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THE COUNTING OF RADIO SOURCES: IS EVOLUTION NECESSARY?

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ABSTRACT

This paper suggests a new and clear-cut way of deciding whether evolutionary effects are necessary to interpret the observed radio-source counts. To this end, a method is outlined for constructing the parent radio luminosity function from a complete sample of radio sources whose redshifts are all known. It is argued that if such a luminosity function can be constructed without evolutionary parameters, then evolutionary effects are not required. The method is illustrated by its applications to two well-known samples of radio sources. It is further shown that the non-evolving luminosity function satisfies the constraint of sky brightness and is consistent with the observed redshift–flux-density plot. Thus it appears from these investigations that evolution is not required either in luminosity or in number density.

I. INTRODUCTION

In the early days of cosmology, it was hoped that the counting of radio sources down to different flux levels would provide a method of discriminating between different cosmological models. This hope has not been realized, largely because the variation in the intrinsic properties of the objects to be counted tends to shroud the cosmological variations. For example, as noted by Hoyle and Burbidge (1970), the plot of $\log z$ ($z =$ redshift of a source) against $\log S$ ($S =$ flux density of the source) gives a scatter diagram. Although this finding was based on a smaller sample, Figs. 1 and 2 given here illustrate the scatter for two complete samples with much larger numbers.

The scatter indicates that the cosmological-distance effect (if indeed it exists!) is smeared out by the wide range of radio luminosities of the sources. Clearly, the radio luminosity function (RLF) plays a critical role in determining the S–z distribution in a typical complete sample. Given a RLF and a particular cosmological model, it is possible to predict the numbers of sources in specified redshift ranges in a complex flux-density-limited sample. The question is, can this process be reversed?

In principle, the answer to this question is affirmative, if the RLF is non-evolving. That is, if there is no z dependence in the RLF, then in a given Friedmann model it is possible to determine the shape of the RLF uniquely from the observed number–redshifts plot in a complete sample. In practice, this has so far not been attempted because there were few complete samples for which most of the redshifts were known. Happily, the situation has improved and, as we shall show here, it is possible to carry out this calculation for at least two samples: (I), the 3CR sample of Bennett (1962), and (II) the 2.7 GHz sample of Wall and Peacock (1985). We will undertake this exercise here.

There are two advantages from such an attempt. First, the RLF can be reliably determined by sampling a large enough volume, and surveys like I and II above do just that. Second, the simplicity of Occam's Razor requires that we first investigate (and rule out if found untenable) the simplest hypothesis, which in this case is that the RLF is non-evolving. If the RLF determined from this hypothesis turns out to be unphysical in the sense to be discussed in Sec. II, we are forced to consider evolutionary models. As the work of Peacock (1985) amply demonstrates, such models require a large number of adjustable parameters and so take away the basic simplicity of the picture. The work outlined here has the advantage of telling us whether the simple (non-evolving) model must necessarily be abandoned in favor of a more complicated (evolving) scenario.

In Sec. II we describe the general mathematical method of inverting the N–z data into an RLF. Then in Sec. III we apply the method of samples (I) and (II). In Sec. IV we subject the RLF to two tests. First, it should not produce excessive radio background. Second, the theoretical z–S plot produced by it should be consistent with the observed one. In the final section we compare our non-evolving models with the evolving ones of Peacock (1985).

II. THE INVERSION FORMULA

a) Basic Assumptions

The formula we derive here is an extension of the earlier work of Burbidge and Narlikar (1976) for nearby sources in a Euclidean space. These authors had found that the number–redshift curve for the 3CR radio galaxies whose redshifts were known in 1975 follows an observed relation

$$\frac{dN}{d\ln z} = \text{constant}, \tag{1}$$

where $N(z) =$ number of sources with redshifts not exceeding z. Assuming a linear Hubble's law, it was then possible to deduce a radio luminosity function

$$f(L) \propto L^{-2.5}, \tag{2}$$

where $f(L)dL =$ number of radio galaxies per unit volume with luminosities in the range $(L, L + dL)$.

Since 1975, there has been considerable progress in the determination of redshifts of the 3CR radio galaxies, with the result that it is possible not only to check the conclusion (2) at the bright end of the RLF but also to determine the RLF exactly.

To this end, we state the basic assumptions of our calculation first.

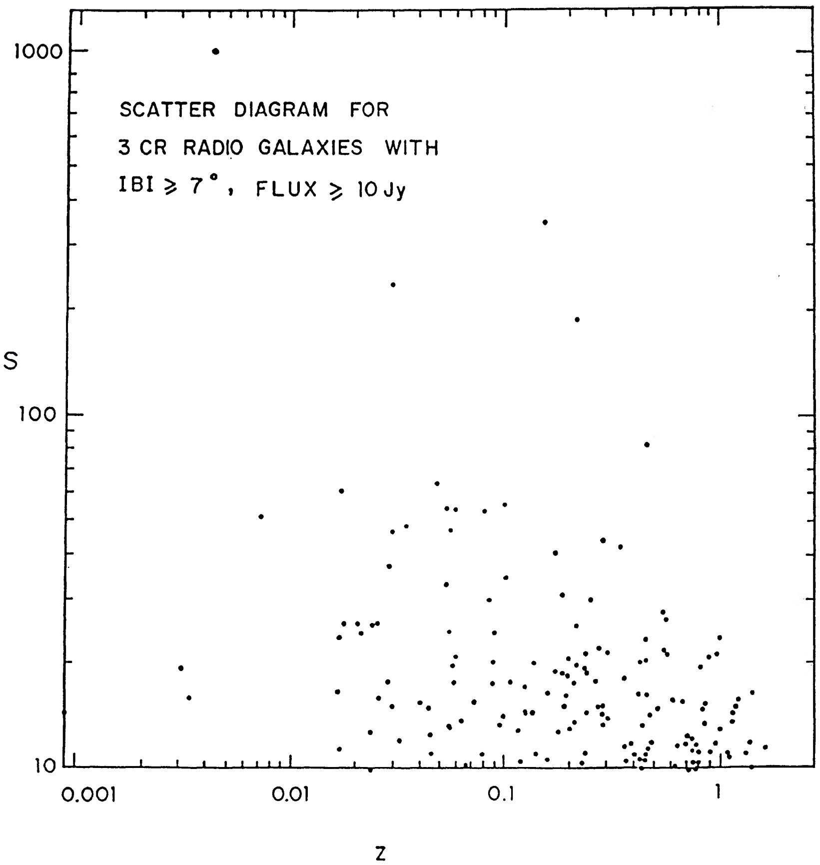

FIG. 1. Scatter diagram obtained by plotting log z against log S for the 3CR radio galaxies with $|b| \geqslant 7°$ and $S \geqslant 10$ Jy.

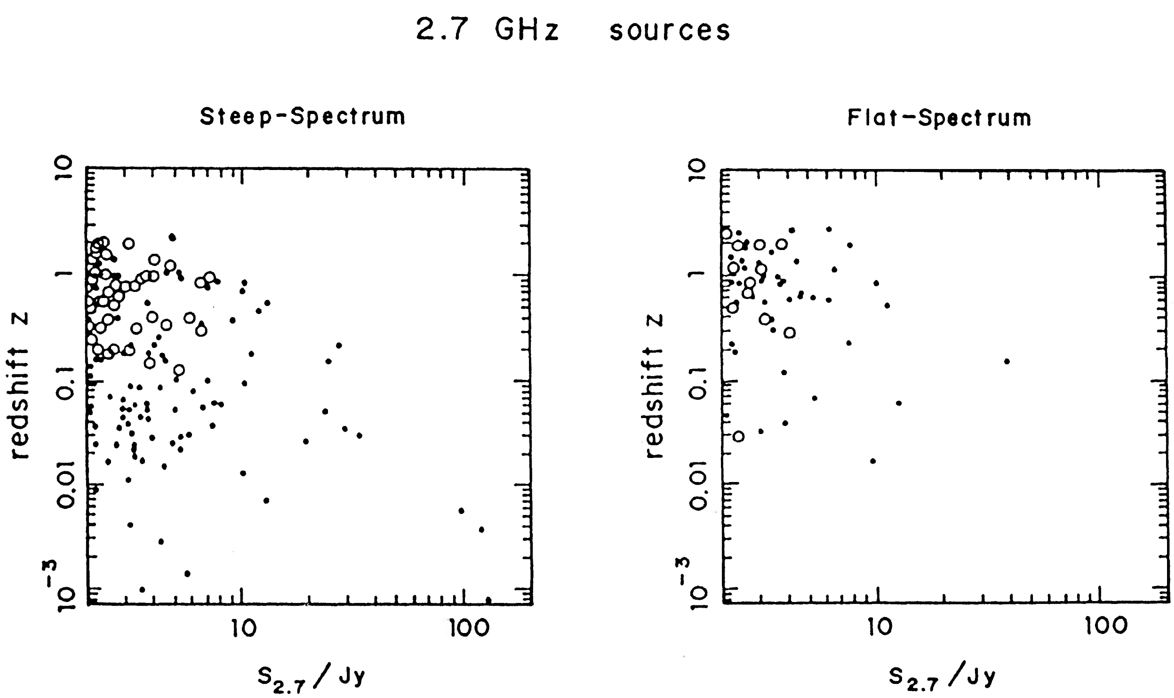

FIG. 2. log $S_{2.7}$/Jy–log z planes for steep- and flat-spectrum sources. Light circles denote estimated redshifts. Figure based on Wall and Peacock (1985).

(i) The cosmological model for doing the calculation is a standard Friedmann–Robertson–Walker model with curvature parameter $k(=1, 0, \text{or } -1)$ and the present deceleration parameter q_0. The present value of Hubble's constant is taken to be $H_0 = 100h_0$ km s^{-1} Mpc^{-1}.

The line element for this model is given by

$$ds^2 = c^2dt^2 - Q^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

in terms of the comoving space coordinates (r,θ,ϕ) and the cosmic time t. For the derivation of cosmological formulas of this section, see Narlikar (1983).

We denote by t_0 the present epoch, and write an overhead dot for a derivative with respect to t. Thus

$$H_0 = \frac{\dot{Q}}{Q}\Big|t_0, \quad -q_0 = H_0^{-2}\frac{\ddot{Q}}{Q}\Big|_{t_0}. \tag{4}$$

(ii) The normalized RLF is denoted by $g(L)$. Let n_0 denote the number of sources per unit proper volume at the present epoch. The function $g(L) \geqslant 0$ is assumed to depend on L only and not on the epoch at which it is measured. We will also assume that as $L \to \infty$, $g(L) \to 0$ in such a way that

$$F(L) = \int_L^\infty g(x)dx \to 0 \quad \text{as } L \to \infty. \tag{5}$$

This condition is physically reasonable and consistent with the requirement $F(0) = 1$. Hereafter, we will assume L to measure the rate at which energy is radiated over a unit bandwidth $\Delta\nu = 1$ Hz.

(iii) In addition to the above condition on $F(L)$, we also require the total sky brightness produced by the source population to be bounded.

(iv) We assume that the survey is limited to all sources in a given solid angle brighter than the flux density S_0, and that the redshifts of all sources are known. Let us assume that for the sample in question dN denotes the number of sources with redshifts in the range $(z, z+dz)$.

In practice, $N(z)$ may be a smooth function which approximates to the above data, with $dN/dz \equiv G(z)$, say.

(v) We will assume that all sources have spectral index $\alpha = 1$. This assumption is not critical to the nature of our discussion, although it does simplify the analytical expressions considerably. For example, if the source at redshift z is radiating energy L per second per unit frequency range with spectral index $\alpha = 1$, it produces a flux density at the observer $S = L/4\pi D^2$, where D is the luminosity distance. For $\alpha \neq 1$, the expression for S gets multiplied by $(1+z)^{1-\alpha}$.

b) Derivation of the RLF

To fix ideas, we will work with a Friedmann model with $k = +1$. For this model $q_0 > 1/2$. Denoting $Q(t_0)$ by Q_0, we get the number of sources per unit coordinate volume at the present epoch as

$$n_0 Q_0^3 = \left(\frac{c}{H_0}\right)^3 n_0(2q_0 - 1)^{-3/2}. \tag{6}$$

Consider the coordinate volume of a shell subtending solid angle Ω at the observer and with the r range $(r, r+dr)$. Then $D = (c/H_0)x$, with

$$x = \frac{1}{q_0^2}\left[q_0 z + (q_0 - 1)(\sqrt{1+2q_0 z} - 1)\right], \tag{7}$$

where the shell $(r, r+dr)$ consists of sources with redshifts in the range $(z, z+dz)$. The coordinate volume of the shell is given by $V(z)\,dz$, where

$$V(z) = \frac{(2q_0-1)^{3/2}x^2}{(1+z)^3\sqrt{1+2q_0 z}}\,\Omega. \tag{8}$$

Multiplying Eqs. (6) and (8) we get the total number of sources in this shell as

$$V(z)n_0 Q_0^3 dz = n_0\left(\frac{c}{H_0}\right)^3 \frac{x^2\Omega}{(1+z)^3\sqrt{1+2q_0 z}}\,dz. \tag{9}$$

Not all these sources will, however, appear in the above survey. The flux-density limit S_0 implies a lower limit on the luminosity L of the sources that appear in the survey:

$$L > 4\Pi\left(\frac{c}{H_0}\right)^2 S_0 x^2. \tag{10}$$

Hence the number of sources with redshifts in the range $(z, z+dz)$ appearing in the sample is

$$dN = n_0\left(\frac{c}{H_0}\right)^3 \Omega \frac{x^2}{(1+z)^3\sqrt{1+2q_0 z}} \times F\left[4\pi\left(\frac{c}{H_0}\right)^2 S_0 x^2\right] dz. \tag{11}$$

Comparison with observations gives us $dN/dz \equiv G(z)$. Thus the problem consists of determining $F(L)$ and hence $g(L)$ from the relation

$$F\left[4\pi\left(\frac{c}{H_0}\right)^2 S_0 x^2\right] = \left(\frac{H_0}{c}\right)^3 \frac{G(z)(1+z)^3\sqrt{1+2q_0 z}}{n_0\Omega x^2}. \tag{12}$$

To solve this problem, define

$$L_0 = 4\Pi\left(\frac{c}{H_0}\right)^2 S_0. \tag{13}$$

The formula (7) can be inverted to give

$$z = q_0 x - (q_0 - 1)\left[\sqrt{1+2x} - 1\right]. \tag{14}$$

Writing $L_0 x^2 = L$, we therefore get

$$F(L) = \left(\frac{H_0}{c}\right)^3 \frac{G(z)(1+z)^3\sqrt{1+2q_0 z}}{n_0\Omega x^2}. \tag{15}$$

On the right-hand side $x \equiv (L/L_0)^{1/2}$ and z is a function of x through Eq. (14). Thus $F(L)$ is explicitly determined. The formula (15) is valid for all values of k and q_0.

At the low-redshift end, the above formula reduces to

$$F(L) \simeq \left(\frac{H_0}{c}\right)^3 \frac{G(q_0 x)}{4\pi n_0\Omega x^2} = \left(\frac{H_0}{c}\right)^3 \frac{L_0 G(q_0\sqrt{L/L_0})}{4\pi n_0 \Omega L}. \tag{16}$$

In the earlier investigation of Burbidge and Narlikar (1976), the formula (1) implied $G(z) = A/z$, $A =$ constant. Therefore, Eq. (16) leads to

$$F(L) \propto L^{-3/2}, \quad g(L) \propto L^{-5/2}, \tag{17}$$

as given by Eq. (2).

c) Validity of the RLF

Before coming to the observed forms of $G(z)$, let us lay down the criterion for deciding whether the solution obtained above is physically valid. For this we need conditions (ii) and (iii) of Sec. II*a*. To illustrate how these constraints can rule out non-evolutionary luminosity functions, consider the hypothetical situation in which the function $G(z) = A/z$ continues up to high redshifts also. It is easy to verify that for $L \gg L_0$,

$$F(L) = BL^{1/4}, \quad B = \text{constant} > 0. \tag{18}$$

Clearly, this result is absurd, since g is negative while F diverges at high luminosities.

Thus it is necessary to check that the RLF obtained is physically meaningful. If it does not turn out to be so, the conclusion must be that evolution in some form or other is necessary. In what follows we will find it convenient to approximate F by a form $L^{-\beta}$, where β varies slowly with L.

III. APPLICATIONS

a) The 3CR Sample

Spinrad *et al.* (1985) have recently updated the optical information on the 3CR catalog. Their list includes 298 sources, of which 195 are radio galaxies, 53 are QSOs, and 38 are unidentified. In addition, there are 12 identifications that are still not confirmed. We shall concentrate on the radio galaxy sample only, leaving aside the QSOs and the unidentified sources. So far as the QSOs are concerned, the situation is as follows. Unlike the radio galaxies, the QSOs do not show a tight Hubble relationship. Further, there are observational studies that may suggest that the redshifts of QSOs are not entirely due to the expansion of the universe (Narlikar 1986). If the QSOs are indeed at cosmological distances, optical studies alone demonstrate evolution (Schmidt and Green 1983). If the QSOs are not at cosmological distances, then they are irrelevant to cosmology and our analysis here cannot be applied to them.

In the radio galaxy sample there are 163 galaxies with $|b| \geqslant 7°, S_0 = 10$ Jy. We have excluded low-latitude objects to prevent nearness to the galactic plane, and the above restriction is in keeping with the earlier work of Burbidge and Narlikar (1976).

The plot of N against $\log z$ is given in Fig. 3, and it may be looked upon as an update of Fig. 1 of Burbidge and Narlikar. It is interesting to note that a linear relationship of the kind given by Eq. (1) exists over considerable spans of $\log z$. One can draw two straight lines across the curve having the equations

$$dN = 54.67\, d(\log z), \quad 0.01585 < z < 0.137, \tag{19}$$

$$dN = 102.5\, d(\log z), \quad 0.137 < z < 1.781. \tag{20}$$

The corresponding integral RLF, given by $F(L)$, is plotted in Fig. 4. Note that it starts off with a dependence $\beta = 1.5$ and flattens to $\beta = 0.45$ at the limit of the sources in the 3CR survey. The luminosity range covered by this RLF is (for $q_0 = 0.5$)

$$3.3 \times 10^{22}\ \text{WHz}^{-1} < L < 2.0 \times 10^{29}\ \text{WHz}^{-1}. \tag{21}$$

Figure 5 shows the plot of β against $\log L$.

It is interesting to note that $F(L)$ flattens towards the upper limit of this range. This flattening cannot, however, go on without limit, since towards the large-redshift end the number count begins to flatten compared to the straight lines of Eqs. (19) and (20). This flattening of number count is not an evolutionary effect. It can be traced to a steepening of the RLF at large L. Thus the overall-consistency requirement of Sec. IIc is satisfied. We will later demonstrate that no abnormal sky-brightness problems arise in this way.

It seems therefore that a non-evolving RLF is able to explain the observed redshift distribution of the radio sources in the 3CR catalog.

b) High-Frequency Sample

Although the 3CR was a pioneering survey and has been studied most thoroughly, it is essentially limited to high-flux-density sources. There have been many surveys extending to fainter flux densities, and these have been at higher frequencies. Most of these surveys are poor in terms of redshift determinations, and so the above technique cannot be applied. It is possible, however, to compare our RLF with that of Peacock (1985) as determined from a survey at 2.7 GHz. This survey (Wall and Peacock 1985) is 73% complete in redshift measurements. For those radio sources for

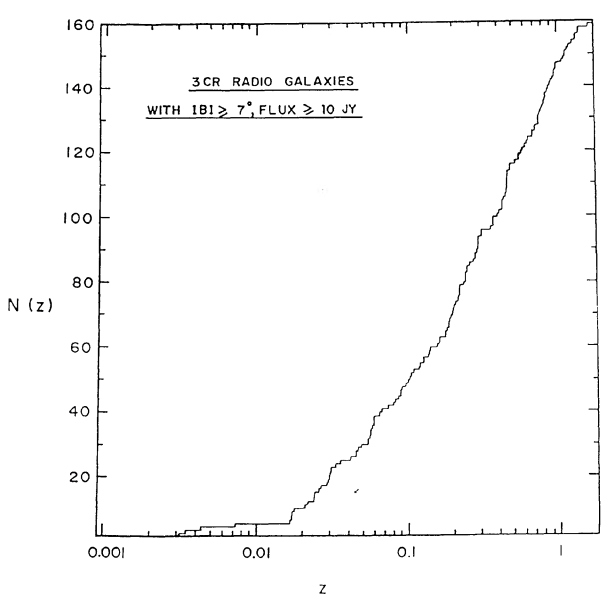

FIG. 3. Plot of $N(z)$ against $\log z$ for 3CR radio galaxies with $|b| \geqslant 7°$, flux $\geqslant 10$ Jy. At high z, the curve shows indications of flattening.

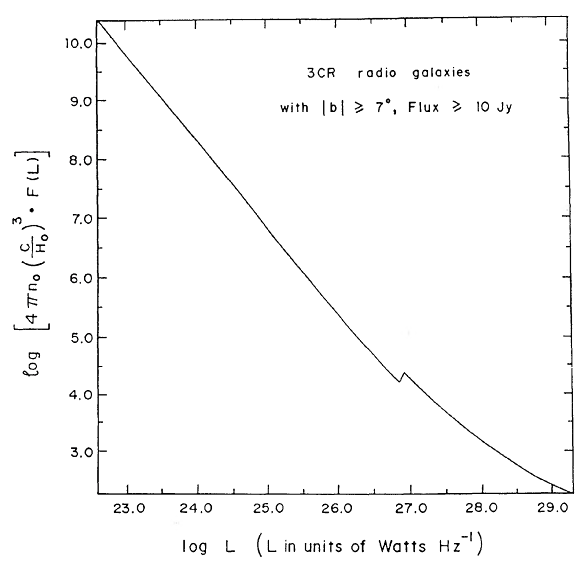

FIG. 4. Plot of $\log F(L)$ against $\log L$ for 3CR radio galaxies with $|b| \geqslant 7°$, flux $\geqslant 10$ Jy. The discontinuity at $\log L \sim 27$ is not physically significant and arises from the two-straight-line approximation of Eqs. (19) and (20).

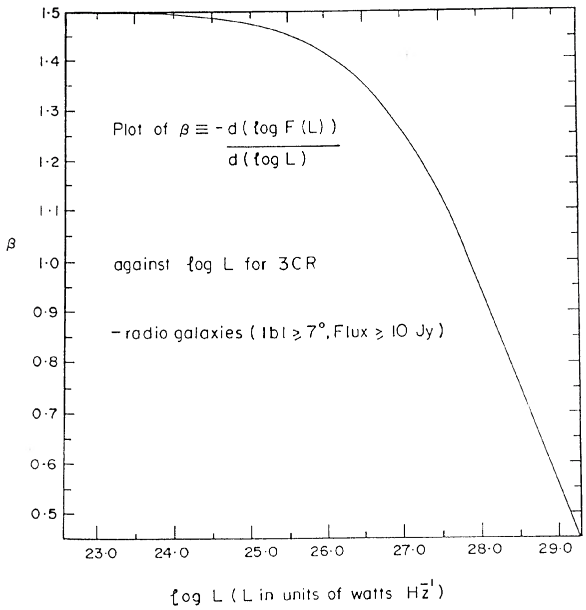

FIG. 5. Plot of $-\beta \equiv d(\log F(L))/d(\log F(L))$ against $\log L$ for 3CR radio galaxies ($|b| \geqslant 7°$, flux$\geqslant$ 10 Jy). The index β steadily decreases from 1.5 for low-luminosity sources to 0.45 for high-luminosity ones.

which redshifts are not known, we follow the estimates given by Wall and Peacock based on their optical magnitudes and Hubble's law. In short, this sample has 233 sources down to $S_0 = 2$ Jy, covering a solid angle $\omega = 9.81$ Sr. Of these, the distribution is as follows:

119 confirmed galaxies,

24 very faint extended objects,

74 confirmed QSOs,

10 stellar-like objects,

6 empty fields.

Although for reasons given in Sec. III*a* we should consider only radio galaxies, for a comparison with Peacock's work we will apply our analysis to the entire sample as well. We will try to fit the N–z curve with a smooth function of the form

$$N(z) = A + B \log z + C(\log z)^2, \tag{22}$$

which is a generalization of Eq. (1) to the quadratic form in $\log z$.

The 233 sources in the entire sample range in z from 0.0008 to 2.852. The range log(0.0008) to log(2.852) was divided into one hundred equal parts, with each bin corresponding to a width of $\Delta \log z = 0.0355$. Since the data give the numbers in each bin, it is possible to make a best 'fit' of the form Eq. (22) by the least-square method. The resulting values are

$$A = 178.64, \quad B = 140.85, \quad C = 27.835. \tag{23}$$

Figures 6 and 7 illustrate the goodness of fit of $N(z)$ with respect to $\log z$ as well as with respect to z. Notice that the fit does not hold well at the extreme high-redshift end. This is to be expected from the steepening of the RLF at high L, which in turn is required by our consistency constraints of Sec. II*a*.

Given the form of $N(z)$, it is now a simple matter to determine the RLF. The result is illustrated in Fig. 8. The RLF at any L in the relevant range can be approximated by a power law:

$$g(L) \propto L^{-\gamma}, \tag{24}$$

where γ varies slowly with L. Table I gives the variation of γ with L, for the Friedmann model $k = 1$, $q_0 = 1/2$. γ briefly rises from its value of 1.98 at $\log L \simeq 24$ to 2.25 at $\log L = 25.5$, then it starts diminishing, falling to 1.79 at $\log L \simeq 29$.

Again, we notice a flattening of the RLF at high L, but one that will not continue for long in view of the flattening of the $N(z)$ curve discussed earlier. Thus there is no conflict with sky brightness. Further, $\gamma(L)$ weakly depends on q_0, the type of cosmology used, and none of the models appears to be ruled out by the consistency of the RLF.

A similar analysis carried out for the radio galaxy sample gives

$$A = 135.32, \quad B = 89.68, \quad C = 14.66, \tag{25}$$

with $\gamma(L)$ varying from 2.03 at $L = 1 \times 10^{23}$ W Hz^{-1} to 1.78 at $L = 4.24 \times 10^{28}$ W Hz^{-1}, for $k = 0$, $q_0 = 1/2$.

In the above analysis, we have not bothered to distinguish between steep-spectrum and flat-spectrum sources. The distinction becomes relevant in the evolutionary scenarios where the two types are assigned different evolutionary properties. So far as our analysis is concerned, a spectral index α changes the condition (10) to

$$L > 4\Pi \left(\frac{c}{H_0}\right)^2 S_0 x^2 (1+z)^{\alpha - 1}. \tag{26}$$

The effect on the analysis of $\alpha \neq 1$ is marginal but can be computed if required. This detail has no bearing on our present investigation, which seeks to answer the question: "Is evolution really necessary?"

c) The Range of Validity of the Analysis

It is important to emphasize here that the forms of $G(z)$ given by Eqs. (19), (20), or (22) are determined purely empirically from the data and, as shown in Figs. 3 and 6, the fits provided by the simpler forms above do not work so well at large $z (\gtrsim 1)$. For example, the rising straight line of Eq. (20) contrasts with the flattening of N with $\log z$ at large z indicated by the data in Fig. 3.

It should be possible to include these effects at high z and determine the form of $G(z)$ more precisely than given here. We have not gone through such an exercise here, partly because there are not many data points beyond $z \gtrsim 1$ to give a reliable fit and also because we wish to demonstrate that even our crude approximations of Eqs. (19), (20), and (22) fare reasonably satisfactorily. For this reason, in a statistical comparison of theory with data, one need not attach much significance to the few points beyond, say, $z \sim 1.2$, where the chosen forms of $G(z)$ do not work well. When surveys rich in high-redshift galaxies become available, the technique can certainly be applied to them with a form of $G(z)$ determined with a greater degree of reliability; and, if $G(z) \propto z^{-1}$ continues to large z, the situation of Eq. (18) would certainly rule out the non-evolutionary hypothesis.

IV. TWO OBSERVATIONAL CONSTRAINTS

a) The Limit of Sky Brightness

The radio luminosity functions derived in Sec. III show a flattening towards the high-luminosity end. This flattening

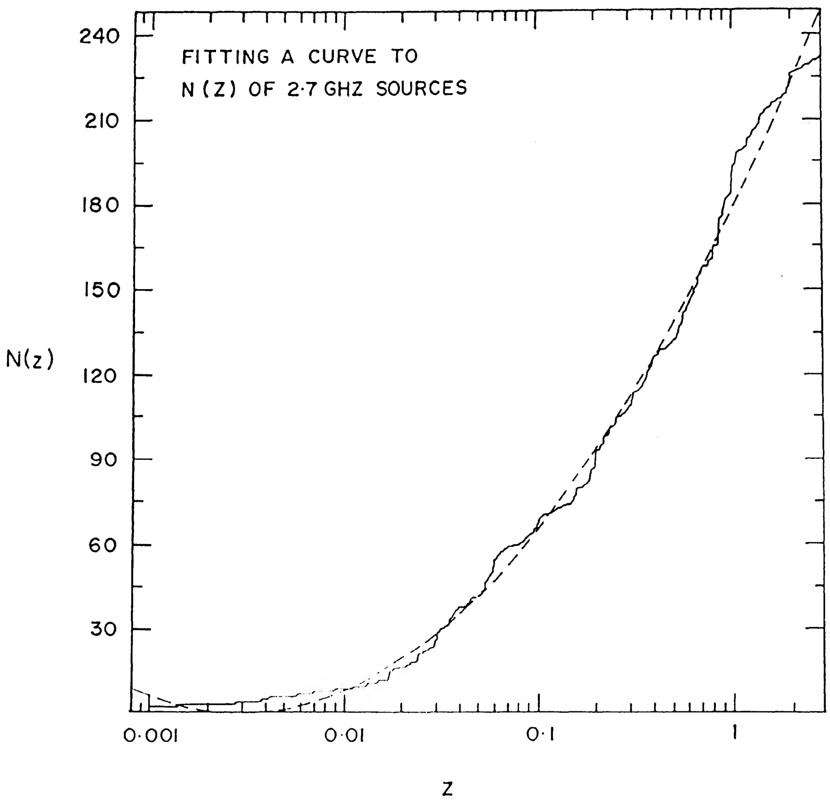

FIG. 6. $N(z)$–log z plot for 2.7 GHz sources (the continuous line is the observational curve, and the dashed line is the least-square fit to the observations).

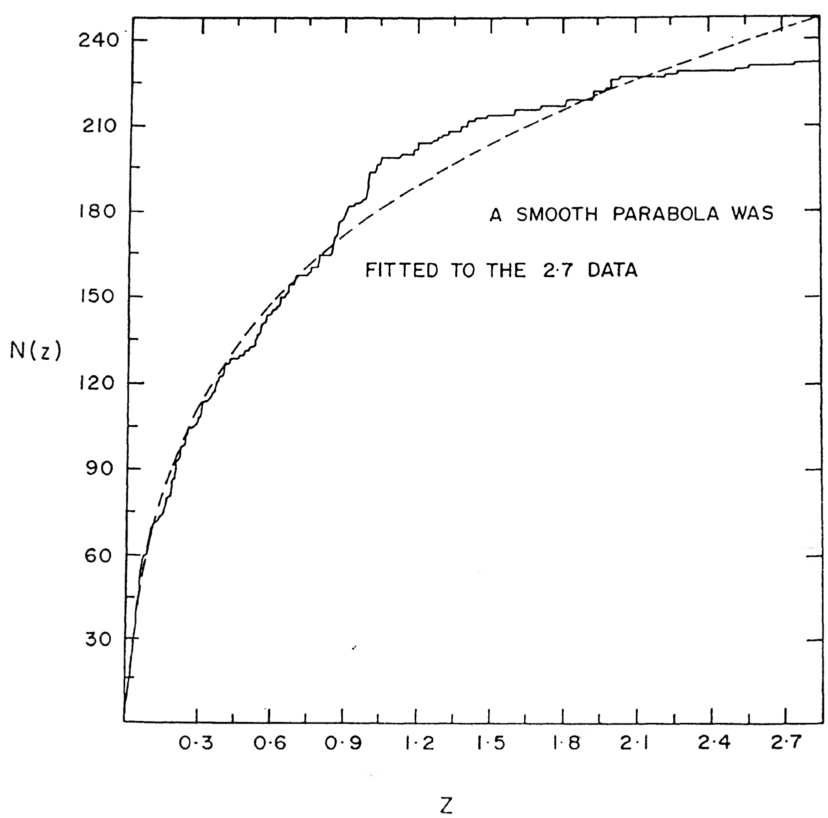

FIG. 7. $N(z)$–z plot for 2.7 GHz sources (the continuous line is the observational curve, and the dashed line is the least-square fit to the observations).

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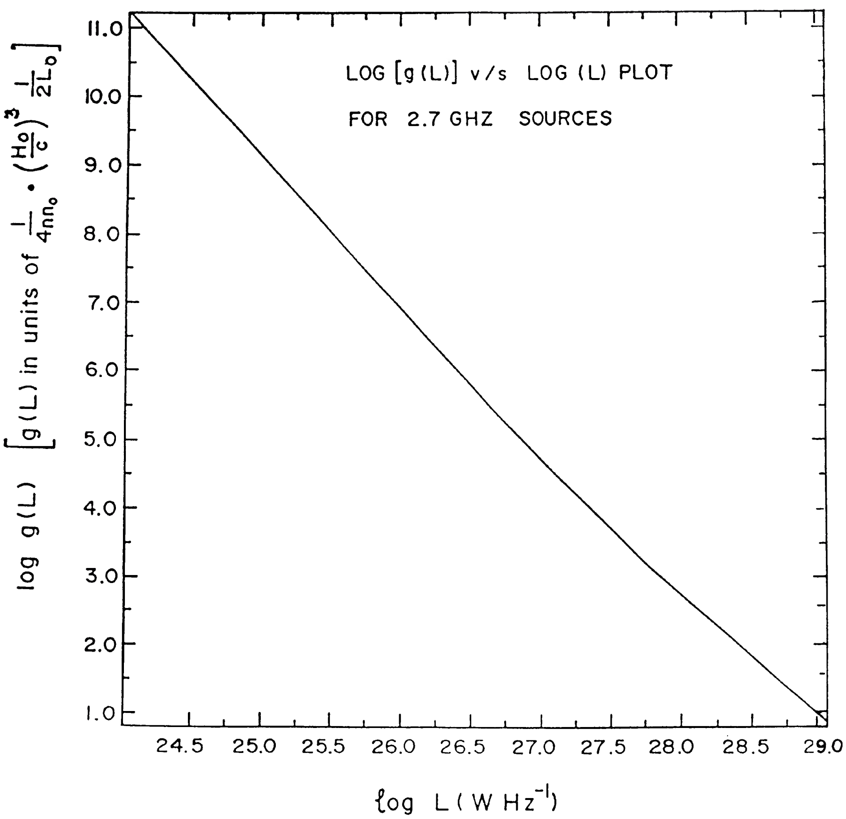

FIG. 8. Plot of $\log g(L)$ against $\log L$ for 2.7 GHz sources.

cannot, however, continue forever, since the number–redshift count curve also begins to flatten at high redshifts. Nevertheless, it is necessary to reassure ourselves that even this intermediate flattening does not produce a 'sky-brightness catastrophe'. Below, we estimate the energy density of the radio background generated by the RLFs calculated in Sec. III.

The total number of sources in the redshift shell $(z, z+dz)$ is given by Eq. (9). The flux density of radiation produced at $r=0$ by sources with $z_1 \leqslant z \leqslant z_2$ and luminosities $L_1 \leqslant L \leqslant L_2$ is therefore

$$S_B = n_0\left(\frac{c}{H_0}\right)\int_{L_1}^{L_2} g(L)L\ dL \int_{z_1}^{z_2} \frac{dz}{(1+z)^3\sqrt{1+2q_0 z}}. \quad (27)$$

TABLE I. 2.7 GHz sources.

$\log_{10} L$	$\gamma = \frac{-d[\log_{10} g(L)]}{d(\log_{10} L)}$
24.02	1.98
24.20	2.07
24.73	2.19
25.11	2.23
25.54	2.25
26.00	2.23
26.30	2.21
26.68	2.16
27.19	2.07
27.57	1.98
28.00	1.88
28.25	1.83
28.76	1.77
29.06	1.79

As before, we have taken the spectral index $\alpha = 1$. Our estimates, in any case, are too crude to require, at this stage, a fine distinction between steep- and flat-spectrum sources.

We will explicitly work out the background flux density S_B for the RLF determined from the 3CR sample. The maximum redshift in this sample is 1.781. Limiting ourselves to the sky brightness generated by all sources out to this redshift and taking L_1, L_2 to be given by Eq. (21), we find that for $q_0 = 0.5$, $S_B = 7.6\times 10^3$ Jy. The value of S_B decreases as q_0 increases: for $q_0 = 0.1$, $S_B = 11.3\times 10^3$ Jy, while for $q_0 = 1$, $S_B = 6.3\times 10^3$ Jy.

A background flux density S_B at $\nu = 178$ MHz would correspond to a total flux of background radiation over the radio-frequency range 10 MHz–30 GHz of

$$\mathscr{F}_B = S_B \times 1.78\times 10^9 \int_{10\,\mathrm{MHz}}^{30\,\mathrm{GHz}} \frac{d\nu}{\nu} \cong 1.4\times 10^{10}\ S_B. \quad (28)$$

In terms of energy density of the radio background, this becomes

$$\xi_B = \frac{1}{c}\mathscr{F}_B \cong 3.5\times 10^{-20}\ \mathrm{erg\ cm^{-3}}. \quad (29)$$

This value is well below the radio-background energy density actually observed at $\leqslant 10^{-18}$ erg cm^{-3} (Narlikar 1983). Of course, to the value (29) we have computed we must add contributions from sources beyond $z \geqslant 1.8$. This contribution will be negligible as the integrand of the z integral in Eq. (27) decreases rapidly for large z.

Had we worked with $\alpha < 1$, we would have obtained somewhat larger values for ξ_B. The value of ξ_B for $q_0 = 0.5$, $\alpha = 0.5$, for example, is higher than the corresponding value for $q_0 = 0.5$, $\alpha = 1$ by less than 20%. This difference hardly matters in the present context.

b) The Redshift–Flux-Density Distribution

A second constraint on an RLF is that it should generate a redshift–flux-density distribution of the source population that is statistically not inconsistent with the observed distribution. It is not expected that theory and observations match exactly since the latter can be subject to statistical fluctuations. Thus, although we started with the integral number–redshift curve, it would be instructive to see to what extent the differential number counts generated by the theory agree with the observed counts in different redshift bins and flux-density bins.

Accordingly, we define by $N(z_1z_2;S_1,S_2)$ the number of sources in the redshift range $z_1 \leqslant z \leqslant z_2$ and the flux-density range $S_1 \leqslant S \leqslant S_2$. Using the notation of Sec. II*b* we find after a simple manipulation

$$N(z_1,z_2;S_1,S_2) = n_0\left(\frac{c}{H_0}\right)^3 \Omega \int_{z_1}^{z_2} \left\{F\left(\frac{S_1L_0x^2}{S_0}\right) - F\left(\frac{S_2L_0x^2}{S_0}\right)\right\} \times \frac{x^2dz}{(1+z)^3\sqrt{1+2q_0z}}, \quad (30)$$

where x is given by Eq. (7). The quantities n_0, c/H_0, and L_0 drop out of the answer if we use Eq. (15). We then get

$$N(z_1,z_2;S_1,S_2) = \int_{z_1}^{z_2} \frac{\{H(\bar{z}_1) - H(\bar{z}_2)\}x^2dz}{(1+z)^3\sqrt{1+2q_0z}}, \quad (31)$$

where

$$H(z) = \frac{G(z)(1+z)^3\sqrt{1+2q_0z}}{x^2} \quad (32)$$

and $\bar{z}_1,\bar{z}_2$ are related to S_1, S_2 by the following relations:

$$\bar{z}_i = q_0x\sqrt{\frac{S_i}{S_0}} - (q_0-1) \times \left\{\sqrt{1+2x\sqrt{\frac{S_i}{S_0}}} - 1\right\}, \quad i=1,2. \quad (33)$$

To apply the χ^2 test for consistency of the theory we have divided the z–S plane into various rectangular bins bounded by z = constant, S = constant. Table II illustrates the procedure for 3CR sources. Here the observed numbers O and the theoretically expected numbers E are given for various rectangular bins bound by z_i,z_{i+1},S_i,S_{i+1}. The redshifts in Table II go as far as 1.252 because the functional form $G(z) \propto z^{-1}$ is not expected to hold at very high redshifts, as seen from Fig. 3.

To ensure stability of the χ^2 test against small-number fluctuation, the required statistical criterion of $O \geqslant 5$ should be followed. This necessarily requires pooling of small bins into large ones. We have tried to ensure this, except for two cases where O = 4. For the bins of Table II, the χ^2 works out at 28.25 for 19 degrees of freedom. The probability of obtaining this by chance is between 0.05 and 0.10. Thus, by standard statistical criteria, the null hypothesis of 'no evolution' cannot be rejected.

TABLE II. z–S data.

z_i	z_{i+1}	S_i	S_{i+1}	$N(z_i,z_{i+1},S_i,S_{i+1})$ Observed, O	Expected, E	$\frac{(O-E)^2}{E}$
0.018	0.504	10.0	11.2	9	18.81	5.12
0.504	1.252	10.0	11.2	6	5.14	0.14
0.018	0.504	11.2	13.0	12	20.01	3.21
0.504	1.252	11.2	13.0	13	5.58	9.87
0.018	0.342	13.0	14.4	11	9.39	0.28
0.342	1.252	13.0	14.4	4	4.81	0.14
0.018	0.342	14.4	15.7	10	7.26	1.03
0.342	1.252	14.4	15.7	6	3.52	1.75
0.018	0.342	15.7	17.8	6	7.95	0.49
0.342	1.252	15.7	17.8	5	4.36	0.09
0.018	0.18	17.8	19.9	5	3.86	0.34
0.18	1.252	17.8	19.9	8	5.03	1.75
0.018	0.342	19.9	24.6	9	7.91	0.15
0.342	1.252	19.9	24.6	8	4.88	1.99
0.018	1.252	24.6	30.4	9	8.92	7.17×10^{-4}
0.018	0.18	30.4	46.9	5	4.4	0.08
0.18	1.252	30.4	46.9	5	6.74	0.45
0.018	1.252	46.9	54.4	4	2.29	1.28
0.018	1.252	54.4	350.0	8	7.18	0.09

c) The Kolmogorov–Smirnov Test

Peacock (1983) has advocated the use of a Kolmogorov–Smirnov test since, in its one-dimensional form, the K–S test is distribution free (Kendall and Stewart 1961). However, as noted by Peacock himself, the two-dimensional K–S test that is needed to test the S–z distribution suffers from certain drawbacks: (i) it is not distribution free and (ii) there is no rigorous analytical derivation available for the statistic to be tested. We have reservations of our own. The K–S test depends critically on monotonicity of data points. In the one-dimensional case, such monotonicity can be uniquely specified and is expressed by cumulative distribution functions. In higher dimensions, one cannot specify this uniquely, which is why such a test cannot even be conceptualized. Indeed, several professional statisticians have advised us that an applicable form of bivariate or multivariate K–S test is not yet available.

Under the circumstances, we have repeated Peacock's exercise *ab initio* for our theoretical distribution. Basically, this involves using a Monte Carlo technique to generate a z–S distribution and then examining the deviation between the theoretical and generated distributions. A K–S statistic Z can be calculated along the lines suggested by Peacock. By producing a large number of Monte Carlo distributions, an empirical distribution for Z is obtained. This distribution tells us the values of Z, which are randomly exceeded with specified probabilities like 1%, 5%, etc. These values can be compared with the Z statistic obtained for theory versus the actual data.

More specifically, we consider the 155 sources in the 3CR catalog with $0.01585 < z < 1.781$. For a sample of size $n = 155$, we generate a Monte Carlo (z,S) plot as per our non-evolving luminosity function. For each data point, we determine the difference between the theoretical and actual fraction of total points lying in the corresponding rectangle, as done by Peacock. Let

$$Z = \text{Max}\{\sqrt{n}\,|\text{theory} - \text{observation}|\}, \quad (34)$$

where in our case

$n = 155.$ (35)

Thus Z can be calculated for each Monte Carlo plot so generated. Figure 9 shows a cumulative distribution of the Z statistic obtained empirically for one thousand Monte Carlo plots corresponding to the $q_0 = 0.5$ Friedmann model. Arrows show the values $Z = 2.41$ and $Z = 2.76$, which are exceeded with probabilities 0.05 and 0.01, respectively. A similar computation of Z for the actual (z–S) plot from the 3CR sample yields a value $Z = 2.47$, which corresponds to a probability (of being exceeded in random fluctuations) of 0.04. Thus the null hypothesis of no evolution is not rejected at the 1% level, although it is marginally significant at the 5% level.

The theory performs better vis-à-vis observation, however, as q_0 is lowered. At $q_0 \cong 0$ (the model considered by Burbidge and Narlikar 1981), the Z value has a probability of being exceeded by chance as high as 10%.

It is clear that the K–S test therefore does not reject the null hypothesis of 'no evolution' as per standard statistical norms. The qualitative performance of the theory in this test is comparable, probability-wise, with that in the χ^2 test.

Considering the caveats stated earlier in deriving the theoretical distribution, for example, the simple and somewhat crude approximation for the observed $G(z)$, the lack of distinction between the flat- and steep-spectrum sources, the present agreement between data and the non-evolutionary hypothesis can certainly be improved further.

V. DISCUSSION

It is worth comparing our non-evolutionary approach with the evolutionary ones which are much more in vogue. The evolutionary hypothesis assumes that the observed source count cannot be explained at all flux levels unless an evolution of number density, luminosity, or both are invoked. The best evolutionary scenario itself has evolved over the years. (See, for example, many references, for example, Longair and Scheuer 1967; Longair *et al.* 1973; Robertson 1978, 1980; Katgert 1980; Wall *et al.* 1980; Peacock and Gull 1981; Van der Laan and Windhorst 1982; Wall and Peacock 1985.) The latest and most comprehensive approach towards an evolutionary model is that of Peacock (1985). We briefly outline his approach first.

In Peacock's analysis, the radio luminosity function $\rho(P,z)$ describes the comoving space density of radio sources of power $P(= L/4\pi)$ at redshift z in the form

$$\log \rho = \sum_{i=1}^{n} \sum_{j=0}^{n-i} A_{ij} x^i(P) y^j(z), \quad (36)$$

where x and y are transformed axes of the P,z plane. The rules of transformation are

$$x(P) = 0.1(\log_{10} P - 20), \quad (37)$$

$$y(z) = 0.1z \text{ for models 1,2,4,5}$$

$$= \log_{10}(1+z) \text{ for model 3.} \quad (38)$$

Thus five models are considered. Further, the coefficients A_{ij} are different for steep-spectrum and flat-spectrum sources. In all, some 30–40 unknown parameters are needed to fit the data. Statistical techniques like the Kolmogorov–Smirnov test are used to determine the best-fit parameters.

It could be argued that because the best-fit parameters imply evolution, the source count implies evolution. However, as we have explained earlier, the information content of the source-count data, i.e., the number redshift distribution of sources in the sample, can be reproduced by a non-evolving RLF. Moreover, as shown in Fig. 10, the $N(z)$ curve determined by us from Peacock's model as per the best-fit values given in his Table A5 does not give as good a fit as the best-fit curve of Fig. 7 on which our RLF is based.

The clue to why a non-evolving RLF is able to reproduce the observed $N(z)$ curve is given by the factor F in Eq. (11). If the radio astronomer were able to pick out arbitrarily faint sources, then he would have counted *all* sources up to any given redshift. In practice, his surveys are flux-density limit-

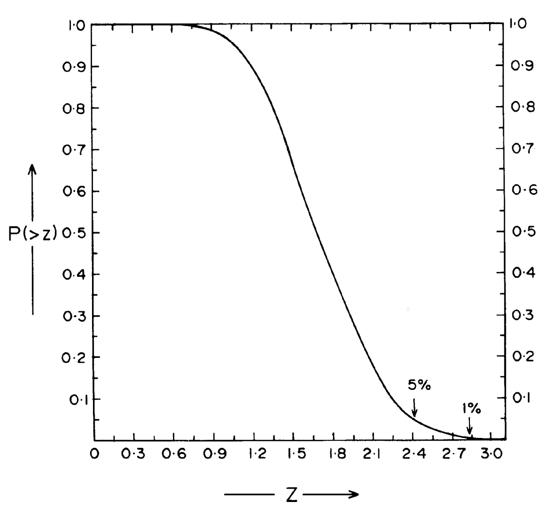

FIG. 9. Empirical cumulative probability distribution of the K–S statistic Z based on one thousand Monte Carlo simulations of the (z–S) plot produced by the non-evolving RLF of Fig. 4.

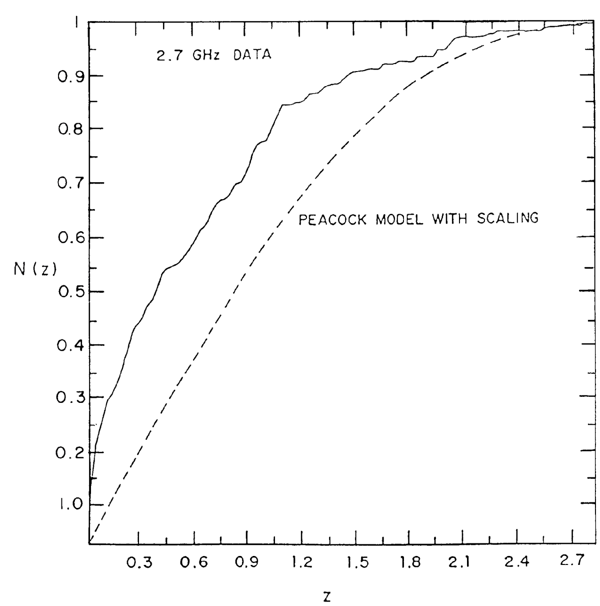

FIG. 10. $N(z)$ curve from Peacock's model (Peacock 1985) against observational $N(z)$. The absolute numbers produced by the luminosity function of Peacock have been scaled to match the maximum value of $N(z)$ at $Z_{max} = 2.852$, the highest redshift in the 2.7 GHz sample. The dashed line has been obtained from Peacock's model.

ed and so he is able to detect only a fraction of his total population. For sources in the redshift range $(z, z + dz)$, this fraction is given by the function F whose argument in Eq. (11) involves the redshift-dependent quantity x. The actual form of dependence of F on z is specified by the RLF. Thus, although the RLF by itself is redshift independent, its effect on F is to produce in it a redshift dependence. The point we wish to make here is that this kind of indirect dependence seems to be enough to explain the observed features of radio-source counts. To carry this argument further, we could say that this apparently evolutionary effect is compatible with the steady-state model.

Because of its non-evolutionary character and fewer parameters, our RLF is more vulnerable to observational tests than the evolutionary one. For, as redshifts get determined for sources in fainter surveys, it should be possible to make the tests of Secs. III and IV again to see whether the non-evolutionary RLF can be sustained. By contrast, if an evolutionary RLF is found to be inconsistent, it can always be made consistent by adding further parameters. The many references to evolutionary approaches cited earlier in this section bear testimony to this fact.

Since the Friedmann universe itself is evolving, one may wonder as to the propriety of discussing a non-evolutionary scenario within such a framework. It is true that only in a non-evolutionary cosmology like the steady-state cosmology is the hypothesis of 'no evolution' a necessary consequence. Indeed, in a later paper we plan to examine the above non-evolutionary RLF within the framework of the steady-state model. Meanwhile, our justification of the present procedure rests on its simplicity and dependence on fewer parameters. As indicated above, the hypothesis of 'no evolution' is more readily testable and hence scientifically more attractive than one that can call on several adjustable evolutionary parameters.

The question of evolution, in general, needs to be investigated further in the light of the new data on other discrete extragalactic objects, e.g., the optical counts of galaxies, the variation of angular size with redshift, etc. We propose to carry out such investigations next. We suspect that the case for evolution is not as unequivocal as is sometimes made out.

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