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Research Note

Was there a big bang?

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Summary. It is shown that within the class of Friedmann-Lemaitre cosmological models the models which avoid a "big bang" (by an appropriate choice of a nonzero cosmological constant) are excluded, because quasars with redshift $z > 4$ are observed, and the density parameter of cold matter is larger than 0.02.

Key words: Cosmology

Assuming Einstein's gravitational field equation with an unspecified cosmological constant and accepting the cosmological principle, i.e. that spacetime is homogeneous and isotropic on a large scale ($\gtrsim 200$ Mpc), one obtains a large family of cosmological models. If one specifies the matter contents to be a non-interacting mixture of cold, pressureless "dust" and hot, relativistic "radiation", characterized by $p=0$ and $p=1/3\varrho$, respectively (p pressure, ϱ energy density) – which is reasonable for a discussion of the overall evolution of the universe if one disregards an inflationary phase – then the possible present states of the models can be coordinatized in terms of the Hubble constant H and three dimensionless parameters

$$\Omega = \frac{8\pi G}{3H^2}\varrho_{\text{dust}}, \quad \omega = \frac{8\pi G}{3H^2}\varrho_{\text{radiation}}, \quad \lambda = \frac{\Lambda}{3H^2} \tag{1}$$

where G denotes Newton's constant of gravitation and Λ Einstein's cosmological constant, respectively.

The space of these cosmological states consists of two four-dimensional regions: an open one which comprises all states belonging to big bang models and a closed one corresponding to those without a big bang. The boundary of the latter consists of the Eddington-Lemaître models which approach the Einstein static universe in the infinite past whereas its interior consists of "bouncing" models (see Ehlers and Rindler, to appear in Montly Notices Roy. Astron. Soc.).

Arguments leading to the conclusion that only big bang models are "realistic" have usually been based on the assumption that the 3 K radiation originated in an early, hot stage with $z > 10^3$; moreover, these arguments do not work for all values of Λ (see, e.g. Hawking and Ellis, 1973). We wish to show here that, within the class of models specified above, models without a big bang can be excluded by weaker assumptions: It suffices to know that quasar redshifts with $z > 4$, assumed to be cosmological, have been observed and that the density parameter of cold matter is larger than 0.02. Thus even if one assumes, with F. Hoyle, that the microwave background radiation is stellar radiation thermalized by interstellar iron needles, our argument requires a big bang.

The proof runs as follows: if we normalize the scale function $a(t)$ of the Robertson-Walker metric such that at present $a(t_0) = 1$, and if we measure time in units of the present Hubble time H^{-1}, then Lemaître's equation for "dust-plus-radiation" models can be written

$$\dot{a}^2 = \omega\,(a^{-2} - 1) + \Omega\,(a^{-1} - 1) + \lambda\,(a^2 - 1) + 1. \tag{2}$$

Now, the scale functions $a(t)$ of all models which do not have a big bang, have a positive lower bound $a_* < 1$ which is determined by the fact $a \to a_*$ if $\dot{a} \to 0$. Equation (2) then implies

$$\lambda = \frac{1 + \omega\,(a_*^{-2} - 1) + \Omega\,(a_*^{-1} - 1)}{1 - a_*^2}. \tag{3}$$

Moreover, for the models in question $\ddot{a}$ tends to a non-negative limit as $a(t)$ approaches its lower bound. This limit vanishes only for a special subclass of models (the Eddington-Lemaître, or A_1-models of Robertson 1933). Therefore, by Eq. (2), one obtains the inequality

$$\lambda \geqq \frac{2\omega + a_*\Omega}{2a_*^4}. \tag{4}$$

Combining (3) and (4) we get

$$\omega\,(1 - a_*^2)^2 + \Omega\, a_*\left(\tfrac{1}{2} - \tfrac{3}{2}\, a_*^2 + a_*^3\right) \leqq a_*^4, \tag{5}$$

with equality holding for A_1-models only. Since both terms on the LHS of (5) are non-negative we can obtain inequalities for Ω and ω separately. If instead of a_* we use the corresponding maximal redshift $z_* = a_*^{-1} - 1$, these inequalities are

$$\Omega \leqq \frac{2}{z_*^2(z_* + 3)}, \quad \omega \leqq \frac{1}{z_*^2(z_* + 2)^2}. \tag{6}$$

The equality sign holds precisely for A_1 dust models and A_1 radiation models, respectively. (The equation for the A_1-dust case was obtained by Crilly in 1968). According to observations (Warren et al., 1987; see also refs. in Shaver, 1987) $z_* > 4$ and $\Omega \geqq 0.05$ (e.g., Peebles, 1986; Metzger and Schmidt-Burgk, 1983; Loh and Spillar, 1986) which is incompatible with (6). Thus, within the stated assumptions, a big bang is unavoidable. A

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similar analysis can be applied to restrict, though not to eliminate, inflectional models, i.e. models which have had a big bang and have a negative deceleration parameter q_0 (see Ehlers and Rindler, to appear).

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