ON THE RELATIVE NUMBERS OF C TYPES AND S TYPES AMONG NEAR-EARTH ASTEROIDS

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ABSTRACT

The S type (high albedo, optically reddish) asteroids appear overabundant among the near-Earth asteroids (NEAs) compared to their abundance in the main belt. The overabundance is most likely an artifact of the unusual viewing geometries at which the NEAs are discovered. We present a model of this discovery bias for the NEAs.

I. INTRODUCTION

The near-Earth asteroids (NEAs) are dynamically distinguished from the main-belt asteroids by their large eccentricities and small perihelia. The orbits of the NEAs are unstable on timescales $\sim 10^7$–10^8 yr, due to gravitational interactions with the terrestrial planets (Wetherill 1974; Shoemaker *et al.* 1979), necessitating a source elsewhere in the solar system. The source of the NEAs is believed to be a combination, in unknown proportions, of (a) extinct comets and (b) main-belt asteroid fragments from the Kirkwood gaps (Wetherill 1988). One way to check for consistency with the asteroidal origin hypothesis is by comparing the compositional distribution of the NEAs with that of the main-belt asteroids.

It is known that the ratio of the number of S type (high albedo, optically reddish) to C type (low albedo, optically neutral) asteroids in the main belt is a function of heliocentric distance (Zellner 1979; Gradie and Tedesco 1982). At the heliocentric distances of Kirkwood gap sources that have been suggested as NEA sources (3:1 resonance, $R = 2.5$ AU; 5:2 resonance, $R = 2.8$ AU), the number of C types (measured down to a given limiting size) is comparable to or greater than the number of S types. By contrast, optical observational data show that the number of S types exceeds the number of C types among the NEAs (Tholen 1984; Zellner *et al.* 1985, Tedesco and Gradie 1987; Veeder *et al.* 1989). Although the observed overabundance of S types among the NEAs is widely suspected to be the result of observational selection effects, no quantitative analysis has been presented to confirm this hypothesis.

The bias inherent in observing the NEAs is largely discovery dependent. Therefore, we have attempted to model the discovery circumstances of the NEAs in order to investigate the observational selection effects involved in NEA observations. In this paper, we suggest that these selection effects are responsible for the apparent overabundance of S types among the NEAs, and we contend that this overabundance is in part a result of the larger phase darkening of the C types over the S types. The large phase angles and preferential phase darkening of the C type NEAs force some of them below the threshold of detectability (in a magnitude-limited survey), thus exaggerating the number of S type NEAs. By comparison, the main-belt asteroids are not much affected by phase darkening, since the range of phase angles attained by these asteroids is small. Previous investigators have either ignored the differential albedo bias between the NEAs and the main-belt asteroids (McFadden *et al.* 1985; Veeder *et al.* 1989) or have multiplied the detected number of C type NEAs by the ratio of albedos (Tedesco and Gradie 1987), without consideration of the important phase darkening or of the size distribution.

Our strategy was to model the selection effects inherent in both observations of main-belt asteroids and observations of NEAs, in order to compare the respective bias effects. As with the main-belt asteroids, the discovery observations of NEAs must suffer from an albedo bias that favors large, high-albedo asteroids over small, low-albedo asteroids. The albedo bias influences the apparent ratio of the number of S types to the number of C types (n_S:n_C) because the S and C types have systematically different albedos [the mean albedo of the S types is about 3–4 times that of the C types (e.g., Bowell and Lumme 1979)]. In a magnitude-limited survey [for example, the ECAS survey (Tholen 1984; Zellner *et al.* 1985)], the smallest detectable C type will always be larger than the smallest detectable S type because of the albedo difference, if both are located at the same heliocentric and geocentric distance. Therefore, a magnitude-limited survey will always over-represent S types relative to C types, since the S types are being counted down to a smaller size than the C types. For this reason, we distinguish between $(n_S{:}n_C)_{\rm apparent}$ (i.e., the ratio of S types to C types when counted down to a given limiting *magnitude*) and $(n_S{:}n_C)_{\rm true}$ (i.e., the ratio of S types to C types when counted down to a limiting *radius*). We can calculate the true n_S:n_C ratio from the apparent n_S:n_C ratio by means of an albedo bias factor, $B_{S:C}$, defined by

$$B_{S:C} = (n_S{:}n_C)_{\rm apparent}/(n_S{:}n_C)_{\rm true}.$$

Physically, the albedo bias is the number by which the apparent n_S:n_C ratio must be divided, in order to obtain the true n_S:n_C ratio (the ratio that would be observed in a size-limited survey). The essential point of this paper is that, for geometrical reasons, the albedo bias for the NEAs is larger than the albedo bias for main-belt asteroids.

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II. BIAS MODELING

The complexity of the viewing geometry in observations of NEAs prevents an obvious analytical solution to the bias problem. A much more suitable approach is the Monte Carlo method, which can be easily adapted to simulate the range of geometrical parameters that describe the NEAs. We use a single Monte Carlo formalism to model the albedo bias of the NEAs and the main-belt asteroids, the crucial difference between the two cases being the different geometric circumstances under which the NEAs and the main-belt asteroids are observed. The differences between the two models are presented in Secs. II*a* and II*b*, while possible weaknesses of the present treatment are discussed in Sec. IV. For definiteness, we present specific physical parameters in the calculations that follow. However, our basic result is not dependent on the particular values of the adopted parameters, except as noted.

a) NEAs

There is no reliable determination of the size distribution of the NEAs. For our purposes, we represent the size distribution of the NEAs by a power law,

$$dN_a = Ka^{-q}\,da,$$

where dN_a is the number of asteroids with radius in the range a to $a + da$, and K is a constant. Observations of main-belt asteroids indicate that the power law index q lies in the range $2.0 \leqslant q \leqslant 4.0$ (Dohnanyi 1969; Hartmann 1969; Ishida *et al.* 1984) with a most probable value $2.5 \leqslant q \leqslant 3.5$. It has been suggested that a single power law index may not describe the entire asteroid population. For example, the C and S types may have different power law indices, or the indices may change with the size range (Chapman *et al.* 1975; Zellner 1979; Ishida *et al.* 1984; Shoemaker 1989, personal communication). However, until more conclusive evidence is found to show otherwise, a power law size distribution with one power law index remains a reasonable first-order approximation. The power law assumption is further supported by the fact that lunar craters produced by impact of NEAs follow roughly a power law size distribution (Shoemaker *et al.* 1979). The calculation described here can easily be repeated using revised size-distribution functions when these are known.

In the model, the radius of an asteroid is randomly selected from the power law distribution, in the range $a_{\min} \leqslant a \leqslant a_{\max}$. For illustrative purposes, we adopt $a_{\min} = 0.5$ km and $a_{\max} = 5$ km, a size range which encompasses the range of radii estimated from NEA observations. Experiments show that there is no significant change in the NEA bias factors when $a_{\min}$ and $a_{\max}$ are arbitrarily changed by factors of 2; we conclude that our results are robust against changes in $a_{\min}$ and $a_{\max}$ at this level. A random number generator is then used to independently select the three components of the geocentric distance Δ, as measured in a geocentric Cartesian frame having one axis along the projected solar radius vector. The allowable range of Δ extends from the surface of the Earth, $\Delta_{\min} = 4.3 \times 10^{-5}$ AU, to the maximum distance at which an S type asteroid with a radius $a_{\max}$ can be detected, $\Delta_{\max}$. Thus, $\Delta_{\max}$ depends on the magnitude limit of the observational survey, m_{lim}. For example, in the specific case of $m_{\text{lim}} = 15.5$, a 5 km radius S type asteroid with geometric albedo $p_S = 0.15$ can just be detected at $\Delta_{\max} = 1.5$ AU. We have compiled a histogram of Δ for the NEAs at the time of their discovery during the period 1983–1988, using the discovery announcements in the IAU Circulars of the same time period [see Fig. 1(a)]. The figure shows that a majority of NEAs are discovered at distances considerably less than $\Delta_{\max}$, with a broad distribution peaked near $\Delta \sim 0.3$–0.5 AU. We show below that the distribution of Δ generated by our model [Fig. 1(b)] is similar to the observed distribution shown in Fig. 1(a).

After Δ has been selected, the heliocentric distance R is calculated from the three components of Δ by assuming the Sun to be located -1 AU away from the Earth along the projected solar radius vector. A histogram of R at the time of discovery has also been compiled from announcements in the IAU Circulars in the period 1983–1988 [Fig. 2(a)]. Though noisy, the histogram shows that NEA discoveries are peaked slightly outside the orbit of Earth. In the 6 yr surveyed by the IAU Circulars, not one NEA was discovered with $R < 1$ AU. We will also show below that the distribution of R generated by our model [Fig. 2(b)] is similar to the observed distribution shown in Fig. 2(a).

The phase angle α is calculated from R and Δ using the cosine formula:

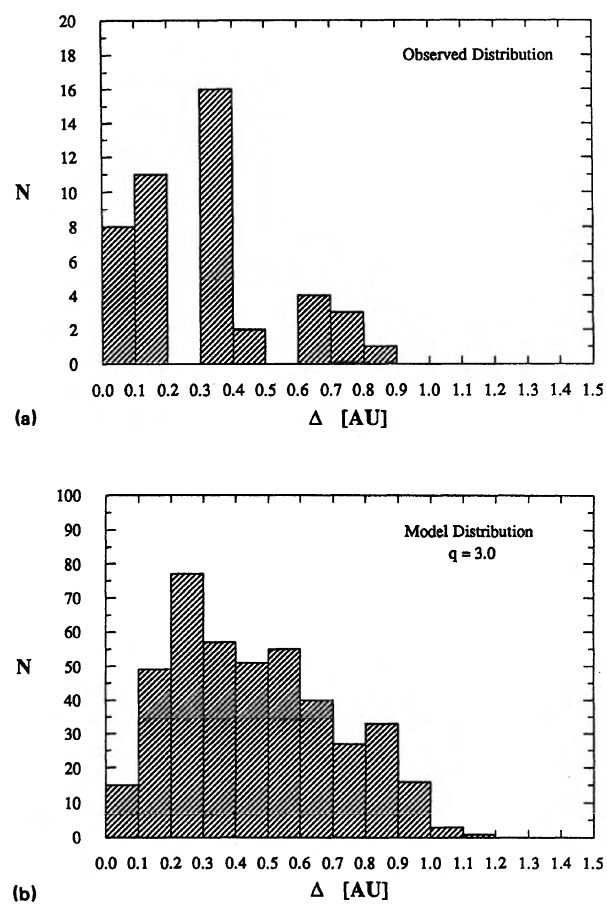

FIG. 1. (a) Histogram of the geocentric distance Δ of NEAs at the time of discovery, compiled from announcements in the IAU Circulars of 1983–1988. (b) Δ histogram of NEAs generated by a Monte Carlo model with $q = 3$ (after phase angle distribution normalization).

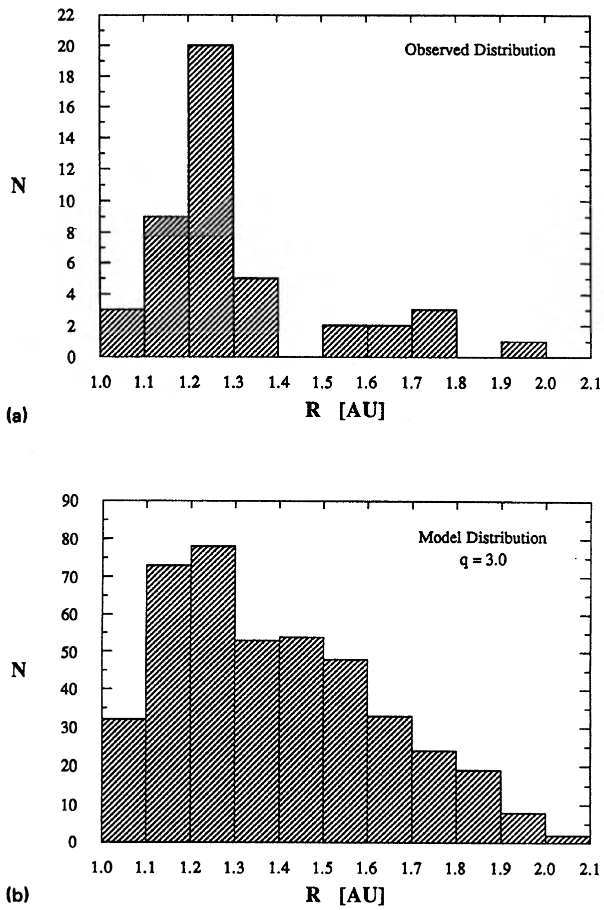

FIG. 2. (a) Histogram of the heliocentric distance R of NEAs at time of discovery, compiled from discovery announcements in the IAU Circulars of 1983–1988. (b) R histogram of NEAs generated by a Monte Carlo model with $q = 3$ (after phase angle distribution normalization).

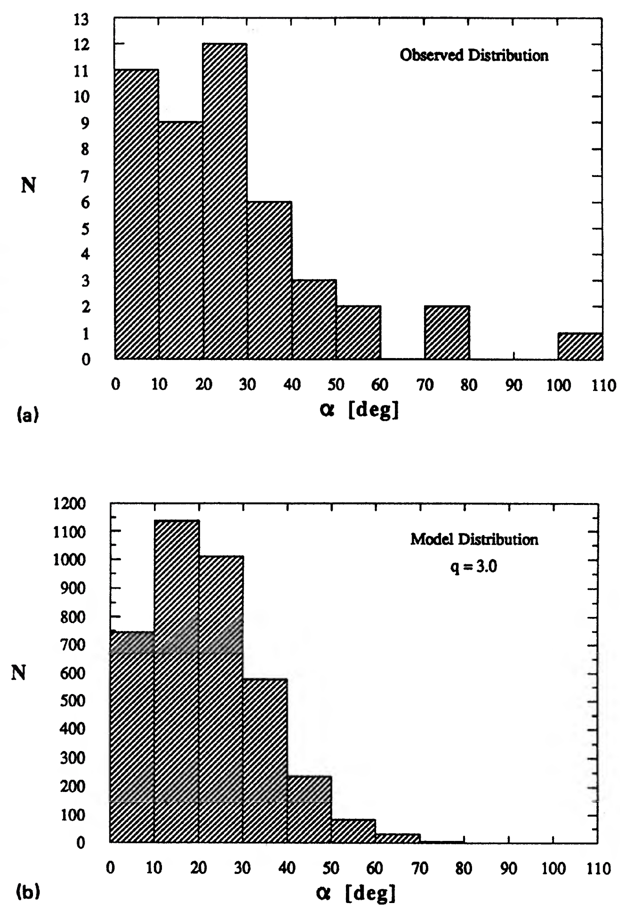

FIG. 3. (a) Histogram of the phase angle α of NEAs at the time of discovery, compiled from discovery announcements in the IAU Circulars of 1983–1988. (b) α histogram of NEAs generated by a Monte Carlo model with $q = 3$.

$$\alpha = \arccos\left(\frac{\Delta^2 + R^2 - 1}{2R\Delta}\right).$$

It is apparent that the NEAs can attain large α, in contrast to the maximum phase angle reached by a main-belt asteroid ($17° \leqslant \alpha_{max} \leqslant 27°$ for $3.5 \geqslant R \geqslant 2.2$ AU). We have compiled a histogram of the phase angles at which the NEAs were discovered during the period 1983–1988, once more using the IAU Circulars [see Fig. 3(a)]. The phase angles at discovery occupy a broad distribution centered on 0° phase angle, with a half-width at half-maximum near 40°. The shape of the α histogram in Fig. 3(a) reflects, in part, the fact that most asteroid observers concentrate their search *near* the opposition point, but not exclusively *at* the opposition point (E. Helin, 1989, private communication). We believe the large width of the phase angle histogram [Fig. 3(a)] is a major reason for the difference between the main-belt albedo bias and the NEA albedo bias.

Knowledge of α and R then allows the computation of the solar elongation ϵ. Asteroids with $\epsilon < 90°$ are discarded, since discovery at such small elongations is unlikely. If the asteroid passes this elongation test, its apparent magnitude m_V is computed from

$$m_V = m_{V(\text{Sun})} - 2.5 \log\left(\frac{pa^2 f}{2.25 \times 10^{16} R^2 \Delta^2}\right),$$

where $m_{V(\text{Sun})} = -26.74$ is the magnitude of the Sun, p is the geometric albedo, and f is the Lumme–Bowell–Harris asteroidal phase function (Minor Planet Circulars 10193–10194, Tedesco 1986):

$$f = (1 - G)\phi_1(\alpha) + G\phi_2(\alpha),$$
$$\phi_1(\alpha) = \exp[-3.33(\tan \alpha/2)^{0.63}],$$
$$\phi_2(\alpha) = \exp[-1.87(\tan \alpha/2)^{1.22}],$$
$$G_C = 0.15 \text{ (for C-types)},$$
$$G_S = 0.25 \text{ (for S-types)}.$$

From the existing literature (e.g., Tholen 1984), we adopt $p_S = 0.150$ for the S type asteroids and $p_C = 0.047$ for the C types. The phase parameter G is closely related to and functionally replaces the linear phase coefficient β that was previously used to fit asteroid photometry (Bowell and Lumme 1979). The Lumme–Bowell–Harris function has the advantage that it accounts for the nonlinearity of the phase curve; it is reportedly applicable in the phase angle range $0° \leqslant \alpha \leqslant 120°$. The G distributions of asteroids are peaked at $G_C = 0.15$ for C types and $G_S = 0.25$ for S types [see the Asteroid II Database in *Asteroids II* (Tedesco 1989)]. Therefore, we follow the practice described in Tedesco (1986) of using these values of G to represent the C and S classes.

The selected asteroid is next subject to a detection criterion. The asteroid is "detected" (counted) if $m_V \leq m_{\text{lim}}$, where m_{lim} is the limiting magnitude in V of the observational survey, and rejected otherwise. For the present model, we adopt $m_{\text{lim}} = 15.5$ as suggested by the magnitudes of the NEAs at discovery and reported in the IAU Circulars. In reality, due to the nonlinearity of photographic plates (the primary tool of NEA discoverers), the probability for discovery of NEAs does not abruptly drop to zero at m_{lim}. Rather, it starts dropping gradually toward zero at magnitude $m < m_{\text{lim}}$ and actually reaches zero at magnitude $m > m_{\text{lim}}$. The angular velocities of the NEA may also help to blur the cutoff at m_{lim} since fast-moving NEAs have a lower discovery probability due to trailing loss. For simplicity, we assume that the main detection criterion is the asteroid's magnitude, and that the NEA discovery probability can be approximated by a step function that is unity at $m \leqslant m_{\text{lim}}$ and zero at $m > m_{\text{lim}}$.

A running count of the number of detected asteroids of each spectral type is recorded until $n_C + n_S = N_{\text{total}}$. Typically, we used $N_{\text{total}} = 10{,}000$, since this number gives acceptable signal to noise in the computed n_S:n_C ratio and, furthermore, can be attained in a reasonable computing time (1–10 cpu hours on a Sun-4 computer). As a final step, the model phase distribution is normalized to the observed phase distribution [Fig. 3(a)], so as to mimic the angular dependence of the discovery observations. The n_S:n_C ratio is re-evaluated following the normalization. Figure 3(b) shows the distribution of phase angles produced by a typical ($q = 3$) model.

The resultant distributions of Δ and R for the NEAs produced by a typical Monte Carlo model (having $q = 3.0$) are shown in Figs. 1(b) and 2(b). The model and actual distributions appear similar [cf. Figs. 1(a) and 1(b), 2(a) and 2(b)], giving us confidence that the model provides a reasonable approximation to the actual circumstances of discovery. Physically, the shape of the histogram in Fig. 1(b) represents competition between the probability that a given NEA will lie within a volume of characteristic size Δ (this probability increases as Δ^3) versus the decreasing likelihood that a distant NEA will be bright enough to satisfy the magnitude selection criterion. Figure 2(b) shows a peak near $R \sim 1.2$ AU, which qualitatively reproduces the peak in the observed distribution at the same R [Fig. 2(a)].

With the model described above, we have calculated the NEA albedo bias for $q = 2.0, 2.5, 2.8, 3.0, 3.5, 4.0$. A summary of the geometric and physical parameters used in the NEA model are listed in Table I.

TABLE I. Sample parameters used in the albedo bias model.

Parameter	NEA Value	Main-Belt Value (3:1 Resonance Region)
m_{lim}	15.5	15.5
a	$0.5 < a < 5.0$ km	$5.0 < a < 50$ km
R	Calculated (see text)	$1.80 \leq R \leq 3.30$ AU
Δ	$4.3 \times 10^{-5} \leq \Delta \leq 1.5$ AU (the maximum distance at which a 5 km S-type asteroid can be seen.)	Calculated from R and α: $\Delta = R\cos\alpha - (R^2\cos^2\alpha - R^2 + 1)^{1/2}$
α	Calculated from R and Δ: $\alpha = \cos^{-1}[(\Delta^2 + R^2 - 1)/2R\Delta]$	$0 \leq \alpha \leq \sin^{-1}(1/R)$
G	$G_S = 0.25$; $G_C = 0.15$	$G_S = 0.25$; $G_C = 0.15$
p	$p_S = 0.150$; $p_C = 0.047$	$p_S = 0.150$; $p_C = 0.047$
q	2.0, 2.5, 2.8, 3.0, 3.5, 4.0	2.0, 2.5, 2.8, 3.0, 3.5, 4.0
N_{total}	10,000	10,000

b) Main-Belt Asteroids

We briefly discuss our calculations performed for asteroids in the main belt. Strictly, there is no need to compute bias factors for the main-belt asteroids, since the main-belt population is sufficiently well known that true (diameter limited) n_S:n_C ratios are already available (e.g., Zellner, 1979), at least for the larger asteroids. However, it is informative to note the important geometric differences which lead to a different (smaller) value of the bias factor in the main belt as compared to the NEA population.

The model for the main-belt asteroids is constructed along similar lines to the NEA model, with differences where appropriate for the main-belt population. The radius of each asteroid is again selected from a power law size distribution. A random number generator then selects the three components of R in a heliocentric Cartesian frame, and selects α according to $0 \leqslant \alpha \leqslant \alpha_{\max}$ (where $\alpha_{\max} = \arcsin 1/R$). The geocentric distance Δ is then calculated from R and α using the cosine formula:

$$\Delta = R \cos \alpha - \sqrt{R^2 \cos^2 \alpha - R^2 + 1}.$$

For the sake of definiteness, we present in Table I the parameters used for asteroids near the 3:1 resonance. The resonance is located at $R = 2.5$ AU. Asteroids with semimajor axes $\leqslant \pm 0.04$ AU from the resonance have perihelia and aphelia contained within the range $1.8 \leqslant R \leqslant 3.3$ AU (*Ephemerides of Minor Planets* 1988), hence we adopt this heliocentric distance range for the present calculation. The albedo biases computed for other regions in the main belt are similar to the specific case discussed here, since the maximum attainable phase angle is not a very strong function of R across the main belt. The majority of the known asteroids in the 3:1 resonance region falls in the size range 5–50 km (*Ephemerides of Minor Planets* 1988). The S and C types in the main belt are also assumed to be represented by the same G and p values as among the NEAs. As in the NEA model, an asteroid is discarded if its elongation is $\epsilon < 90°$, and if its apparent magnitude satisfies $m_V > m_{\lim}$ (the main detection criterion is again assumed to be the asteroid's magnitude). Using this model, we have calculated the main-belt albedo bias for the cases $q = 2.0$, 2.5, 2.8, 3.0, 3.5, 4.0.

III. RESULTS

The albedo bias factors $B_{S:C}$ resulting from the application of the model to the NEA and main-belt populations are presented in Table II. For the several power law models of the NEAs summarized in Table II, we find $5 \leqslant B_{S:C} \leqslant 6$. Thus, the apparent ratio of S type to C type asteroids among the NEAs must be reduced by a factor 5–6 in order to obtain the true ratio. The table also shows that $B_{S:C}$ for the NEAs is greater than or equal to $B_{S:C}$ for the main belt, for all q in the range $2 \leqslant q \leqslant 4$. This means that the S type asteroids should be even *more* over-represented in observations of the NEAs than they are in observations of the main-belt asteroids, qualitatively consistent with the apparent overabundance of S types among the NEAs. Since the true asteroid distribution has already been determined for the main belt (Zellner 1979; Gradie and Tedesco 1982; Gradie *et al.* 1989), the novel result here is the size of the selection effect among the NEAs, which must be considered in future observations of this group of asteroids.

IV. DISCUSSION

For the purpose of comparison, we list in Table III the n_S:n_C ratios in the main belt, according to the available optical surveys and the *IRAS* survey (Matson 1986). The *IRAS* survey suffers from subtle biases of its own (e.g., Spencer *et al.* 1989); however, it is not subject to the same albedo bias which afflicts magnitude-limited optical surveys, and so it presumably better reflects the true main-belt population. The n_S:n_C ratio from the *IRAS* data is similar to the ratio $[(n_S{:}n_C)_{\text{true}} \sim 0.6]$ deduced long ago by Zellner (1979) and others. We are reassured that our bias correction of the raw ECAS survey for the main-belt asteroids (assuming

TABLE II. Model bias correction factors.

Power law index q	$B_{S:C}$ (NEA)	$B_{S:C}$ (main-belt)	$B_{S:C}$ (NEA) / $B_{S:C}$ (main-belt)
2.0	5.61 ± 0.14	2.15 ± 0.05	2.61 ± 0.09
2.5	5.38 ± 0.13	2.58 ± 0.06	2.09 ± 0.07
2.8	5.73 ± 0.14	3.10 ± 0.07	1.85 ± 0.06
3.0	5.68 ± 0.14	3.41 ± 0.08	1.67 ± 0.06
3.5	5.65 ± 0.14	4.27 ± 0.11	1.32 ± 0.05
4.0	5.88 ± 0.15	5.50 ± 0.15	1.07 ± 0.04

$q = 3.0$) yields a ratio $(n_S{:}n_C)_{\rm true} = 0.6{:}1$, consistent with those reported by other sources. The bias factors for the main-belt asteroids are in fact similar to the factors computed analytically (neglecting phase darkening) for a magnitude-limited survey at each size distribution index q. This is as expected, since the effects of phase darkening are relatively small in the main belt.

The computed true $n_S{:}n_C$ ratios for the NEAs are also listed in Table III. The listed errors in the ratios for the NEAs are lower limits to the true errors, calculated according to Poisson statistics. In view of the uncertainties due to the small number of classified NEAs, the corrected NEA $n_S{:}n_C$ ratios are close to the true $n_S{:}n_C$ ratios in the main belt. The dramatic apparent overabundance of S types among the NEAs is thus plausibly explained as an artifact of the discovery bias described above.

Several, probably significant uncertainties exist in our calculation of the NEA bias factors. Particularly important uncertainties exist regarding the parametrization of the observational search strategies employed by the discoverers of NEAs. For example, the discoveries of NEAs depend on the work of several wide field observers who use different types of instrumentation. It is most likely that a single limiting magnitude does not fully characterize the discovery circumstances of those bodies (see Sec. II *a*). The different limiting magnitudes translate into different size ranges for the detectable NEAs, which in turn influence the distributions of R and Δ in which these objects are detected. It is not possible to exactly model the inhomogeneous observational techniques that characterize the diverse programs that have contributed to the discovery of NEAs. The similarity between the model and the actual distributions of Δ and R (Figs. 1 and 2) suggests that the basic characteristics of the current observational search strategies have been captured in the model, but there is room for improvement. It would be helpful to have an improved account of the search strategies used by the discoverers of NEAs, from which more detailed models of these search strategies can be constructed.

A second class of uncertainties stems from incomplete knowledge of the physical parameters of the asteroids, particularly of the NEAs. Recent work by Veeder *et al.* (1989) emphasizes the difficulty involved in establishing the physical properties of the NEAs from limited data. For example, the geometric albedos of the NEAs are rarely known with confidence, since, even when the scattered and thermal emissions are measured, lack of knowledge of the visual and infrared phase coefficients prevents extrapolation to 0° phase. The G parameter of the Bowell–Lumme–Harris phase function is determined from asteroid photometry as a function of phase angle. The validity of their phase function is largely untested at very large phase angles, since most asteroid photometry is obtained at small phase. Uncertainties in the relative phase functions of the C and S asteroids may lead to values of the bias factors different from the ones calculated here. In addition, too few NEAs have been measured to be sure that their size distribution conforms closely to a power law. Another likely complication is that some NEAs could be misclassified as a result of the neglect of phase reddening. It is known that asteroidal colors become redder with larger phase angles (Bowell and Lumme 1979; Lumme and Bowell 1981). Although it is a small effect (typically ~0.001–0.003 mag/degree) among main-belt asteroids, the NEAs can reach sufficiently large phase angles ($\alpha \geqslant 100°$) that the phase reddening may be significant (0.001 mag/deg$\times 100° = 0.1$ mag) and could induce false classification of NEAs, if the classification is based on asteroidal colors

TABLE III. Relative numbers of S types and C types.

Source	NEAs		Main-Belt (3:1 Resonance)	
	Apparent $n_S{:}n_C$	True $n_S{:}n_C$	Apparent $n_S{:}n_C$	True $n_S{:}n_C$
Tedesco and Gradie (1987)	24 : 5	0.8 ± 0.4 : 1 *	---	---
Veeder *et al.* (1989)	20 : 2	1.8 ± 1.3 : 1 *	---	---
TRIAD	---	---	---	0.6 : 1
Gradie and Tedesco (1982)	—	—	—	0.7 : 1
Gradie *et al.* (1989)	—	—	—	0.7 : 1
IRAS	---	---	0.8 : 1	0.8 : 1
ECAS	11 : 1	1.9 ± 1.9 : 1 *	1.9 : 1	0.6 : 1 *

* The bias correction was carried out by the authors of this paper, using the bias factors calculated with $q = 3.0$ (see Table 2).

taken at large α. (It should be noted here that although the majority of the NEA *discovery* phase angles are less than 45° [see Fig. 1(a)], NEAs are frequently *observed* subsequently at much larger α, where phase reddening can be important.) The present model could be improved if the physical parameters of the NEAs, especially the phase function and the size distribution, were known with greater confidence. Presumably, future observations will lead to this improvement.

A final caution concerns the distinction between bias effects that afflict the discovery of NEAs and those that afflict classification of NEAs. The bias factors presented here were calculated on the assumption that S and C types are equally likely to receive taxonomic classification once discovered. We know of no evidence to invalidate this assumption. However, it is possible that selective classification of detected NEAs may result in bias factors different from (larger than) those presented here. In view of these potential uncertainties, we regard the computed bias factors only as a guide to the corrections which must be applied to the apparent statistical distributions of C type and S type NEAs. It seems clear that there is presently no observational basis for the claim that the true $n_S:n_C$ ratio among the NEAs is different from the main-belt ratio. In future observational programs, it will be important to correct for all differential phase effects such as phase darkening and/or phase reddening before calculating the true distribution of spectral types among the NEAs.

V. CONCLUSIONS

By using a Monte Carlo approach to model the albedo bias of the NEAs and the main-belt asteroids, we find:

(1) The calculated bias factors for the NEAs are in the range $5 \leqslant B_{S:C} \leqslant 6$, and are large enough to account for the apparent overabundance of S types among the NEAs as reported by various magnitude-limited surveys.

(2) The NEA bias factors are larger than those for the main-belt population, because the NEAs can be (and are) discovered at larger phase angles than the main-belt asteroids, and hence suffer more from differential phase darkening.

(3) There is no compelling observational evidence for a difference in the ratio of S type to C type asteroids between the NEAs and the main belt.

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