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Atmospheric and internal refraction in meridian observations

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Summary. Various strong evidence has been obtained for Internal Refraction, INR, in the tube of meridian telescopes, and an efficient technique for eliminating it has been demonstrated. This effect has passed unnoticed until recently although it is usually as large as 1.0 arcsec and quite variable, and it has presumably dominated all measurements of the so-called horizontal flexure. Even up to 3″ INR has been observed in a vertical circle.

An INR of 1.″0 in a horizontal meridian tube must be caused by a vertical temperature gradient about 4 K/m (positive towards zenith), but a satisfactory explanation for this gradient has only partly been found.

Through the removal of INR a much better determination of atmospheric refraction becomes possible, since the two effects are inseparably confounded.

A difference between the refraction constants in north and south is interpreted as a tilt of the atmospheric layers on La Palma by typically 36″.

Key words: astrometry – instruments – Earth's atmosphere

1. Introduction

The error in declination due to the telescope tube is usually called flexure and the observed zenith distance z is corrected according to the equation

$$z_{\mathrm{corr}} = z_{\mathrm{obs}} + F \sin(z) \quad (1)$$

where F is the horizontal *flexure coefficient*, or often, in less precise language, the *flexure*. Sometimes a vertical flexure corresponding to a $\cos z$ term in (1) has been discussed.

Internal Refraction (*INR*) or *Tube Refraction* has long been a concern for horizontal meridian circles, but now its existence has been demonstrated in three *classical meridian circles* and in a vertical circle. It can affect the observed declinations typically by $1'' \sin z$, i.e. with a similar dependence on zenith distance as horizontal flexure, but it is quite variable in size during a night and from night to night, and it is not expected to follow strictly a $\sin z$ law.

A thorough discussion of mechanical flexure, thermal flexure and INR has recently been given by Høg and Miller (1986) hereafter called Paper II, so that we can be rather brief here. In Paper II it was shown that F at the USNO 6 inch meridian circle is a function of $\dot{T}$, *the rate of change of temperature*. T is measured in the air near the meridian circle. The total flexure coefficient is a function $F(\dot{T})$ of the form

$$F = a_F + b_F \operatorname{abs}(\dot{T}) \quad (2)$$

where the constants have been determined to be $a_F = 0.''40$ and $b_E = 0.''50$ per K/hr at the USNO instrument for a certain period. Values for other instruments are given in Sect. 2.2.

A simple and efficient equipment to eliminate the INR by circulation of the air in the tube was proposed in Paper I (Høg, 1984) and has now been tested on the Carlsberg Automatic Meridian Circle (CAMC) on La Palma. It made the flexure term constant within 0.″09 rms or less. For a description of the CAMC see Helmer and Morrison (1985) or Helmer (1984).

This paper is divided as follows. The evidence of INR is presented in Sect. 2. Horizontal flexure without INR is discussed in Sect. 3, and the determination of atmospheric refraction and flexure in Sect. 4, followed by a discussion in Sect. 5.

2. Evidence of INR

The available evidence for INR in the CAMC shall be presented extensively in logical sequence in order that meridian astronomers may get away as soon as possible from a discussion whether INR exists, so that a fruitful discussion of the properties of INR can start and so that meridian groups would be encouraged to remove INR from their instruments too.

In view of the importance of such a large error source in all extant meridian observations and in view of the scepticism often expressed about the existence of a significant INR it is not superfluous to present all this evidence, although even a small part of it would be fully convincing.

2.1. *INR in the CAMC*

Direct evidence for INR in the CAMC is obtained when switching the ventilation on and off, Sect. 2.1.1. A correlation of INR with temperature differences is found in Sect. 2.1.2, but no correlation with the temperature rate has been found, cf. Sect. 2.1.3.

2.1.1. Direct demonstration

In a series of measurements the zenith distance, z, of a collimator was observed two times per minute. The zenith distance is reckoned positive to the north. Figure 1a and 1b show such observations for both collimators, plotted with arbitrary zero point.

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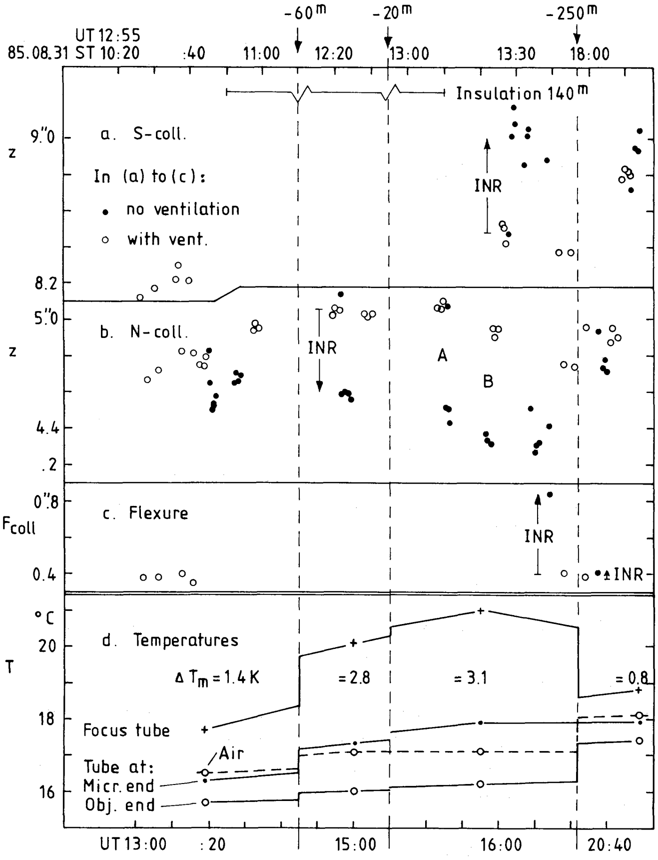

Fig. 1. Zenith distance, z, of S- and N-collimators (a, b), the flexure F_{coll} (c), and different temperatures (d), plotted as functions of time. INR is the difference between the zenith distances with and without ventilation. See Sect. 2.1. – During a period of 140 min a blanket wrapped around the micrometer caused it to heat-up more than usually, thus increasing the INR. – The pavilion was closed throughout

With intervals of 5–10 min the ventilation was switched on or off and a number of observations were made. The INR is the difference between zenith distances with and without ventilation. The sign of the observed INR corresponds, in fact, to the air temperature increasing vertically upwards inside the tube.

Some conclusions on the time constants of the system follow from the two series of measurements at A and B at the N-collimator in Fig. 1b. The seven and six points in each series are spaced at 30 s, and each point is the result of 29 s integration time. After the three circles at A the ventilation is switched off, but the first following dot shows the same z because the air still rotates. At the second dot the INR is in full action, hence about 30 s or less is needed to establish the full thermal gradient. At B the first circle is as free from INR as the following two, hence the INR is completely removed less than 10 s after switching on the ventilation.

The size of the INR is correlated with some tube temperature gradients. The measurements are discussed in detail in the next Section.

The INR was directly observed at three stars. The declination of the star was observed without ventilation for 16 s before meridian passage. Then the ventilation was switched on and another 16 s observation obtained. It appeared that the INR at the three stars was in fair accordance with the INR at the collimator at $\sin z = 1$, as was expected.

The INR at Nadir, $\sin z = 0$, was observed to be less than 0.″10, i.e. in fact zero, as expected.

Special tests where the motion of cigar smoke was watched through the objective lens showed a quick rotation close to the objective, slowing down to about 2 seconds per revolution at the cube of the telescope. It was concluded that the vertical gradient is established in about 30 s in a horizontal tube, and that probably the transverse gradient in an inclined tube takes equally long time to build up. It is therefore believed that all meridian observations of stars are affected by a thermal gradient i.e. by an INR, corresponding nearly to the equilibrium value, irrespective of the motion from one star to the next. The telescope motion does not significantly diminish the INR since the new gradient is established so soon after the telescope comes to rest that the star observation will already be seriously affected.

Tests using less air flow showed that the present ventilation system supplies ample amounts of air. This appeared both from smoke watching and from observations of INR. Hence, we are sure that the INR is zero when ventilation is running.

It has been predicted from earlier experiments on a collimator (Paper I, Sect. 4.1) that the seeing effect or image motion caused by tangential ventilation in a conventional meridian circle would be entirely negligible. This is confirmed by the present experiments on the main tube of the CAMC.

2.1.2. Correlation with temperature differences

The INR observed in Sect. 2.1.1 is caused by a vertical temperature gradient in the tube. Such a gradient would arise from temperature differences on the tube and should even be proportional with such differences. Therefore, a number of temperatures on the tube and of the air were measured with a hand-held electronic thermometer with 0.1 K repeatability. The telescope was in horizontal position.

Four of the temperature differences shall be mentioned here, two relating to the air and two to the telescope tube. Since such temperatures are important for understanding the instrumental effects, but are never quoted in literature we shall give them here.

The air inside the tube at the objective was typically 0.5 K warmer than outside the tube in the night with open pavilion, Fig. 2b, while there was no difference with closed pavilion. This is easy to understand in view of the fairly constant temperature with closed and well insulated pavilion, and in view of the cooling by night with open pavilion.

The transverse (vertical) gradient of the air inside the tube could not be measured by our equipment since the places to measure could not be reached. Furthermore, the expected gradient of 0.2 K per 10 cm for a typical INR of 0.″5 could not have been measured with certainty. A large axial gradient, i.e. an increase of air temperature of typically 0.5 K to 1.0 K was, however, found inside the tube from objective to micrometer. There is, therefore, no reason to question the existence of the transverse gradient.

The two temperature differences relating to the tube itself are ΔT_m from the steel tube to the micrometer flange, and ΔT_t on the steel tube, from the objective to the micrometer end, cf. Figs. 2b and 3.

Typically, $\Delta T_m = +2$ K was found within only 20 cm from the micrometer with closed pavilion, but only 0.5 K with open pavilion, cf. Fig. 3b.

Typically, $\Delta T_t = +0.5$ to 1.0 K was found, as shown in Fig. 3a. The fairly good correlation between ΔT_t and ΔT_m seen in the

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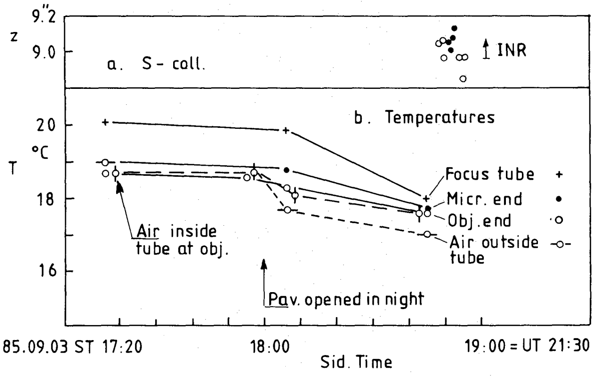

Fig. 2. Temperatures and INR. (a) INR at the S-collimator in the evening. (b) change of temperatures after opening the pavilion in the evening. After an hour all telescope temperatures lie in a range of only 0.5 K compared with 1.5 K before opening. The air outside the tube is about 0.5 K colder

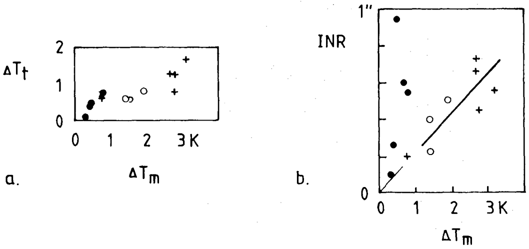

Fig. 3. Correlation between temperature differences along the telescope tube (axial) and the INR. Circles (0) and crosses (+) are with closed pavilion, crosses being with the micrometer heated or cooled. Dots (.) are with open pavilion in the night. ΔT_t is the difference on the steel tube: micrometer end minus objective end. ΔT_t is micrometer flange minus steel tube at micrometer end (Sect. 2.1.2)

figure is a good reason for considering henceforth only the larger of the differences, i.e. ΔT_m, since this must be the major physical cause of the two.

In fact the large size of $\Delta T_m \simeq 2$ K pointed to the micrometer as the main heat source, hence the main source of INR at the CAMC. This seems to explain why a preliminary study showed no correlation between observed flexure and the rate of change of temperature. Such a $(\dot{T}, F_{\text{coll}})$ correlation has been clearly found (Paper II) at a USNO meridian circle before the experiments started on La Palma in 1985.

If the micrometer is the main source of INR a correlation $(\Delta T_m, \text{INR})$ would be expected. Such a test was carried out and confirmed our expectation. The micrometer was wrapped around with an insulating blanket so that its temperature and ΔT_m started to increase as caused by the power dissipation in the micrometer. The temperatures obtained on one day (85.08.31) are plotted in Fig. 1d, and it appears that temperatures, ΔT_m and INR do in fact increase during the insulation lasting 140 minutes.

The value at $\Delta T_m = 0.8$ K was obtained with "*cooled*" micrometer. The blanket was removed and the electric power to the micrometer was switched off for some time, thus allowing it to cool, Fig. 1d.

The sets of $(\Delta T_m, \Delta T_t, \text{INR})$ plotted in Figs. 3a and 3b were collected from 5 days with closed pavilion and from 4 nights with open pavilion. Figure 3a shows the fairly good correlation between ΔT_m and ΔT_t, thus confirming that the micrometer is indeed the main heat source. Figure 3b shows a good correlation between ΔT_m and INR when the pavilion is closed (circles and crosses) but, quite surprisingly, not when the pavilion is open in the night (dots). Perhaps the wind and its direction, radiation to the sky and $\dot{T}$ are additional parameters affecting the INR when the pavilion is open.

But we definitely conclude from the dots in Fig. 3b that the micrometer heat cannot cause the INR that is observed with open pavilion.

Our studies terminated here for lack of time, and because we already had eliminated the INR by the ventilation.

2.1.3. Temperature rate and INR for the CAMC

The correlation found between INR and various temperature differences allows us to conclude that heating by the micrometer is the main cause of INR when the pavilion is closed, but *not* when it is open in the night. Therefore, the $\dot{T}$ dependence found in Paper II might be expected in the night, and this question will be discussed here.

Some plots of $\dot{T}$ versus F_{coll} were prepared in 1985, but to our surprise showed a large scatter. Much of this scatter has probably been caused by an alignment error (ALE) which was then unknown. No new $(\dot{T}, F_{\text{coll}})$ plots with correction for the alignment error are available, but an even more instructive view at the data is provided by the diagrams in Fig. 4, showing F_{coll}, T_{air} and T_{pier} as functions of time for 23 consecutive nights in July 1985. F_{coll} has been corrected for the alignment error.

A close correlation between T_{air} and T_{pier} is obvious, and we shall henceforth consider only T_{pier} and F_{coll}, since T_{pier} must be similar to the temperature of the telescope, which is supposed to be responsible for the $\dot{T}$ causing the INR.

The typical behavior of T_{pier} is a drop by 2–3 K during the night which becomes slower during the night. Hence abs$(\dot{T})$ is decreasing and F_{coll} should be decreasing according to Eq. (2).

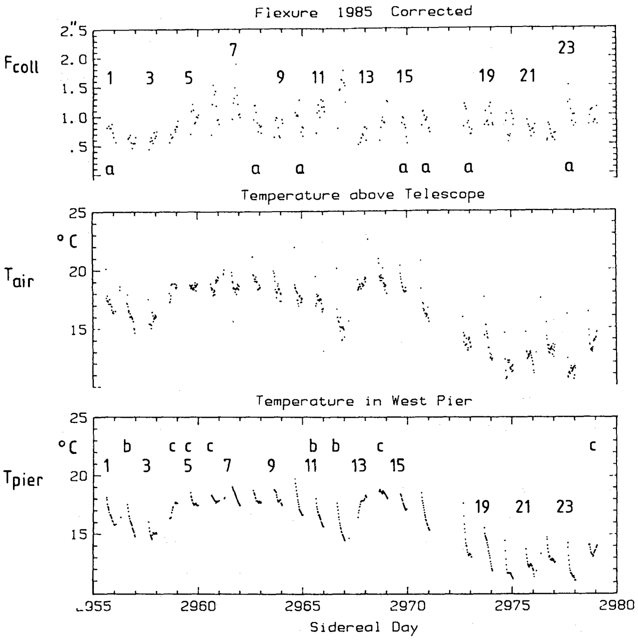

Fig. 4. Flexure and temperatures for 23 nights without ventilation are shown, but no close correlation between F_{coll} and T or $\dot{T}$ has been found. (a) F_{coll}, (b) T of air in pavilion, (c) T of West pier

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Seven nights with clearly decreasing $F_{\rm coll}$ are labelled *a* in Fig. 4 and have in fact the typical run of $T_{\rm pier}$.

Some other nights with equally typical $T_{\rm pier}$ labelled *b* show, however, a different run of $F_{\rm coll}$, not fitting Eq. (2).

The nights with almost constant $T_{\rm pier}$ are labelled *c*. Some of them (4 and 14) show increasing $F_{\rm coll}$, others (5 and 6) show a large variation of $F_{\rm coll}$, not in agreement with Eq. (2).

Finally, the nights with largest average $F_{\rm coll}$ (6, 7, 12 and 23) do not all have a large abs($\dot{T}$).

In conclusion, less than half of the nights agree with Eq. (2) and many nights are directly contradicting. Hence, other physical causes than $\dot{T}$ must also be taken into account to explain the $F_{\rm coll}$ and its variations. Probably the remaining part of the alignment error is not a sufficient explanation.

It may be worthwhile to test Eq. (2) on the CAMC with new data, since recently the main micrometer has been insulated from the telescope tube and the alignment error has been cured. Such a test would require observation of at least a dozen nights without ventilation and representing the different possible runs of T shown in Fig. 4.

2.2. *Evidence of INR in other instruments*

(a) The first published note about INR in a *meridian circle*, known to us, is due to van de sande Bakhuyzen (1921) who discusses internal refraction as a theoretical possibility. He expects an effect of the order 0.″1 to 0.″2 (see his p. 19), but no experimental evidence of INR is given. Having realized the possibility of INR he selects in a subsequent study only such measurements of flexure that could be expected to suffer the least from INR, especially those made with open pavilion.

(b) The existence of INR in a *horizontal tube* was supposed by Høg (1973). It has been studied by Kirijan et al. (1982) and in Paper I.

(c) Evidence of INR in the telescope of a *classical meridian circle*, the USNO 6 inch, was found in Paper II in the dependence between the flexure, F, and the time rate of temperature, $\dot{T}$, expressed by (2), see Sect. 1, and Table 1, items 1a and 1b.

(d) The same sort of evidence has been found by Lazorenko (1984) in a vertical circle

(e) and by Yoshizawa (1986) in a meridian circle, see Table 1, items 3 to 4b. It is noted from Table 1 that b_F is much larger in two of the instruments, namely 1″ per K/hr in Kiev and Tokyo, and smaller, but quite variable over years in the USNO instrument.

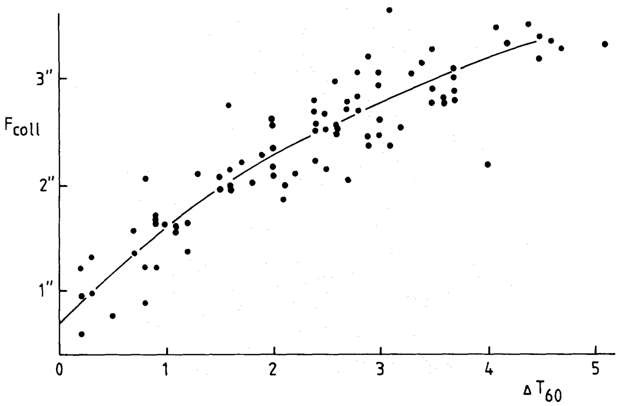

Fig. 5. Flexure versus the temperature shock ΔT_{60} during the first 60 minutes after opening the pavilion of the Kiev vertical circle. (Adapted and reproduced with permission from Lazorenko, 1984)

Since the study by Lazorenko (1984) is a technical report and written in Russian it is not easily available. We shall, therefore, with the kind help of Dr. Lazorenko, give a summary and reproduce a figure. The flexure of the Wanschaff vertical circle at Kiev, Golosejevo, during night observations has been studied by Lazorenko (private communication to EH in 1986, and in a technical report 1984). He found a relation between the flexure and the thermal shock, i.e. the drop of air temperature in the pavilion when it is opened in the evening. The physical reason for this dependence was suggested to be "a sum of real flexure and refractional obstacles in pavilion and instrument."

The observed increase of flexure was 1″ to 3″. Lazorenko measured $\Delta T_t = T_0 - T_t$, where T_0 is the air temperature in the closed pavilion and T_t is the temperature after t minutes. The closest correlation with flexure was found for $t = 60$ min where the correlation coefficient was $r \simeq 0.9$ for the relation

$$F = 0.''70 + 0.''95\,\Delta T_{60} - 0.''08\,\Delta T_{60}^2 \qquad (3)$$

as illustrated in Fig. 5. The shock ΔT_t is a strongly non-linear function of t, and it therefore means a very rough simplification

Table 1. Flexure depending on rate of temperature, $\dot{T}$, for four instruments, cf. Sect. 2.2

Instrument	a_F	b_F	Time
1a) USNO 6 inch meridian circle:			
Period W550, 1963 to 1971	0.″05	−0.″10	day and night
1b) Period W650, Sep. 1977 to Mar. 1981	0.40	+0.50	day and night
2) CAMC, 1985, without ventilation	(0.9)	?	night
3) Kiev, Golesejevo, vertical circle	0.70	+0.95	night
4a) Tokyo, photoelectric meridian circle	0.99	+0.69	day
4b) same instrument	0.96	+1.00	night

Explanation: The flexure coefficient is $F = a_F + b_F$ abs ($\dot{T}$), where $\dot{T}$ is in (K/hr). Note that two different values for b_F, the $\dot{T}$ coefficient, have been found at the USNO meridian circle, cf. Paper II, Section IV h. Approximate values of b_F = 1.″0 per K/hr have been found at the instruments in Kiev and Tokyo. At the CAMC a clear dependence on $\dot{T}$ was not found

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when we adopt $\Delta T_{60} = \dot{T}$ for the sake of comparison in Table 1 by means of (2). Neglecting the fairly small square term, this corresponds to $a_F = 0\farcs70$ and $b_F \simeq 0\farcs95$ per K/hr in our equation (2).

Equation (5) in Paper II gives the shift Δy due to INR, i.e. ΔF, as functions of T' (in K/m) the gradient perpendicular to the beam inside a telescope of focal length L (in m)

$$\Delta y = 0\farcs10 T'L \tag{4}$$

Hence, assuming $\Delta y = 2''$ and $L = 2$ m for the Kiev instrument we obtain $T' = 10$ K/m, corresponding to 1 K per 10 cm across the air inside the telescope. This should be directly observable with suitable thermometers.

3. Horizontal flexure without INR

Two methods of eliminating INR seem fairly simple to apply to an existing meridian circle: ventilating the tube, or filling it with Helium. Two other methods probably don't offer any advantage over the two first, and they require a new and expensive construction of the tube: evacuation of the tube, or giving it double walls with spiralling air, see Appendix A. Results with ventilation are given here.

3.1. F_{coll} during the night with INR and without

The flexure term $F_{\rm coll}$ is determined several times each night by means of the collimators. Such values are plotted in Fig. 6.a1 for 3 nights without ventilation and in Fig. 6.b1 for 3 nights with ventilation. The corresponding air temperatures $T_{\rm air}$ are plotted below in a2 and b2. (Correction for the ALE discussed in Appendix B has been applied to $F_{\rm coll}$).

Without ventilation $F_{\rm coll}$ appears to vary in a total range of $1\farcs1$ around a mean value $1\farcs2$. The variation decreases with ventilation to only $0\farcs35$ and the mean value decreases by $0\farcs7$. This decrease is in agreement with the typical INR about $0\farcs6$ shown by the dots in Fig. 3b.

The beneficial effect of the ventilation is obtained in spite of the larger variations of air temperature during the three nights with ventilation.

Table 2. The six nights of Fig. 6

Date SD	Ventilation	Wind m/s	Wind Direction	$F_{\rm coll}$	$F_{\rm FK5}$
2964	No	1–3	180°	$0\farcs93$	$0\farcs60$
2965	No	1–2	270	1.09	1.13
2966	No	0–1	150	1.48	1.59
2992	Yes	2–3	90	0.55	−0.20
2993	Yes	5–10	0	0.44	−0.31
2994	Yes	1–3	180	0.48	−0.18

direction N is 0°, E is 90°

Mean values of wind speed and direction and of flexure for the 6 nights are given in Table 2.

3.2. Flexure during 1985

The systematic changes of flexure throughout the whole year 1985 appear from Figs. 7c and 7d where the mean value of $F_{\rm coll}$ for each night is plotted, and the $F_{\rm FK5}$ obtained from the FK5 stars each night by Eq. (6).

Figure 7d (bottom) shows a seasonally constant flexure $F_{\rm FK5}$ during Period No. 1 decreasing by $\Delta F = 1\farcs3$ to another, also a constant value during the remaining part of the year. The systematic decrease is due to the use of ventilation which at the same time diminishes the night-to-night scatter from $0\farcs6$ to $0\farcs25$ rms.

Figure 7c (above) showing a mean value of $F_{\rm coll}$ per night gives a similar overall picture of the effect of ventilation as $F_{\rm FK5}$. Some confusion arises, however, from the middle period where the value is between those of Periods No. 1 and 3. This is most probably due to imperfect ALE corrections in Period No. 2, cf. Appendix B. The scatter of $F_{\rm coll}$ after $t_{\rm ALE}$ is only $0\farcs09$, showing fully the improvement by ventilation, while the systematic changes before $t_{\rm ALE}$ are ascribed to the imperfect ALE correction.

It is noted that the zero-point of $F_{\rm coll}$ has an unknown systematic error due to the deficiency of the collimator micrometer.

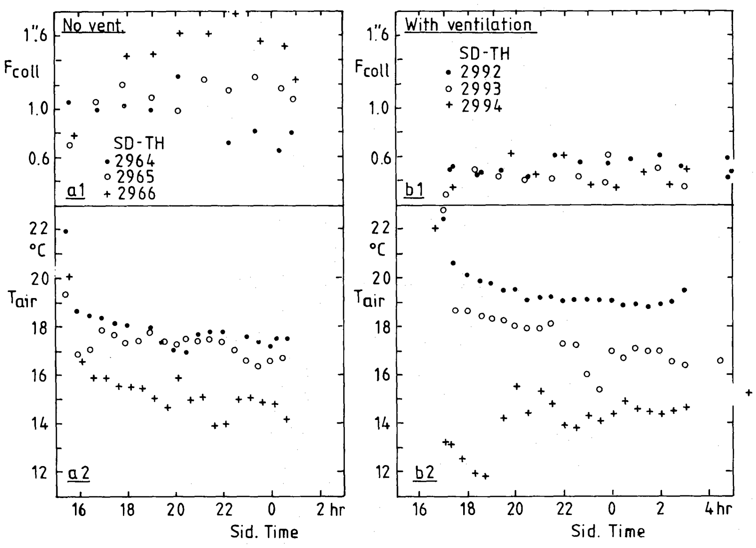

Fig. 6. Flexure variation during the night shown for 3 + 3 nights. Large variation with INR (a1), and very small variations without INR (b1). Air temperatures are plotted below

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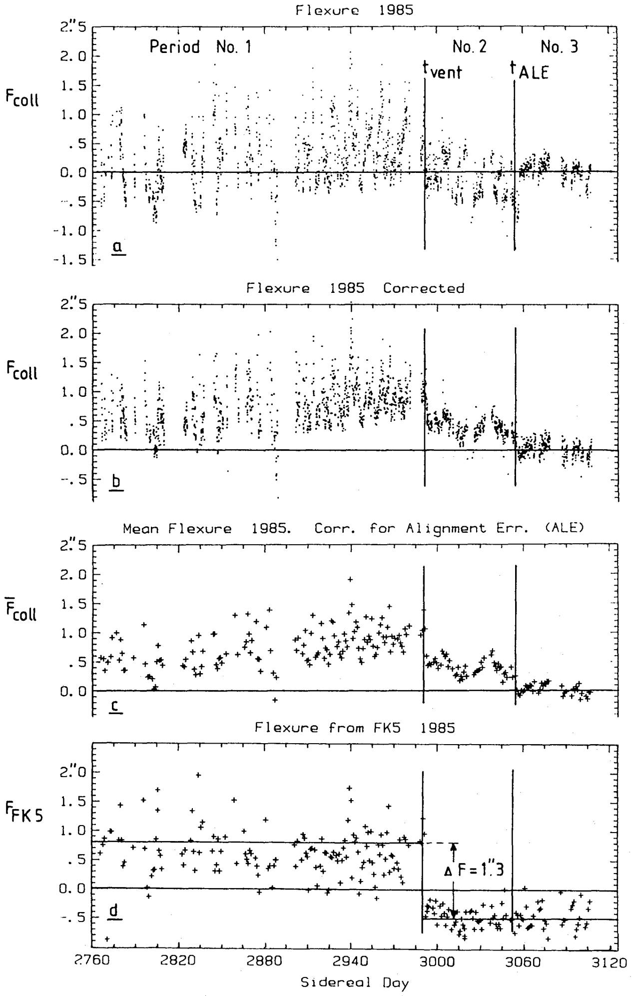

Fig. 7. Flexure observed during 1985. (a) Individual observed values of $F_{\rm coll}$, (b) the same, but after correction for the alignment error, ALE, explained in Appendix B. (c) A mean value of $F_{\rm coll}$ per night from (b). (d) Flexure $F_{\rm FK5}$ obtained from the FK5 stars each night

Hence, the fact that the $F_{\rm coll}$ is close to zero in the last interval, but not in the middle should not be overinterpreted.

4. Atmospheric refraction and flexure

Two flexure coefficients are determined for a given night, the mean value of $F_{\rm coll}$ by means of collimators, and $F_{\rm FK5}$ by means of the FK5 stars. They were plotted in Fig. 7 as function of time, and their correlation will be discussed henceforth. It will be shown in Sect. 4.2 that the refraction correction R_m for a night is much more accurately determined when INR is eliminated. Finally, a tilt of the atmospheric layers on La Palma is determined.

4.1. F_{FK5} *versus* F_{coll}

The observed zenith distances are fitted to the FK5 system. This is done for every night separately by means of some 80 FK5 stars distributed fairly uniformly throughout the night and on zenith distances in the interval $-84° < z < +84°$. A least squares solution is obtained giving a zero-point, Z_0, a *correction, ΔR, to the assumed refraction constant*, and the flexure term $F_{\rm FK5}$. The corresponding observation equation is in its simplest form

$$Z_0 \times 1 + \Delta R\,{\rm tg}\,z + F_{\rm FK5}\sin z = z - z_{\rm obs} \equiv \Delta z \tag{5}$$

where z is the zenith distance calculated from the FK5 position, proper motion, refraction etc., and z is defined positive north of zenith and negative to the south ($z = \delta - \varphi$).

In practice, a solution for separate refraction-constant-corrections R_N and R_S to the north and south of zenith were made by the observation equation

$$Z_0 \times 1 + R_N H(z)\,{\rm tg}\,z + R_S H(-z)\,{\rm tg}\,z + F_{\rm FK5}\sin z = \Delta z \tag{6}$$

where $H(z)$ is the Heaviside function $= 1$ for $z > 0$ and $= 0$ for $z \le 0$.

A weight p as function of zenith distance was applied

$$p = (\cos z)^{0.9} \tag{7}$$

which is a fairly good approximation of σ^{-1} where $\sigma(z)$ is the s.d. of an observation. It would have been more conventional to apply a weight proportional to σ^{-2}, but this has in fact not been used here.

We define the mean refraction-constant-correction

$$R_m = 0.5(R_N + R_S) \tag{8}$$

and the refraction-constant-difference

$$R_d = 0.5(R_N - R_S) \tag{9}$$

We have with good approximation

$$\Delta R \cong R_m \tag{10}$$

if the stars are fairly equally distributed to the north and south of zenith, and we will henceforth also call R_m briefly the *refraction correction.* In reality, about twice as many FK5 stars are observed south of zenith as to the north, but we do not believe this changes our conclusion. The values of Z_0 and $F_{\rm FK5}$ obtained by (6) are approximately the same as would be obtained by (5).

The flexure term $F_{\rm FK5}$ has already been compared with $F_{\rm coll}$ in Figs. 7c and 7d with respect to time variations. It is even more interesting to see whether the two flexure values obtained each night are correlated, hence Fig. 8a.

Without ventilation (dots) a weak, but marked correlation is present, especially when the largest values are taken into account. The weakness of this correlation shows that the flexure obtained by the collimators is not a very good measure for the real flexure affecting the star observations. Furthermore, the variation of $F_{\rm FK5}$ appears to be larger than that of $F_{\rm coll}$, which indicates that INR does *not* follow a sinz dependence, but is larger at the stars than corresponding to the F measured at the collimators. With ventilation in action, however, there is no correlation at all.

The scatter of the flexures in two of the three periods of observation appears from Figs. 8a, and was discussed in 3.2. For Period No. 3 (circles) we see that $F_{\rm FK5}$ scatters much more than $F_{\rm coll}$ and it will be shown in the following section that the larger scatter of $F_{\rm FK5}$ is due to the structure of the observation equation (5), and not due to e.g. uncertainty of FK5 positions, or a remaining variation of instrumental flexure. It follows from the small scatter 0.″09 rms of $F_{\rm coll}$ in Period No. 3 that the remaining instrumental flexure is very constant when ventilation is applied.

Figure 8a shows most strikingly that all observations of flexure without ventilation have been completely dominated by INR

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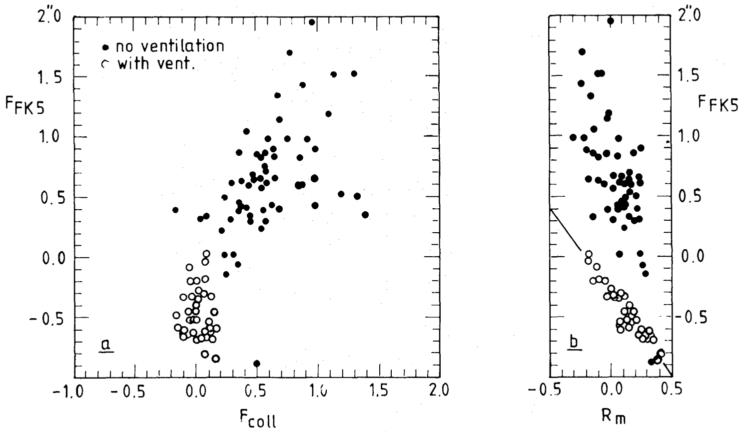

Fig. 8. Correlation of F_{FK5} with F_{coll} and with refraction for periods No. 1 and 3 (dots and circles, respectively. (a) F_{FK5} versus F_{coll}, one point per night. (b) F_{FK5} versus the refraction correction R_m for the same nights. The drastic improvement caused by removing INR appears from the circles. Further explanation in Sections 4.1 and 4.2

and its variations. This is true for the CAMC and presumably for all other meridian and vertical circles. Hence, a *mechanical* component of flexure cannot be studied before removing INR. This is at present only achieved at the CAMC.

4.2. F_{FK5} *versus the refraction correction* R_m

The terms of refraction correction and flexure determined by (5) are correlated because the coefficients (tg z and sin z) have a similar shape in the interval of z where the stars are observed. A correlation between R_m and F_{FK5} does in fact appear in Fig. 8b.

The correlation is much more pronounced with ventilation than without, the reason being the large real variations of flexure due to INR without ventilation. Due to this large variation we cannot in a quantitative way explain for the correlation found without ventilation.

Without INR, however, the situation is so simple that a quantitative explanation of the correlation can be given. The (negative) correlation, in Fig. 8b is expressed by the straight line

$$F_{FK5} = -0''.30 - 1.4R_m \tag{11}$$

The same correlation and parameters within 0".05 and 0.10, respectively, are in fact found in Period No. 2 (not shown here).

Let us consider a modification of the observation Eq. (6) which is numerically equivalent, but contains different parameters:

$$Z_0 \times 1 + R_d \,\mathrm{tg}(\mathrm{abs}(z)) + R_m(\mathrm{tg}\, z - r \sin z) + F_F \sin z = \Delta z \tag{12}$$

where R_d and R_m are defined by (8) and (9), r is an arbitrary constant, and F_F is defined by

$$F_F = F_{FK5} + rR_m \tag{13}$$

The observation equation (12) is numerically equivalent to (6) through (8), (9), and (13) and will give exactly the same residuals in a least-squares solution. The advantage of (12) is that the last two terms will be uncorrelated (be orthogonal) by a choice of r satisfying the equation

$$\sum p \sin z(\mathrm{tg}\, z - r_{\mathrm{neq}} \sin z) = 0 \tag{14}$$

since the left hand side of this equation is the element outside the diagonal in the normal equations for a night expressing the correlation between the two last terms of (12).

We have found a value of $r_{\mathrm{neq}} = 1.9$ from (14) with a standard deviation of 0.2. An observed value $r_{\mathrm{obs}} = 1.4 \pm 0.1$ is obtained from Fig. 8b by means of (11) and (13) which, furthermore, give $F_F = -0''.30$. The discrepancy between r_{neq} and r_{obs} is statistically significant, but is presently not understood. Any real, physical variation of the true mean refraction correction, R_m, must have its origin in the atmospheric air outside the telescope. It, therefore, cannot be correlated with any true, physical variation of the instrumental flexure. The correlation shown by the circles in Fig. 8b must, therefore, be due to the structure of (5). Therefore, the *true* refraction correction and the *true* flexure must be constant within less than 0".09 rms since this is the standard deviation of the *estimated* values around the line in the figure.

For use in Sect. 4.3 we give an equivalent form of (12) containing a parameter s in the second term

$$Z_c \cdot 1 + R_d(\mathrm{tg}(\mathrm{abs}(z)) - s) + R_m(\mathrm{tg}\, z - r \sin z) + F_F \sin z = \Delta z \tag{15}$$

where

$$Z_c = Z_0 + sR_d \tag{16}$$

The advantage of (15) is that all four terms can be made nearly orthogonal by proper choice of r and s. When the terms are orthogonal (uncorrelated) the uncertainty of a parameter can be reliably judged from the s.d. obtained only from the diagonal in the inverse normal equations, since the coefficients outside the diagonal are zero. If we choose approximately $r = r_{\mathrm{neq}} = 1.9$ and $s = 1$, we obtain a diagonal normal equations matrix and, consequently, also a diagonal inverse matrix.

It is noted that even and odd functions in z, say the first and the last in Eq. (5), i.e. 1 and sin z, are not quite orthogonal because the distribution of FK5 stars is not quite symmetric about $z = 0$.

4.3. *North-south difference in refraction*

The refraction difference $R_d = 0.5(R_N - R_S)$ has been studied, and an interpretation as zenith refraction, i.e. tilt of layers is proposed. It appears as expected that this study is independent of INR.

The mean value of R_d is $-0''.076$ using ventilation, according to Table 3, and, as expected, nearly the same without ventilation. The variation of R_d from night to night is expressed by the s.d. 0".076 with ventilation, and nearly the same without.

The s.d. of R_S and R_N from the LSQ solutions by (6) is about 0".050, hence a s.d. of $0.050 \cdot 2^{-0.5} = 0''.035$ for R_d would be expected in a solution with (12), since R_S and R_N are orthogonal.

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Table 3. North-south difference of atmospheric refraction (unit arcsec)

	No ventilation				With ventilation			
	n	R_S	R_N	R_d	n	R_S	R_N	R_d
Mean value	19	–	–	−0.090	33	–	–	−0.076
s.d. direct				0.078				0.076
s.d. by Eq. (6)	19	0.045	0.043	0.031	34	0.056	0.041	0.035
s.d. of s.d.						0.027	0.019	

In short, a night to night variation of R_d of size 0.″076 rms is indicated, which is significantly larger than the formal error of 0.″035 rms.

The term R_d is perhaps due to a zenith refraction, i.e. to a common tilt about an east-west axis of the atmospheric layers causing the refraction. The tilt may be given as the zenith distance $z = u$ having zero refraction.

The observation Eq. (6) should then be replaced by the non-linear equation

$$Z \cdot 1 + R(\mathrm{tg}(z-u) - tg\, z) + R_m \mathrm{tg}(z-u) + F \sin z = \Delta z \tag{17}$$

containing the unknown parameters Z, u, R_m and F and the refraction constant $R = 46'' = 0.000223$ rad on La Palma (and 60″ at sea level). Since u is small a linearized form of Eq. (17) will be adequate. We have the linear approximation of the term

$$\mathrm{tg}(z-u) \simeq \mathrm{tg}\, z - u(1 + \mathrm{tg}^2 z) \tag{18}$$

and hence, to a first order approximation taking $uR_m(1+\mathrm{tg}^2 z) \simeq 0$ we replace Eq. (17) by

$$Z \cdot 1 + R_u(1 + \mathrm{tg}^2 z) + R_m \mathrm{tg}\, z + F \sin z = \Delta z \tag{19}$$

where

$$R_u = -uR \tag{20}$$

An equivalent form would be

$$Z_0 \cdot 1 + R_u(\mathrm{tg}^2 z - s) + R_m(\mathrm{tg}\, z - r \sin z) + F_F \sin z = \Delta z \tag{21}$$

where the parameters s and r may be chosen freely.

$$Z_0 = Z + (s+1) R_u \tag{22}$$

and

$$F_F = F + rR_m \tag{23}$$

Equation (21) will obtain four nearly orthogonal terms if we choose r and s to be approximately

$$r = 1.4 \tag{24}$$

and

$$s = 2 \tag{25}$$

The second term in Eqs. (15) and (21) for $s = 1$ and 2, respectively, is shown in Fig. 9, by a full and dashed line, respectively. A value $R_d = -0.''05$ was used as typical, since the variation of R_d has an s.d. 0.″076, cf. Table 3. The dashed line corresponds to $R_u = -0.''008$ which has been found on average from solutions using the observation equation (19) and using FK5 stars up to abs(z) $\simeq 82°$. All observations of 1986 were used. The scale value in arcsec makes obvious that the difference between the two curves is inconspicuous, less than 0.″05, for zenith distances from −81° to +81°.

For larger z the difference between the curves rapidly increases, but the large uncertainty at these zenith distances will make it impossible to decide between the two representations by means of observations there.

The average inclination for 1986 is obtained by (20) giving

$$u = -R_u/R = +0.''008/46'' = +36'' \tag{27}$$

and a variation from night to night also of 36″ rms, cf. Table 3.

Equation (27) with $u > 0$ means that the layers are on average tilted towards north, i.e. making the angle betweenn the layers and the terrain surface at the La Palma site slightly smaller than if the layers were horizontal.

Significantly larger values of u are found at northerly winds than at other directions, but u remains mainly positive.

Furthermore, the first three quarters of 1986 show a larger average u than the last quarter of the year, the difference being about 30″.

In conclusion: The north-south difference in refraction is small but most probably it is real. The interpretation as a tilt of the atmospheric layers of typically 36″ towards north should be studied. A study of individual residuals at very large zenith distance will not be conclusive because of the large uncertainties. More promising is a study of systematic differences at all zenith distances, which must be based on long series of observations covering one or several years.

Finally, a remark on monitoring and catalogue production: For the monitoring of each night an orthogonal observation equation should be used so that the significance of parameter variations may be judged by direct comparison with the standard deviations, undisturbed by correlations.

Probably no significant time dependent terms exist. Hence, Eq. (15) with four parameters and $r = 1.6$ and $s = 1$ may be used without further terms.

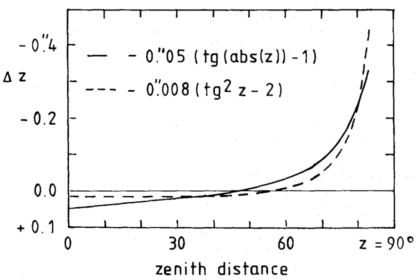

Fig. 9. The correction for north-south difference in refraction. Full curve: Interpretation as different refraction constants. Dashed curve: interpretation as a tilt of atmospheric layers of about 36″

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For catalogue production, however, Eq. (15) is not optimal, since the parameters F_F, andR_m, but not Z_c and R_d are constants over long intervals of time, i.e. months or years. Mean values should, therefore be derived and be used for the catalogue, lest the instrumental system becomes unnecessarily deformed by local irregularities in the FK5 catalogue. Furthermore, the few nights showing coarsely deviating parameters would be more correctly reduced by the mean parameters.

5. Discussion

5.1. *Influence of INR on the accuracy*

A preliminary study of the raw differences Δz used in Eq. (5) between the zenith distances $z_{\rm obs}$ measured in the instrumental system and the z derived from the FK5 shows two things. The average differences are much smaller without INR and secondly the night to night scatter has improved considerably, except near zenith where it remains the same. Hence, prior to ventilation the raw declination residuals were completely dominated by the INR which even changed considerably from night to night. The improved stability of the instrumental system allows other phenomenae to be studied, like the influence of wind direction and wind speed.

For the differential positions the standard deviation of the least squares fit to the FK5 has been studied. The s.d. of unit weight σ_δ, i.e. reduced to zenith by means of (6) and (7) is plotted against σ_α in Fig. 10. The plots contain (a): 35 nights just prior to $t_{\rm vent}$, the time when ventilation was installed, and (b): 35 night just after $t_{\rm ALE}$ when the alignment error was cured, hence an unbiased selection of nights affected by INR and not affected. Neglecting the two outlying nights with $\sigma > 0\farcs25$ we obtain from 34 + 34 nights the average σ_α, σ_δ and the correlation coefficient r, with INR

$(\sigma_\alpha, \sigma_\delta, r) = (0\farcs159, 0\farcs168, 0.28)$

and without INR

$(0\farcs166, 0\farcs149, 0.49)$

The s.d. of a sample σ is about $0\farcs03$.

It appears that σ_δ is 6% larger than σ_α with INR, and 9% smaller without INR. Hence, removal of INR has improved the s.d. of a declination observation by about 15 percent. This improvement does, however, not affect the internal mean error of the derived star positions. It is therefore concluded that the smaller s.d. is due to a closer fit to FK5 i.e. to an improvement of the systematic errors of the differential declinations.

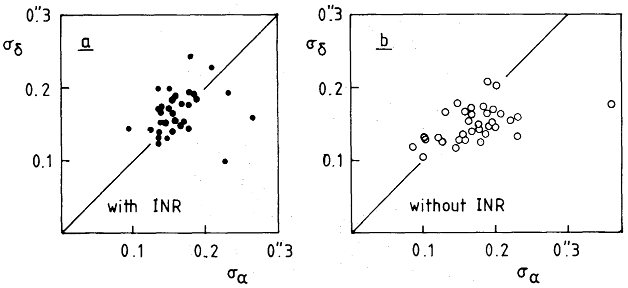

Fig. 10. The standard deviations of an observation determined from the fit to FK5 stars are nearly equal in α and δ, and the removal of INR has diminished σ_δ by 15 percent compared with σ_α

5.2. *Historical note*

It may wonder that INR has not been discovered before when it amounts to as much as 1 arcsec. Two reasons can be given, firstly, the effect follows approximately a sin z law and is therefore not easily distinguished from a flexure of mechanical origin. Secondly, the study of meridian techniques in terms of basic physical terms (temperature, elasticity etc.) tends to be neglected, which is quite understandable in view of the many other more urgent technical problems to be solved at a meridian circle. The senior author (EH) therefore wants to acknowledge his gratitude to G. van Herk who has so often at scientific meetings urged us to study temperatures around the instrument. His own very first temperature measurements were taken at the Brorfelde instrument in 1954. A very extensive study of this sort is due to Yoshizawa (1986).

In 1970 van Herk drew his (EH's) attention to the danger of INR in the proposed Glass Meridian Circle and this lead him to propose tangential ventilation (Høg, 1973). But even van Herk has not really thought of INR in classical meridian circles, as he said in 1986 when mentioning the paper by Bakhuyzen (1921).

5.3. *Conclusion*

When INR is eliminated the *Atmospheric Refraction* can be studied much better and, in fact, the refraction constant obtained from FK5 stars in a night agrees with the calculated refraction constant from meteorological data within $0\farcs10$ rms or less.

The question of correcting extant meridian observations for INR has been discussed in Paper II where a correction $\Delta\delta_\alpha$ with amplitude $0\farcs035$ and 24 hours period was predicted. But in view of the great variability of INR for a single instrument and from one instrument to another very careful studies are required to obtain reliable systematic corrections, if it is at all feasible.

Appendix A: elimination of INR

Four methods to eliminate INR in a telescope tube have been proposed as mentioned in Sect. 3 and some of them have been tested. The choice of a method in any particular case should be based on practical engineering considerations rather than on basic physical arguments. Hence, we have preferred the simple tangential ventilation instead of e.g. constructing a vacuum telescope. Refraction would certainly be eliminated in vacuum, but the resulting pressure forces on tube optics and micrometer imply that some engineering problems will have to be solved lest the astrometric accuracy would be impaired. According to C. Kühne (1987, priv. comm.) the extra cost would be only 10000 DM, if a vacuum tube were introduced at the time of manufacture of a meridian circle. Astrolabes and Photo Zenith Telescopes with vacuum are in successful operation in China as described by Hu Ningsheng (1984).

Kirijan et al. (1982) have studied refraction anomalies in the horizontal tube of the Pulkovo Horizontal Meridian Circle and they have diminished the INR by spiralling air in a double-wall tube. This method is designed to keep the inner wall iso-thermal,

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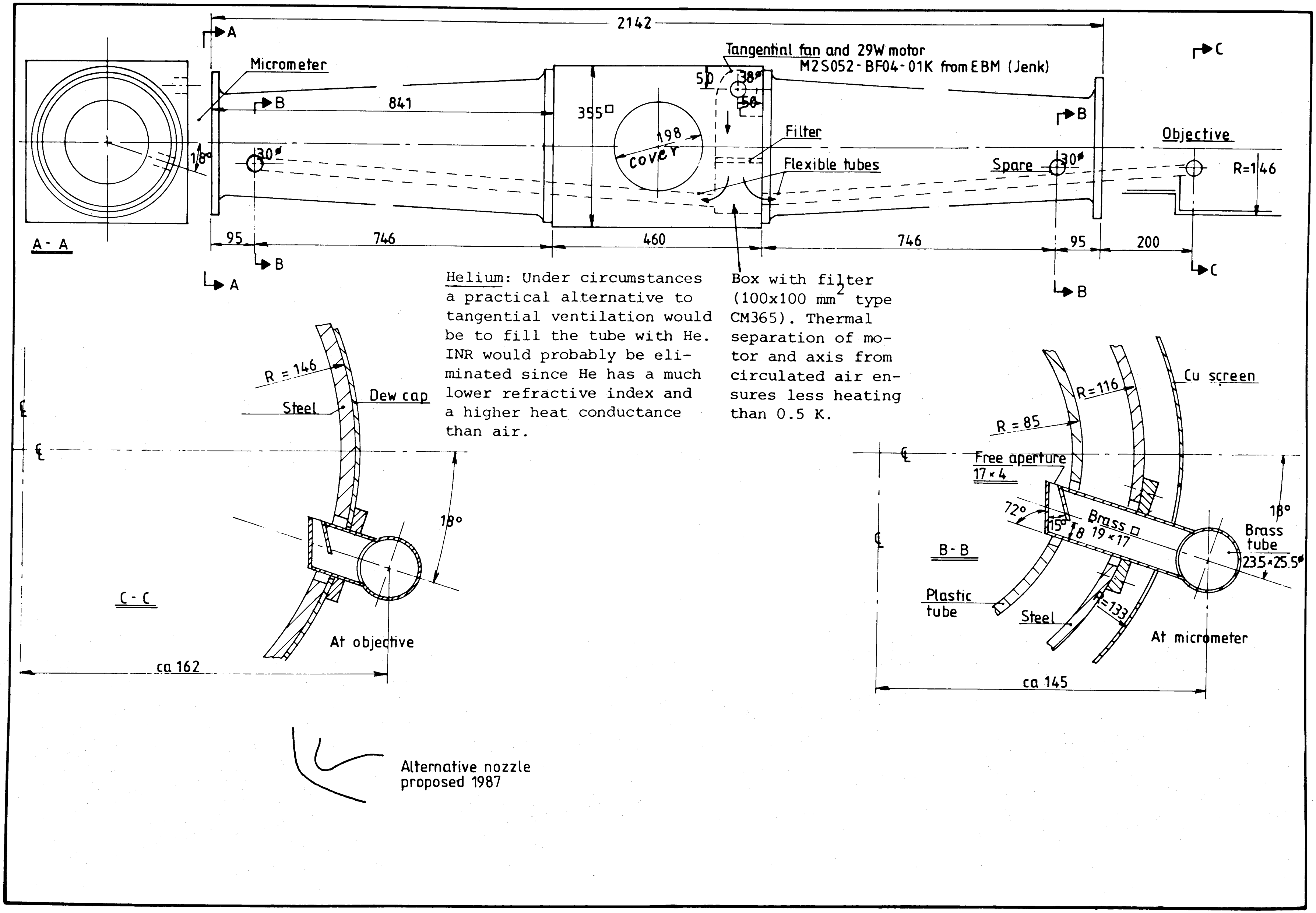

Fig. 11. The tangential ventilation system installed on the CAMC since Sep. 1985. Lower half: Tangential injection of air at the objective (left) and at the micrometer (right). Upper half: The air spirals through the smooth inner plastic tubes and it is filtered before it is recirculated through the two outer tubes

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but this is not enough to eliminate the vertical gradient caused by the dropping temperature in night and the resulting heat transfer between walls, optics and the inner air. Hence, an INR depending on $\dot{T}$ would still be expected in the double-wall system.

Filling the tube with helium would probably eliminate INR since He has a much lower refractive index than air and a much higher heat conductance. The indices of refraction are $n = 1.000036$ for Helium and 1.00029 for air. The focal length will accordingly decrease by $0.00025 \times f \simeq 0.7$ mm. The heat conductances are 14.22 and 2.41 in ($10^{-2}\,\mathrm{W\,m^{-1}\,K^{-1}}$), respectively. Helium filling should be fairly simple to install on a modern meridian circle. The openings in the cube through which alignment is made must be provided with optical windows, and tube and optics must be tight lest too much Helium would be wasted. Means for equilibrations of pressure inside and outside of the telescope should probably be provided, as well as a continuous supply of Helium since this gas easily diffuses out of the system.

Tangential ventilation is probably the simplest of all methods. It was proposed by Høg (1973) and tests on a collimator tube have been reported in Paper I. It was shown that no seeing disturbances would be introduced. The resulting system design for the CAMC is shown in Fig. 11. (Detailed drawings are available from EH). The idea is to give the air a low rotation about the optical axis, thus eliminating the thermal gradient. This is achieved by tangential injection of about 1 l/s at low pressure through a nozzle of $4 \times 17\,\mathrm{mm}^2$ aperture at each end of the telescope. The air is flowing to the nozzles through flexible plastic tubes on the outside of the telescope, and the fairly large diameter of these (25 mm) is required to diminish the loss of pressure. The air is recirculated through a coarse filter, but the filter does not seem to be important. The system has a *thermal separation* of the motor (29 W power consumption) and its axis from the circulated air, lest it should be heated and cause seeing disturbance. Less heating than 0.5 K is achieved. The telescope is provided with *smooth inner tubes*, lest the air rotation about the optical axis would be stopped too quickly. A smooth tube fits into each half of the telescope tube, and each of them ends in the side of a perpendicular tube inside the cube through which the alignment is made. The latter tube has a hole with connection to the intake of the fan. The motor is stopped while the covers of the cube automatically open for taking alignment, so that only little new air will be sucked into the tube. The new air could contain dust which might settle e.g. on the slit plate in the micrometer. The motor is actually running, also in daytime, when no observations are being performed, but this is not really necessary.

After 18 months of continuous running no problems of e.g. dust or wear-down of components have been encountered. The satisfactory results of the ventilation are reported in the main text.

Two possible improvements should be mentioned. The filter and hence the filter box are probably dispensable, at least on La Palma, since very little dust appears to be collected. The present nozzle used at the CAMC has high pressure gradients inside and at the exit, and therefore high turbulent energy dissipation. The alternative nozzle (Fig. 11, lower left) is more efficient with its smooth inside and expanding exit, so that even less air flow would suffice to maintain the rotation of air about the optical axis. Hence, a less powerful tangential fan would be adequate (but we have found none available on the Danish commercial market).

Appendix B: confusion by alignment error

The evidence obtained from FK5 stars, Sect. 4, was at first confused by two other effects namely by a fault in the collimator micrometer giving an Alignment Error (ALE), and by atmospheric refraction. Hence, three variable effects had to be disentangled: The INR, the ALE, and the atmosperic refraction. We succeeded in eliminating the INR by ventilation, correcting for the ALE, and improving the determination of atmospheric refraction.

Two months after the ventilation was installed a serious fault in the Collimator Micrometer (CM) located at the N-collimator was found which had corrupted most previous alignments of the collimators, and hence the resulting flexures, F_{coll}.

The alignment is made by measurement of the vertical position in the CM of the image of the point source in the S-collimator. This observed vertical position and the zenith distances of the point sources in N- and S-collimators (cf. Figs. 1a and 1b) are combined to derive F_{coll}. It was found that the vertical position suffered from a systematic error being a function of the vertical position itself. A possible cause of the error is nonlinearity of the V-slits of the CM, which may be either intrinsic or be caused by a dust grain on the slits. The cause cannot easily be identified in the present CM, since disassembly is not a trivial operation. In a future CM under design optical means will be provided to look at the slits from behind to see if they are straight and clean. Furthermore, the V-slits will be evaporated on glass while the present ones are real slits in a thin metal plate, hence being perfect traps for dust or insects. (It is also important to ensure that the pinhole receives un-vignetted illumination).

The variation of the ALE with the vertical position (y) was determined by measuring the flexure using a series of different y-settings. The flexure was found to vary as much as 1.″1 and unfortunately the y-setting preferred untill then, was at one of the steepest parts of the curve. A daily routine was introduced to keep the y-setting at a rather flat part of this curve. (The new range of y was only 5″ centered on $y = 35''$ as compared to the old range of 17″ centered on $y = 30''$). Using this calibration it is possible to reduce the variation of F_{coll} introduced by the ALE, but it was not possible to determine the actual value of the ALE and therefore not possible to get the correct zero-point for F_{coll}.

Figure 7a shows all values of F_{coll} observed in 1985 without correction for the ALE. The two times $t_{\mathrm{vent}} = \mathrm{SD}2992.5 = 85.08.16$ when ventilation was started and $t_{\mathrm{ALE}} = \mathrm{SD}3057.5 = 85.10.20$ when the ALE variation was cured, divide the year in three intervals No. 1, 2, and 3. The scatter or variation is four times larger in the first interval than in the last, due to the INR and ALE in the first interval. The rather large scatter still remaining in the middle interval shows that the ALE causes about the same amount of scatter as the INR.

In Fig. 7b all values of F_{coll} in 1985 have been corrected for the ALE. This correction is quite successfull as appears from the small scatter now obtained also in the middle interval. Hence, the remaining scatter in the first interval must be completely dominated by INR, and corresponds to a standard deviation of 0.″6.

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