

THE SUNGRAZING COMET GROUP. II

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Received 15 June 1989; revised 7 August 1989

ABSTRACT

Our previous work is updated with the inclusion of the data on the 15 sungrazing comets that appear to have been detected by the SOLWIND and SMM coronagraphs during 1979–1989. These objects are almost certainly Kreutz sungrazers, and the SMM objects belong to what we have defined as Subgroup I. A case is now also made for the association of the SOLWIND objects with this subgroup. Our earlier evolutionary analysis is improved by utilizing orbits defined with respect to the barycenter of the solar system. This shows that there is a prevailing direction to the evolution, with nodal longitudes tending to increase and perihelion distances to decrease with time, and that the survival of 1963 V, for example, was due to the favorable position of Jupiter when this comet came to perihelion. Our earlier conclusion that the Subgroup II members 1882 II and 1965 VIII separated from each other at their previous perihelion passage is clearly confirmed, and it is shown that 1945 VII could also have separated at the same time. A corresponding genetic relationship is now shown to be very probable for Subgroup I members 1843 I and 1880 I, and it is suggested that 1963 V, which has a revolution period that is more than twice as long as this pair, separated from the pair's immediate progenitor a revolution earlier; if this is the case, the previous perihelion passage of the combined 1843 I/1880 I object must have occurred in late 1487 or (less probably) in mid-1463, but there are no recorded observations of comets at those times. With the exception of comet 1970 VI, which is now defined to be the single member of Subgroup IIa, it thus appears that all the observed Kreutz sungrazers originated in one or the other of two comets that must have appeared around the year 1100 (one of them presumably being the comet of 1106). Although the earlier study suggested that the separation of the progenitors of the two subgroups took place as long as 10–20 revolutions ago, a more rapid evolution seems likely, and a moderately plausible case can be made for the genesis of the whole system, including Subgroup IIa, in the Aristotle–Ephorus comet of –371.

I. INTRODUCTION

One of the most curious contributions to cometary astronomy during the past ten years has been the apparent detection by the spaceborne SOLWIND and Solar Maximum Mission (SMM) coronagraphs of as many as 15 comets (as of mid-1989) in the vicinity of and rapidly approaching the Sun. Apart from papers by Michels *et al.* (1982) and Sekanina (1982) on the first SOLWIND comet, and by Sheeley *et al.* (1982) on the second and third, publication of information on these objects in the technical literature has been restricted to the *IAU Circulars*. In each case this publication included a brief note by the present writer to the effect that the comet concerned was probably a member of the Kreutz sungrazing group, which means that the perihelion distance is less than about 0.02 AU, the revolution period is a few centuries, and the perihelion longitude and latitude are confined to the region $L = 282^\circ 0' \pm 0^\circ 4'$, $B = +35^\circ 2' \pm 0^\circ 2'$ (equinox 1950.0, which is used for all the results in this paper). None of the 15 seems to have survived its perihelion passage, and except for the possible introduction of faint emissions of Si II and Ni II into coronagraph spectrophotographic records obtained at Lomnický štít 10 hr after the perihelion passage of SOLWIND 1 (Chochol *et al.* 1983), none of them was detected from the ground.

The SOLWIND and SMM discoveries would more than double the number of Kreutz sungrazers available at the time of the writer's earlier investigation of the sungrazers (Marsden 1967, hereafter referred to as Paper I), even when the most uncertain earlier members are included. Ever since the breakup of the most spectacular member, 1882 II, into individual fragments having revolution periods $P = 6.7, 7.7, 8.8$, and 9.6 centuries (Kreutz 1891), it has been logical to

assume that the observed Kreutz sungrazers were the result of successive fragmentation of earlier members as they passed perihelion. In Paper I it was suggested that the distribution of the orbits of the Kreutz sungrazers required progressive fragmentation over as many as 10–20 revolutions about the Sun, although the apparent existence of two distinct subgroups of the comets implied that only two of the early fragments had consistently survived. We proved beyond all reasonable doubt that comets 1882 II and 1965 VIII, which belong to Subgroup II, must have separated from each other at their immediately previous perihelion passage, presumably around the early part of the 12th century (and possibly observed as the comet of 1106). The relationship among the members of Subgroup I, which have significantly smaller perihelion distances and lines of nodes some 20° ahead of the Subgroup II comets, was less clear, but we concluded that these objects must all have been produced over at most one revolution more.

In an oral presentation (Marsden 1966) of Paper I, the writer showed a slide listing the probable and possible Kreutz sungrazers and suggesting that the next one would appear in 1970. This "prediction" was, of course, simply made from the observation that comets 1882 II and 1965 VIII had been preceded by members 1880 I and 1963 V and the former followed by 1887 I (with 1945 VII then the only such comet observed between 1887 and 1963). Although 83 years just happens to be the duration of seven revolutions of Jupiter and the Sun about their center of mass, there is no entirely logical reason why this cycle should govern the actual appearances of the comets (especially when different subgroups are involved), and the prediction was certainly not intended to be taken seriously. Nevertheless, the next Kreutz

sungrazer—the only one since 1965 to have been observed from the ground—was comet 1970 VII! Tempting though it may be to speculate on the odd temporal distribution, the SOLWIND and SMM discoveries suggest that members may in fact be coming back to the Sun more or less continuously.

At the time Paper I was written it was usual to describe cometary splitting in terms of velocity of separation. That was before the modern spate of studies on the effects of non-gravitational forces on cometary motions. Such studies, considered in terms of the Whipple (1950) icy-conglomerate model, led to an appreciation that all comets should experience, to a greater or lesser extent, a radial acceleration outward from the Sun. In a brilliant follow-up to this work, Sekanina (1977) pointed out that it was therefore reasonable to describe cometary splitting in terms of a differential radial acceleration (and an initial separation velocity that is usually essentially zero). In the case of 1882 II, and also for the two components of 1965 VIII, he found differential accelerations on the order of 10^{-5} of the solar gravitational attraction and separation times within a couple of hours of perihelion passage. Since a large nongravitational acceleration implies a large relative mass loss, it follows that the most long-lived and substantial nucleus of a comet that has split will tend to be that with the shortest revolution period. Such is undoubtedly the case with 1965 VIII, the nuclei of which have calculated values of $P = 8.8$ and 10.6 centuries, but indications are that comet 1882 II is a partial exception to the rule (which has otherwise been demonstrated for some 20 comets), its principal nucleus in all probability being the one with $P = 7.7$ centuries.

The other relevant development since Paper I is Sekanina's (1978, 1984) attempt to determine the orbit of comet 1887 I. This comet is unique in that, although it was widely observed in the southern hemisphere from 10 to 20 days past perihelion, it had nothing that could be interpreted as a head. By assuming the head to be on the sunward extension of the great circle through the tail axis and forcing the Kreutz values of L and B , Sekanina derived a parabolic orbit with perihelion distance 0.004 83 AU and remarked that the tail was undoubtedly a synchrone of material released upon the demise of the head around perihelion passage; the solar radius is 0.004 65 AU, and the smallest perihelion distance of a comet known to have survived is 0.005 06 AU (in the case of 1963 V).

II. THE SOLWIND AND SMM COMETS

The basic information provided for the SOLWIND comets consisted of a separation from the center of the Sun and a position angle reckoned from the solar north pole. The conversion to 1950.0 right ascensions α and declinations δ was made by the author, and the positions collected in Table I have in some instances been slightly revised from what was published in the *IAU Circulars*. The data for the SMM comets were provided directly in the form of 1950.0 right ascensions and declinations, and these are simply reprinted for convenience in Table II, which also includes the two SMM observations of the comet SOLWIND 5.

The three brightest of the space-coronagraph comets seem to have been SOLWIND 1, SMM 5, and SMM 7, all of which became brighter than magnitude -4 . SOLWIND 6 was somewhat fainter, and SOLWIND 2, SMM 2, SMM 4, and SMM 9 fainter still. At magnitude $+1$, SMM 6 was perhaps the faintest of the 15 objects. All must have brightened up

TABLE I. SOLWIND observations and residuals.

UT	α	δ	$\Delta\alpha$	$\Delta\delta$
1979 XI (SOLWIND 1) <i>IAUC</i> 3640				
Aug. 30.789	10 ^h 26.41	+08° 44.8'	-1.1	-0.8
30.796	10 26.75	+08 45.2	+0.2	-1.2
30.802	10 27.18	+08 47.5	(+3.4	+0.3)
30.809	10 27.31	+08 48.3	+1.6	+0.2
30.816	10 27.39	+08 49.5	-1.1	+0.5
30.856	10 29.04	+08 54.6	-0.1	0.0
30.867	10 29.64	+08 56.7	+1.7	+0.4
30.885	10 30.25	+09 00.6	-1.4	+1.4
1981 I (SOLWIND 2) <i>IAUC</i> 3718				
Jan. 26.860	20 43.77	-19 20.6	+0.9	+0.1
26.867	20 43.59	-19 19.0	+0.5	+0.2
26.874	20 43.59	-19 18.3	(+2.7	-0.6)
26.881	20 43.42	-19 16.2	(+2.5	0.0)
26.888	20 43.11	-19 14.0	+0.3	+0.7
26.927	20 42.04	-19 05.9	-1.5	+0.1
26.934	20 41.92	-19 04.7	-0.6	-0.3
26.941	20 41.92	-19 03.6	(+2.0	-0.8)
26.947	20 41.68	-19 01.3	+0.9	+0.1
26.954	20 41.43	-18 59.7	+0.1	0.0
26.994	20 40.13	-18 49.4	-1.2	+0.5
27.001	20 39.83	-18 48.2	(-2.2	-0.1)
27.008	20 39.81	-18 47.5	+1.0	-1.2
27.014	20 39.56	-18 45.7	+0.5	-0.9
27.021	20 39.21	-18 42.9	-0.8	0.0
1981 XIII (SOLWIND 3) <i>IAUC</i> 3719				
July 19.999	07 45.48	+19 38.1	-0.8	-0.1
20.030	07 46.12	+19 44.7	-0.4	+0.9
20.069	07 47.02	+19 50.4	+0.9	-0.7
20.096	07 47.59	+19 56.6	+0.5	+0.2
20.135	07 48.51	+20 04.6	+0.4	+0.2
20.162	07 49.13	+20 09.8	-0.6	-0.5
1981 XXI (SOLWIND 4) <i>IAUC</i> 4230				
Nov. 3.999	14 25.71	-16 58.6	0.0	-3.4
4.038	14 26.54	-16 51.0	-0.2	-0.4
4.105	14 28.10	-16 42.1	+1.2	+0.3
4.171	14 29.56	-16 33.7	+0.9	-0.1
4.238	14 31.21	-16 22.9	+2.5	+1.1
4.304	14 32.45	-16 11.0	-2.1	+2.6
4.371	14 33.99	-16 00.0	-3.6	+1.5
1983 XX (SOLWIND 6) <i>IAUC</i> 4229				
Sept. 24.863	11 54.42	-00 49.9	-2.0	-1.2
24.870	11 54.76	-00 49.1	+0.1	-0.9
24.878	11 54.97	-00 46.8	-0.3	+0.9
24.885	11 55.32	-00 47.1	+1.9	+0.2
24.892	11 55.46	-00 47.0	+0.8	-0.2
25.062	12 00.92	-00 31.7	-0.6	+1.9
1984 XII (SOLWIND 5) <i>IAUC</i> 4230				
July 28.302	08 21.83	+18 30.4	-0.1	+0.5
28.309	08 21.95	+18 31.5	-1.4	+0.1
28.316	08 22.18	+18 33.0	-1.2	+0.1
28.324	08 22.18	+18 34.1	(-4.8	-0.5)
28.331	08 22.72	+18 37.9	-0.2	+1.8
28.368	08 24.03	+18 44.0	+1.1	-0.1
28.375	08 24.26	+18 45.5	+1.0	-0.2
28.383	08 24.50	+18 47.2	+0.4	-0.3
28.390	08 24.73	+18 48.9	+0.2	-0.2
28.397	08 25.08	+18 50.3	+1.6	-0.4
28.435	08 26.52	+18 58.8	+1.7	-0.6
28.443	08 26.55	+18 59.1	-2.4	-2.1

The references are to the discovery announcements on the *IAU Circulars*. The (O-C) residuals are from the orbits in the upper half of Table III.

TABLE II. SMM observations and residuals.

	UT	α	δ	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha'$	$\Delta\delta'$
1984 XII			(SOLWIND 5)			IAUC 4230	
July	28.373	08 ^h 24 ^m .04	+18°50'.4	(-1.3	+5.2)	-0.8	+0.4
	28.439	08 26.88	+19 02.4	(+4.7	+2.1)	+0.5	-0.3
1987 XXII			(SMM 1)			IAUC 4621	
Oct.	5.909	12 38.60	-04 57.0	-0.1	+0.2	+1.4	-4.6
	5.974	12 40.92	-04 50.4	+0.1	-0.3	+0.8	-4.2
1987 XXV			(SMM 2)			IAUC 4621	
Oct.	17.814	13 22.68	-09 38.4	-0.2	+0.3	+2.5	-4.7
	17.879	13 24.63	-09 31.2	+0.2	-0.4	+1.0	-5.2
1988l			(SMM 3)			IAUC 4648	
June	27.640	06 19.80	+22 46.2	+0.5	-0.9	-0.6	-0.3
	27.651	06 19.96	+22 49.8	+0.1	-0.1	-1.0	+0.4
	27.662	06 20.04	+22 52.8	(-2.0	-0.2)	-3.0	+0.3
	27.693	06 20.80	+23 02.4	-0.2	+0.9	-1.1	+1.4
	27.705	06 21.00	+23 06.6	(-1.3	+1.6)	-2.2	+2.1
	27.717	06 21.36	+23 09.0	-0.6	+0.4	-1.4	+0.9
	27.722	06 21.56	+23 13.2	(-0.1	+2.9)	-0.9	+3.3
1988m			(SMM 4)			IAUC 4660	
Aug.	21.700	09 56.92	+12 01.8	-0.1	+0.3	+0.3	-0.3
	21.736	09 58.48	+12 05.4	+0.2	+0.3	+0.4	-0.2
	21.753	09 59.28	+12 06.0	-0.1	-0.8	-0.3	-1.2
1988n			(SMM 5)			IAUC 4668	
Oct.	11.884	13 02.76	-07 31.2	-0.1	+0.5	-0.2	+2.2
	11.901	13 03.32	-07 30.0	+0.1	+0.1	-0.2	+1.8
	11.907	13 03.44	-07 30.0	-0.8	-0.4	-1.2	+1.2
	11.919	13 03.88	-07 28.2	+0.2	+0.3	-0.2	+1.8
	11.955	13 05.12	-07 25.2	+0.5	-0.3	0.0	+1.0
	11.972	13 05.68	-07 23.4	-0.1	-0.4	-0.7	+0.7
1988q			(SMM 7)			IAUC 4692	
Oct.	24.683	13 52.16	-12 21.6	-0.1	+0.2	0.0	0.0
	24.749	13 54.08	-12 12.6	+0.1	-0.2	+0.2	-0.5
1988p			(SMM 6)			IAUC 4684	
Nov.	18.177	15 33.16	-19 52.8	-0.6	0.0	+2.5	+0.2
	18.249	15 34.36	-19 36.6	+0.1	+0.4	+2.3	-1.6
	18.260	15 34.56	-19 34.8	+1.6	-0.5	+2.8	-2.9
1989m			(SMM 8)			IAUC 4793	
June	2.403	04 36.36	+21 22.2	-0.3	+0.6	-5.7	0.0
	2.415	04 36.36	+21 24.6	-0.5	0.0	-5.7	-0.6
	2.421	04 36.36	+21 25.8	-0.6	-0.3	-5.8	-0.8
	2.468	04 36.56	+21 39.0	+0.5	-0.1	-4.1	-0.7
	2.479	04 36.60	+21 42.0	+0.3	-0.5	-4.1	-1.1
	2.485	04 36.64	+21 44.4	+0.5	+0.2	-4.0	-0.4
1989q			(SMM 9)			IAUC 4815	
July	8.644	07 03.92	+22 08.4	+0.6	-0.3	+1.0	+1.0
	8.656	07 04.16	+22 11.4	0.0	0.0	+0.4	+1.1
	8.692	07 05.04	+22 19.2	-1.2	-0.7	-1.6	-0.1
	8.703	07 05.52	+22 23.4	+0.3	+0.5	-0.3	+0.9
	8.709	07 05.68	+22 25.2	-0.1	+0.8	-0.8	+1.2
	8.720	07 06.12	+22 27.0	+0.3	-0.3	-0.8	-0.1

The references are to the discovery announcements on the *IAU Circulars*. The first set of (O-C) residuals is from the orbits in the upper half of Table III, and the second set is from the lower half.

tremendously as they approached the Sun, and it seems that it would have been quite impossible to detect them in a dark sky beforehand. The upper half of Table III shows some new attempts to determine parabolic orbits for these comets on the assumption of Kreutz L and B . Least-squares fits have been made only for the time of perihelion passage T (in UT and used here also to identify the comets), the perihelion distance q (in AU), and the argument of perihelion ω , although the resulting values for the longitude of the ascending node Ω and the inclination i are also shown. The $(O - C)$ residuals $\Delta\alpha$, $\Delta\delta$ for these solutions are given in Tables I and II, the figures for the omitted observations being shown in parentheses.

It is clear enough that the observations can be adequately satisfied by Kreutz-type orbits, but some of the results, notably for SOLWIND 4 and 5 (the latter from the SOLWIND observations only), are particularly uncertain. From the values of the angular elements one gets the impression that the first three SOLWIND comets belong to Subgroup II, although q for SOLWIND 1 is too small. The poor quality of the SOLWIND 1 observations led Sekanina (1982) to conclude that this object's perihelion distance might be as small as $0.35R_{\odot}$; however, as pointed out by Weissman (1983), to have q as small as this makes little sense, for there is no way in which the planetary perturbations would have permitted the comet to avoid striking the sun at its *previous* perihelion

passage! The orbit solutions for the SMM comets are more consistent and suggest that these are all members of Subgroup I. It is disturbing that the SOLWIND and SMM observations of SOLWIND 5 are not particularly consistent with each other, a point to which we shall return later.

Some readers may be worried by the assumption that the SOLWIND and SMM objects are Kreutz sungrazers. In this connection we make reference to Figs. 1(a) and 1(b), which show the tracks in the sky of objects moving in the orbit of the Subgroup I comet 1963 V from 6 to 1 hr before perihelion at 10-day intervals throughout the year. Figures 2(a) and 2(b) show the corresponding tracks for the Subgroup II comet 1965 VIII. The calculated tracks for the first 13 coronagraph objects are shown in Figs. 3(a)–3(c), the heavy bars indicating the time spans and positions actually covered by the observations. As expected, none of these objects was observed to approach the Sun from the north, and agreement of Figs. (3) with Figs. (1) or (2) for the appropriate dates of the year is really very good. The fact that all 15 objects can be made compatible with Kreutz sungrazers rather strongly suggests that they all *are* Kreutz sungrazers.

The SMM objects are interesting also because of the rather short intervals between some of their appearances. SMM 2 followed SMM 1 and SMM 7 followed SMM 5 by only 12–13 days. The orbits computed for the members of each of these pairs are rather similar. More to the point, the orbit of SMM 1 (with a change in T , of course) satisfies the two observations of SMM 2 with residuals $+0.4$, $+0.8$ and -0.4 , -0.5 , and the SMM 7 observations agree with the SMM 5 orbit to $+0.4$, -1.4 and $+0.8$, -1.3 . It is clear that there must be a particularly close relationship between SMM 1 and 2 and between SMM 5 and 7 and probably among all nine SMM comets.

If two fragments of a sungrazer are only 12 days apart typically eight centuries after breakup at the previous perihelion passage, the relative radial acceleration at splitting would be only about 4×10^{-10} of the solar gravitational attraction, or 2 pm/s². The resulting separation velocity could therefore not exceed the mutual escape velocity for several years. Fragments separating at very small differential accelerations must either remain in physical contact or must orbit around each other, becoming independent comets much later. By curious chance, the discoveries of SMM 1 and 2 closely coincided with those of comets 1988e and 1988g, the first definite case of a related pair of long-period comets that were *not* sungrazing. In this case the components were separated by 76 days after 13 millennia, but the corresponding radial acceleration of 100 pm/s² would again have significantly delayed the acquisition of escape velocity (Marsden 1988). While an orbiting pair is possible for the 1988e/1988g situation, the smaller relative accelerations and the need for as many as nine bound sungrazer fragments argue more in favor of a physical-contact hypothesis, with tidally weakened sections at perihelion eventually leading to true separation.

III. HELIOCENTRIC AND BARYCENTRIC ORBITS

Paper I included a demonstration of the indirect perturbations on the heliocentric orbits of sungrazing comets as Jupiter and the Sun move about their common center of mass. We hinted at the utility of starting from orbits referred to the barycenter of the solar system but did not then pursue this approach to the problem. We did take this approach in the

TABLE III. SOLWIND and SMM orbits.

	T	q	ω	Ω	i
Direct Kreutz solution					
1979 Aug.	30.949	0.00480	67.67	344.30	141.45
1981 Jan.	27.100	0.00792	65.43	341.41	140.67
1981 July	20.336	0.00612	68.43	345.26	141.70
1981 Nov.	4.532	0.00450	77.68	356.87	143.84
1983 Sept.	25.188	0.00753	78.58	357.98	143.98
1984 July	28.556	0.01541	56.67	329.74	136.38
1987 Oct.	6.069	0.00538	80.59	0.46	144.25
1987 Oct.	18.027	0.00627	82.63	2.96	144.46
1988 June	27.784	0.00516	85.88	6.96	144.70
1988 Aug.	21.819	0.00591	82.25	2.50	144.43
1988 Oct.	12.072	0.00513	88.08	9.65	144.78
1988 Oct.	24.864	0.00579	86.14	7.28	144.71
1988 Nov.	18.369	0.00590	91.08	13.32	144.79
1989 June	2.578	0.00557	84.72	5.54	144.63
1989 July	8.772	0.00462	91.80	14.20	144.78
Assumption of 1963 V original orbit					
1979 Aug.	30.941	0.00424	89.35	10.89	144.62
1981 Jan.	27.082	0.00418	86.38	7.28	144.62
1981 July	20.357	0.00422	85.57	6.31	144.60
1981 Nov.	4.539	0.00426	85.13	5.80	144.59
1983 Sept.	25.129	0.00460	83.67	4.14	144.52
1984 July	28.483	0.00477	83.80	4.34	144.56
1987 Oct.	6.063	0.00500	85.77	6.75	144.57
1987 Oct.	17.999	0.00500	85.79	6.77	144.57
1988 June	27.783	0.00496	86.25	7.32	144.58
1988 Aug.	21.811	0.00495	86.35	7.44	144.58
1988 Oct.	12.068	0.00494	86.45	7.55	144.59
1988 Oct.	24.863	0.00494	86.47	7.58	144.59
1988 Nov.	18.338	0.00493	86.52	7.64	144.59
1989 June	2.572	0.00489	86.90	8.09	144.59
1989 July	8.769	0.00489	86.99	8.21	144.59

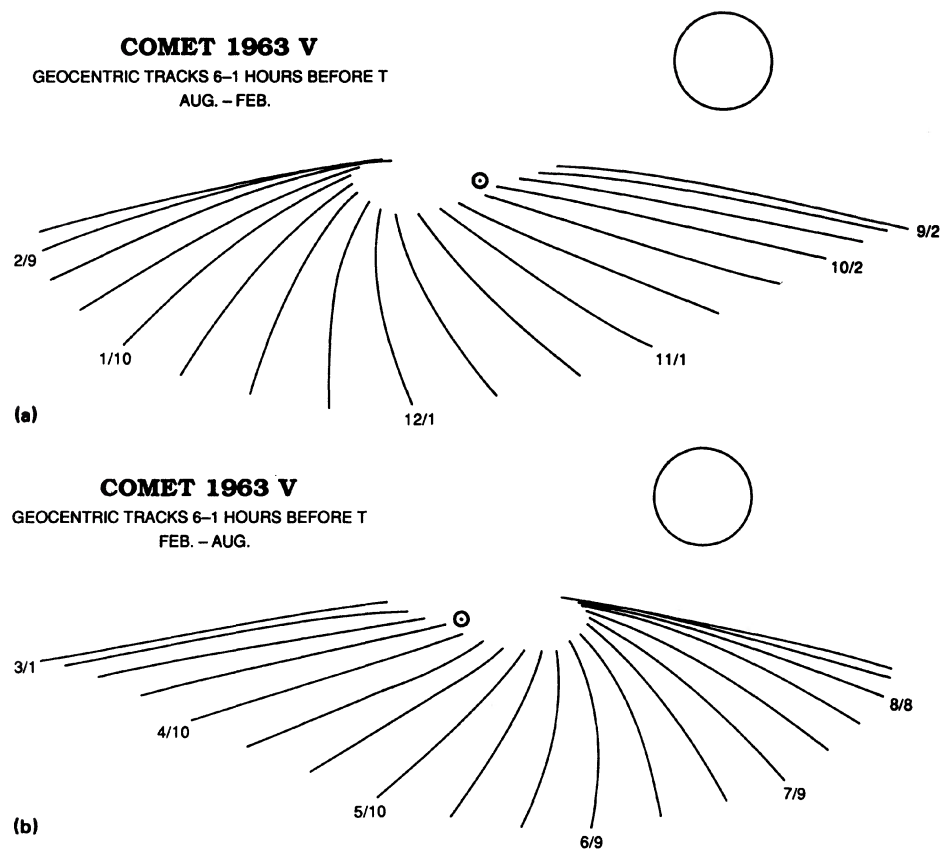


FIG. 1. Geocentric tracks for objects moving in the orbit of comet 1963 V from 6 to 1 hr before perihelion at 10 day intervals throughout the year. North is up and the large circle shows the Sun's apparent size. The tracks from August to February are shown in (a), and those from February to August in (b).

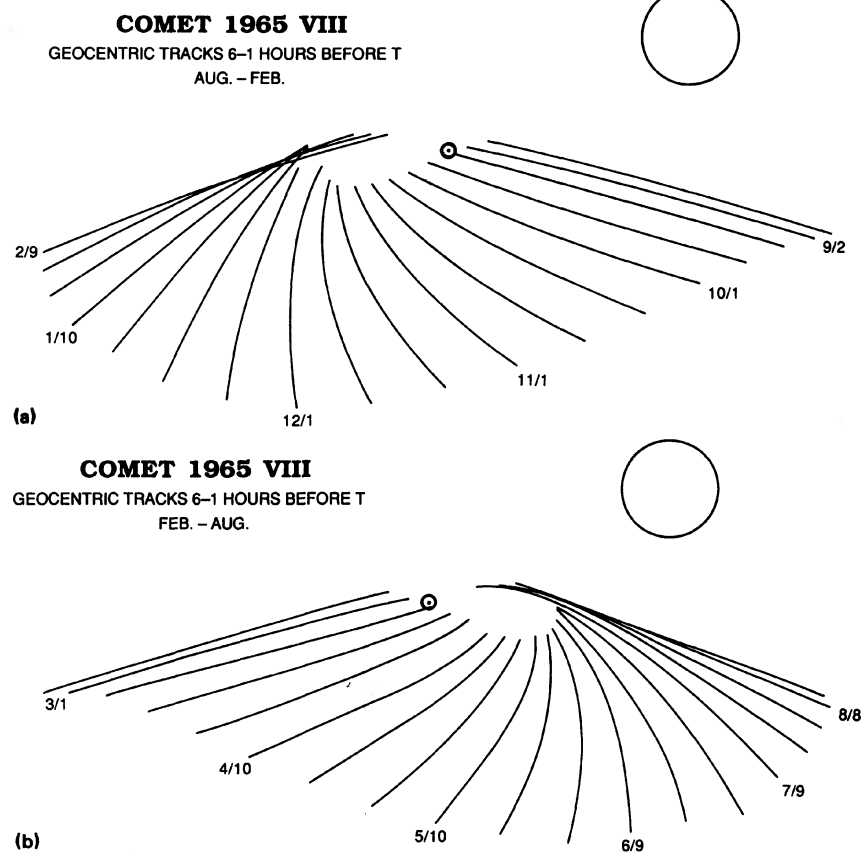


FIG. 2. Geocentric tracks (cf. Fig. 1) for comet 1965 VIII.

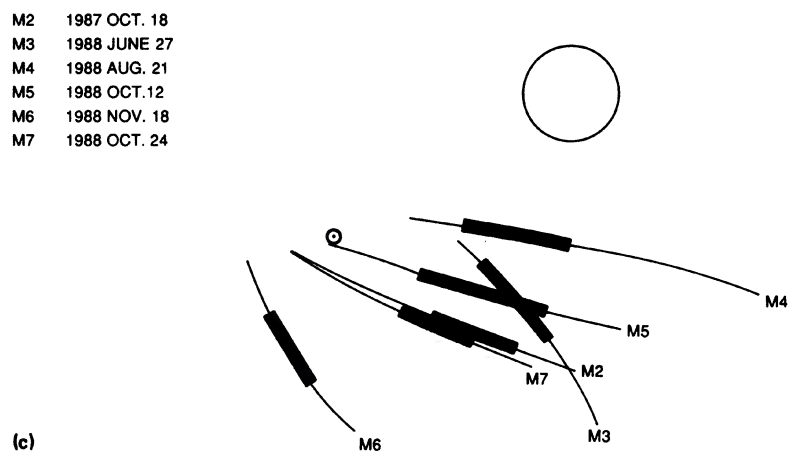
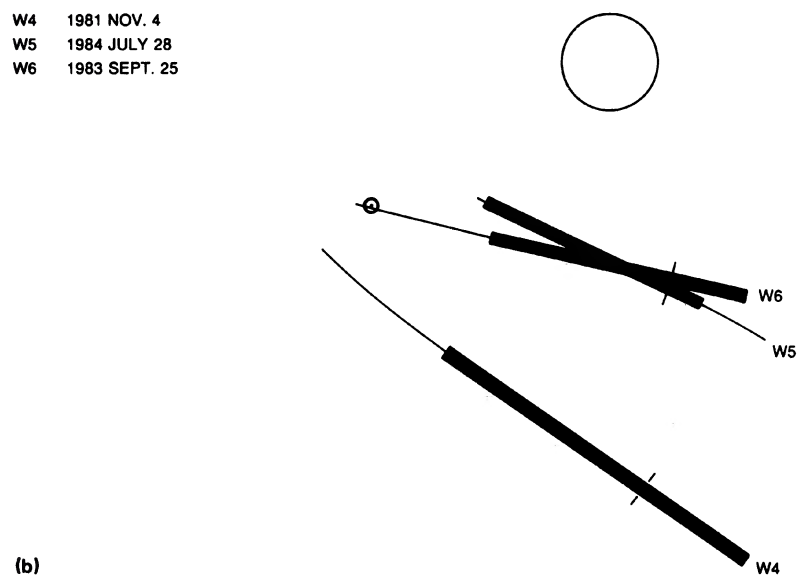
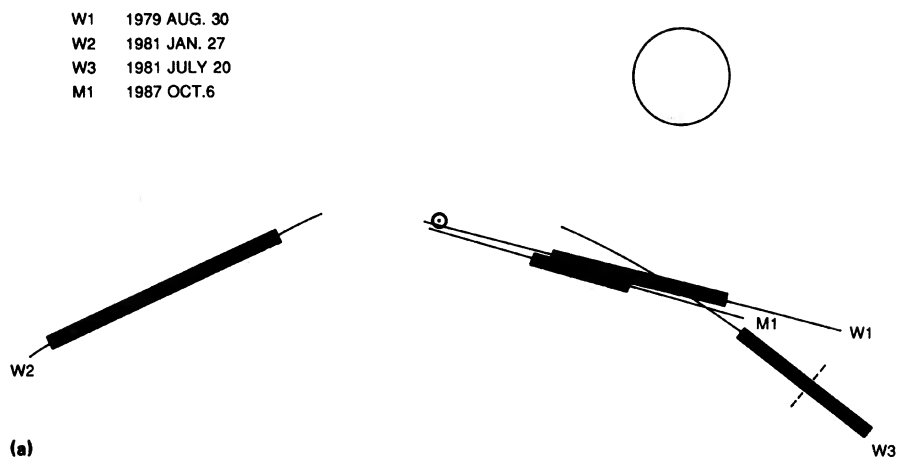


FIG. 3. Observed tracks for the SOLWIND (W) and the first seven SMM (M) comets. North is up and the large circle shows the Sun's apparent size. The solid sections represent the span actually covered by the observations, the thinner lines through these sections the interval from 6 to 1 hr before perihelion (from the orbits in the upper part of Table III). Some of the SOLWIND objects were observed more than 6 hr before perihelion, and the 6 hr point is then indicated by a short line perpendicular to the tracks. SOLWIND 1-3 and SMM 1 are shown in (a), SOLWIND 4-6 in (b), and SMM 2-7 in (c).

1988e/1988g example, the possibility that these two long-period comets separated from each other at their immediately previous perihelion passage being conveniently examined by simply comparing their "original" orbits. As in "Oort Cloud" studies, an original orbit is derived by running the heliocentric orbit osculating at a date near perihelion passage back to when the comet was beyond the outer planets and then referring the orbit to the barycenter. Although in such studies the interest is really only in $1/a$, the reciprocal of the semimajor axis, that element is of rather minimal interest when one is examining cometary splitting.

The left-hand side of Table IV lists heliocentric orbits (again with the comets identified by perihelion date) for the eight Kreutz sungrazers of the 19th and 20th centuries that could be observed well separated from the Sun. The orbits, which are osculating at a date near T , are rounded from the figures in the standard catalog of cometary orbits (Marsden 1986). Four of the orbits are taken to be exactly parabolic, and the others are ellipses with eccentricities in excess of 0.9999 (i.e., $1/a > 0.01 \text{ AU}^{-1}$). The corresponding original orbital barycentric elements, indicated with a subscript zero, are shown in the right-hand columns; in each case these are derived for an epoch some 8000 days before perihelion, the heliocentric distances then being some 40–45 AU.

The essential coincidence of the original orbits of the 1882 and 1965 entries is evident. The difference is not quite the same as was given in the middle column of Table X of Paper I, which was obtained by the more accurate procedure of actually integrating the orbits over a complete revolution (with T forced to be identical at the joint earlier return of the two comets) and comparing the resulting heliocentric data. As was discussed in Paper I, that difference is comparable to the difference between the orbits of the individual components of the 1965 comet after it had split.

Of course, only a very slight difference in the circumstances of the splitting of a Kreutz sungrazer at perihelion has an enormous effect on the times when the fragments return to perihelion. It is therefore useful to examine the different heliocentric orbit that would result if a comet came back to perihelion at a different time. The broken line in the upper part of Fig. 4 shows the differences $P - O$ (heliocentric osculating near Perihelion *minus* Original barycentric) in Ω for the 1965 VIII orbit as T varies through more than two revolutions of Jupiter. The differences for ω and i show similar variations, and although there is some asymmetry in the sense that positive values of the variations in the angular elements prevail, and the effects of the other planets render

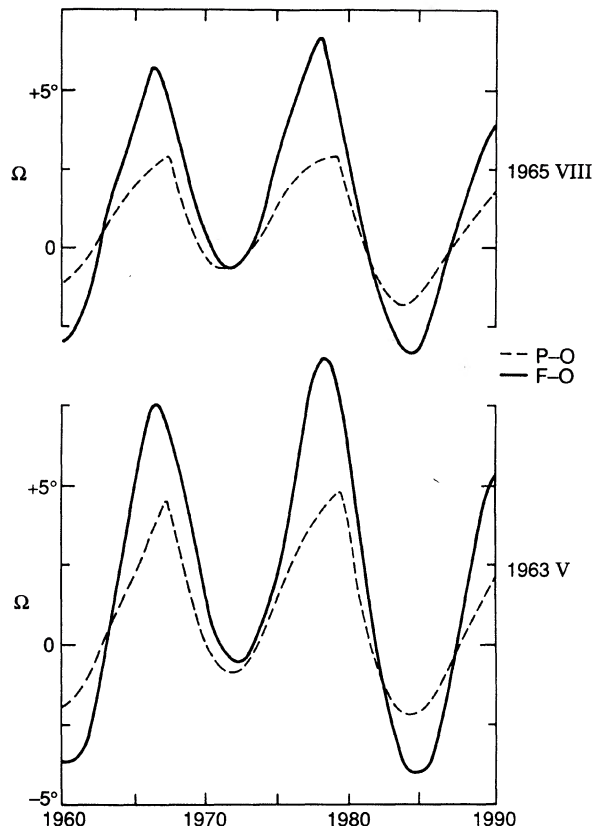


FIG. 4. The broken curve shows the difference $P - O$, the solid curve the difference $F - O$ in Ω for the 1965 VIII and 1963 V orbits as a function of perihelion time during 1960–1990.

the repetition from one Jupiter cycle to the next rather imperfect, the differences are also very roughly in agreement with Eqs. (11) in Paper I (with $\lambda_1 = 0$ in 1963 and $\beta \sim 0$)—which allow only for the indirect Jupiter perturbations. The broken line in the upper part of Fig. 5 shows the differences $P - O$ for q , and we can see that, as noted in Paper I, q is 90° out of phase with the angular elements and reaches a maximum for a cometary perihelion date that roughly coincides with Jupiter's. The $1/a$ perturbations are essentially in phase with those in q , and we can also note that L has maximum and minimum values (282.35 and 282.12 , respectively)

TABLE IV. Orbital elements for the best-established sungrazers.

T	$1/a$	q	ω	Ω	i	$1/a_0$	q_0	ω_0	Ω_0	i_0
1843 Feb. 27.911	+0.01561	0.00553	82.64	2.83	144.35	+0.01565	0.00513	83.70	4.00	144.43
1880 Jan. 28.118	0.0	0.00549	86.24	7.08	144.66	-0.00007	0.00506	85.35	5.88	144.66
1882 Sept. 17.724	+0.01202	0.00775	69.59	346.96	142.00	+0.01230	0.00766	67.34	344.12	141.41
1887 Jan. 11.934	0.0	0.00483	83.51	3.88	144.38	+0.00084	0.00518	82.43	2.63	144.29
1945 Dec. 27.965	0.0	0.00752	72.06	350.50	141.87	+0.00088	0.00807	71.43	349.80	141.73
1963 Aug. 23.956	+0.01071	0.00506	86.16	7.24	144.58	+0.01065	0.00473	85.61	6.48	144.59
1965 Oct. 21.184	+0.01089	0.00779	69.05	346.30	141.86	+0.01115	0.00767	67.35	344.13	141.41
1970 May 14.486	0.0	0.00888	61.29	336.32	139.07	+0.00073	0.00917	61.77	336.98	139.26

The left-hand columns, rounded from Marsden (1986), give the heliocentric values of the orbital elements for an epoch close to perihelion; four of the orbits are exact parabolas. The right-hand columns give the corresponding values (T being omitted) for the original orbits referred to the barycenter of the solar system.

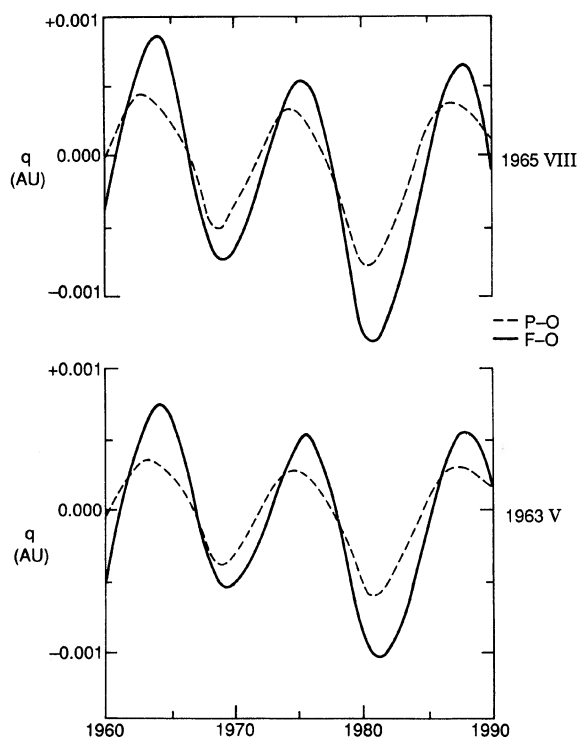


FIG. 5. The broken curve shows the difference $P - O$, the solid curve the difference $F - O$ in q for the 1965 VIII and 1963 V orbits as a function of perihelion time during 1960–1990.

around the time of maximum and minimum q ; the range of B is from $+35^\circ 25$ to $+35^\circ 10$.

The lower parts of Figs. 4 and 5 show the corresponding perturbations for the 1963 V orbit. While there are obvious similarities to the 1965 VIII case, the variation is now enhanced in the case of Ω (and ω) but slightly diminished in the case of q . For 1963 V the range of L is from $281^\circ 97$ to $281^\circ 74$ and of B from $+35^\circ 34$ to $+35^\circ 25$. Particularly interesting is the fact that the largest positive variation in q was attained in 1963, the year the comet was actually observed. With original values q_0 (which is, of course, the comet's minimum distance from the barycenter of the solar system but reckoned 8000 or more days earlier) scarcely larger than $1R_\odot$, any objects that may have separated from 1963 V at its previous perihelion passage would obviously hit the Sun if they were to return at a time when the $(P - O) q$ variation is negative, and extensive dissipation (as happened in the case of comet 1887 I) might be expected for small to moderate positive q variations too.

Differences $F - P$, where F is the “future” barycentric

orbit, can, of course, be calculated in the same way. As it happens, these differences are somewhat similar to and only slightly out of phase with the differences $P - O$, so the total change $F - O$ also shows the same trends. These total changes in Ω and q for the two comets for assumed perihelion dates during 1960–1990 are indicated by the solid lines in Figs. 4 and 5. The curves are representative of other time intervals also, and it follows that repeated fragmentation of the members of the Kreutz group as they keep returning to perihelion at random times results in more of an advance than a retrogression of Ω and a tendency for q generally to decrease, suggesting that all the members will eventually hit the Sun. Statistically, one therefore expects the Subgroup II orbits to be more nearly similar to that of the original Kreutz progenitor than the Subgroup I orbits are. That the changes in Ω and q are out of phase can be expected to complicate the evolution, so the fact that the subgroups can be delineated in both quantities suggests that the observed comets have evolved from each other rather quickly. Granted also that the individual perturbations are generally larger than was indicated in Paper I, it seems unlikely that the progenitors of the subgroups could have separated from each other more than about half a dozen revolutions ago.

IV. SUBGROUP II

In addition to 1882 II and 1965 VIII, Table IV suggests that 1945 VII and 1970 VI are members of Subgroup II.

a) Comet 1945 VII

Comet 1945 VII was observed only on five consecutive nights some two weeks before perihelion, and in Paper I we noted that the observation on the fourth night seemed to be inferior (to the extent of more than $20''$) in relation to the others; this observation was therefore omitted from the parabolic orbit solution shown in Table IV, a solution that gives $L = 282^\circ 87$, $B = +35^\circ 98$.

It is instructive to constrain the orbit of comet 1945 VII to satisfy the Kreutz values of L and B , and in this case one can obtain the parabolic solution labeled A in Table V. The right-hand columns again show the corresponding original orbit, and the $(O - C)$ residuals are given in the columns labeled A in Table VI. Somewhat surprisingly, one can get a considerably better fit if the orbit is allowed to depart (if only slightly) from a parabola, and orbit B is a solution also for the eccentricity (or $1/a$). The residuals for orbit B suggest that it might be the third observation, rather than the fourth, that is the least satisfactory, but the first observation may not be particularly good either. Orbit C is a compromise solution in which $1/a$ is assumed to be exactly 0.01 AU^{-1} , and given the obvious uncertainty of the measurements the residuals must be regarded as acceptable.

Comparison with Table IV indicates that the original val-

TABLE V. Orbital elements for comet 1945 VII.

	T	$1/a$	q	ω	Ω	i	$1/a_0$	q_0	ω_0	Ω_0	i_0
A	1945 Dec. 27.956	0.0	0.00696	$67^\circ 77$	$344^\circ 42$	$141^\circ 48$	+0.00089	0.00750	$67^\circ 29$	$343^\circ 91$	$141^\circ 35$
B	1945 Dec. 27.992	+0.01797	0.00701	68.36	345.18	141.67	+0.01882	0.00754	67.84	344.61	141.53
C	1945 Dec. 27.976	+0.01000	0.00699	68.10	344.84	141.59	+0.01087	0.00752	67.60	344.30	141.45
D	1945 Dec. 27.980	+0.01016	0.00713	67.84	344.66	141.55	+0.01134	0.00767	67.35	344.13	141.41

Orbit A is an osculating parabolic orbit assumed to have Kreutz L and B . Orbit B is the corresponding general solution. Orbit C has assumed $1/a$. Orbit D has the same original values as comet 1965 VIII.

TABLE VI. Observations and residuals for comet 1945 VII.

UT	α	δ	A		B		C		D	
			$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$
1945 Dec. 11.04687	15 ^h 09 ^m 03 ^s .12	-65°12'30".4	+0'.5	+44".2	+9'.7	+5".0	+5'.6	+22".4	+10'.8	+5".1
12.04691	15 38 54.61	-63 59 42.4	-9.4	+15.9	-3.5	-3.1	+6.1	+5.3	-2.5	-2.8
13.07189	16 06 37.35	-62 21 45.3	-13.9	-12.0	-15.6	-11.7	-14.8	-11.8	-15.2	-11.4
14.07000	16 30 33.45	-60 27 04.2	+17.0	-11.5	+4.9	+5.8	+10.3	-1.9	+4.3	+5.6
15.06885	16 51 24.18	-58 18 00.7	+29.1	-29.1	+4.8	+3.7	+15.6	-10.9	+2.8	+2.5

The observations are topocentric and were made at the Boyden Observatory, Bloemfontein. The fourth observation is a reconstruction from the data in Paper I, because the original values have been lost. The (O-C) residuals are with respect to the four orbits shown in Table VII.

ues for these new solutions are really rather similar to the original orbits for comets 1882 II and 1965 VIII, suggesting that comet 1945 VII may have been produced with these other objects at their perihelion passage in the 12th century. Orbit D in Table V was made to be compatible with the original orbit of comet 1965 VIII (with $1/a$ appropriately larger, a shift of no more than 0".005 in ω and the solution otherwise only for T), and the corresponding residuals in Table VI are as satisfactory as one can expect.

As noted in Paper I, the positions of comet 1945 VII were measured in 1952 by A. G. Mowbray. Since the positions seem never to have been published, we take the liberty of including them in the left-hand columns of Table VI.

b) Comet 1970 VI

It is not clear that comet 1970 VI is really a member of Subgroup II, but the object is obviously related much less closely to Subgroup I. The orbit determination is certain enough that one cannot contrive to make the original orbit identical with that of comet 1965 VIII (for we already have $L = 282^\circ 25'$, $B = +35^\circ 08'$), and to change the orbit from a parabola also makes little difference. One must reluctantly conclude that this comet separated from the 1882 II/1945 VII/1965 VIII parent at an earlier stage, and we shall therefore now define it to be the sole known member of Subgroup IIa.

V. SUBGROUP I

Since comets 1843 I and, particularly, 1963 V have orbits that are rather reliably determined, we can say quite definitely, as we did in Paper I (specifically in Table XI), that these comets did *not* have identical original orbits. Most of the comets listed in Table IV were observed only after perihelion, and none of them was at all spectacular before perihelion. If there are other objects traveling in the original orbit of comet 1963 V, they can easily escape detection—from the ground, at any rate—unless their approaches to the Sun are very well timed. Table IV suggests some possible resemblance between the original angular elements for the 1880 and 1963 entries, but the difference in the case of q is unacceptable.

a) The SMM Comets

As noted in Sec. II, the SMM comets of 1987–1989 presumably belong to Subgroup I. The progression in the values

of ω and Ω shown in the upper half of Table III is in the direction compatible with the assumption of a common original orbit, but the range expected from Fig. 4 is much smaller.

The lower half of Table III includes the perihelion-osculating heliocentric orbits that these comets would have had (parabolas being assumed, and a solution therefore made only for T) if their original orbits were the same as that of comet 1963 V. The residuals $\Delta\alpha'$, $\Delta\delta'$ from these solutions are given in the final columns of Table II. Although these are excellent for SMM 4, 5, 7, and 9, the fit is just tolerable for SMM 1 and 2 and downright disappointing in the case of SMM 8. The original orbit of comet 1843 I gives a somewhat better representation in these poor cases, but the declination residuals remain between $-2'$ and $-3'$. On the other hand, fitting the 1843 I orbit to SMM 5 and 7 leaves systematic declination residuals of $+3'$ to $+5'$. For the remaining objects the 1963 V and 1843 I fits are comparable.

b) The SOLWIND Comets

The 1963 V original orbit also gives an excellent fit to the two SMM observations of SOLWIND 5; the residuals are included in Table II, and the osculating orbit is also in the lower half of Table III. As could be anticipated from our earlier remark on the consistency of the SMM and SOLWIND observations, the latter show systematic residuals, amounting up to $-7'$ in declination for the earliest observations. This declination inconsistency can be removed if one set of observations is rotated about the center of the Sun by 4° . In view of the manner in which the SOLWIND right ascensions and declinations were constructed, it is probably reasonable to apply this adjustment as a rotation by $+4^\circ$ to them, rather than one of -4° to the SMM data.

Attempts to fit the 1963 V original orbit to the other SOLWIND comets understandably yield systematic residuals, but there is a surprising consistency from comet to comet, with the early declinations of SOLWIND 2, 3, and 6 all yielding residuals in the range $-6'$ to $-8'$. For SOLWIND 4 (which was observed for a considerably longer time than the others) the early declination residuals amount to $-13'$ and for SOLWIND 1 to as much as $-20'$. A similar rotation by $+4^\circ$ gives very good agreement with the 1963 V original orbit in the case of SOLWIND 4, and the best fits for SOLWIND 6 and 3 are accomplished with rotations by $+3^\circ$ and $+2^\circ$, respectively. The SOLWIND 2 observations

would require rotation by a rather different value, -3° , although it should be noted that this is the sole SOLWIND comet that appeared during the half of the year that the solar north pole is to the west, rather than to the east, of the northernmost point of the Sun in the terrestrial equatorial system. In the case of SOLWIND 1 the rotation required is a rather more excessive $+12^\circ$.

Anyway, in each case, the relevant osculating elements are included in the lower half of Table III, and the adjusted positions α' , δ' and resulting residuals are shown in Table VII. So while one is entitled to have some reservations about this modification of the SOLWIND data, it seems not at all out of the question that the SOLWIND comets, like the SMM comets, are Subgroup I members closely related to each other and perhaps specifically also to comet 1963 V.

c) Frequency of Appearance

It is difficult to estimate the completeness of the space-coronagraph cometary detections of the past decade. For perhaps six months in 1980 and for a little over a year beginning in mid-1984 both the SOLWIND and SMM coronagraphs were operating. Only one comet was noted during the whole of this time, but it was clearly recorded by both instruments.

There is thus a reasonable indication that the 1987–1988 bonanza (with perhaps only slightly reduced activity in 1989) is a real effect. Unfortunately, it will not be immediately possible to verify this, as SMM will re-enter the Earth's atmosphere later in 1989. It is disappointing that none of the sungrazers (except for the bright object 1965 VIII) has been reported as directly detected by a ground-based coronagraph.

There is a chance of detection during a total solar eclipse, the frequency of which must place *some* bound on the frequency of the sungrazers. The perhaps somewhat optimistic rate of a member every two months and the presumption that a comet (or its tail) would be detected during the four hours before perihelion passage imply that such an object should be detectable 1/360 of the time. One might make a case for sure detection over an interval of more like 12 hr, which would imply a detectable object 1/120 of the time. There are about 70 total solar eclipses per century. Just one presumed Kreutz sungrazer has been observed during a total solar eclipse: "Tewfik," observed on 1882 May 17. In Paper I we suggested that Tewfik was a member of Subgroup I (even though the best Subgroup II member appeared only four months later), so it appears that with the addition also of comet 1887 I there were three observed Subgroup I members (one much by chance) during an interval of only seven years.

The SMM comets alone show that Subgroup I is quite populous, and on adding the 1880–1887 trio and considering that the SOLWIND comets might also belong, we obtain the impression of quite prolific activity, although it is unclear whether this goes on all the time. Subgroup II, on the other hand, perhaps yields more isolated members, and although comets 1965 VIII and 1970 VI appeared less than five years apart, we have shown that they were not very intimately related. The kind of partial breakup that prevents true separation in the vicinity of the Sun is perhaps just a Subgroup I phenomenon.

As has been noted, the very existence of the subgroups implies that many of the fragments do not survive. In the past it has certainly been the author's impression that these

TABLE VII. Adjusted SOLWIND observations.

	UT	α'	δ'	$\Delta\alpha'$	$\Delta\delta'$
1979 XI		(SOLWIND 1)		Rotation $+12^\circ$	
Aug.	30.789	10 ^h 26 ^m .19	+09°04.5	-1.4	+0.4
	30.796	10 26.53	+09 04.0	-0.2	-0.6
	30.802	10 26.98	+09 05.0	+3.2	0.0
	30.809	10 27.13	+09 05.4	+1.5	-0.2
	30.816	10 27.22	+09 06.4	-1.1	+0.3
	30.856	10 28.92	+09 06.7	-0.4	-2.5
	30.867	10 29.55	+09 07.1	+1.6	-2.9
	30.885	10 30.21	+09 09.1	-1.6	-2.3
1981 I		(SOLWIND 2)		Rotation -3°	
Jan.	26.860	20 43.90	-19 14.1	+3.0	-1.5
	26.867	20 43.72	-19 12.7	+2.5	-1.4
	26.874	20 43.71	-19 12.0	+4.7	-2.0
	26.881	20 43.54	-19 10.1	+4.0	-1.4
	26.888	20 43.22	-19 08.1	+1.5	-0.7
	26.927	20 42.12	-19 01.3	-1.6	-1.5
	26.934	20 42.00	-18 59.9	-1.0	-1.5
	26.941	20 42.00	-18 58.8	+1.5	-1.8
	26.947	20 41.75	-18 56.6	+0.1	-0.8
	26.954	20 41.50	-18 55.3	-0.9	-0.9
	26.994	20 40.17	-18 46.1	-3.3	-0.1
	27.001	20 39.86	-18 45.1	-4.5	-0.6
	27.008	20 39.84	-18 44.5	-1.4	-1.6
	27.014	20 39.58	-18 42.9	-2.0	-1.8
	27.021	20 39.23	-18 40.4	-3.3	-0.3
1981 XIII		(SOLWIND 3)		Rotation $+2^\circ$	
July	19.999	07 45.31	+19 42.9	-4.5	-1.7
	20.030	07 45.97	+19 49.2	-2.9	-0.2
	20.069	07 46.88	+19 54.5	-0.3	-1.1
	20.096	07 47.47	+20 00.5	+0.6	+0.4
	20.135	07 48.41	+20 08.1	+2.4	+1.2
	20.162	07 49.04	+20 13.0	+2.7	+1.3
1981 XXI		(SOLWIND 4)		Rotation $+4^\circ$	
Nov.	3.999	14 25.20	-16 49.4	-0.9	-3.3
	4.038	14 26.06	-16 42.5	-0.7	-0.6
	4.105	14 27.68	-16 34.9	+1.4	-0.7
	4.171	14 29.18	-16 27.8	+1.6	-1.6
	4.238	14 30.88	-16 18.4	+3.8	-1.0
	4.304	14 32.18	-16 07.6	-0.2	+0.3
	4.371	14 33.78	-15 57.9	-1.0	-0.9
1983 XX		(SOLWIND 6)		Rotation $+3^\circ$	
Sept.	24.863	11 54.32	-00 43.3	-1.8	-1.0
	24.870	11 54.67	-00 42.7	+0.2	-0.8
	24.878	11 54.88	-00 40.6	-0.3	+0.8
	24.885	11 55.24	-00 41.1	+1.8	-1.1
	24.892	11 55.37	-00 41.1	+0.4	-0.6
	25.062	12 00.89	-00 29.7	(-9.6	-2.0)
1984 XII		(SOLWIND 5)		Rotation $+4^\circ$	
July	28.302	08 21.69	+18 37.6	(-3.0	0.0)
	28.309	08 21.81	+18 38.6	(-4.1	-0.2)
	28.316	08 22.05	+18 39.9	(-3.5	0.0)
	28.324	08 22.06	+18 41.0	(-6.8	-0.3)
	28.331	08 22.62	+18 44.3	(-1.7	+1.8)
	28.368	08 23.96	+18 49.2	(+0.6	+0.1)
	28.375	08 24.20	+18 50.5	(+0.6	+0.1)
	28.383	08 24.44	+18 52.0	(-0.1	+0.2)
	28.390	08 24.68	+18 53.5	(-0.4	+0.3)
	28.397	08 25.04	+18 54.6	(+0.9	+0.1)
	28.435	08 26.52	+19 01.8	(-1.8	-0.2)
	28.443	08 26.55	+19 02.1	(-7.3	-1.4)

Rotation indicates the amount by which the position angles were adjusted. The (O-C) residuals are from the orbits in the lower half of Table III.

fragments just gradually disperse as they move away from the Sun, and there is also a strong implication of this in Sekanina's extensive work on cometary splitting. The coronagraphic observations of the past decade make it clear, however, that many tiny fragments can and do survive right up to the time of their next perihelion passage.

d) Comets 1843 I and 1880 I

Could there perhaps be a common parent for comets 1843 I and 1880 I? The orbits in Table IV show quite good agreement in q_0 , but there is a difference of ~ 1.9 in Ω_0 . It is not really appropriate to try to compare the orbits, however, for one is an ellipse with $P = 5.1$ centuries and the other is a parabola. Kreutz (1901) calculated parabolas for both comets, and the increase in nodal longitude of 0.8 for his parabolic solution for 1843 I goes a long way toward diminishing the discordance.

But the orbits are not really parabolic, and Kreutz also made a solution for each comet on the assumption that $P = 8.0$ centuries. Certainly, such a solution does absolutely no injustice to the observations of comet 1880 I, and the fit to 1843 I is acceptable unless excessive weight is given to the daytime observations in Maine and Mexico on the day after perihelion passage. Our very similar heliocentric orbits (ignoring the daytime observations) for $P = 8.0$ centuries are given on the first two lines (labeled A) of Table VIII, and the corresponding barycentric orbits are shown too. The fact that Ω_0 for 1880 I is now only 0.25 larger than that for 1843 I suggests that we should make solutions for shorter periods.

The pair of entries labeled B in Table VIII were selected—not entirely at random—from a small range of solutions that give values of Ω_0 for the two comets that agree to the last figure quoted, and orbits C have identical values of ω_0 . The agreement for the other elements is in each case very good, and one can scarcely doubt that 1843 I and 1880 I are related in the same manner that 1882 II and 1965 VIII are related. For these two 1843 I solutions the values of P are ~ 3.8 and 3.6 centuries, respectively, and for comet 1880 I the P were, of course, taken to be ~ 0.37 century longer. Separation was therefore presumably at some time during the second half of the 15th century. These 1843 I values of P are shorter than Kreutz' basic value by about twice the probable error he quoted and are therefore in reasonable agreement with the daytime observations.

The orbits labeled D were calculated on the assumption that the two comets had $P = 1.7$ and 2.1 centuries, thereby permitting coincidence with the comet of 1668; it is clear that there has now been an overcorrection, with Ω_0 for 1880 I now more than 0.4 smaller than for 1843 I. In any case, orbit D will not satisfy the daytime observations of 1843 I and yields residuals of more than $10''$ for the nighttime normal places when the daytime data are ignored. Further, computation of (heliocentric) q for the previous value of $T = 1668$ March 1 yields 0.00488 and 0.00482 AU; survival may have been possible for a sufficiently large comet, but at best the fate of comet 1887 I would be more likely.

e) Relationship of 1843 I/1880 I with 1963 V

If comet 1843 I had $P = 3.6$ to 3.8 centuries, the parent of it and 1880 I can be supposed to have had a somewhat similar revolution period (on the grounds that 1843 I was the principal component, both from its more spectacular appearance and from the fact that its period was shorter than that of 1880 I), so it should have had its previous perihelion passage in the late 11th or early 12th century. The orbit determination given in Paper I and an independent computation by Sekanina (1967) show that comet 1963 V has a rather well-determined period that is certainly included in the range 8.4 – 9.2 centuries, and it could thus have been at perihelion around the same time.

It is therefore intriguing to examine the feasibility of the hypothesis that 1963 V separated at that time from the parent of 1843 I and 1880 I—for convenience, we shall give this putative parent the name "Combo"—the latter pair then having gone around the Sun twice before they were observed and in less time than 1963 V went around once!

From Tables IV and VIII we have that the differences in the original orbital elements, in the sense 1843 I – 1963 V, are

$$\begin{aligned}\Delta q_0 &= +0.00044 \text{ AU} & \Delta \Omega_0 &= -2.70 \\ \Delta \omega_0 &= -2.09 & \Delta i_0 &= -0.19 \\ & & & \text{(orbits B)}\end{aligned}$$

and

$$\begin{aligned}\Delta q_0 &= +0.00045 \text{ AU} & \Delta \Omega_0 &= -2.75 \\ \Delta \omega_0 &= -2.14 & \Delta i_0 &= -0.20 \\ & & & \text{(orbits C)}.\end{aligned}$$

TABLE VIII. Orbital elements for comets 1843 I and 1880 I.

	T	$1/a$	q	ω	Ω	i	$1/a_0$	q_0	ω_0	Ω_0	i_0
A	1843 Feb. 27.914	+0.01160	0.00549	82.80	3.03	144.38	+0.01164	0.00509	83.91	4.25	144.47
	1880 Jan. 28.097	+0.01160	0.00554	85.13	5.78	144.52	+0.01152	0.00511	84.16	4.50	144.50
B	1843 Feb. 27.908	+0.01904	0.00557	82.49	2.64	144.32	+0.01907	0.00517	83.52	3.78	144.40
	1880 Jan. 28.085	+0.01801	0.00556	84.55	5.10	144.44	+0.01793	0.00515	83.55	3.78	144.41
C	1843 Feb. 27.907	+0.01991	0.00557	82.45	2.60	144.31	+0.01994	0.00518	83.47	3.73	144.39
	1880 Jan. 28.083	+0.01875	0.00557	84.47	5.01	144.42	+0.01866	0.00515	83.47	3.68	144.40
D	1843 Feb. 27.896	+0.03199	0.00565	81.96	2.04	144.21	+0.03200	0.00526	82.87	3.03	144.29
	1880 Jan. 28.066	+0.02795	0.00560	83.60	3.99	144.29	+0.02786	0.00519	82.54	2.60	144.25

Orbits A are assumed to have $P = 8.0$ centuries. Orbits B are constrained to have identical original Ω_0 and orbits C identical ω_0 . Orbits D are constrained so that the previous combined perihelion time of the pair would have been on 1668 Mar. 1.

Since the original orbit of 1843 I can be equated with the future orbit of Combo, and the hypothesis of common parenthood for Combo and 1963 V requires that they should have the same original orbit (except for $1/a_0$), we have merely to search for possible dates in the latter half of the 15th century when the differences $F - O$ for Subgroup I were equal to the above values.

Inspection of the 20th-century $F - O$ values for 1963 V in Figs. 4 and 5 indicates that this condition would have been approximately met only in 1962 and 1986, i.e., about a year before Jupiter passes perihelion; during the intervening Jupiter revolution the competing influence of Saturn on the opposite side of the Sun prevented the variations in Ω (and ω) from attaining sufficiently negative values. Before undertaking the necessary computations for the 15th century, we can note that 42 mean revolutions of Jupiter take 4.982 centuries. Since 17 revolutions of Saturn take 5.008 centuries (and, for that matter, six revolutions of Uranus and three of Neptune take 5.041 and 4.944 centuries, respectively), we can guess that the only solutions during the interval of interest would be around 1463–1464 and 1487–1488. Such indeed seems to be the case. Other maximum negative $F - O$ values were only -1.5 in 1450, -0.5 in 1474, and -2.3 in 1497; although the last of these is tolerably close to the required value, the q variation was then too small, and it did not rise to $+0.0004$ AU until that in Ω had changed to -1.8 .

We have confirmed these solutions by direct numerical integration. Table IX shows the results of the computation of three variant orbits (with changes only in initial $1/a$) for each of orbits B (which coincide closely with the 1463–1464 solution) and orbits C (likewise for the 1487–1488 solution), starting from the perihelion-osculating orbits in Table VIII. In each case the first line shows osculating elements for an 1843 I variant at the time of the 15th-century T indicated. We note that q would have been slightly larger than that observed for comet 1963 V, so survival would seem assured. The second line gives the corresponding result for 1880 I, with T forced to have the same value. The third line shows the result of integrating Combo back another revolution. The fourth line gives the figures for 1963 V, the earlier T forced to be identical with that of Combo.

In the first computation of each trio there is tolerable agreement in q , but Ω for 1963 V is systematically more than 0.5 in excess of that for Combo. The Ω difference is smaller for the second computation of each trio, but there is still a significant excess. This excess is reduced in the third computation, but the difference in q is now becoming apparent. This effect is noticeably larger for orbits B than for orbits C, and it is also the case that orbits B show somewhat more of a discordance between the 15th-century figures for 1843 I and 1880 I. The most consistent computation of the six appears to be the last, with Combo separating into 1843 I and 1880 I at $T = 1487$ October 29, although some time during the following six weeks—until the date in the fifth computation, say—would be satisfactory.

There are no records of observations of sungrazing comets in 1463 or 1487–1488—or, indeed, at any time during this whole time interval under consideration. A Kreutz sungrazer with $T = 1487$ October 29 could well have escaped detection, for by the time one could expect to see a bright postperihelion tail in morning twilight, the comet would have been well south of -30° declination and moving rapidly southward. Few, if any, potential observers located out-

TABLE IX. Hypothetical osculating elements.

T	q	ω	Ω	i	
Orbits B					
1463 Aug. 17	0.00509	84.61	5.08	144.52	1843 I
	0.00508	84.66	5.11	144.53	1880 I
1066 Dec. 16	0.00509	83.11	3.07	144.34	Combo
	0.00503	83.39	3.71	144.37	1963 V
1463 June 28	0.00512	84.66	5.14	144.53	1843 I
	0.00511	84.72	5.17	144.54	1880 I
1069 Jan. 29	0.00525	84.88	5.17	144.50	Combo
	0.00515	85.03	5.65	144.52	1963 V
1463 Mar. 21	0.00518	84.74	5.22	144.53	1843 I
	0.00517	84.80	5.25	144.54	1880 I
1072 Feb. 13	0.00497	86.45	7.13	144.64	Combo
	0.00478	86.39	7.38	144.66	1963 V
Orbits C					
1488 Jan. 25	0.00508	84.54	5.01	144.51	1843 I
	0.00506	84.56	5.00	144.52	1880 I
1112 Nov. 13	0.00471	82.44	2.33	144.27	Combo
	0.00472	82.64	2.85	144.29	1963 V
1487 Dec. 12	0.00510	84.63	5.11	144.52	1843 I
	0.00508	84.65	5.10	144.52	1880 I
1113 Nov. 17	0.00481	83.16	3.19	144.35	Combo
	0.00479	83.19	3.51	144.36	1963 V
1487 Oct. 29	0.00512	84.70	5.20	144.53	1843 I
	0.00511	84.72	5.19	144.53	1880 I
1114 Dec. 4	0.00493	83.68	3.81	144.39	Combo
	0.00487	83.54	3.93	144.39	1963 V

The three groups in the upper half correspond to one of the solutions (Orbits B) discussed in Sec. V (e), those in the lower half to the other (Orbits C), which appears to be more probable solution.

side north-temperate latitudes were then recording astronomical observations. If T is put later in the year, a record would be even less likely. Observations before perihelion would have been a little easier from a geometrical point of view (at least with an October perihelion date), but as already mentioned, these comets then tend to be considerably less spectacular. Somewhat more difficult to explain is the absence of daytime records of the object within, say, 12 hr of perihelion, but longitude coverage would have been limited, and poor weather at the few strategic sites is a distinct possibility. Absence of observations is another reason to reject the 1463 solution, for with $T = 1463$ March 21 (for example), postperihelion evening observations at north-temperate latitudes should certainly have been made. To minimize the possibility of nighttime (but not daytime!) detection, the comet would have had to come around the middle of the year. The alternative solution of $T = 1463$ June 28 would have kept the comet within 30° of the Sun until August 10, and although then at declination -2° it would still have been well south of the Sun and rather awkward to detect from the northern hemisphere.

The elements, especially the perihelion dates, given in Table IX for the earlier passage of Combo and 1963 V should certainly not be taken literally. Only the *differences* in the elements have any meaning, and these depend entirely on what happened at Combo's intervening return. Because of the nongravitational effect of the breakup into 1843 I and

1880 I, it is impossible to establish the revolution period of Combo. Nongravitational effects are undoubtedly also involved in the case of 1963 V, although these would presumably be smaller than the uncertainty of the gravitational orbit solutions. The only real requirement is, of course, that q be large enough for the comet to survive perihelion, and since 0.004 73 AU now becomes the *future* value q_f , this means that Jupiter was not close to its own perihelion. Although the comet of 1106 has been mentioned as the possible parent of 1882 II and 1965 VIII, it would also therefore be a viable candidate for the parent of Combo and 1963 V. Heliocentric elements are given in Table X for both parenthood assumptions. In Paper I the comet of 1075 was mentioned as the possible earlier appearance of 1963 V. This is not possible, since a more careful computation shows that q would have been only 0.004 59 AU.

f) Other Members

It seems logical to consider that the original orbits of all three Subgroup I members of the 1880s are identical, and taking orbit C for 1880 I we obtain the osculating values for Tewfik and 1887 I in Table X (with solutions of course made for T). This Tewfik orbit satisfies Trépied's (1882) observation (as given by Kreutz) within 30", a significant improvement over the 0:07 residual mentioned in Paper I. Had it been a larger object, Tewfik could have survived. The result for 1887 I is really quite similar, especially for the critical element q , to what Sekanina deduced in a quite different manner, and the ephemerides from the two orbits for the 10-day arc covered by the observations agree within 10'.

Finally, there is the comet of 1668 and the other possible Kreutz sungrazers of the late 17th century. Table X includes two hypothetical orbits for the 1668 comet, the first on the assumption of $P = 5.6$ centuries and separation from Combo and 1963 V in the 12th century, and the second with $P = 1.8$ centuries and origin with 1843 I and 1880 I. The latter possibility seems less likely, in spite of the larger q , for if the comet survived, it could have been "used" as the parent of comet 1843 I in the first place—a scenario we have already gone to pains to discredit. In Paper I we mentioned that the comet of 1695 possibly belonged to Subgroup I, and orbits based on the assumptions adopted for comet 1668 are presented here. Even for the shorter period, q would have been too small for postperihelion observation as other than a synchronic like the 1887 I phenomenon, so association of this object with the sungrazers is, as Kreutz remarked, very questionable. It is

not possible to say much about comets 1689 and 1702a, except to repeat that Kreutz membership is doubtful for the former but just possible for the latter (although the subgroup cannot really be assigned).

VI. RELATIONSHIP OF THE SUBGROUPS

Figures 6(a) and 6(b) illustrate our proposed recent evolution for the Kreutz sungrazers. For most of the time the various orbits are represented by vertical lines, the barycentric values of Ω in Fig. 6(a) and of q in Fig. 6(b). The horizontal segments show the changes from the original (preperihelion) to the future (postperihelion) values. The perihelion-osculating heliocentric values are indicated by black circles at the appropriate point on each horizontal line, the sizes of the circles being roughly proportional to the spectacularity of the comets. Future orbits are not shown for the comets that were not observed after perihelion passage. Although some of our assumptions are certainly open to criticism, we feel that a reasonable case has been made for tracing the 25 comets shown to just three earlier objects in a tolerably unequivocal manner.

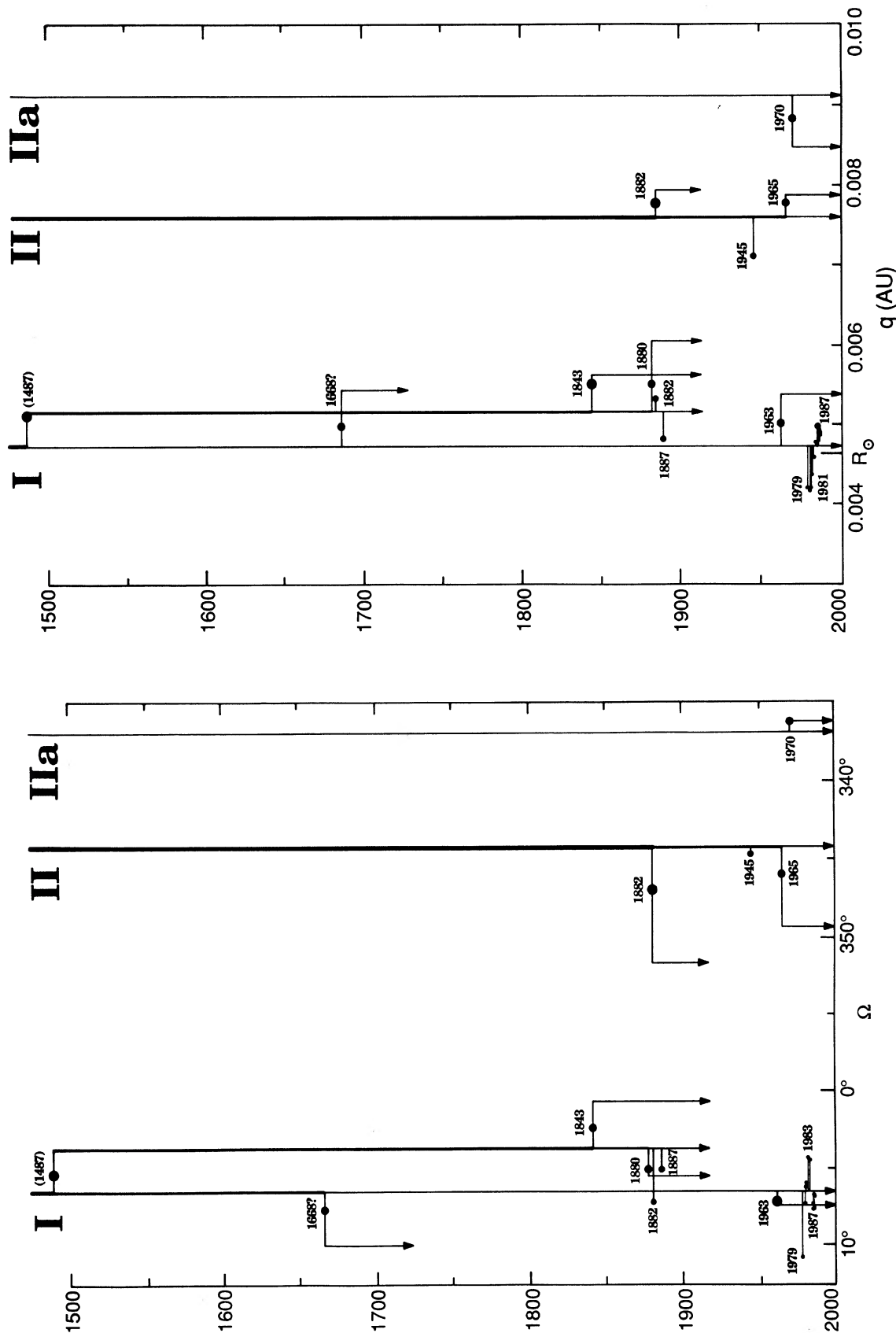
Much of this evolution of Subgroups I, II, and IIa is also illustrated in the q versus Ω plot presented as Fig. 7. The vertical lines that represented the barycentric orbits in Figs. 6(a) and 6(b) now appear as points. The point of convergence in the bottom left-hand corner of this figure is therefore the common barycentric orbit of Combo (1487), 1963 V, and (possibly) 1668 after the separation around the year 1100, and the lines leading away from this point extend to the respective barycentric orbits after the 1487, 1963, and 1668 perihelia. For a perihelion matching Jupiter's perihelion, as in 1963, the line is directed upward, and subsequent progression is counterclockwise. The 1487 future orbit becomes, of course, the original orbit for the sungrazers of 1843 and 1880.

The line in the upper right-hand section of the figure likewise shows the transition from the original to the future orbit of comet 1970 VI. Beneath it appear the transitions for 1882 II and 1965 VIII, again with a common point. Osculating elements are not illustrated, but they would generally be points near the center of (but not in fact generally on) the straight lines.

It is interesting to speculate on the even earlier evolution. Since the precise perihelion date of the Subgroup I parent is unknown, the curve illustrates the locus of the points corresponding to the previous barycentric orbit if this occurred

TABLE X. Miscellaneous osculating elements.

T	q	ω	Ω	i	
1106 Feb. 5	0.00797	68°.19	345°.09	141°.69	1882 II/1965 VIII parent
1106 Feb. 5	0.00499	86.42	7.36	144.64	Combo/1963 V parent
1882 May 17.463	0.00534	86.16	6.99	144.49	Origin with 1880 I
1887 Jan. 11.936	0.00481	84.61	5.02	144.47	Origin with 1880 I; synchronic
1668 Mar. 1.349	0.00508	86.61	7.77	144.58	Origin with 1963 V
1668 Mar. 1.340	0.00552	84.68	5.26	144.43	Origin with 1843 I/1880 I
1695 Oct. 23	0.00444	87.18	8.30	144.62	Origin with 1963 V; not viable
1695 Oct. 23	0.00486	85.01	5.52	144.47	Origin with 1843 I/1880 I; probably not viable



during the Jupiter revolution 1102–1114. The previous line segment would then be directed from a point on this curve to the main intersection already mentioned. The lower half of the curve is unacceptable because the heliocentric q at perihelion would have been less than $1R_{\odot}$. As already noted, an 1106 date is viable, but the pre-1106 barycentric orbit would then be even further removed from Subgroups II and IIa, thereby pushing the date of the original separation of the comets further back in time. Perihelion passage in 1114 or, better still, 1101 would clearly speed up the evolution.

It is quite likely that the perihelion date was even earlier, and the broken-line segment XI (for 11th century) leads back simply to a hypothetical optimal original value. Obviously, there could have been further fragments at this 11th-century splitting, and if one of them were a substantial component with a revolution period shorter than that of Combo, this would presumably have been the parent's previous revolution period. For simplicity we shall suppose that the previous period was again basically that of comet 1843 I, and this would yield a previous perihelion passage early in the eighth century. The broken-line segment VIII again assumes

perihelion at an optimal time, and the hypothetical process is repeated for segments IV and I, which take the barycentric orbit of the Subgroup I progenitor back before the first century to the circle shown in the vicinity of $q = 0.008$ AU, $\Omega = 339^{\circ}$. In keeping with Figs. 4 and 5, we note that as q gets larger, more of the change in Ω is transferred to the change in q .

Turning now to Subgroup II, we maintain that it is still reasonable to consider the 1106 comet as the parent, and the resulting original orbit (which could be approximately derived simply by translating the Subgroup I curve to the Subgroup II region) can then be connected to the circle at $q = 0.008$ AU, $\Omega = 339^{\circ}$ with only one more link. This would represent a perihelion passage in the fourth century, again on the assumption that the period remains at roughly the 7.7 centuries of the principal component of 1882 II.

Finally, we need to tie in Subgroup IIa. Unfortunately, there is no information whatever about the revolution period of comet 1970 VI, but if this is even roughly similar to those of the genuine Subgroup II members, the original orbit should be connectable to the circle in Fig. 7 in just two

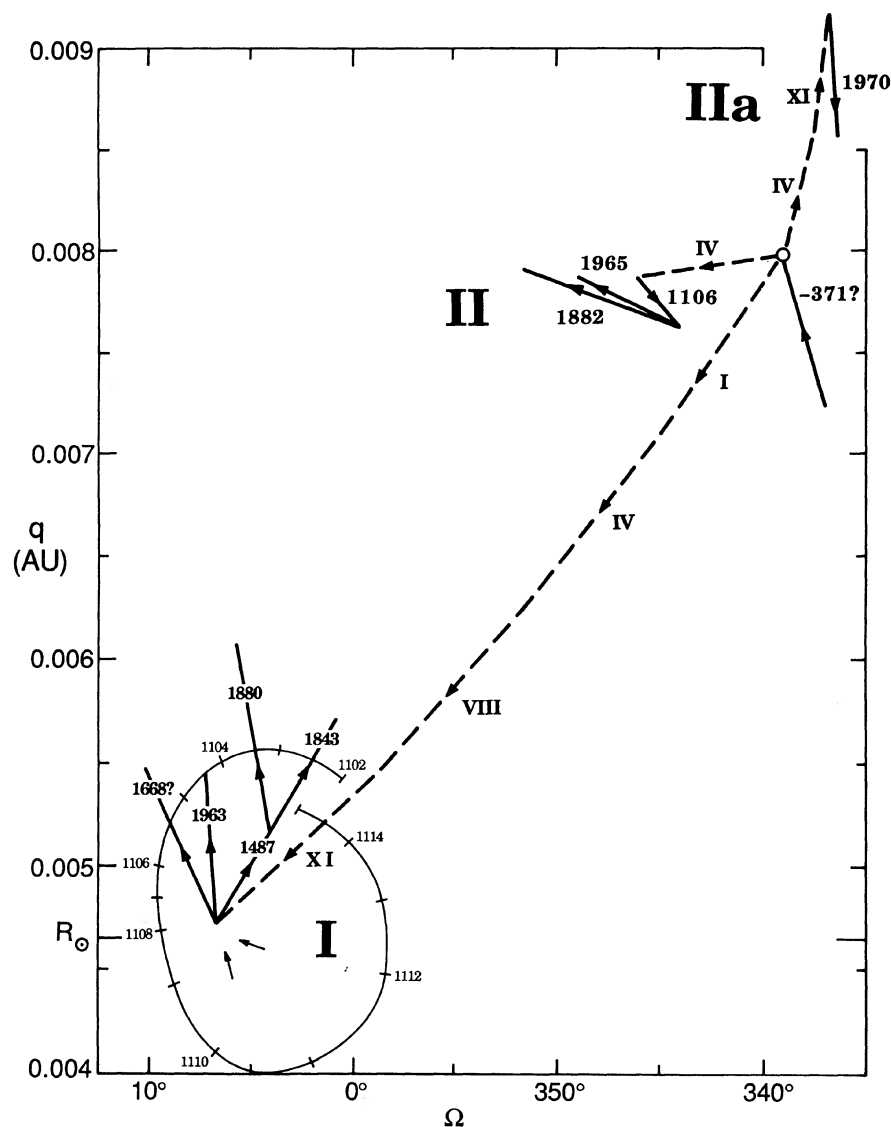


FIG. 7. Plot of q versus Ω . This shows the vertical-line orbits of Fig. 6 as points and the horizontal connections as directed lines. In the section involving Subgroup I, the curve illustrates the locus of original orbits for perihelion time assumed in the interval 1102–1114. The broken lines indicate possible evolutionary tracks, these being traced back to a hypothetical circle at $q = 0.008$ AU, $\Omega = 339^{\circ}$, speculated as the then-future orbit of the Kreutz group progenitor, perhaps the comet of -371 .

links—and it is. Comet 1106 was presumably not the previous apparition of comet 1970 VI.

The circle can therefore be made to represent the common future orbit of the fragments of the original progenitor of all the Kreutz sungrazers, and this object could have appeared in the fourth century BC. This might conceivably have been the celebrated comet of — 371 (or thereabouts) observed by Aristotle and others. The original orbit of the progenitor would then be roughly at the beginning of the segment shown. The — 371 comet was definitely a sungrazer of some type—viz. the orbit attributed to Pingré that was tabulated by Galle (1894)—and an unsubstantiated claim by Ephorus indicates that he saw it split.

The scenario outlined in the last few paragraphs is obviously extremely speculative, and the innumerable free parameters that even it entails make it inappropriate to attempt a more rigorous computation. The early evolution is clearly a complex random-walk problem, and the common destination of the backward journeys begun in Fig. 7 is unknown. It is unlikely, however, that the true solution involved excessive excursions from the diagonal path shown, or the orbits of the known sungrazers would fill the q – Ω space more completely. The Kreutz progenitor presumably made ap-

proaches to the Sun before the fourth century BC (with $P \sim 4$ centuries?), and there is little reason to suppose that the fragmentation process was not already occurring then—even if there were only one persistent survivor. The alternative of a — 371 origin but more steps in the process (using intervening ancestors with shorter revolution periods) seems less likely as so few of the early sungrazers seem to have been observed. This is a problem with *any* hypothesis for the evolution of the system, however, and we are painfully aware that the sequence outlined here requires that six of the eight intermediaries that apparently passed perihelion unrecorded during the first 15 centuries of our era ought to have been truly spectacular objects.

It is a pleasure to thank Chris St. Cyr and his colleagues for the enthusiasm with which they have reported and measured the comets detected by SMM. The pioneering discoveries of Don Michels, Neil Sheeley, and their colleagues involved with SOLWIND are also gratefully appreciated. This investigation has also benefited from discussions with Zdenek Sekanina over the years.

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