

A COMPARISON OF DIGITAL CENTERING ALGORITHMS

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ABSTRACT

Synthetically produced star images have been used to investigate five one-dimensional centering algorithms for a wide range of observing conditions. This work confirms the earlier photographic results of van Altena and his collaborators in finding that the Gaussian fit and median routines are good centering methods, while the moment and derivative-search routines are found to be inferior. A modification to the moment analysis that suppresses the sky background below a prechosen threshold level is found to be the best centering method when the sky-background level is significant. Detailed discussions of the advantages and disadvantages of each of the methods considered herein are given as well as a discussion of the real-time image-processing techniques used by the USNO 7 in. transit circle.

I. INTRODUCTION

In order to achieve even greater accuracies, the trend in modern measuring techniques has moved from the use of analog centering devices toward the processing of digitized image data with a chosen centering algorithm. Major papers dealing with the processing of digitized star images are those of van Altena and Auer (1975), Chiu (1977), Auer and van Altena (1978), and Chiu, van Altena, and Stetson (1979) (hereafter, these papers will be collectively referred to as VAACS), and theoretical aspects of digital image processing have also been reported by Lindegren (1978), King (1983), and Lee and van Altena (1983). The centering algorithms considered by van Altena and his collaborators (VAACS) consisted of various forms of the Gaussian functional fit, moment, median, and the derivative search. Of these methods, their choice consists of fitting a one-dimensional symmetric Gaussian profile with a flat background to each of the observed marginal distributions. They also conclude that two-dimensional centering methods are much slower than the processing of marginal distributions and only slightly more accurate. The pioneering effort made by VAACS in image processing has led to a wide range of applications to astrometry (e.g., Monet and Dahn (1983), van Altena *et al.* (1986), and Cudworth and Raucher (1987), to name a few).

The purpose of this paper is to further investigate these digital centering algorithms. In addition to the algorithms considered by VAACS, this paper will also discuss a modification to the moment analysis wherein a star image can be trimmed at a prechosen threshold above the background-sky level, a method used by Pratt (1977) and others with great success. As will be subsequently discussed, this procedure offers a significant gain in accuracy, particularly for weak images, over the traditional treatment of the moment. Other centering methods which are intended for the processing of large amounts of data at low precision (e.g., those described by Herzog and Illingworth (1977), Pratt (1977), and Tyson and Jarvis (1979)) will not be included in this discussion.

As a means of investigation, this paper will compare the various centering routines under consideration by their ability to process synthetically generated star images accurately. Following the recommendation made by VAACS, only one-dimensional methods (that is, those that process marginal distributions) will be considered in this paper. The use of simulations is extremely useful in that synthetic star images can be created to cover just about any type of situation,

thereby enabling detailed investigations to be made, and synthetic data also allow for the various centering techniques to be investigated free from the influence of instrumental factors. It is important that conclusions based solely upon the analysis of simulated star data not be overinterpreted so, whenever possible, the results of this paper will also be compared with the empirical results presented by VAACS. Throughout this paper, the detector producing the digital data will be assumed to be linear and governed by Poisson statistics, which includes the numerous pulse-counting detectors with two-dimensional digital outputs that are available today. The results of this paper are also applicable to the processing of photographic plates with unsaturated images, since the statistics of photographic grains mimic that of Poisson noise (see Lee and van Altena (1983) and the references cited therein for more details).

Model image results for each of the centering routines under consideration in this paper will be presented covering a wide range of seeing conditions, background-sky levels, and star-image brightnesses. Each of these centering methods will be evaluated in terms of accuracy, speed, convergence properties, and the ability to detect either faint images or to deal with bright sky backgrounds. The final consideration is important for badly darkened first-epoch photographic plates and for the ability to observe bright stars during the daytime in fundamental work (see Sec. VI). Sections II and III of this paper will describe the simulations and the centering algorithms considered herein; Sec. IV will evaluate each of the centering routines in terms of the criteria given above; and Sec. V will investigate systematic errors caused either by the star image being too close to a scan-array boundary or by not being sufficiently resolved by the pixel spacings. Finally, a discussion of the real-time image-centering techniques and procedures in current use with the USNO automated 7 in. transit circle will be given in Sec. VI.

II. THE GENERATION OF SYNTHETIC STAR IMAGES

In this paper, the model chosen for a star image consists of a two-dimensional circular Gaussian with a flat sky background and smeared with Poisson noise. The adjustable parameters include (1) the image position (x_c, y_c) within the scan box, (2) the rectangular scan-box dimensions (n_x, n_y) , (3) the aperture size a in arcseconds, (4) the step spacing s between adjacent pixels in arcseconds, (5) the seeing (FWHM) ϕ in arcseconds, (6) the total number of counts N within the image, and (7) the sky-background level b ex-

pressed in units of counts arcsec^{-2} . The above notation and specified units of measure will be used throughout this paper. In order to keep this investigation tractable, it will be assumed that the aperture is square in shape, and the step interval in each coordinate direction is identical. Moreover, only the fully sampled case defined when the aperture is equal to the step size ($a = s$) will be considered. Undersampling of the image ($a < s$) is inefficient, and Chiu (1977) finds no gain in accuracy by oversampling. The distribution function of counts at any point (ξ, η) within the scan area is given by the following equation, where R is the rms half-width of the image ($1 \text{ FWHM} = 2.36R$), and the other parameters are as defined above.

$$F(\xi, \eta) = b + \left(\frac{N}{2\pi R^2} \right) \exp \left[-1/2 \left(\frac{\xi - x_c}{R} \right)^2 \right] \times \exp \left[-1/2 \left(\frac{\eta - y_c}{R} \right)^2 \right]. \quad (1)$$

The count level within each pixel is produced by numerically integrating the stellar image $F(\xi, \eta)$ over the aperture area, which is centered on the pixel center. A two-dimensional trapezoidal rule and a 5×5 grid pattern were used for all integrations. Tests conducted with finer grid spacings for the integration gave the same results but took prohibitively longer to execute. Once the integration is completed, the count level within each pixel is then smeared with Poisson noise. Figure 1 illustrates a scan containing two synthetic star images. These routines are part of an image-generating and processing package developed for the USNO 7 in. transit-circle program.

III. DIGITAL CENTERING ALGORITHMS CONSIDERED

In the following, a description of each of the digital centering algorithms under consideration in this paper will be presented. Included are the Gaussian functional fit, moment,

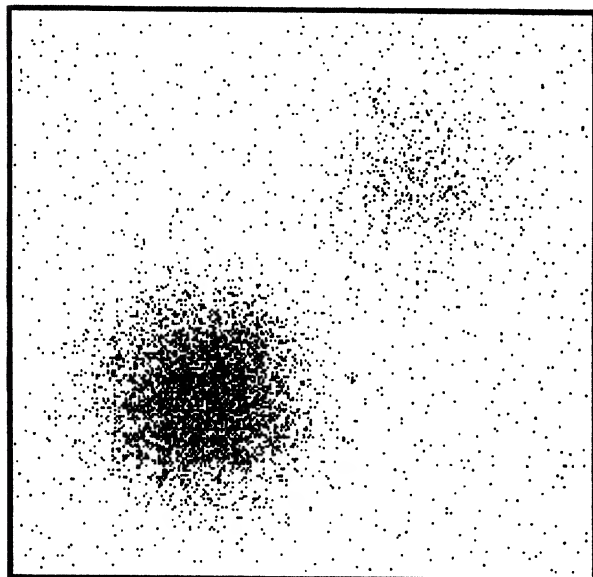


FIG. 1. Illustration showing two artificially produced star images of differing brightnesses that are representative of those used within this study. The scan size is 30×30 pixels, and a dot representation has been used to show the count level within each pixel.

modified moment, median, and derivative search. Only the one-dimensional representations for each of these methods will be considered; they are one dimensional in the sense that an array of count data $I(x, y)$ can be reduced to a one-dimensional distribution of counts by forming marginal distributions in each coordinate direction. For the x marginal, each marginal array entry is defined by

$$M(x) = \sum_y I(x, y). \quad (2)$$

A similar expression exists for the y marginal and, as a matter of convenience, only the x coordinate version of equations will be given throughout this paper, with the understanding that the y coordinate representations are completely analogous. Procedures common to all of the centering algorithms will be discussed in the following section.

a) Symmetric Gaussian

According to VAACS, this is the recommended algorithm for the processing of digitized photographic images. It consists of fitting a symmetric Gaussian function with a flat sky background to each of the coordinate marginal distributions. The basic equation of condition is given below:

$$M(x) = B + H \exp \left[-(x - x_c)^2 / 2R^2 \right], \quad (3)$$

where B is the background level as defined in the marginal (note that $B = n_y b$, where n_y is the array cross dimension used in creating the marginal distribution), and H is the central image height in the marginal distribution, which can also be expressed in terms of the total number of counts N included in the image with the relation

$$H = N / (2\pi)^{1/2} R. \quad (4)$$

This algorithm is a parametric approach and requires a solution for the four parameters expressed in Eq. (3) which can be easily linearized with a first-order Taylor's expansion and solved by iterative least squares. As recommended by Auer and van Altena (1978), the initial values for these parameters were estimated from the separate marginal distributions, and convergence was defined at the point when the maximum change in the x_c value was under $\pm 0.001 \text{ arcsec}$.

b) Moment

In this method (e.g., see Auer and van Altena 1978), the image centroid is set equal to the first moment of the marginal distribution, which has been corrected for the contribution due to the background level. The basic equation is given below:

$$x_c = \sum_x x [M(x) - B] / \sum_x [M(x) - B]. \quad (5)$$

Following Auer and van Altena, the mean of the outer marginal endpoints can be used to calculate the background level B , and this method for determining the background level will be used throughout this paper unless otherwise indicated. A good discussion of moments and their application to astrometry has been given by Strobie (1980). The moment algorithm, as defined above, was investigated by van Altena and Auer (1975) and Auer and van Altena (1978) and found to be less accurate than the functional fit to a symmetric Gaussian. In particular, they found that this method was sensitive to count levels in the background and particularly so for badly centered star images. The fact that this method shows this form of sensitivity is not surprising since, like the

arithmetic mean, points located away from the image center are weighted in the solution by the square of their distance from it. Hence, a poor determination for the background level, the presence of several large count levels in the background, or a badly centered image within a scan could cause a serious degradation of the centering process. More about this will be discussed in Secs. IV and V. In order to keep this algorithm from being confused with the following one, it will sometimes in the following discussions be referred to as the "traditional moment."

c) Modified Moment

This paper will also consider a modified version of the moment analysis, just described, that is much less sensitive to noise in the sky-background level. In this method, most of the count data in the background are effectively eliminated by defining a threshold level above that set by the background and then only considering those count levels in the data array that are above this threshold in the centering process. Specifically, once the background level b is determined in the $I(x, y)$ data array, each pixel in the array is thresholded according to the following precepts:

$$\begin{aligned} I(x, y) &= I(x, y) - b & \text{if } I(x, y) \geq T, \\ I(x, y) &= 0 & \text{if } I(x, y) < T, \end{aligned}$$

where T is the chosen threshold level. In this paper, the threshold was chosen at the 3σ level above that of the smoothed background. For images governed by Poisson statistics, this threshold is just $kb^{1/2}$, where k is a factor ($k < 1$) defined by the method of smoothing used. It is important that the sky-background level be determined quite accurately in order for this method to work, and this paper finds that if the sky level is defined as the mode of the histogram of counts in the data array, then even images subjected to very high sky levels can be well centered (see Bijaoui (1980) for a discussion of using the mode to define the sky level). After the array is thresholded in this fashion, essentially eliminating 99.7% of the background-sky counts, then Eq. (5) can be used to determine the centroid in each coordinate. The tradeoff in this method is that some of the count data in the image wings will be lost through the thresholding process. Tests made herein with simulated star data show that this loss will have a negligible impact on the accuracy of the moment centering, except possibly for very skewed star images associated with very bright background levels. As will be subsequently discussed, this modification of thresholding the image before initiating the centering analysis offers a very significant improvement over that of the traditional moment method.

d) Median

The median is a fractile method, wherein the center of an image x_c is defined to be that value of x that divides the starcount data into two equal halves. This centering method is not sensitive to wild points in the marginal data and, according to Chiu (1977), is a good method for the centering of faint star images. In terms of procedures, after the background level is subtracted from each element of the marginal distribution, a cumulative distribution of counts $Q(x)$ can be subsequently defined that starts from the first marginal element. The median can then be defined as that value of x for which $Q(x) = \frac{1}{2} Q(x_{\max})$. Since the distribution function is defined empirically, interpolation must be used to define the

value of the median. A weakness of this centering routine is its sensitivity to the chosen method for the interpolation, and numerical experiments conducted in this study indicate that the process of interpolation is sensitive to both the size of the pixel spacing (that is, to the resolution of the image into pixel elements) and to the amount of noise associated with the count data in the central part of the image. Linear interpolation can give very large systematic errors. Instead, Chiu (1977) has used a quadratic polynomial interpolation for determining the median but, even with this refinement, this paper finds that the centering errors caused by this method of interpolation can still be as large as ± 0.04 pixels. In this paper, a cubic interpolation will be used throughout which is quite accurate for the star images under consideration.

e) Derivative Search

This centering algorithm is described by Stetson (1979) and is recommended by VAACS for the segmentation of large scans into subarrays containing individual star images. The following methodology is largely taken from him. Once the marginal is formed, a smoothed derivative $G(x)$ can be determined at each of the marginal grid points by using the following definition for a numerical derivative:

$$\begin{aligned} G(x) &= \frac{1}{2} [M(x+1) - M(x-1)] \\ &+ \frac{1}{4} [M(x+2) - M(x-2)]. \end{aligned} \quad (6)$$

The array of derivatives is then searched for its maximum and minimum values, and a line is fit by least squares to the array points between these two limits. For Gaussian-shaped images, the x intercept of this line is the image centroid, and the fitting interval includes those marginal points within $\pm R$ units of the image center. Because this method only uses the central part of the star image, it is of little use for the centering of saturated images. Chiu *et al.* (1979) found this method to be less accurate than that of a Gaussian functional fit, but because of its greater speed, found it nevertheless useful for the preprocessing of large amounts of image data.

IV. COMPARISON OF THE DIFFERENT CENTERING ALGORITHMS

Besides the choice for a centering algorithm, some discussion is also needed to describe the procedures followed in conjunction with the use of these algorithms. First of all, various techniques for smoothing the digital data before applying the centering routine were tried. For well-defined stellar images, it was found that smoothing the data offered only a marginal gain in positional accuracy, which agrees with Chiu's (1977) analysis of photographic stellar images, but on the other hand, this paper finds that smoothing did seem to offer some gain when the images were quite weak or subjected to very high background-sky levels. After some numerical experimentation with differing smoothing techniques, the smoothing method adopted for use in this paper was chosen as the three-channel binomial filter described by Bevington (1969), which was generalized for the two-dimensional case. This low-pass filter is quite fast and can be further accelerated with vector-manipulating firmware.

Moreover, the trimming of images, that is, the rejection of count data far away from the image center, is very important for several of the centering methods. Chiu *et al.* (1979) recommend truncating the count data further away than 2–3 times the image half-width R from the image center. As will be described in the next section, this is a reasonable conclu-

sion, and in this paper the trimming will be done throughout at the 2.5 times level. The use of trimming requires that each centering solution be iterated until the trimming process is convergent, and tests conducted in this paper found that convergence was usually reached after about 2–4 iterations with little difference between the individual methods. Only the Gaussian algorithm provides for a direct determination of the image half-width. For the other methods, the width was estimated from a second-order moment analysis of the marginal data which agreed very well with the corresponding Gaussian result. As an alternative to trimming, the various robust weighting schemes described by Coleman *et al.* (1980) and the Pierce routine discussed by Chauvenet (1960) were also investigated, and the numerical results were quite clear. The robust schemes offered no clear improvement in accuracy over the simple process of trimming. On the other hand, they often considerably lengthened the amount of time needed to determine a centroid, since they generally required many iterations before reaching convergence. Stigler (1977) has reached a similar conclusion about the use of robust weighting schemes.

With these procedures, the centering routines under discussion in this paper and the following conditions:

Counts in image $N = 30\text{--}300,000$ counts,
 Seeing $\phi = 1, 2,$ and 4 arcsec,
 Sky level $b = 1, 10, 100,$ and 1000 counts arcsec $^{-2}$,

were used to generate a grid of centering results as gleaned from synthetically produced stellar images. Common to all of these numerical results, the chosen array box size was 40×40 pixels, both the aperture and step size were set to $\frac{1}{2}$ of the image FWHM, and each image was displaced from the

box center by 10% of the array size in order to evaluate any effect caused by decentering. These scanning parameters are similar to those used by VAACS. Each result for a star image is based upon the average of 25 simulations smeared with Poisson noise. A reliable image centroid could not be determined for some of the weaker images and for some images that were subjected to very high sky-background levels. For the conditions in this section, the extreme seeing and sky-level cases are shown graphically in Figs. 2–5. The in-between cases included in this study turn out to be just smooth gradations between these extremes, and in order to limit the amount of discussion, they will not be discussed in the following at any length. As might be expected, Figs. 2–5 show the well-known result that the centering accuracy and the ability to detect weak stellar images is degraded by both poor seeing conditions and high background levels.

The results of the numerical calculations show that two of the centering algorithms are decidedly inferior to the others. As shown in all of the figures, the derivative-search routine is significantly less accurate than the other methods and becomes increasingly less so for progressively weaker images or for higher background levels. The method is particularly sensitive to noise in the central part of the image. Stetson (1979) uses this method for segmenting stellar images in crowded fields, but in light of the above findings, its usefulness for high-precision astrometry is quite limited. Moreover, the traditional-moment centering routine has several drawbacks. For low sky levels ($b < 10$ counts arcsec $^{-2}$), it is comparable to the best of the centering routines (see Figs. 2 and 3). However, as noted by Auer and van Altena (1978) in their study of photographic stellar images and confirmed in this study, the moment is very sensitive to count levels in the sky background. In particular, increased sky levels ($b > 10$ counts arcsec $^{-2}$) tend to shift the image systemati-

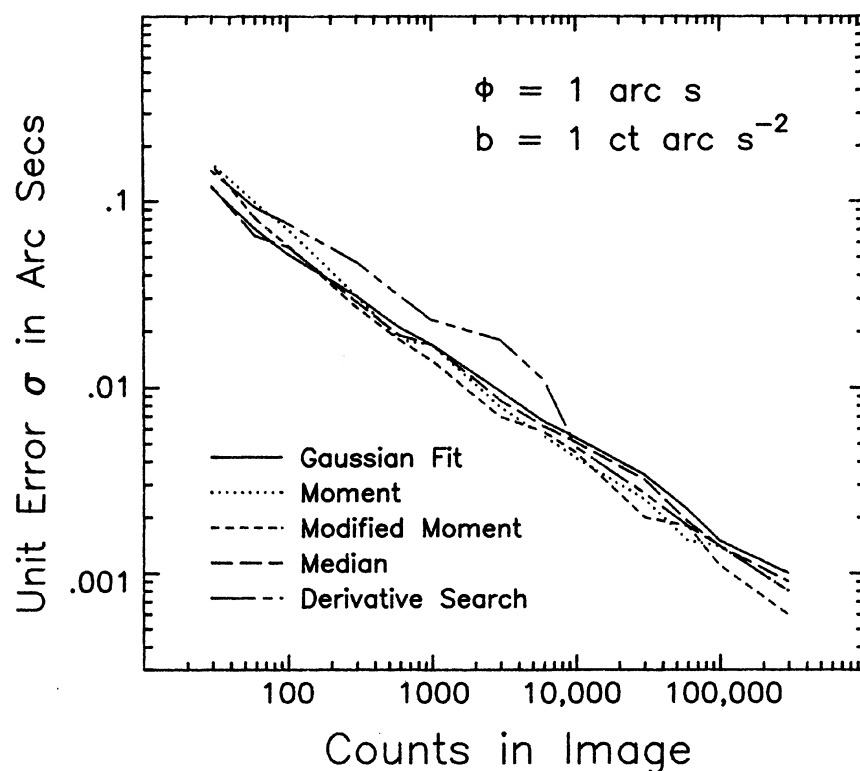


FIG. 2. Expected mean centering error for good seeing ϕ and a negligible sky background b for the methods considered in this paper.

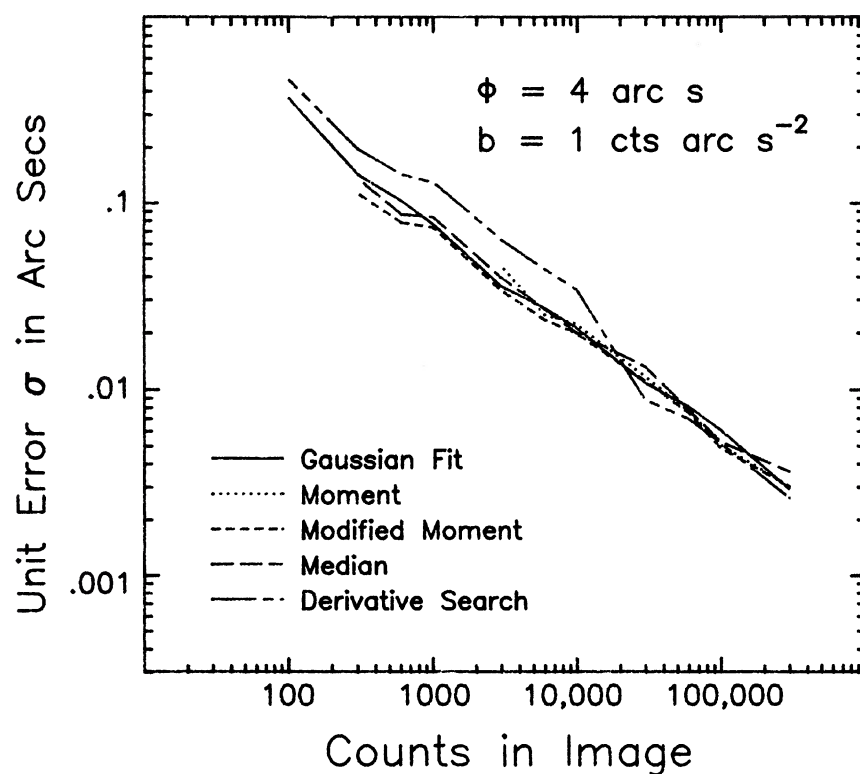


FIG. 3. Expected mean centering error for poor seeing ϕ and a negligible sky background b .

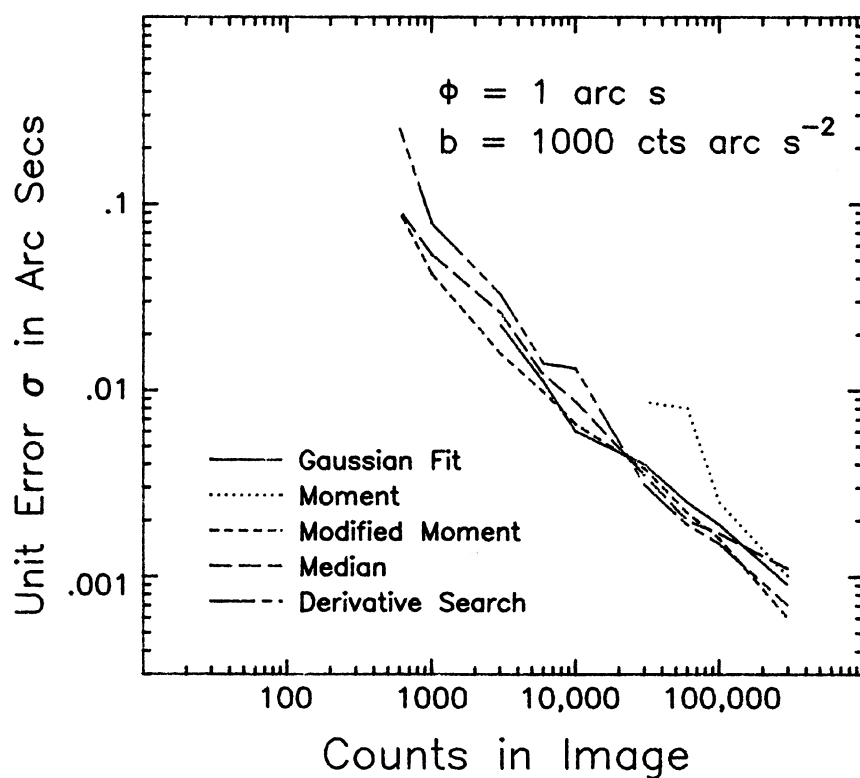


FIG. 4. Expected mean centering error for good seeing ϕ and a bright sky background b .

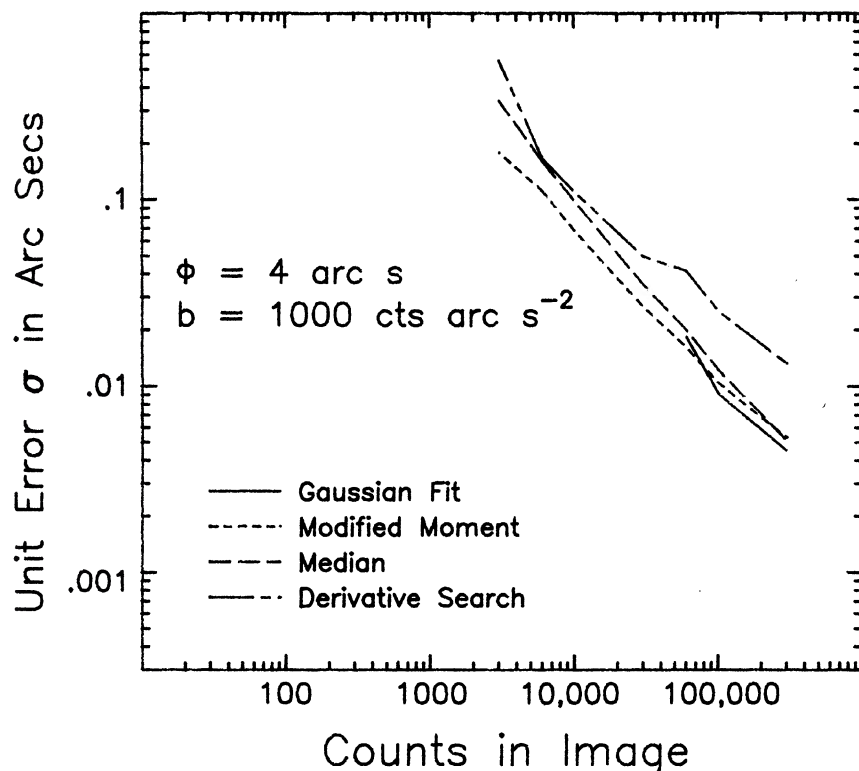


FIG. 5. Expected mean centering error for both poor seeing ϕ and a bright sky background b . Because the moment method did so poorly under these conditions, it is not plotted in the figure.

cally toward the center of the scan box. For the completely sky-dominated case, the centroid will just be the center of the scan box irrespective of the location of the star image. In Figs. 2–5, the results for the moment analysis are only plotted for those cases where this systematic trend does not seriously compromise the accuracy of the method and, as seen, this method has almost no usefulness for significant sky-background levels or for poor seeing conditions. The moment is definitely inferior to other centering methods that can be used for high-precision astrometry.

The remaining three algorithms (the Gaussian fit, modified moment, and the median) are good centering methods with some important differences. For high S/N images or low sky-background levels ($b < 10 \text{ counts arcsec}^{-2}$), all three methods give a similar accuracy, but for increased sky levels, differences become apparent, as illustrated in Figs. 4 and 5. As seen, the median routine becomes less accurate than that of the modified moment, with an increased contribution from the sky background—particularly for faint images—and this may also be marginally true for the Gaussian-fit routine as well. Increased sky-count levels add noise to the marginal distributions, which in turn degrades the centering process. Since the modified moment eliminates most of the sky contribution by the process of thresholding (see Sec. III), the superiority of this method over the others for significant sky levels probably results. Considering the above discussion concerning the two different moment-centering methods, it is quite clear that the elimination of counts in the sky background by the process of image thresholding can make a very significant improvement.

The moment can only center faint images effectively so long as the sky-background level is very low, and the seeing is quite good. All of the other centering algorithms considered do well at detecting stellar images so long as the background levels are not very significant ($b < 10 \text{ counts arc}$

sec^{-2}). For increasing levels of background, the traditional moment rapidly becomes of little use, and the Gaussian method loses much of its effectiveness when the background level becomes high. The Gaussian is the standard used nowadays in image-processing applications for astrometry, and this paper suggests that it is not necessarily the best method for all conditions. For example, numerical simulations indicate that if a star image is not well defined in its marginal distribution because of the contribution made by sky noise, then the least-squares solution for the Gaussian image-profile parameters can become ill defined, and the solution can actually diverge. Other methods can better handle this particular situation.

In terms of overall speed, the fastest methods are the moment, median, and the derivative-search methods, as indicated from this study. These routines take about 0.2–0.4 s to process an image in both coordinates, although it should be pointed out that these times will vary with both the scan size and the type of computer used. In a relative sense, the modified-moment routine is about 2.7 times slower than the aforementioned routines, largely because of the overhead time required for the thresholding of the image, but nonetheless it is still a relatively fast method. The Gaussian routine taking about 8.5 times longer to process an image is by far the slowest of the routines considered in this paper, and with a HP 1000 F-series computer the processing time for it can sometimes exceed 6 s for the fainter images.

V. POSITIONAL ERRORS CAUSED BY DECENTERED OR POORLY RESOLVED IMAGES

a) Decentered Stellar Images

Each of the centering algorithms considered in this paper was evaluated for the influence of positional errors caused either by the stellar image being too close to one of the scan

TABLE I. Positional error caused by decentered images. The spacing is the distance between the image center and a scan boundary in units of the FWHM of the image profile.

Spacing	Gaussian Fit	Moment	Modified Moment	Median	Deriv. Search
(FWHM)		(arc s)		(arc s)	
0.00		0.802	0.574	0.500	
0.25	-0.540	0.490	0.305	0.247	
0.50	-0.028	0.277	0.119	0.094	0.019
0.75	-0.008	0.172	0.034	0.026	0.013
1.00	-0.001	0.136	0.003	0.005	0.002
1.25	0	0.099	0	0	0
1.50	0	0.061	0	0	0
1.75	0	0.033	0	0	0
2.00	0	0.020	0	0	0
2.25	0	0.004	0	0	0
2.50	0	0	0	0	0
3.00	0	0	0	0	0

boundaries or by being poorly resolved into pixel elements. The evaluations were made from data generated from numerical simulations, and each case will be discussed in the following. To evaluate the positional error caused by a star image being poorly centered within a scan box, the image was placed at different distances from a box boundary (ranging from being well within the box to where 50% of the image fell outside of it), and at each distance the associated positional error caused by the decentering was determined. The numerical results are presented in Table I and shown in Fig. 6. The distances used in this discussion are expressed in units of the image FWHM, i.e., the seeing disk.

As seen from the figure, each of the centering algorithms is affected by a positional error as increasing amounts of the image profile are positioned outside of the scan area. As a matter of convention, positive positional errors indicate shifts of the calculated image center toward the center of the scan box and negative errors away from the box center. Only the Gaussian-fit centering routine shows a negative effect, which seems to be caused by the calculated image radius R being increasingly overestimated as more and more of the image falls outside the scan box. The numerical results further indicated that if more than 25% of the image profile falls outside the box, then the Gaussian solution will generally diverge, and, in particular, noisy image profiles are more likely to do so. This is why Fig. 6 shows a cutoff at 0.25 FWHM for the Gaussian-fit method. The derivative-search algorithm also shows the presence of a cutoff. Since this method only uses the part of the image under the FWHM (see Sec. III), the calculation for it can become unstable once any of the image within 0.5 FWHM of its center falls outside of a scan boundary. Due to their nonparametric natures, the remaining two moment- and median-centering algorithms are not subjected to a cutoff and will always produce a centroid, albeit the positional error can be high, so long as some of the image remains within the scan box.

Each centering algorithm shows a differing positional error as the distance between the image center and a scan boundary is reduced. As Fig. 6 shows, the traditional-moment analysis is the most sensitive to positional errors caused by decentered images and, in addition, the trend shown in the figure for this method is strongly influenced by the count level in the sky background. The case shown is for a sky level of $b = 5$ counts arcsec^2 , which steepens sharply if the sky level is increased. VAACS have likewise found from studies of photographic images that decentering can seriously degrade the accuracy of the moment-centering method. The

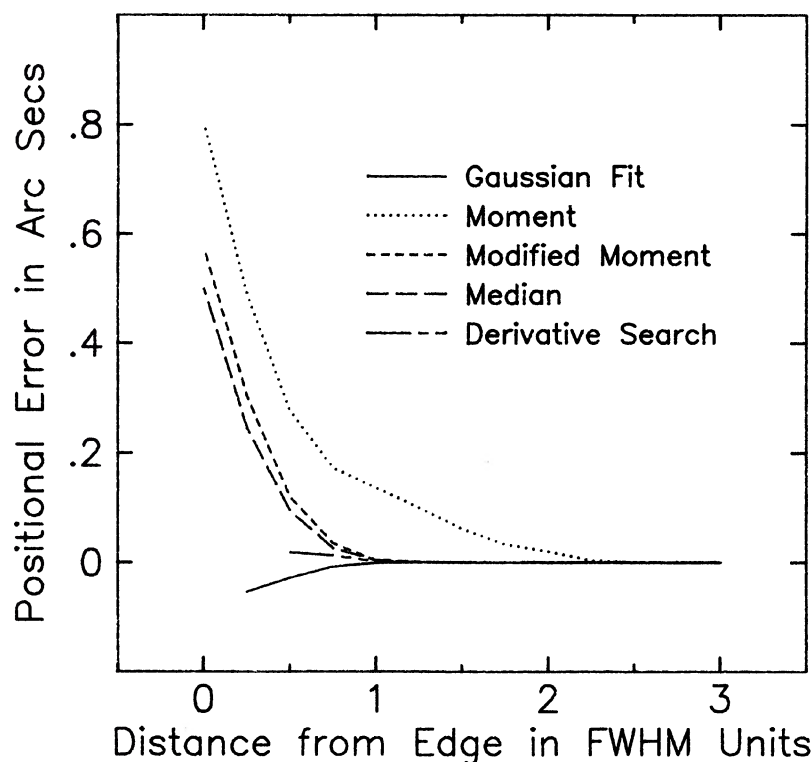


FIG. 6. Positional error caused by star images being too close to a scan-box boundary. Positive errors reflect image shifts toward the center of the scan box. The moment-centering algorithm gives the worst results.

TABLE II. Maximum positional error caused by the step size.

No. Steps under FWHM	Gaussian Fit	Moment	Modified Moment	Median	Deriv. Search
		(arc s)	(arc s)	(arc s)	
0.75		0.070	0.071	0.262	0.330
1.00		0.011	0.011	0.122	0.198
2.00	0	0	0.001	0.008	0.132
4.00	0	0	0.001	0.001	0.062
6.00	0	0	0.002	0.001	0.035
8.00	0	0	0.002	0.001	0.026
12.00	0	0	0.002	0.001	0.015
16.00	0	0	0.002	0.002	0.011

modified-moment and median centering routines give comparable positional errors, as seen in Fig. 6, and of all the centering methods discussed in this paper, the Gaussian and derivative are the least sensitive to decentering errors up to the point where cutoffs become important. Unlike the others, both of these methods give very high weight to the central part of the image in the centering process and therefore are not strongly affected once one of the image wings is truncated. Except for the inferior moment analysis, all of the other centering methods are not subjected to appreciable positional errors caused by decentering so long as the image center is always greater than 1.25 FWHM ($3R$) from a scan boundary, and this result is in agreement with the 2–3 R spacing recommended by VAACS. As a result, this paper recommends that the minimal spacing between an image center and a scan-box boundary never be less than 1.25 FWHM units if high-precision astrometric work with any of the centering routines under discussion in this paper is to be done.

b) Poorly Resolved Stellar Images

Positional errors caused by the image profile being poorly resolved were also investigated from numerical simulations. The resolution is defined by the step size (the spacing between adjacent pixels), and in the following discussion it will be expressed in terms of a dimensionless unit consisting of the number of steps under the FWHM of the image profile. In many practical applications, a large step size would be advantageous in terms of an increased speed of operation, and this discussion will investigate how far this process can go before serious positional errors begin to manifest themselves. The numerical tests were made for a wide range of image resolutions, and since the positional error is sensitive to where the image center falls within a given pixel, a series of calculations were made with the image center being shifted across a pixel dimension in successive $\frac{1}{20}$ of a pixel increments. The result for each series was a sawtoothlike pattern of positional errors. Table II gives the maximum positional errors encountered in each of these series, and these errors are also shown in Fig. 7 for each of the centering methods. As might be expected, these errors become more important with increasingly poorer resolution of the stellar image.

The derivative centering algorithm is the most sensitive to resolution effects and needs a resolution exceeding 16 steps under the FWHM in order to be comparable with the other centering methods. The results for the other methods are consistent with one another. All four centering routines are not appreciably affected by positional errors so long as there are at least $n > 4$ steps under the FWHM of the image profile, which is in agreement with the usage made by VAACS in their analysis of photographic stellar images. Each of these routines is affected to differing degrees for resolutions under $n < 4$. The Gaussian method is completely unaffected until about $n = 2$, whereupon its least-squares solution for the profile parameters becomes underdetermined. The median,

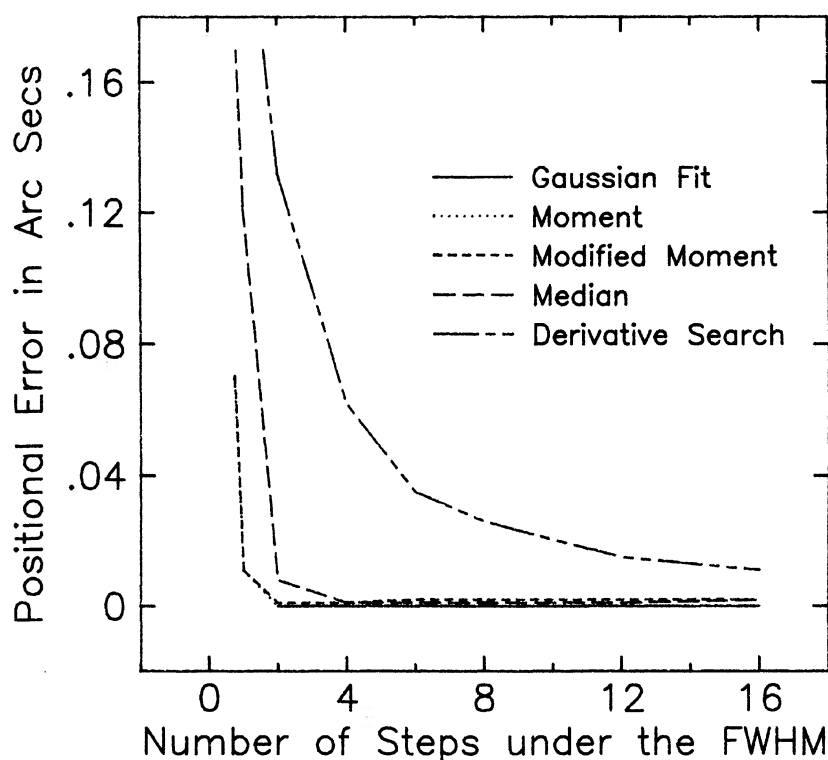


FIG. 7. Positional error caused by star images being poorly resolved into pixel elements. The two moment-centering methods overlap in the figure.

next to the derivative-search method, gives the largest positional errors for $n < 4$, and this seems to be related to the fact that the median method has problems in interpolating for the image center if the cumulative marginal distribution (see the discussion in Sec. III) is poorly resolved. Both moment-centering techniques give virtually identical results for poorly resolved stellar images. The results of this paper recommend that there be at least $n \geq 4$ steps under the FWHM of the image profile if high-precision astrometry is to be done. Lower resolutions are possible in survey work where overall accuracy can be compromised for increased speed of operation. Since the derivative-search algorithm needs very high image resolution, it is again not recommended for high-quality work.

VI. REAL-TIME IMAGE PROCESSING WITH THE USNO 7 IN. TRANSIT CIRCLE

The USNO 7 in. transit circle is currently observing from New Zealand some 3400 FK5 and 20 600 International Reference Stars (IRS) south of declination $\delta = +30^\circ$ as part of a large observing program that is both differential and fundamental in nature (see Hughes (1982) for details). Observations of solar system objects and of instrumental constants are also made on a continuing basis. The central aim of the IRS program is to provide a well-determined position and proper motion for a reference star in each square degree of the sky. Observations of the northern sky are being made concurrently with the USNO 6 in. transit circle located in Washington, DC.

The 7 in. transit circle has been extensively refurbished and automated since its last major undertaking. The micrometer uses an F4012RP ITT image-dissector scanner along with a filtered MA-2 photocathode as its detector and a linear imaging device with a wide dynamic range and low dark-count level at ambient conditions, and on-line telescope control and image processing is controlled with a HP 1000 F-series computer and associated peripherals. Hughes *et al.* (1986) give a fuller description of the instrument and its operation.

Several factors strongly influenced the choice of a star-centering algorithm for use in this program. Since the observing, done intensively both night and day, generates large volumes of digital data that need to be rapidly processed in order to control various operations of the telescope and to monitor its performance, the image processing has to be done in real time. Consequently, the centering routine has to be accurate, fast, and highly convergent. Moreover, the routine must acquire and observe stars with very low S/N ratios in order to reach the fainter stars in the IRS program. As a part of the fundamental program, it must also make observations of stars during the day that are subjected to both low S/N ratios and to very high sky-background levels. (Stars are observed to within 7° of the Sun.) As an example, Fig. 8 shows a digital scan for one of the day stars observed in the program, and even fainter stars than this one have to be observed during the day on a continuing basis. Representative nighttime star images are shown in Fig. 1. Based upon these considerations and the discussion given in Secs. IV and V, the choice for a centering algorithm for use in this particular observing program is quite clear. The modified-moment analysis best meets these requirements and has produced very satisfactory results during the past 4 yr of its usage in the 7 in. observing program. The methods for image smoothing and trimming are as described in Sec. IV. Stars as faint as

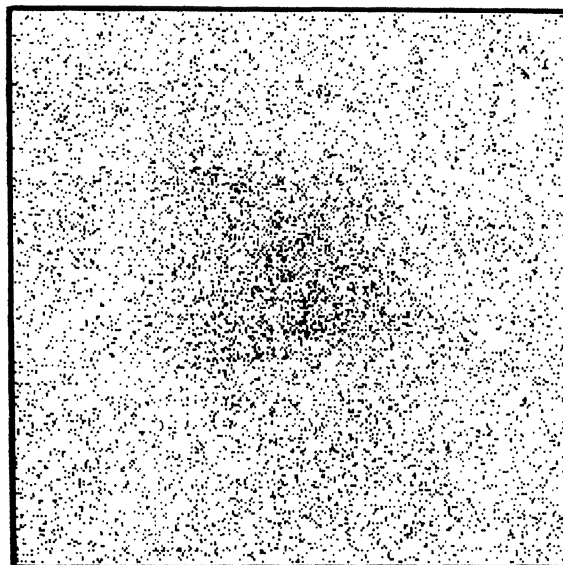


FIG. 8. Image of the southern star σ Scorpii (3.0 mag) observed during the day with the USNO 7 in. transit circle. Only the stellar image and the scatter in the sky background is shown. The field size is 10×10 arcsec, and the integration time was 4.2 ms.

$V = 11.5$ mag can be observed during the night, and others as faint as $V = 5.0$ mag can be observed during the day. Both cases represent significant gains over that typical for traditional visual observing. The combined image-processing time to centroid both coordinates is about 150 ms, and the internal accuracy in each coordinate is at the ± 0.05 arcsec level, which is small in comparison to that introduced by atmospheric motions (0.2–1.0 arcsec at our site) and to other sources of error that are common in transit-circle astrometry.

Each star observation begins with a 30×30 pixel (18×18 arcsec) large area scan in order to acquire the object, and a local maximum search routine similar to that described by Herzog and Illingworth (1977) is then used to acquire the correct star within the scanned area. If not acquired from the initial scan, then subsequent scans involving larger box sizes and longer integrations are tried until either the star in question is found or sufficient time has elapsed since the start of the observation to warrant its termination. Although not needed in the current observing program, provisions exist for the centroiding and processing of multiple star images within the same scan. Once acquired, each object is observed for about 60 s of time, and each observation is made up of between 15 and 100 scans, depending on the magnitude of the star. The overhead for the real-time image processing is about 8% of the available time for the observation. The image positional data and the clock time are stored after the processing of each image, and mean values of the atmospheric seeing, sky transparency, and the starcount rate are also stored. As described by Stone (1988), these count rates can be used eventually to derive photoelectric V magnitudes for the some 24 000 stars included within the observing program. As an illustration, Fig. 9 shows the computed declination relative to the mean during a transit of the southern star ν Octantis. The observation was made with the 7 in. transit circle, and virtually all of the scatter is caused by atmospher-

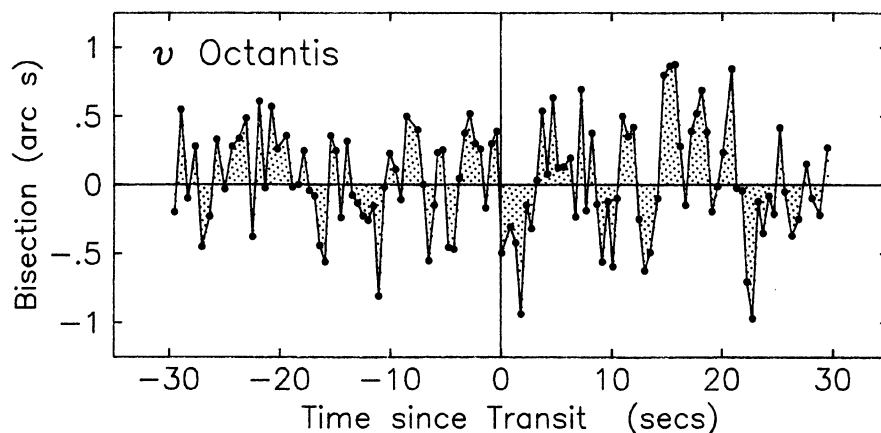


FIG. 9. Illustration showing a star observation made with the USNO 7 in. transit circle. The star is ν Octantis (5.6 mag) and its relative position in declination is shown in the course of its transit. All the image processing was done in real time, and essentially all of the shown scatter is due to atmospheric motion.

ic image motion. The plot in right ascension, suitably corrected for diurnal motion, is very similar.

The 7 in. transit circle can operate for long periods of time without supervision and can automatically respond to variations in sky conditions. Often, several of the initial choices for the scanning parameters have to be modified in the course of an observation in order to respond to changes in the sky transparency, seeing conditions, and the behavior of the instrumental system. In order to control this operation in real time, a diagnostic 16 bit control word is returned with each processed scan, and the bit locations in this word are used to identify specific problems that require corrective action before the onset of the next scan. Table III gives a description of the specific conditions monitored in the 7 in. observing program. For example, if bits 0, 7, and 8 are set, then the next scan will be executed with a longer integration time and a larger scan-box size, and the telescope will be shifted in position in order to clear the overlap situation. Only data without any bits set in the status word are saved.

Based upon experience obtained from the analysis of observational data and from simulations, the present study finds that the optimal observing scan parameters (those for which the corrections strive for) consist of an integration time for which the star image is well defined ($S/N > 7$ and $C > 100$ counts), a step size of 25% of the image FWHM in each coordinate, a scan area that is well centered on the object, and a box size equivalent to 4 FWHM in each coordinate. The last value is larger than the minimal value dis-

cussed in Sec. V in order to allow for possible image motion encountered in the course of a scan. A new set of optimized scanning parameters is produced with the completion of each scan, but rather than immediately implement them, experience has shown that it is better to update the scanning parameter with a running average of previous determinations in order to smooth out the effect caused by rapidly varying sky conditions which can sometimes occur. In the 7 in. program, each new determination of a scanning parameter is given a 50% weight in computing the updated value used in the subsequent scan. Largely because of these procedures, the telescope operates very efficiently under all possible sky conditions, and very little editing of the observed positions is needed.

The modified-moment analysis is used in the 7 in. astrometric program to centroid stars, Mercury, Uranus, Neptune, and 13 asteroids in the program. Observations of extended sources, such as the Sun and Jupiter, are also made; however, since each of these extended sources requires its own particular centering algorithm, no further discussion will be given for them in this paper. Besides celestial objects, observations of fiducial wires mounted within the telescope's micrometer and the collimating telescopes are also made to relate the celestial and instrumental coordinate systems to one another. The positions of the four micrometer reference wires are measured repeatedly in the course of each observation. Figure 10 shows a marginal distribution for one of the wires mounted in the 7 in. micrometer. After some experimentation with differing centering routines, including complicated two-dimensional schemes that determine the wire center from the wire-edge pixels, this study finds that if a wire marginal distribution is inverted, then an image very similar to that of a star marginal can be created (see Fig. 10), which can be well centered with a modified-moment analysis. This is the method used for processing wire images with the 7 in. transit circle, and the application of this centering algorithm to wire images is both very fast and accurate. An individual wire image can be measured to about $\pm 1.5 \mu\text{m}$ with one scan, and most of this error is contributed by the detector itself. Additional scans are always taken in order to reduce this error to an insignificant amount. Hence, the same centering routine can be used to process both star and wire images.

VII. SUMMARY

Synthetically produced star images have been used to investigate the properties of five one-dimensional centering al-

TABLE III. The diagnostic status word.

Bit No.	Meaning
0	S/N ratio is too small (Weak image).
1	Star is badly centered in the scan.
2	Centering solution did not converge.
3	Image is poorly resolved into pixels.
4	Star not present in acquisition scan.
5	Scan exceeds data buffer size.
6	No acceptable image was found.
7	Star overlaps with a micrometer wire.
8	Bad seeing conditions.
9	Sky level is not flat (Clouds present).

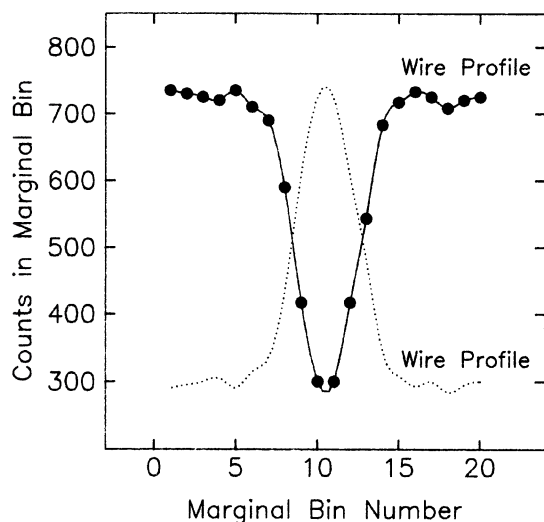


FIG. 10. Profile (marginal distribution) for one of the reference wires mounted within the USNO 7 in. transit-circle micrometer. The inverted image profile can be centered as if it were a star image (see text).

gorithms under a wide range of both atmospheric seeing and background-level conditions. The algorithms under study were those of the Gaussian fit, moment, median, derivative search, and a modification to the moment analysis that suppresses the sky background through a process of image thresholding. This study concurs with the studies made by van Altena and his collaborators, who found that the Gaussian-fit centering algorithm is a very good method for centering the normal range of star images. However, it does have some faults, namely, it is not very good at centering weak stellar images subjected to high background levels, it is the slowest of the routines considered herein, and if a star image happens to fall very near to a scan boundary, its solution can actually diverge.

The moment is definitely an inferior centering technique, since it cannot effectively cope with significant sky-background levels, the presence of which can cause both the acci-

dental error of centering to become high as well as producing a systematic shift of the image center toward the center of the scan box. The median, as pointed out by Chiu (1977) and confirmed in this study, is a good method for the centering of faint astronomical images, although the synthetic results presented in this paper do find that this method is somewhat sensitive to the manner chosen for interpolating for the image center, particularly when the image is poorly resolved into pixels. The derivative-search centering algorithm is another inferior method and, like the moment, is not recommended for doing precision astrometry outside of a very narrow range of conditions. This method suffers from a number of serious defects. It is consistently the least accurate of the methods considered in this paper, requires that the stellar image be finely resolved into pixel elements for precision work, and also breaks down if the image is located near a scan boundary.

A modified-moment analysis, wherein pixel information below a defined threshold level is rejected from the centering process, was found, like the Gaussian fit and the median, to be a very good centering routine for dealing with the normal range of stellar images, and it is the best routine of the ones considered in this paper for dealing with high sky-background levels. This particular algorithm has been used with great success in the current USNO 7 in. transit-circle southern observing program. Although not investigated in this paper, it is suspected that the Gaussian-fit and median centering routines might also benefit with the application of image-thresholding techniques when dealing with large sky levels. Investigation was also made into positional errors caused by the stellar image being either decentered within the scan box or being poorly resolved into pixel elements. It is recommended that the image center be kept away from a scan boundary by a spacing exceeding 1.25 times the FWHM of the image profile, and the resolution under the image profile in each marginal distribution should be at least 4 pixels. All in all, the synthetic-star results presented in this paper are in good agreement with the photographic results presented by van Altena and his collaborators (VAACS). The centering of wire images can also be done very effectively by inverting the wire profile and then using a modified-moment analysis on it.

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