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COSMOLOGY WITH A TIME-VARIABLE COSMOLOGICAL "CONSTANT"

P. J. E. PEEBLES AND BHARAT RATRA
Joseph Henry Laboratories, Princeton University
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ABSTRACT

If the potential $V(\phi)$ of the scalar field that drove inflation had a power-law tail at large ϕ, $V \approx \phi^{-\alpha}$, the mass density, ρ_ϕ, associated with ϕ would act like a cosmological constant that decreases with time less rapidly than the mass densities of matter and radiation. If ρ_ϕ were appreciable at the present epoch it could help reconcile the low dynamical estimates of the mean mass density with the negligibly small space curvature preferred by inflation.

Subject headings: cosmology — early universe

I. INTRODUCTION

One way to reconcile the dynamical estimates of the cosmological density parameter, that consistently yield $\Omega \approx 0.1$–0.4 (Peebles 1986; Brown and Peebles 1987), with the requirement of the inflation picture that the present universe have negligibly small space curvature (Guth 1981) is to assume that $(1 - \Omega)$ is balanced by a cosmological constant, Λ (Peebles 1984). The two problems with this are that we must assume that the contributions of matter and Λ to the present expansion rate are coincidentally similar, and that the required value of Λ defines a characteristic energy,

$$\epsilon = \left[\frac{3(1-\Omega)H_0{}^2\hbar^3c^5}{8\pi G}\right]^{1/4} = 0.003(1-\Omega)^{1/4}h^{1/2}\ \mathrm{eV}\ , \quad (1)$$

($H_0 = 100h$ km s^{-1} Mpc^{-1}) that does not seem natural, (e.g., Wilczek 1984). The first is unattractive but certainly does not rule out the model. We might avoid the second problem by assuming that the effective Λ term is decreasing slowly toward the natural value, $\Lambda = 0$. This could happen if the inflaton energy density that drove inflation had been converted in part to entropy at the end of inflation, leaving a part that decreased much more slowly. It might then happen that at the present epoch the inflaton energy density has come to dominate the more rapidly decreasing mass density in baryons.

This *Letter* describes a cosmology based on a Λ-like term that decreases with time because the potential energy of the inflaton field ϕ has a power-law tail at large ϕ. Time-dependent Λ-like terms have been discussed for the purpose of obtaining a tolerably small net value of Λ from a quantum field theory (Dolgov 1983; Witten 1983; Banks 1985; Barr 1986 and references therein), but as far as we are aware there has previously only been a discussion of the effect that such a term would have on the estimates of the age of the universe (Olson and Jordan 1987). We ignore here the problem of making the asymptotic value of Λ small and we assume that the scalar field does not couple to ordinary, light (with masses much smaller than the Planck mass), matter so as to avoid violating the equivalence principle (Ratra and Peebles 1987). To be consistent with inflation we assume that the universe has zero space curvature. The model is described in the next section. Our analysis of the cosmological tests for the model is described in § III.

II. THE MODEL

We adopt units so $\hbar = c = 1$, and we write the action for the model as

$$S = \int d^4x\left[\sqrt{-g}\,\frac{m_p{}^2}{16\pi}\left(-R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{\kappa}{2}m_p{}^2\phi^{-\alpha}\right) + \mathscr{L}\right], \quad (2)$$

where $m_p = G^{-1/2}$ is the Planck mass, $\mathscr{L}$ is the Lagrangian density for ordinary matter and radiation, and ϕ is the classical scalar field whose stress-energy tensor acts like a time-variable Λ. The term with constants $\kappa > 0$ and $\alpha > 0$ is the assumed approximation to the potential energy $V(\phi)$ at large ϕ. If ϕ drives inflation then $V(\phi)$ must be large (and fairly flat, to approximate a constant Λ; Fischler, Ratra, and Susskind 1985) at smaller ϕ and ought to have a steep part to cause production of entropy. Equation (2) is a classical phenomenological action; we expect the quantum mechanics of such an action to be plagued with the usual problem of nonrenormalizability. For further discussion of this model and of the case where V has an exponential tail, see Ratra and Peebles (1987).

For a homogeneous cosmological model with zero space curvature equation (2) yields

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\kappa\alpha}{2}m_p{}^2\phi^{-(\alpha+1)} = 0\ ,$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_p{}^2}(\rho + \rho_\phi)\ ,$$

$$\rho_\phi = \frac{m_p{}^2}{32\pi}[(\dot{\phi})^2 + \kappa m_p{}^2\phi^{-\alpha}]\ , \quad (3)$$

where dots denote derivatives with respect to t, ρ is the mass density in ordinary matter and radiation, ρ_ϕ is the contribution from the ϕ field, and $a(t)$ is the cosmological scale factor.

In linear perturbation theory spatial gradients in ϕ act like particles with very low mass that cannot bind to a nonrelativistic gravitational potential well that is much smaller than the horizon, H^{-1} (Ratra and Peebles 1987). Thus dynamical studies of groups and clusters of galaxies with size $\ll H^{-1}$

cannot detect concentrations of ρ_ϕ. In this sense ρ_ϕ acts like a cosmological constant, although ρ_ϕ varies with time.

To avoid affecting the usual theory of nucleosynthesis we assume that $\rho_\phi \ll \rho$ at $a(t)$ much less than the present epoch, a_0. If $a \propto t^n$ then the stable solution to equation (3), when $\rho_\phi \ll \rho$, satisfies

$$\frac{\rho_\phi}{\rho} = \left[\frac{a(t)}{a_1}\right]^{4/[n(\alpha+2)]} . \tag{4}$$

Under a wide range of initial conditions $\phi(t)$ approaches this power-law solution with increasing time (it is a time-dependent fixed point; Ratra and Peebles 1987). Equation (4) indicates that if $\alpha > 0$, ρ_ϕ decreases but less rapidly than ρ so at late times $\rho_\phi > \rho$. Equation (4) also defines the characteristic epoch, a_1, at which the dominant mass density switches from ordinary matter to the ϕ field.

We shall assume that at $a \approx a_1$, ρ is dominated by nonrelativistic matter, so $\rho \propto a^{-3}$. Then, at $a \ll a_1$,

$$\phi = \left[\frac{2}{3}\alpha(\alpha+2)\right]^{1/2} \left(\frac{a}{a_1}\right)^{3/(\alpha+2)} , \tag{5}$$

and

$$\rho(a_1) = \frac{m_p^{\,4}}{16\pi}\kappa\left(\frac{\alpha+2}{\alpha+4}\right)\left[\frac{2}{3}\alpha(\alpha+2)\right]^{-\alpha/2} . \tag{6}$$

Since $\Omega \gtrsim 0.1$, a_1 must be fairly close to a_0. This coincidental similarity between the contributions of nonluminous and ordinary matter to the present expansion rate appears, in one form or another, in most nonluminous matter models—for instance, if the axion were to be the nonluminous matter, the axion mass (i.e., a parameter of the axion potential) must have a specified dependence on the present baryon density. Although in their present form such relations appear to render models of nonluminous matter rather unattractive, they are, perhaps, best interpreted as reflecting our present lack of knowledge of the microphysical relation between nonluminous and ordinary matter.

III. THE CLASSICAL COSMOLOGICAL TESTS

We obtain $a(t)$ by numerically integrating equations (3) from $a \ll a_1$, with initial conditions from equation (5). The Hubble and density parameters are

$$H = \frac{\dot a}{a}, \quad \Omega = \frac{8\pi G\rho}{3H^2} = \frac{\rho}{\rho + \rho_\phi} . \tag{7}$$

The time parameter, $H_0 t_0$, where t_0 is the time from $a = 0$, is shown as a function of Ω and α as the solid lines in Figure 1. Most recent discussions of H_0 and t_0 (de Vaucouleurs 1986; Hesser *et al.* 1987; Tammann 1987; Thielemann *et al.* 1987; Winget *et al.* 1987) put the product at $0.6 \lesssim H_0 t_0 \lesssim 1.4$. In the limit $\alpha \to 0$ our model is equivalent to the usual constant Λ model, where $H_0 t_0$ is given by equation (4) of Peebles (1984). The effect of increasing α at fixed Ω is to reduce $H_0 t_0$. This may be an interesting effect, depending on how the extragalactic distance scale puzzle is resolved.

Since we are assuming flat space sections, the coordinate distance is

$$r = \int_t^{t_0} \frac{dt}{a(t)} , \tag{8}$$

and the bolometric distance modulus of a galaxy at redshift z differs from the distance modulus at the same redshift in the Einstein–de Sitter model, where the coordinate distance is r_e, by the amount

$$\Delta m = 5 \log_{10}\left(\frac{r}{r_e}\right) . \tag{9}$$

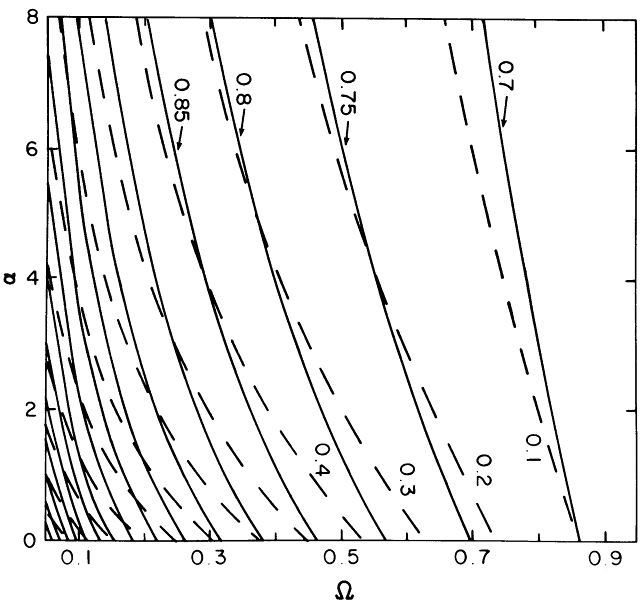

FIG. 1.—Contours of fixed time parameter, $H_0 t_0$ (*solid lines*), and distance modulus $\Delta m(z)$ (eq. [9]) relative to the Einstein–de Sitter model at $z = 1.5$ (*dashed lines*). The vertical axis is the power-law index α in the potential for the inflaton field, ϕ (eq. [2]). The horizontal axis is the density parameter (not including the inflaton part). The contour intervals are $\delta(H_0 t_0) = 0.05$ and $\delta(\Delta m) = 0.1$.

The dashed line in Figure 1 show contours of fixed Δm at $z = 1.5$, which is near the high-redshift end of the z-m diagram of Spinrad and Djorgovski (1987). Their results suggest that $\Delta m \approx 0.5$ at $z \approx 1.5$; it may be significant that we can reconcile $\Delta m \approx 0.5$ with $\Omega \approx 0.2$ by assuming $\alpha \approx 4$.

For conserved objects the count per unit increment of redshift is

$$\frac{dN}{dz} \propto z^2 A(z), \quad A(z) = \frac{H_0^{\,3} a_0^{\,2} r^2}{z^2}\frac{a}{\dot a} . \tag{10}$$

The solid lines in Figure 2 are contours of fixed A at $z = 0.7$, which is near the high-redshift end of the $A(z)$ test of Loh and Spillar (1986). Loh (1987) estimates $0.14 \lesssim A \lesssim 0.25$ at $z = 0.7$. With zero space curvature and a constant Λ ($\alpha \to 0$ in Fig. 2) this would require $\Omega \gtrsim 0.8$. It may be encouraging that if $\Omega \approx 0.2$, as suggested by the dynamical tests, and $\alpha \approx 4$ our model predicts $A \approx 0.36$, which is not so far from the Loh bound.

The angular sizes of objects of fixed linear size vary inversely with $H_0 a_0 r(z)$. In our model $H_0 a_0 r(z)$ is greater than in the Einstein–de Sitter model and less than in the constant Λ case so our model does better than the Einstein–de Sitter case but less well than the constant Λ model at fitting the z-dependence of angular sizes of radio sources (Kellermann 1972; Wills 1979).

We consider, finally, the growth of density inhomogeneities in linear perturbation theory. On scales much less than the horizon, perturbations to ϕ are relativistic and so may be

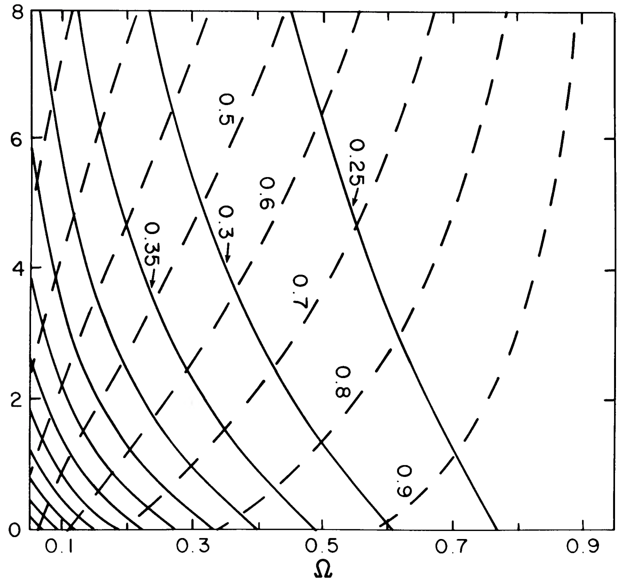

FIG. 2.—Contours of fixed $A(z)$ (eq. [10]) (*solid lines*) and normalized density perturbation growth factor Δ (eq. [12]) (*dashed lines*). The contour intervals are $\delta(A) = 0.05$ and $\delta(\Delta) = 0.1$.

ignored. (The equations of relativistic linear perturbation theory are discussed in Ratra and Peebles 1987.) The density contrast $\delta = \delta\rho/\rho$ in ordinary matter satisfies (Peebles 1980, § 10)

$$\ddot{\delta}(t) + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho\delta\ . \qquad (11)$$

At $a \ll a_1$, $\delta \propto a$, so a convenient normalization is

$$\Delta(\Omega, \alpha) = \frac{\delta(t_0)}{(1 + z_i)\delta(t_i)}\ , \qquad (12)$$

where $\delta(t_0)$ is the present value and $\delta(t_i)$ the value at z_i with $a_i \ll a_1$. Then Δ is the factor by which the growth of linear density fluctuations has fallen below the Einstein–de Sitter case. The dashed lines in Figure 2 shows contours of constant Δ in the (Ω, α) plane. The peculiar velocity field associated with the growing linear density perturbation $\delta(\boldsymbol{x}, t)$ is

$$\boldsymbol{v} = \frac{Hfg}{4\pi G\rho}, \qquad f = \frac{a}{\delta}\frac{d\delta}{da} = \frac{\dot{\delta}}{H\delta}\ ,$$

$$\boldsymbol{g} = G\rho a \int d^3x'\, \delta(\boldsymbol{x}')\frac{(\boldsymbol{x}' - \boldsymbol{x})}{|\boldsymbol{x}' - \boldsymbol{x}|^3}\ . \qquad (13)$$

The factor f is nearly independent of α and is tabulated for $\alpha = 0$ in Peebles (1984). Given $\delta(\boldsymbol{x})$, f is the factor by which the velocity is changed from the Einstein–de Sitter case. If $\Omega \approx 0.2$, $f \approx 0.36$, which is about sufficient to reconcile the relatively large galaxy density fluctuations and small relative peculiar velocities observed on scales ~ 1 to 10 Mpc (Peebles 1986).

IV. DISCUSSION

The predictions of this model for the classical cosmological tests approach the conventional constant Λ model as $\alpha \to 0$ and the Einstein–de Sitter model as $\alpha \to \infty$. We have remarked that our model may be helpful in reconciling a small density parameter, $\Omega \approx 0.2$, with the results for the classical cosmological tests, but we emphasize that the observational data are quite tentative and that the main purpose of the discussion is to indicate what constraints may soon be feasible.

At large α, where the predictions for the classical tests approach the Einstein–de Sitter model, the growth of linear density fluctuations is suppressed by the stiff ϕ energy (dashed lines in Fig. 2). At $\Omega \approx 0.2$ and $\alpha \approx 4$ the growth factor is down a factor of ~ 2.3 from that of the Einstein–de Sitter model, which does not seem very serious considering our limited understanding of how galaxies and clusters of galaxies formed. The baryon isocurvature model (Peebles 1987*a*, *b*) seems to be well adapted to this cosmology. As discussed by Peebles (1987*a*), Efstathiou and Bond (1987), and Gouda, Sasaki, and Suto (1987), one may need scattering by plasma at redshifts $z \approx 20$ to suppress small-scale fluctuations in the microwave background, but this seems reasonable because in the isocurvature model galaxies form at high redshift.

The motivation from astronomy for this cosmology is the small dynamical estimates of Ω. These estimates may be biased low if galaxies cluster more strongly than mass (White *et al.* 1987 and references therein). However, if the nonluminous mass has low pressure, galaxies would have to form at relatively low redshifts to prevent gravitational instability from removing the bias, which does not naturally fit the observations (Peebles 1987*c*). In our model, galaxies trace mass (excluding ρ_ϕ) and so can form at high redshift. We can assume this mass density is dominated by baryons, which we are sure exist, and nucleosynthesis theory indicates are present in about the abundance needed to satisfy the dynamical tests (Yang *et al.* 1984).

The motivation from particle theory is the wish to eliminate the unreasonably small characteristic energy ϵ (eq. [1]) and to find a relation between the energy density that drove inflation and the present Λ (if there is one). The former succeeds only partially. At the end of inflation when the present entropy in the primeval fireball is supposed to have been created the net mass density is $\rho_R = m_R{}^4$ and the power-law form for $V(\phi)$ extrapolated back to this epoch is

$$V_R \approx \rho_R \left(\frac{m_1}{m_R}\right)^{8/(\alpha+2)}\ , \qquad (14)$$

where $m_1 \approx \epsilon$ (eq. [1]). For commonly quoted numbers $m_1/m_R \lesssim 10^{-25}$. If $\alpha \to 0$, $V_R/\rho_R \leq 10^{-100}$. This enormously small number is one of the conceptual problems with a cosmological constant. If $\alpha \approx 6$ we get $V_R/\rho_R \lesssim 10^{-25}$, which is still exceedingly small but somewhat closer to other known small numbers in particle physics—for example, the ratio of the electroweak to grand unification scale is $\sim 10^{-13}$.

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P. J. E. Peebles and Bharat Ratra: Joseph Henry Laboratories, Department of Physics, Princeton University, Princeton, NJ 08544