virtually 100% efficiency. They can be darkened to 70% reflectivity by a relatively small amount of contaminants (10^{-4} by weight; Clark 1982; Lucey and Clark 1985), which may then determine the observed spectral features. It is not unusual for 1–20 μm “dirty” ice spheres with desirable reflectivity (we tried H_2O and NH_3 plus silicate contamination) to produce of order 5% polarization of the disk light. Larger, observed polarization may require some nonsphericity of grains (which was discussed by Wolstencroft and Walker (1987) in the context of the asymmetry of the disk image). Of considerable importance for the *IRAS* flux inversion is the fact that water ice exhibits a strong, broad absorption maximum around 47 μm , which manifests itself mainly in the 60 μm band, as well as a narrower one near 12 μm . It shows little absorptivity (emissivity) in the 25 μm band (unless grains are larger than 15 μm , but then the disk size problems set in). Ammonia ice, on the other hand, has a number of emissivity peaks, most prominent at 25 μm and 10 μm , but little emissivity at 60 μm . It is therefore extremely hard to reproduce the *IRAS* fluxes using pure ice of a single kind only. However, silicates and/or carbonaceous material needed to lower the visible albedo to about 70% may, at least in principle, fill the emissivity gaps because silicates have along with the 10 μm feature another peak at 20 μm which affects observations in the 25 μm band.

Certainly, many mixtures of the above mentioned contaminated ices can be found to satisfy the present observational constraints (and produce emissivity laws more or less similar to eq. [8]). We are, thus, faced at present with an acute non-uniqueness problem. The situation could vastly be improved

by disk spectrophotometry in the near-IR. For instance, both ammonia and water ices have strong absorption features near 3 μm (2.97 μm and 3.1 μm , correspondingly), which may help decide between the grain compositions proposed here. An attractive feature of icy grain composition is the possible interpretation of the inner disk clearing as the ice sublimation zone. The two most stable ices, water and ammonia ice, sublime quickly at temperatures in excess of about 150 K (see, e.g., Lamy 1974), which coincides well with the disk temperature at half-clearing radii in our models. If this is so, the inner disk boundary may in fact be steeper than that inferred from ME inversion based, by necessity, on a restricted number of data points.

V. CONCLUSIONS

While the outer and inner disk regions can, to a certain extent, be modeled separately as the *IRAS* spectrum does not convey much information on the low temperature solid material around β Pic, and coronagraphs by necessity mask the regions containing hotter material, there is always an overlap region between the hot inner and cool outer disk, corresponding roughly to $T = 30$ –60 K. Matching coronagraph and far-IR data pertaining to this region determines the self-consistent solution for the grain area per unit volume in the whole disk. Neither *IRAS* data alone or coronagraphic images alone can provide us with this self-consistency, or give reliable albedo estimates.

We investigated a grid of theoretical models reproducing coronagraphic images and established relatively tight constraints on the model parameters. In particular, we were able to reject unambiguously a constant characteristic disk thickness (see § II). Similarly, the lack of a strong asymmetry in the coronagraph images implies a nearly edge-on viewing geometry of the disk, i.e., $i_{\text{tilt}} < 10^\circ$, or a flat scattering function (or both). The maximum tilt $i_{\text{tilt}} \simeq 14^\circ$ requires the asymmetry factor of the phase function to be approximately between -0.3 and 0.3 . The disk has a nearly constant opening angle ($\beta \approx 1$) out to at least 250 AU from the star. Half of the scattering material is located closer than about 7° from the disk ecliptic, if the disk is seen edge-on (such viewing geometry seems to be supported by the more recent observations of the outer disk regions by Smith and Terrile 1987). Should the disk be tilted, however, the opening angle decreases proportionally to ζ_6 with a growing disk tilt (cf. Table 1, models I and I5–I14).

We were able to construct self-consistent, single-component models using both large-grain models and midsize grains with the far-IR emissivity inversely proportional to the wavelength. Only the edge-on or slightly inclined models ($i_{\text{tilt}} < 10^\circ$) with large grains are capable of producing in a natural way the (still controversial) neutral extinction of the direct starlight. The structure of edge-on, large-particle models, in particular the surface density decreasing inside 15 AU and the radial density profile in the ecliptic plane close to r^{-3} , compare favorably with the previous models of Diner and Appleby (1986).

However, large-grain models predict small (<1") effective size of the disk in *N* and *Q* near-IR bandpasses, which seems to contradict recent observations (D. Backman, private communication). If these results are correct, midsize grain models M7–M30 (cf. Table 3) are favored. In these models, the normal optical thickness of the disk peaks at a model-dependent radius of 25 to 52 AU, leaving the center of the disk free of dust grains. The predicted extinction of direct starlight in this case is less than 0.03–0.07 mag, unless the inner disk

parts have a significantly lower z/r ratio than the outer, directly observable ones. The integrated area of the grains out to 500 AU in our models is typically equal to 10^{30} cm². With a smallest grain radius of 1 μ m, the minimum mass of the disk turns out to be $M_{\min} = 10^{26}$ g. The actual disk mass can easily be many orders of magnitude larger.

A model-independent lower limit on the visible grain albedo is 0.5; actual albedos in models M7–M30 range from 0.66 to 0.76, classifying the disk material as very bright by solar system standards. No unique determination of particle composition is possible at this stage. We identified two plausible composition/size models, one involving few-micron-sized feldspar or olivine grains, possibly contaminated by a small amount of material having UV absorption band shortward of the U bandpass, and a second involving “dirty” water and/or ammonia ice frost with particle sizes between about 1 and 15 μ m. In both cases, the contaminants may be of carbonaceous nature, as postulated recently by R. J. Terrile (private communication). Further observations aimed at revealing the possible near-IR spectral features of the grains around β Pic would be crucial in elucidating grain composition. If grains are icy, the inner disk clearing may represent the ice sublimation zone.

It is tempting to speculate that the particulate disks around Vega and Fomalhaut, which have been detected in the far-IR *IRAS* measurements (Aumann *et al.* 1984) but which defy detection in visible scattered light (Smith and Terrile 1987), are dominated by grains that are poor scatterers but good absorbers, as compared with shiny β Pic particles.

Accurate spatially resolved far-IR measurements carefully corrected for diffraction/seeing effects are needed to better con-

strain the spatial structure of the disk, and clarify whether it can be modeled, as it was done in this paper, as a single-component system having the same spatial distribution of the sources of the scattered starlight and the thermal radiation. The above assumption would have to be dropped, and an additional, dark species of grains located in a larger disk introduced in case such a measurement would give FWHM of the 60 μ m band image exceeding $\sim 8''$, or $\sim 3''$ in either 10 or 20 μ m bands. Notice that this would necessitate a *larger* albedo of the coronagraphically observed population of grains (and, hence, possibly exclude their silicate composition).

In a recent review Wolstencroft and Walker (1987) first raised the problem of the interpretation of an apparent absence of submicron-sized particles in the disk. A noticeable optical thickness of the disk and an opening angle of order 10° should result in relatively frequent and energetic collisions between the disk particles, naturally producing large quantities of small grains. This, and other problems connected with the disk dynamics and origin, are to be the subject of subsequent papers.

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REFERENCES

- Artymowicz, P. 1988, *Ap. J. (Letters)*, **335**, L79.
 Aumann, H. H. 1985, *Pub. A.S.P.*, **97**, 885.
 Aumann, H. H., *et al.* 1984, *Ap. J. (Letters)*, **278**, L23.
 Buitrago, J., and Mediavilla, E. 1986, *Astr. Ap.*, **162**, 95.
 Campins, H., Rieke, G. H., and Lebofsky, M. J. 1985, *A.J.*, **90**, 896.
 Clark, R. N. 1982, *Icarus*, **49**, 244.
 Crown, D. A., and Pieters, C. M. 1987, *Icarus*, **72**, 492.
 Diner, D. J., and Appleby, J. F. 1986, *Nature*, **322**, 436.
 Dohnanyi, J. 1969, *J. Geophys. Res.*, **74**, 2531.
 Draine, B. T., and Lee, H. M. 1984, *Ap. J.*, **285**, 89.
 Gillett, F. C. 1986, in *Light on Dark Matter*, ed. F. P. Israel (Dordrecht: Reidel), p. 61.
 Gradie, J., Hayashi, J., Zuckerman, B., Epps, H., and Howell, R. 1987, in *Proc. of the 18th Lunar and Planetary Conference* (Part I), p. 351.
 Hobbs, L. M. 1986, *Ap. J.*, **308**, 854.
 Jura, M. 1975, *A.J.*, **80**, 227.
 Kondo, Y., and Bruhweiler, F. C. 1985, *Ap. J. (Letters)*, **291**, L1.
 Koornneef, J. 1983, *Astr. Ap.*, **128**, 84.
 Lamy, P. L. 1974, *Astr. Ap.*, **35**, 197.
 Leinert, C., Link, H., Pitz, E., and Giese, R. H. 1976, *Astr. Ap.*, **47**, 221.
 Lucey, P. G., and Clark, R. N. 1985, in *Ices in the Solar System*, ed. J. Klinger *et al.* (Dordrecht: Reidel), p. 155.
 Narayan, R., and Nityananda, R. 1986, *Ann. Rev. Astr. Ap.*, **24**, 127.
 Paresce, F., and Burrows, C. 1987, *Ap. J. (Letters)*, **319**, L23 (PB).
 Schiffer, R., and Thielheim, K. O. 1981, *Astr. Ap.*, **116**, 1.
 Skilling, J. 1984, *Nature*, **309**, 748.
 ———. 1986, in *Maximum Entropy and Bayesian Methods in Applied Statistics*, ed. J. H. Justice (Cambridge: Cambridge University Press), pp. 156 and 179.
 Smith, B. A., and Terrile, R. J. 1984, *Science*, **226**, 1421.
 ———. 1987, *Bull. AAS*, **19**, 829.
 Titterton, D. M. 1985, *Astr. Ap.*, **144**, 381.
 Vidal-Madjar, A., Hobbs, L. M., Ferlet, R., and Albert, C. E. 1986, *Astr. Ap.*, **167**, 325.
 Wagner, J. K., Hapke, B. W., and Wells, E. N. 1987, *Icarus*, **69**, 14.
 Weissman, P. R. 1984, *Science*, **224**, 987.
 Wolstencroft, R. D., and Walker, H. J. 1987, *Phil. Trans. Royal Soc. London*, in press.
 Yon, S. A., and Pieters, C. M. 1987 in *Proc. of the 18th Lunar and Planetary Conference* (Part I), p. 581.

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