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Letter to the Editor

Tidal evolution in the Neptune-Triton system

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Summary: Neptune's moon Triton is spiralling in towards the planet due to tides raised on both Neptune and Triton. Dissipation from tides on Triton will arise when either its orbital eccentricity or spin-axis obliquity is non-zero. Triton's current obliquity may lie close to either 0° (state 1) or 100° (state 2), corresponding to the two stable Cassini extrema of its rotational Hamiltonian. The Kaula tidal formalism is used to model the past and future evolution of the system in both states. For nominal parameters ($Q_N = 10^4$, $Q_T = 10^2$) in state 1, Triton will reach Neptune's Roche limit in ~3.6 Gyr with a decrease in its orbital inclination from the present 159° to ~ 145°. In state 2, substantial heating due to obliquity tides leads to significantly different orbital evolution. In this case, Triton's inclination will increase to an end-point of 180° in 10^7 – 10^8 yr, at which time the satellite will make a transition to state 1. Triton will then evolve in to Neptune's Roche limit in ~1.4 Gyr. Extrapolation into the past suggests that Triton's orbit has always been retrograde, with an inclination of at least ~ 125°. For Q_T = 100, any initial eccentricity would have damped to the present upper limit of 5×10^{-4} in ~ 200 Myr. If Triton was captured at an earlier epoch, then any measurable current eccentricity is most probably due to cometary impacts, rather than representing a tidal "relic".

Keywords: Planets and Satellites: Neptune, Triton—Cassini states—Tidal heating—Orbital Evolution

1. The Neptune–Triton system

Triton orbits Neptune at an inclination $i = 159 \pm 1.5°$ with semimajor axis a=14.1 R_N (Harris, 1984), where $R_N = 2.52 \times 10^4$ km is Neptune's stratospheric equatorial radius (French *et al.*, 1985, and refs. therein). Neptune has only one other known satellite, Nereid, which is relatively small (radius 525 km for a geometric albedo of 0.04) and in an inclined (27.5°), highly eccentric (0.756), and distant (~ 220 R_N) orbit (Cruikshank and Brown, 1986). Nereid's gravitational effects on the rest of the system are negligible. The present Neptune–Triton system is thus remarkably uncomplicated from a dynamical point of view.

Many investigators have speculated on the reason for Triton's retrograde orbit. Lyttleton (1936) suggested that both Pluto and Triton originated as prograde satellites of Neptune, only to experience a catastrophic gravitational interaction. McCord (1966) showed that tidal evolution could have brought Triton inward from a near-parabolic orbit, lending plausibility to a capture origin for the satellite. Harrington and Van Flandern (1979) suggested Triton's peculiar orbit and Pluto's supposed "escape" were caused by an encounter with a massive "rogue" body, a conclusion contested by Farinella et al. (1980). Finally, McKinnon (1984) combined momentum and energy constraints with an improved knowledge of Pluto's mass to show that all Pluto–Triton interaction scenarios for the origin of Triton's retrograde orbit are untenable, leaving the capture hypothesis as the most likely possibility.

To assess this hypothesis, and to predict the future of the Neptune–Triton system, requires modeling the evolution of Triton's orbit. McCord (1966), following MacDonald's (1964) tidal formalism, conducted the first such investigation and concluded that Triton will reach Neptune's Roche limit within 10^7–10^8 yr. However, this result depended on a specific dissipation factor for Neptune of Q_N=10^2–10^3, now considered to be improbably small. (Peale (1988) has argued that the Q of Uranus must be at least 4.6×10^3, with values of ~ 10^5 entirely possible; the Q's of Jupiter and Saturn are at least 10^5 and perhaps as great as 10^6 (Goldreich and Soter, 1966).)

Szeto (1981) argued that, despite Triton's low eccentricity ($e < 5 \times 10^{-4}$; Harris, 1984), the orbital effects of tides raised on Triton ("Triton tides") dominate those due to tides on Neptune ("Neptune tides") by a factor ~ 10^3. His analysis, however, neglected to treat correctly the effect of Triton's orbital inclination on the satellite tides. A proper treatment first requires a determination of Triton's as yet unobserved spin-axis obliquity. This may be calculated from the theory of Cassini states (Sec. 3), but a prerequisite to both these and the subsequent tidal evolution calculations is the choice of a self-consistent set of physical parameters for the Neptune–Triton system.

2. Choice of a self-consistent parameter set

The Kaula perturbation equations (Kaula, 1964, 1966; Burns, 1986) used to calculate orbital evolution in Secs. 6 and 7 require knowledge of a number of interdependent physical parameters. Three of these, the initial values for the evolving orbital elements (semimajor axis a, inclination i, and eccentricity e) are known, or at least bounded (see Sec. 1). We also require values for the two bodies' masses, M_N and m_T, radii R_N and r_T, Love numbers k_N and k_T, and specific dissipation factors Q_N and Q_T. Of these quantities, only $M_N = 1.02 \times 10^{29}$ g (Harris, 1984) and R_N are well established. We now attempt to determine a "best" choice for the remaining values.

Alden (1943) estimated m_T from observations of Neptune's barycentric wobble. His measurements yielded $m_T/M_N = (1.28 \pm 0.23) \times 10^{-3}$. However, this value is difficult to reconcile with more recent radiometric (Lebofsky et al., 1982; Morrison et al, 1982) or speckle interferometric (Bonneau and Foy, 1986) measurements of Triton's radius. We choose Lebofsky et al.'s (1982) radius estimate, r_T=1750 ± 250 km, a value consistent with that of Morrison et al. (1982), although above the upper bound of Bonneau and Foy's (1986) speckle result. If Alden's (1943) mass ratio is correct, this radius requires Triton's density, ρ_T, to be 5.9 g cm^{-3}, an unreasonable value. Instead, we arbitrarily set ρ_T=2.0 g cm^{-3}, which, with our adopted radius, gives m_T/M_N=4.4×10^{-4}, about 1/3 of Alden's value.

Triton's second-order Love number, $k_T = 3/(2 + 19\mu/g\rho_T r_T) = 0.12$, where g is Triton's surface gravity and we adopt the rigidity of water ice, μ=4×10^{10} dyne cm^{-2} (Proctor, 1966). Harris (1984) calculates Neptune's second-order Love number k_N using a moment of inertia factor, γ_N, calculated from an interior model. However, the resulting value k_N=0.2 is in fact inconsistent with his own value for the second-order coefficient in the harmonic expansion of the gravitational potential J_2, given the relation $k_N = 3J_2/q$, where q=$\omega_N^2 R_N^3/GM_N$ and ω_N is Neptune's rotational angular velocity. In this paper we adopt the view that the values of physical parameters used in our calculations, though unavoidably somewhat uncertain, must be internally consistent. We therefore follow Dermott's (1984) iterative approach towards finding a set of "best", mutually consistent values.

The procedure is as follows. We choose an observational value for either the oblateness, f, or rotation period, P_N. Suppose for clarity that we choose P_N = 18.2 hr (Harris, 1984, and refs. therein). We then guess an initial "seed" value for γ_N, and calculate Neptune's obliquity ϵ_N, from the equation $\sin\epsilon_N = (m_T a^2 n \sin i)/(\gamma_N M_N R_N^2 \omega_N)$, where n is Triton's orbital mean motion and $\omega_N = 2\pi/P_N$. This equation simply states that the pole of the invariable plane is given by the orientation of the total angular momentum vector of the Neptune–Triton system, so that the horizontal component ($\sin \epsilon_N$) of Neptune's rotational angular momentum must be equal and opposite to the horizontal component ($\sin i$) of Triton's orbital angular momentum, where ϵ_N and i are both measured relative to the invariable plane. This allows us to calculate Neptune's J_2 via $J_2 = (4/3)(a/R_N)^2(T_T \sin i)/[T_P \sin 2(i + \epsilon_N)]$, where T_P and $T_T = 2\pi/n$ are Triton's orbital precession period and orbital period, respectively. Harris (1984) gives $T_P/\sin i$ = 1738 yr. This estimate of J_2 allows us to calculate k_N and f, via the equations $k_N = 3J_2/q$ and $2f = 3J_2 + q$. Finally, f may be substituted into the Darwin–Radau relation, $\gamma_N \approx 2/3 - (4/15)(5q/2f - 1)^{1/2}$, to yield a new value for γ_N. In practice, iterations of these equations converge, independent of the initial choice of γ_N, to better accuracy than justified by observational uncertainties in fewer than five cycles. An analogous procedure can be followed if f = 0.021 (Lellouch et al. 1986; Hubbard et al. 1987) is taken as the given observational quantity. Since it is unclear at present whether observations of P_N or f are more reliable, our final adopted values are determined by averaging the results of the two iterations. We find P_N=17.0 hr, f = 0.018, ϵ_N=0.86°, $J_2 = 3.8 \times 10^{-3}$, k_N = 0.46, and γ_N = 0.25. Of these values, ϵ_N, J_2, and γ_N change by $\lesssim$10% with the choice of either P_N or f as the independent variable in the iteration, whereas k_N increases from 0.40 to 0.53 as P_N ranges from 15.8 hr (for f = 0.021) to 18.2 hr. Increasing m_T to Alden's value increases ϵ_N to ~ 2.5°, but has little effect on other parameters.

It is true that the procedure outlined above may also suffer from defects. The value for P_N (and therefore q) is via lightcurve variations in CH_4 absorption bands (Harris, 1984), or motions of visible cloud features (Hammel and Buie, 1988), while f is determined by stellar occultations (Hubbard et al., 1987). These parameters are thus appropriate to levels in Neptune's upper atmosphere. The equations for k_N and γ_N, however, depend on the rotation rate in the deep interior. We are thus implicitly assuming that the interior and atmospheric periods are the same, or at least very similar. Bearing in mind the facts that Uranus' atmospheric rotation period was found from Voyager observations to vary substantially with latitude, and to differ by over an hour from the magnetic field (presumably interior) rotation period (Stone and Miner, 1986), we recognize that a definitive set of parameters must await the Voyager/Neptune encounter in August 1989.

Finally, values must be chosen for the dissipation factors, Q_N and Q_T. Lower bounds on Q for Jupiter, Saturn and Uranus can be derived from the positions of their innermost satellites, on the assumption that these satellites began their outward orbital evolution from their primary's Roche limit 4.6 Gyr ago (Goldreich and Soter, 1966). Upper bounds on Q for Jupiter and Saturn are implied by requiring a tidal origin for their satellite commensurabilities. Neither approach is possible for Neptune, because Triton is orbiting retrograde and thus evolving inwards, and of course the system has no commensurabilities. Values of Q estimated via these techniques for the other giant planets range from $\sim 10^4$ to 10^6 (Burns, 1986). We choose $Q_N = 10^4$ as a plausible minimum value. Estimates of Q for satellites composed of rock/ice mixtures typically range from 10 to 400 (Squyres et al., 1985). We choose $Q_T = 100$. Timescales for orbital evolution scale linearly with Q (see Secs. 6 and 7), so the effects of different choices are easily deduced.

3. Triton's possible Cassini obliquities

Tides raised in a synchronously rotating satellite do not result in any significant changes in the orbital angular momentum, due to the very small angular momentum reservoir in the satellite's spin. Triton's spin angular momentum $L_{spin} \sim m_T R_T^2 n \sim 10^{-5} L_{orb}$, where $L_{orb} = m_T a^2 n$ is Triton's orbital angular momentum. We can thus safely neglect changes in L_{orb} due to Triton tides. If the satellite's orbit is eccentric, however, then the variation of tide height with orbital distance, coupled with an oscillation in the bulge orientation due to variations in the instantaneous orbital angular velocity can lead to substantial energy dissipation, which must ultimately deplete the orbital energy (cf. Yoder, 1982). A similar dissipation of tidal energy will occur for a circular, inclined orbit, if the spin axis of the satellite is not perpendicular to the plane of the orbit. In most cases, the satellite's obliquity is small, and the resulting dissipation is negligible (cf. Yoder and Peale, 1981; Szeto 1983), but Triton may be an exception to this rule.

Triton's obliquity must satisfy Cassini's (1693) third law, which in its modern form (Colombo, 1966) states that a satellite's spin axis and orbit normal remain coplanar with the normal to the invariable plane during their mutual precessional motion. Such coprecession is possible only for values of Triton's obliquity θ (measured from the orbit normal) that satisfy:

$$(3/2)\gamma(\theta)\sin\theta\cos\theta + (3/8)\beta(\theta)\sin\theta(1-\cos\theta) + \Omega\sin(\theta - i) = 0, \quad (1)$$

(Peale, 1969), where $\Omega = T_T/T_P$ and $\gamma = (C-A)/C$ and $\beta = (B-A)/C$ are Triton's moment of inertia differences. For a synchronously rotating satellite with nonzero obliquity, the hydrostatic moment differences are given by (Jankowski *et al.*, 1989):

$$\gamma(\theta) = (5\zeta/32)(5 + 6\cos\theta + 21\cos^2\theta) \quad \text{and} \quad \beta(\theta) = (15\zeta/16)(1+\cos\theta)^2, \quad (2)$$

where $\zeta = (M_N/m_T)(r_T/a)^3$. Eqs.(1) and (2) may be solved for θ by successive approximation. In general, two or four solutions exist, depending on the ratios γ/Ω and β/Ω. These solutions are called Cassini states. For the present day Triton with a=14.1 R_N, i=159°, and M_N, m_T, and r_T as above, there are two possible stable states: state 1 with $\theta = -0.26°$, and state 2 with $\theta = 100°$. We label these states S1 and S2, respectively. Given hydrostatic moments, ρ_T is the only poorly-constrained parameter appearing in eqs.(1) and (2); as ρ_T varies from 1 to 4 g cm^{-3}, θ in S1 or S2 changes by less than ~0.5 and 5 degrees, respectively, resulting in negligible changes in tidal heating (Sec. 5). Triton's unusually large inclination leads to a substantial probability that the satellite may be found in S2, although most other (low inclination) satellites are known to occupy S1 (Jankowski *et al.*, 1989).

4. Tides on Neptune and Triton

Orbital evolution of Triton will occur due to both Neptune and Triton tides. We model the effects of Neptune tides (i.e., tides raised on Neptune by Triton) by Kaula's perturbation theory (Kaula, 1964; 1966), in which the secular perturbations of Triton's motion are given by sums over individual terms arising from a Fourier-decomposition of Triton's tidal potential. Thus, the time derivative of the semimajor axis is given by

$$\dot{a} = 2\sum_{lmpq} K_{lm}(a)[F_{lmp}(i+\epsilon_N)]^2[G_{lpq}(e)]^2(l-2p+q)\sin\epsilon_{lmpq}, \quad (3a)$$

$$K_{lm}(a) = k_l na(m_T/M_N)(R_N/a)^{2l+1}(2-\delta_{m0})(l-m)!/(l+m)!, \quad (3b)$$

where the sum is taken over $l = 2, 3, ..., 0 \le m \le l, 0 \le p \le l$, and $-\infty \le q \le +\infty$. In eq.(3a), the $F_{lmp}(i+\epsilon_N)$ and $G_{lpq}(e)$ are polynomials in $\sin(i+\epsilon_N)$ and power series in e, respectively, which are tabulated for values through $l = 4$ by Kaula (1966). i and ϵ_N are measured relative to the invariable plane, so that $(i+\epsilon_N)$ is the inclination of Triton's orbit relative to the plane of Neptune's equator. In eq.(3b), k_l is the l^{th}-order Love number; we have designated k_2 above by k_N. ϵ_{lmpq} is the phase lag (due to dissipation) associated with the tidal term $[lmpq]$. We shall assume the phase lags to be small, and equal for all tidal components, so that $\sin\epsilon_{lmpq} \approx \epsilon_{lmpq} = Q_N^{-1}\text{sign}[m\omega_N - (l-2p+q)n]$. This amounts to assuming a frequency-independent Q. Expressions similar to eq.(3a) hold for $\dot{e}$ and di/dt. This formalism has recently been summarized and discussed by Burns (1986).

One might hope that we could also apply the same formalism to model Triton tides. Indeed, it is frequently asserted in the literature that such satellite tidal effects may be calculated by interchanging satellite and planetary parameters in eqs.(3), and multiplying by the mass ratio M_N/m_T (MacDonald 1964; Kaula 1964; Burns 1986). This simple prescription fails, however, for several reasons (Yoder 1982; Szeto 1983). First, in the case where the inclination is non-zero, the argument $i+\epsilon_N$ of the F_{lmp} functions must be replaced by θ, the satellite's obliquity. Second, the satellite's equatorial plane, unlike the invariable plane, is non-inertial, so that corrections must be introduced into the equation for di/dt due to satellite tides, incorporating appropriate averaging over the satellite's precessional cycle. Third, and most importantly, for a synchronous satellite such as Triton is certain to be (Jankowski *et al.*, 1989), there are additional non-tidal torques due to the triaxial figure of the satellite. In effect, the Kaula expressions only take into account effects due to the satellite's elastic tidal bulge and do not include effects due to the satellite's permanent figure distortion. The net effect of these "permanent" torques is to cancel any non-zero tidal torque due to a finite orbital eccentricity (or inclination) (Yoder 1982). As indicated in Sec. 3 above, there can be no significant transfer of angular momentum into or out of the orbital motion associated with satellite tides, but the Kaula expressions alone can be shown not to conserve orbital angular momentum for a synchronous satellite.

To avoid these difficulties, we describe the action of Triton tides by appealing to angular momentum and energy considerations. Although Triton tides result in negligible changes in the orbital angular momentum, L_{orb}, this quantity itself is not constant because of J_2-induced precession. Precession of Triton's orbit will not, however, alter the component perpendicular to the invariable plane, $L^z_{orb} = L_{orb}\cos i$. Thus L^z_{orb} may be treated for practical purposes as though it were strictly conserved, although solar torques on Triton will induce even the invariable plane normal to precess on a time scale of 10^6 years. Setting $L^z_{orb} = \sqrt{GM_N a(1-e^2)}\, m_T \cos i$ equal to a constant and differentiating, we obtain the relation:

$$\dot{a}/a = 2e\dot{e}/(1-e^2) + 2\tan i\,(di/dt) = 2a\dot{E}_{orb}/GM_N m_T, \quad (4)$$

where we have used $E_{orb} = -Gm_T M_N/2a$. Thus we can relate $\dot{a}$, $\dot{e}$, and di/dt due to satellite tides to the tidal dissipation of energy in Triton, since $\dot{E}_{orb}$ must equal the energy lost by tidal heating: $\dot{E}_{orb} = -\dot{E}_{tidal}$.

5. Tidal heating of Triton

In general, tidal heating is due to both eccentricity and obliquity effects. A tidal heating model taking both of these into account has been developed by Peale and Cassen (1978). For an incompressible homogeneous satellite, and keeping only the $l = 2$ terms in the Kaula expansion, they find

$$\dot{E}_{tidal} = K_T(a)\sum_{mpq}(2-\delta_{0m})\frac{(2-m)!}{(2+m)!}[F_{2mp}(\theta)]^2[G_{2pq}(e)]^2 X_{2mpq}, \quad (5)$$

where $K_T(a) = k_T(1+k_T)^{-1}GM_N^2 na^{-1}(r_T/a)^5$, $X_{2mpq} = (2-2p+q-m)\sin(\epsilon_{2mpq})$, and the assumed frequency-independent phase lags are given by $\sin(\epsilon_{2mpq}) \simeq Q_T^{-1}\text{sign}[(2-2p+q-m)+(2-2p-m)\dot{\omega}_T/n]$. ($\dot{\omega}_T$ is the time derivative of the argument of pericenter of Triton's orbit.) For small eccentricity and obliquity, eq.(5) may be approximated by

$$\dot{E}_{tidal} \simeq (3/2)K_T(a)Q_T^{-1}(7e^2 + \sin^2\theta), \quad (6a)$$

while for small e and $\theta \simeq 90°$,

$$\dot{E}_{tidal} \simeq (15/8)K_T(a)Q_T^{-1}. \quad (6b)$$

For Triton, e is currently $< 5\times10^{-4}$ and, since both Neptune and Triton tides will only act to reduce e even further, $e \approx 0$ should remain an excellent approximation throughout all future orbital evolution. In this case, eq.(4) may be rewritten as

$$\tan i\; di = da/2a, \quad \text{with} \quad \dot{a} = -(2a^2/GM_N m_T)\dot{E}_{tidal} \quad (7)$$

where $\dot{E}_{tidal}$ is given by eq.(6a) or (6b), depending on whether Triton is in S1 or S2, respectively. Of course, Triton's *past* orbit may well have been eccentric; this case will be discussed further in Sec. 7.

The tidal heating implied by eqs.(6) has been discussed for a variety of Triton radii and compositions by Jankowski *et al.* (1989). Tidal heating in Cassini state 1 ($\theta \approx 0°$) is several orders of magnitude smaller than radiogenic heating. However, heating in S2 ($\theta = 100°$) is substantial; eq.(6b) gives $\dot{E}_{tidal} = 1.3 \times 10^{21}$erg sec^{-1}, a result which scales as $(\rho_T/2.0 \text{ g cm}^{-3})^2(r_T/1750 \text{ km})^7(14.1R_N/a)^{15/2}(100/Q_T)$. Here we take $k_T \approx 0.12(\rho_T/2.0\ g\ cm^{-3})^2(r_T/1750\ km)^2$, an approximation good to $\sim$10%.

In fact, the heating rate of Triton in S2 for the parameter values adopted in this paper is inconsistent with infrared observations of Triton (Jankowski *et al.*, 1989), by a factor ~ 6. This argues either that Triton is in state S1 rather than S2, or that one or more of the parameters (e.g., Q_T) used in our calculation is incorrect. In particular, the strong dependence of $\dot{E}_{tidal}$ on r_T means that a Triton smaller by several hundred kilometers in radius may, in fact, occupy S2 without violating any present observational data.

6. The future of Triton

To numerically calculate orbital evolution in the Neptune–Triton system, we integrate the changes in Triton's orbital elements due to Neptune tides (eqs.(3)) for $\dot{a}$, and the analogous equations for $\dot{e}$ and di/dt; see, *e.g.*, Burns, 1986) and sum these with those due to Triton tides, from eqs.(6) and (7). The size of the timestep used in the integration is varied so that a never changes by more than 0.1% in any iteration. When summing over $lmpq$ in eqs.(3) and analogous equations, only terms with $l = 2$ are kept. This is an acceptable approximation, as $K_{lm}(a) \propto (R_N/a)^{2l+1}$ (see eq.(3b)), so that the $l = 3$ terms are each a factor $(14.1)^2$ $\sim$200 smaller than the corresponding $l = 2$ term. Of course, this approximation becomes worse as Triton evolves in towards Neptune, but in practice orbital evolution occurs so quickly within $\sim 10R_N$ that little error is introduced into evolutionary timescales by excluding the higher order terms. This was verified explicitly by initial numerical runs in which all terms through $l = 3$ were included; these terms were later dropped to minimize computing time. For reasons outlined in Sec. 7, we restrict our calculations to the small-e regime, and have accordingly truncated the G_{lpq} expansion to $|q| \leq 2$. Higher order terms are of order e^3, or smaller. At each iteration, new values are also computed for ϵ_N and $P_N = 2\pi/\omega_N$ by angular momentum conservation, and for θ by iteration of eqs.(1) and (2). The results of our integrations are shown in Figs. 1 and 2.

We consider first the evolution of Triton's orbit forwards in time, assuming Triton lies in state S1 (the solid line in Fig. 1). In this case, the evolution of Triton's semimajor axis is due almost entirely to Neptune tides, and in fact is dominated by a single semi-diurnal term. This can be shown explicitly by considering eq.(3a). For small e, $G_{lpq} \propto e^{|q|}$ (Burns, 1986), so only the terms $[lmpq]$=[2m00], [2m10], and [2m20] can contribute substantially to the sum. (Our integration takes $e = 5 \times 10^{-4}$ as the current value for the eccentricity.) The multiplicative factor $(l - 2p + q)$ in eq.(3a) renders the contribution from the [2m10] term zero. Additionally, each G_{lpq} is multiplied by the sum of appropriate F_{lmp} terms; examination of these for $(i + \epsilon_N) \sim 160°$ reveals that only the term F_{222} makes a significant contribution. Thus, orbital evolution in S1 due to Neptune tides is due almost entirely to the [2220] term. Eqs.(3) can be written out explicitly for this term, and the result integrated to yield a timescale for orbital evolution between an initial distance a_0 and a final distance a due to the action of Neptune tides alone:

$$\tau_N \approx (2/39)(a_0^{13/2} - a^{13/2})(M_N/m_T)(Q_N/R_N^5 k_N)(GM_N)^{-1/2}. \quad (8)$$

Similarly, from eqs.(4) and (6a), we can derive a timescale τ_T for semimajor axis evolution in S1 due to Triton tides. Dividing τ_N by τ_T gives

$$\tau_N/\tau_T \approx (k_T/k_N)(Q_N/Q_T)(\rho_N/\rho_T)^2(R_N/r_T)(7e^2 + \sin^2\theta_1), \quad (9)$$

where $\rho_N = M_N/(4/3)\pi R_N^3$ is Neptune's mean density and we have added a subscript to θ as a reminder that this expression refers only to state S1, i.e., small Triton obliquity. For our nominal parameter values, and with $e = 5\times10^{-4}$ and $\theta_1 = -0.26°$, we find that $\tau_N/\tau_T = 4.9\times10^{-3}$, so that orbital evolution timescales in S1 are a factor $\sim$200 longer for Triton tides than for Neptune tides. To this accuracy, the effect of Triton tides on semi-major axis evolution in S1 is indeed negligible, unless Neptune's Q exceeds 10^6. Note that, with the present eccentricity, the major contribution to Triton's tidal dissipation arises from the small state 1 obliquity, so our setting of $e = 0$ does not appreciably affect our results. Eq.(8) may be rewritten as

$$\tau_N \sim 3.6\ (2.0\ g\ cm^{-3}/\rho_T)(1750\ km/r_T)^3(Q_N/10^4)\ \text{Gyr}, \quad (10)$$

where we display the dependence of τ_N on those parameters most likely to change after the Voyager/Neptune encounter. τ_N can then be appropriately rescaled. This approximate analytic result is in close agreement with the numerical integration shown in Fig. 1, which includes all terms in the full expansion, as well as Triton tides.

An analogous procedure can be used to study the future evolution of Triton's inclination in state S1. The effect of Triton tides is calculated in this case by setting $\dot{e}$=0 in eq.(4), to yield eq.(7). Neptune tides again dominate, but by only a factor of $\sim$14 with our nominal parameters. There are three large Neptune tidal terms of comparable magnitude : the semi-diurnal tide [2220], and two diurnal terms [2110] and [2120]. The diurnal terms are approximately equal, but opposite in sign, leaving the semi-diurnal term as the principal contributor. In consequence, the combined effect is not sensitive to our assumption of a frequency-independent Q. (Indeed, a similar inspection of the signs and magnitudes of the leading terms in $\dot{a}$ (eqs.(3)), $\dot{e}$, and $\dot{E}_{tidal}$ (eq.(5)) reveals them to be insensitive to this assumption as well.) Summing these three terms, we obtain

$$(di/dt)_N \approx -(3/4)\sin i(k_N/Q_N)(m_T/M_N)(R_N/a)^5 n \quad (11)$$

Combining this result with the dominant term in $\dot{a}$ and integrating gives:

$$\sin i \approx (a_0/a)^{1/4}\sin i_0, \quad (12)$$

which predicts a reduction in inclination from 159° to $\sim$ 146° as a decreases to 2.5 R_N. The numerical integration in Fig. 2 shows a somewhat smaller reduction in i, due to the effects of Triton tides.

Satellite tides act to drive $\sin i$ towards 0, in the case of Triton driving the inclination toward 180° (and Triton's obliquity toward 0°), in the direction opposite to Neptune tides. (For a prograde satellite, both planet and satellite tides drive i towards 0.) This may be understood physically by noting that energy dissipation in the satellite must decrease the semimajor axis, thus decreasing the total orbital angular momentum (for $e = 0$). Since the z component of angular momentum remains unchanged, the orbital plane must move towards the invariable plane. Satellite-tide driven orbital evolution then ceases when the inclination reaches either 0° or 180°, in which situation the Cassini obliquity goes to zero and energy dissipation due to obliquity tides ceases. This scenario is exactly analogous to the damping of eccentricity by radial tides. For small obliquities, we find from eqs. (6) and (7)

$$(di/dt)_T = -(3/2)(\sin^2\theta_1/\tan i)(k_T/Q_T)(M_N/m_T)(r_T/a)^5 n. \quad (13)$$

Whether Neptune or Triton tides dominate di/dt is thus of qualitative, as well as quantitative, significance. Since Neptune tides dominate this evolution by only a small margin, it is possible that modest changes in parameter values (*e.g.* Q_N, Q_T, or r_T) from those used here could reverse the sense of inclination evolution shown in Fig. 2. Even with our nominal parameters, Triton tides would have dominated di/dt if Triton's semi-major axis ever exceeded $\sim$ 50 R_N, where the Cassini obliquity $\theta_1 = 1.0°$.

Let us now consider the consequences for orbital evolution if Triton is in Cassini state 2. We follow a procedure analogous to that used to derive eqs.(8) and (9), using eq.(6b) rather than (6a) to evaluate $\dot{E}_{tidal}$. The timescale for orbital evolution of a due to Triton tides will be shortened by a factor $\sim (4/5)\sin^2\theta_1$ (neglecting the small e^2 term in eq. (6a)), which for $\theta_1 = -0.26°$ is $\sim 1.6 \times 10^{-5}$. Triton tides will thus dominate the evolution of a in S2 by a factor $\sim$ 300. The resulting timescale is given by

$$\tau_T \approx (8/195)(a_0^{13/2} - a^{13/2})(m_T/M_N)(Q_T/r_T^5 k_T)(GM_N)^{-1/2}. \quad (14)$$

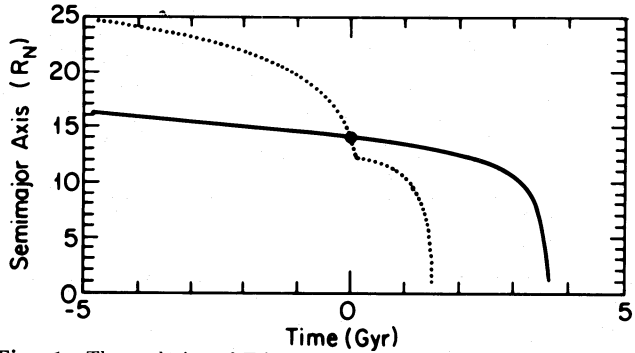

Fig. 1. The evolution of Triton's semimajor axis as a function of time, for Cassini states 1 (solid curve) and 2 (dotted curve), for $Q_T = 10^2$ and $Q_T = 10^3$, respectively. For both cases, $Q_N = 10^4$. Past orbital evolution is for the case $e \equiv 0$. The case $e \neq 0$ is discussed in Sec. 7 of the text. The current system is indicated by the filled circle, at 14.1 R_N. The abrupt change of slope in the dotted curve at 9×10^7 yr corresponds to the transition to state 1 which occurs at $i = 180°$ (see text).

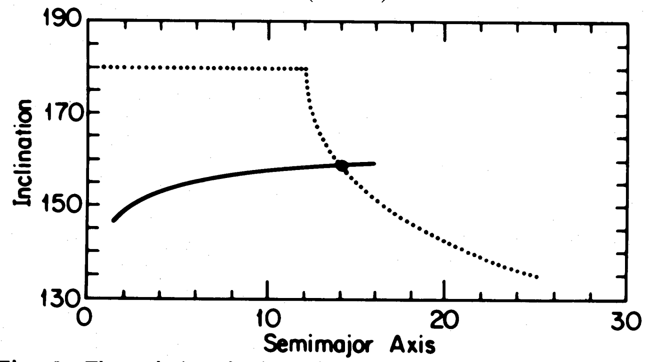

Fig. 2. The evolution of Triton's inclination as a function of semimajor axis, with $e \equiv 0$, for Cassini states 1 (solid line; $Q_T = 10^2$) and 2 (dotted line; $Q_T = 10^3$). $Q_N = 10^4$. The current system is indicated by the filled circle, at $a = 14.1R_N$ and $i = 159°$. In state 2, i reaches its maximum value of 180° at $a = 12.3R_N$, after which a transition to state 1 occurs.

As might be expected, in S2 Triton tides also dominate the evolution in inclination, in this case by a factor ~ 4000. From eq.(7), for small e:

$$a/a_0 = (\cos i_0/\cos i)^2, \tag{15}$$

and the inclination is rapidly driven to 180°, as described above. For Triton, satellite tides in S2 will have driven i to 180° when $a = 12.3$ R_N. Eq.(14) shows that, for Q_T=100, the time required for a to decrease to 12.3 R_N in S2 is only 8.6×10^6 yr.

When Triton's inclination reaches 180° in S2, an apparently paradoxical situation arises: $\dot{E}_{tidal}$ remains finite, since $\theta_2 \simeq 90°$, but no more energy can be removed from the orbit without reducing L^z_{orb}. Formally, di/dt diverges at this point. This puzzle is resolved by a consideration of the stability of state S2 when $i = 180°$. Libration about this state is possible only within a narrow zone bounded by the "critical parabola" (cf. Peale, 1969, or Jankowski *et al.*, 1989). As i approaches 0° or 180°, the critical parabola shrinks towards zero width, restricting the zone of stability for state 2. Physically, this means S2 will become unstable to small external perturbations (e.g., impacts), which will then drive Triton out of S2. The obliquity will then rapidly damp (on a tidal despinning timescale of $\sim 10^4$ yrs) to S1. Thereafter, Triton will remain in S1, evolving inwards according to eq.(8), with i remaining equal to 180°. From eq.(8), we estimate that Triton will reach Neptune's Roche limit following this scenario in $\sim 3.6\ (12.3/14.1)^{13/2}$ Gyr = 1.4 Gyr.

As discussed in Sec. 5, the rate of tidal heating required by Q_T=100, given the other parameters chosen here, lies above observational limits. Thus, if Triton is in S2, the $\sim 10^7$ yr timescale derived above for Triton's orbit to reach $i = 180°$ is probably unrealistically short. To simulate more plausible parameters, we have in fact performed our numerical integrations for S2 with $Q_T=10^3$, giving an acceptable heating rate and a timescale of $\sim 10^8$ yr for Triton's orbit to damp to 180°. The results of these numerical integrations are shown as dotted lines in Figs. 1 and 2. Once again, the full integrations are in good agreement with the approximate analytical results derived above.

Of course, should Triton actually lie in S2, this analysis implies that we are presently observing this satellite in the final $\sim 10^7 - 10^8$ yr of its orbital evolution before it reaches a perfectly retrograde ($i = 180°$) orbit. Given that ρ_T, r_T, and Q_T are not sufficiently well constrained to rule out S2 on the basis of excessive tidal heating alone, this unlikely result is perhaps the strongest argument against Triton currently occupying S2. Otherwise, it would seem that we are particularly privileged to observe a large satellite at such an unusual time in its orbital history.

7. Triton's past orbital evolution

Our modeling of Triton tides via eq.(4) does not allow us to simultaneously treat the evolution in eccentricity and inclination. Without a more detailed model of satellite tidal torques, we can only set either $\dot{e}$ or di/dt to 0, and then determine the change in i or e, respectively, due to $\dot{E}_{tidal}$. For the future orbital evolution of Triton, this poses no serious problem, as e is currently small, and will remain so. In tracking the possible course of past evolution, we consider two simple limiting cases.

First we assume that e has always been small, so that $\dot{E}_{tidal}$ drives changes in i only. This allows us to put bounds on the *maximum* change in i that may have occurred in the past. The resulting evolutionary paths are also shown in Figs. 1 and 2, for either choice of Cassini state. Timescales for the past evolution of a in Fig. 1 are very well approximated by eq.(8) for S1, and eq.(14) for S2. Variations in i are consistent with eqs. (12) and (15), respectively. "Initial" semi-major axes are $16R_N$ for S1, and $25R_N$ for S2, the increase in the latter case being due to the great efficiency of satellite obliquity tides when $\theta \simeq 90°$. Fig. 2 shows that Triton's past inclination could not have differed very much from what it is today, changing by only $\sim 1°$ in S1, and by $\sim 25°$ in S2. Although the S2 results are calculated for $Q_T = 10^3$, they change relatively little if Q_T is reduced by an order of magnitude. In this case, eq.(14) shows that a at ~4.6 Gyr ago would increase by only a factor $10^{2/13} = 1.43$, or to $\sim 36R_N$. From eq.(15), we see that this would corespond to an initial inclination of 126°. It thus appears that, barring catastrophic or non-gravitational effects, Triton has always had a retrograde orbit, with an inclination in the range of 120° – 160°.

What if we do permit e to evolve? From eq.(9), we see that Triton tides will dominate Neptune tides in semi-major axis evolution if $e \gtrsim 0.03$. At this point, the past evolution would diverge substantially from that shown in Fig. 1. The Kaula tidal formalism, however, is not well suited to simulating orbital evolution in the case of large eccentricities. There is a question as to whether the sums over q in eqs.(3) and (5) even converge for $e > 0.66$ (Kovalevsky and Sagnier, 1977; but see Szeto and Lambeck, 1982 for a contrary view), so it is likely that the results cannot be trusted for eccentricities greater than this value. Moreover, even for moderate eccentricities the sums in eqs.(3) and (5) converge very slowly, so that truncation may introduce spurious results. Additional errors can be introduced in the evaluation of the individual G_{lpq}s, by failing to retain sufficient terms in their power series expressions. Szeto and Lambeck (1982) provide several illustrative examples, showing that terms up to e^5 must be evaluated even for $e = 0.3$ in some cases, and that the sums over q must be extended to $|q| \simeq 8$ for $e = 0.3$, and to $|q| \simeq 16$ for $e = 0.5$. The alternative tidal formulation of MacDonald (1964) appears better suited to handling large–e situations, and was used by McCord (1966) to study the possible early evolution of Triton from a near-parabolic orbit. MacDonald's formulation, however, is somewhat unphysical in its treatment of the tidal phase lags for highly eccentric orbits, and although it does conserve orbital angular momentum, it is deficient in its evaluation of satellite tidal dissipation. Even in the small–e limit, it predicts an $\dot{E}_{tidal}$ which is only 3/7 of the Peale and Cassen (1978) result.

Even without detailed calculations, it is possible to derive at least one important result. When Triton tides are dominant, and assuming di/dt may be neglected, eqs. (4) and (6a) may be combined to yield (for small e): $e = e_0 \exp[-(t-t_0)/\tau_e]$, where $\tau_e = (2/21)(Q_T/k_T)(m_T/M_N)(a/r_T)^5 n \approx 3.0\times10^7(Q_T/100)$ yr at Triton's current distance. Thus, for Q_T=100, e is damped with a 30 Myr time constant. Simultaneously, a is reduced slowly, so as to conserve $L_{orb} \propto (a(1-e^2))^{1/2}$. This result allows a crude estimate of the time it would have taken Triton to decay from a large eccentricity orbit; we find that Triton's orbit could have decayed from $e = 0.2$ to the present upper limit of 5×10^{-4} in ~200 Myr. Over this same period, a would have been reduced insignificantly, from 14.7 R_N to 14.1 R_N. (Recent calculations by Goldreich et al. (private communication) suggest that a period of this order may in fact suffice to damp even an initially near-parabolic orbit, provided that the initial periapse satisfies the above angular momentum constraint.)

However, since we have no more than an upper bound on Triton's present eccentricity, it is impossible to say when in Triton's past this rapid evolution took place. Indeed, Triton's primordial eccentricity may by now have decayed below a level where it is possible, even in principle, to say when this evolution occurred. This is because Triton will currently have some very small steady-state eccentricity due to random cometary impacts. Using Burn's (1977) equation for the change in e due to an applied perturbative force, a comet of mass m_c and impact velocity v_c striking Triton will perturb e by $\Delta e \sim [(1-e^2)^{1/2}/na](m_c/m_T)v_c$. Using estimates of typical cometary mass, velocity, and frequency of collision at Neptune (Ip and Fernández, 1988), and performing the appropriate gravitational scaling, we find that cometary collisions should give Triton a steady-state (random walk) eccentricity $e \sim 10^{-9}$. As only ~400 Myr would be required for Triton's orbital eccentricity to decay from its current upper bound of 5×10^{-4} to $\sim 10^{-9}$, it seems likely that Triton's primordial eccentricity has long since decayed below this level of comet-induced "noise", so that the timing of Triton's past orbital decay may now be impossible to reconstruct.

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