

Atomic time scales for pulsar studies and other demanding applications

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Summary. The International Atomic Time TAI is produced in near real time and definitive when issued in the Annual Report of the Bureau International de l'Heure. Although it satisfies almost all the needs for an accurate time scale, there are some cases where better approximations to an ideal atomic time are required. By combining TAI and the data of the primary cesium standards of the National Research Council of Canada (NRC) and of the Physikalisch-Technische Bundesanstalt of FRG (PTB), we have established for the interval 1976.0–1987.0 an improved atomic time scale. The new scale is usually very close to TA(NRC) and TA(PTB), laboratory time scales, which can also be employed directly for the most demanding application, for some stated intervals.

Key words: atomic time – pulsars

1. Introduction

The International Astronomical Union (IAU) in 1976 has defined an ideal form of the atomic time under the denomination “Terrestrial Dynamical Time”, by Recommendation 5 of Commissions 4, 8, and 31. We will adopt here the abbreviated designation “Terrestrial Time”, TT, for this ideal time, as suggested by Guinot and Seidelmann (1987), in order to avoid the ambiguities of the adjective “dynamical”. TT is the proper time of an ideal cesium clock at the geocenter, but with a frequency offset so that the second of TT is equal to the SI second at the geoid level.

The practice of astronomy requires realizations of TT, which will be designated as TT(XXX), where XXX is an identifier. For instance, using the International Atomic Time TAI, one can write:

$$TT(\text{TAI}) = \text{TAI} + 32.184 \text{ s}.$$

The realization TT(TAI) satisfies almost all the needs: its departure from TT being probably less than $10 \mu\text{s}$ since 1977. It has the advantage of being definitive, TAI being never revised. It is therefore recommended to use it, for unification sake, except in the cases where a better approximation to TT is needed.

The need of a better time scale appears now with the study of millisecond pulsars. We have therefore established an improved time scale TT(BIPM87) which is disseminated as corrections to TAI,

$$TT(\text{BIPM87}) - \text{TAI}$$

computed at 10-day intervals. This scale, contrarily to TAI, can be revised, and future versions with different identifier will be

prepared to prolong and improve it. The interval considered in this paper is 1976.0–1987.0.

2. Principles of the establishment of TT(BIPM87)

The Bureau International de l'Heure, BIH, has established regularly TAI until end 1987. On the 1st of January 1988, the responsibility of this task has been taken over by the Bureau International des Poids et Mesures, BIPM, without changing the algorithm. A “free atomic scale”, EAL (échelle atomique libre), is first derived from the data of numerous atomic clocks, up to 180 clocks in 1987. Most of these clocks are industrial cesium standards. In the long term, EAL has the characteristics of a frequency random walk; it is nevertheless highly stable as it will be shown later. But it has been observed that the EAL frequency has an annual fluctuation with respect to the primary frequency standards, with a peak to peak amplitude of $1 \cdot 10^{-13}$ in 1976–1978 (Guinot and Azoubib, 1980), which has somewhat decreased since then. We believe that the seasonal variation is mostly due to EAL, not to the primary standards, for the following reasons.

(a) The biases of the primary standards are frequently evaluated, while those of the numerous industrial standards participating in EAL are not.

(b) The primary standards working as clocks (primary clocks) at the National Research Council (Canada), NRC, and at the Physikalisch-Technische Bundesanstalt (FRG), PTB, do not exhibit annual differences with respect to each other.

(c) Somme industrial cesium clocks show unquestionable seasonal variations.

Until 1977 January 1, TAI was equal to EAL. On 1977 January 1, 0h TAI, the TAI normalized frequency was lowered by $10 \cdot 10^{-13}$ in order to ensure the agreement of the TAI unitary interval with the best realization of the SI second at sea level. Subsequently, TAI was derived from EAL by applying a frequency offset which is modified by steps of $2 \cdot 10^{-14}$ when necessary, in order to maintain the agreement with the frequency of the primary standards. Both the primary clocks and the frequency standards operating from time to time participate in this frequency steering. But no attempt was made to correct the annual term of EAL.

Thus, the defects of TAI are, in addition to the frequency offset prior to 1977.0.

- long term frequency fluctuations of a few units of 10^{-14} , due to the uncertainties of the primary standards and to the imperfection of the frequency steering;

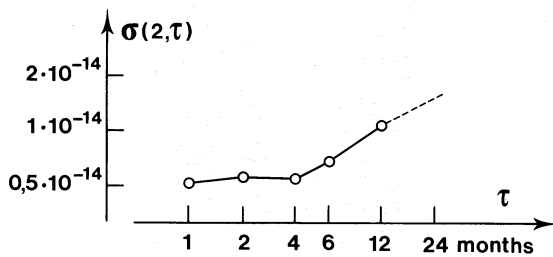


Fig. 1. Stability of EAL, assuming that the number and quality of the clocks are maintained at the 1984 level, and that the annual component is removed

- a seasonal fluctuation with a peak to peak amplitude which has decreased from about $1 \mu\text{s}$ in 1977 to about 350 ns in 1986.

We have tried to correct these defects by making use of the stability of EAL and the accuracy of the primary clocks NRC-CsV, PTB-CS1 and PTB-CS2.

3. Establishment of TT(BIPM 87)

3.1. Data

(a) The scale TAI is available in the BIH Annual Report, as corrections to the readings of clocks and time scales. The relationship between EAL and TAI is also given by these reports.

(b) The long term stability of EAL is evaluated in the following way. The initial data are the two-month mean rates with respect to EAL of the n clocks which participate in the establishment of TAI. Let us consider first the stability for two-month samples ($\tau = 2$ months). Consider a clock j and, for the consecutive two-month intervals I_i and I_{i+1} , the rate samples $B_{i,j}$ and $B_{i+1,j}$. For this clock, one computes

$$\sigma_j^2 = \frac{1}{2} (B_{i,j} - B_{i+1,j})^2.$$

An estimate of the Allan variance $\sigma^2(2, \tau = 2 \text{ months})$ for EAL is then computed from the average for the clock ensemble ($j = 1, \dots, n$) of the σ_j^2 , taking into account the BIH statistical weights of the clocks. In other terms, the Allan variance is estimated from an ensemble average instead of the usual time average. The procedure, of course, cancels the common deterministic frequency changes. Similarly the variance can be estimated for multiples of two months samples. The stability of EAL shown by Fig. 1 has been obtained by a study of 1983–1984 data; with more recent data the stability is slightly better.

(c) The primary cesium standard NRC-CsV (Mungall et al., 1973) began to provide frequency calibrations in 1973. Since 1975 May 1, it works in a quasi-continuous mode, as a primary clock. The overall uncertainty (one sigma) of its normalized frequency was estimated to $5.3 \cdot 10^{-14}$ (Mungall et al., 1976) and subsequently confirmed (Mungall and Costain, 1977). An intermittent electronic trouble seems to have perturbed the frequency of NRC-CsV at the end of 1984 and the beginning of 1985: we have not used its data from 1984 May 9 to 1985 April 24. Since 1975 December 28, 0h UTC, the time scale TA(NRC) is produced by NRC-CsV, after correction of the gravitational frequency offset due to the altitude. TA(NRC) has therefore the second at sea level as unitary scale interval, as TAI.

(d) The primary cesium standard PTB-CS1 has been put into operation in 1969 (Becker, 1977). In 1977 the overall uncertainty (one sigma) of its normalized frequency was estimated to

$2.6 \cdot 10^{-14}$, then, in 1980, to $1.1 \cdot 10^{-14}$ (BIPM Com. Cons. Def. Seconde, 10, p. 54). But a more conservative estimate of $3 \cdot 10^{-14}$ was given later (Bauch et al., 1986). From 1969 to July 1978, the frequency of the time scale TA(PTB) was calibrated several times per year using PTB-CS1. These calibrations were transferred to TAI and EAL. As from July 1978, PTB-CS1 has worked as a clock (as NRC-CsV). The gravitational correction is applied. Since 1980, TA(PTB) is provided by a secondary atomic clock, phase adjusted to the readings of PTB-CS1. We have:

$$1980 \text{ Jan. 1} - 1980 \text{ Dec. 31} \quad \text{TA(PTB)} - \text{PTB-CS1} = 2.000 \mu\text{s} + \varepsilon$$

$$1981 \text{ Jan. 1} - \quad \quad \quad \text{TA(PTB)} - \text{PTB-CS1} = 363.400 \mu\text{s} + \varepsilon,$$

the readings of PTB-CS1 having been corrected by $-361.400 \mu\text{s}$ on 1981 Jan. 1 (no time step of TA(PTB)). The small quantity ε reaches sometimes 30 ns , but is usually much less. For instance, in 1986, its quadratic mean was 14 ns .

(e) The primary clock PTB-CS2 (Bauch et al., 1987) has been put into operation in 1985, with an overall uncertainty of $2.2 \cdot 10^{-14}$ (Bauch et al., 1986). Its data are available at BIPM since August 1986.

(f) An external check of the stability of EAL and of the primary clocks consists in studying their normalized frequency differences since they are practically independent: each of the primary clock contributes less than 1% to EAL. Samples over one year have been selected, to be free of the annual effects. Table 1 gives the observed frequency differences with respect to EAL, and Table 2, the corresponding square root of the Allan variance. Unfortunately, the trouble of NRC-CsV prevents use of the recent data for this clock. With PTB-CS1 the data are compatible with stabilities over one year of both PTB-CS1 and EAL in the range $0.5 - 1.0 \cdot 10^{-14}$, thus with the estimation in (b) and Fig. 1.

3.2. Algorithm

TT(BIPM 87) is evaluated from 1976.0 to 1987.0. In order to use the frequency calibrations, TT(BIPM 87)–EAL has been established by integration of frequency samples, taken over two-month intervals so that the noise due to the time comparisons be sufficiently low.

(a) The two-month frequency samples are obtained by simple averaging of the data of the above mentioned standards. The number of standards is variable, depending on the intervals of availability and validity. Let us call Δf the average

$$\Delta f = f(\text{EAL}) - f(\text{standards})$$

(normalized frequencies), and t_c the central dates of these samples (Fig. 2).

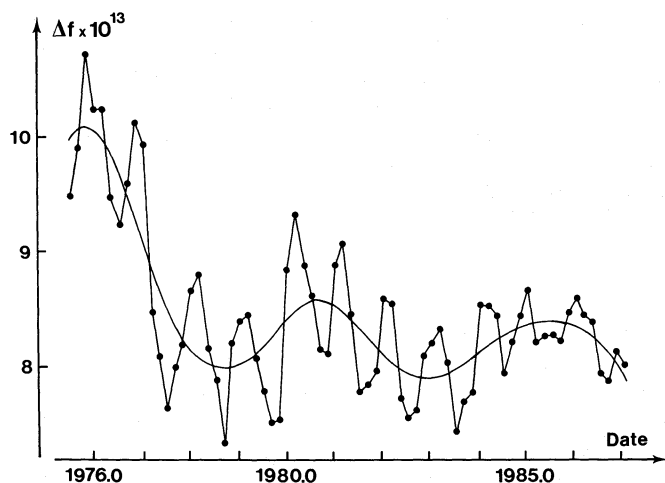
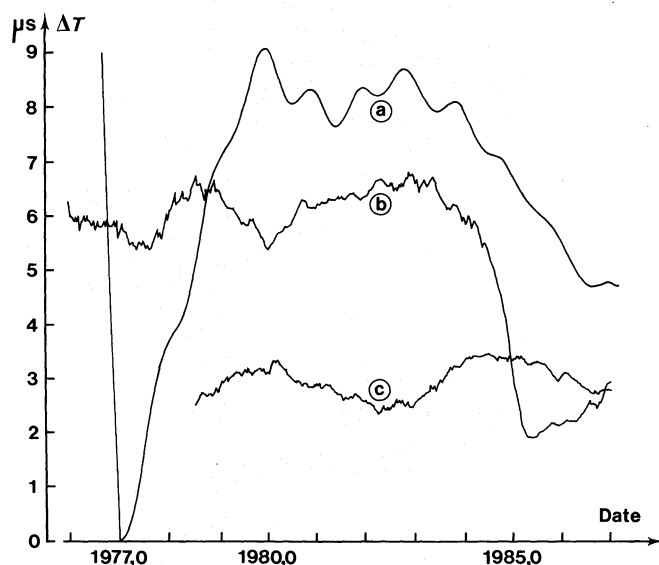
(b) The next step is iterative. Let us assume that

$$\Delta f = \Delta f_a + \Delta f_e + \Delta f_s,$$

where Δf_a is an annual component due to EAL, Δf_e a long term variation of the EAL frequency, Δf_s a variation due to the instability of the standards for averaging time of 1 to 2 years. An estimate $\widehat{\Delta f_e}$ of Δf_e is obtained from a filtering of $\Delta f - \widehat{\Delta f_a}$, where $\widehat{\Delta f_a}$ is an estimate of Δf_a . We have used the Vondrak's smoothing technique (Vondrak, 1969, 1977) with a frequency cutoff of about $1/900$ days and with equal weight for all the Δf . The initial values of $\widehat{\Delta f_a}$ have been taken as zero. The next approximations to Δf_a result from moving least square fits over 6 consecutive $\Delta f - \widehat{\Delta f_e}$. The adopted value for a central date t_c is the average of the adjusted values for the six fits covering the date t_c . Three iterations are used. Figure 2 shows the final values of $\widehat{\Delta f_e}$.

Table 1. Normalized frequency differences between EAL and the primary clocks NRC-CsV and PTB-CS1

Interval	$y(\text{EAL} - \text{NRC-CsV})$ ($\times 10^{14}$)	$y(\text{EAL} - \text{PTB-CS1})$ ($\times 10^{14}$)
1975 Sept. 4 – 1976 Aug. 29	93.55	
1976 Aug. 29 – 1977 Sept. 3	87.75	
1977 Sept. 3 – 1978 Aug. 29	85.55	
1978 Aug. 29 – 1979 Sept. 3	77.70	82.36
1979 Sept. 3 – 1980 Aug. 28	84.76	84.71
1980 Aug. 28 – 1981 Sept. 2	84.16	83.46
1981 Sept. 2 – 1982 Aug. 28	80.20	80.54
1982 Aug. 28 – 1983 Sept. 2	77.72	81.64
1983 Sept. 2 – 1984 Aug. 27	76.81	82.69
1984 Aug. 27 – 1985 Sept. 1	73.50	83.33
1985 Sept. 1 – 1986 Aug. 27	84.30	82.18
1986 April 19 – 1987 April 24	81.92	80.84

**Fig. 2.** $\Delta f = f(\text{EAL}) - f(\text{standards})$, raw and smoothed values. The smoothing is the last iteration for Δf_e (see the text)**Fig. 3.** ΔT represents $\text{TT}(\text{BIPM87}) - E - 32.184 \text{ s}$, where E is: (a) TAI; (b) $\text{TA}(\text{NRC}) + 25 \mu\text{s}$; (c) $\text{PTB-CS1} - 355.400 \mu\text{s}$ for 1978–1980; $\text{PTB-CS1} + 6 \mu\text{s}$ for 1981–1987**Table 2.** Stability over one year of the frequency differences between EAL and the primary clocks NRC-CsV and PTB-CS1

Interval	$\sigma_y(2, \tau = 1 \text{ yr}) \times 10^{14}$	
	$y(\text{EAL} - \text{NRC-CsV})$	$y(\text{EAL} - \text{PTB-CS1})$
1978–1986	–	1.18
1978–1984	2.73	1.34
1981–1986	–	0.71
1982–1986	–	0.69

(c) The quantity $-\widehat{\Delta f_e} - \widehat{\Delta f_a}$, which should be freed of the instabilities of the standards represented by Δf_e , is integrated, which produces a time scale T under the form of corrections $T - \text{EAL}$. These differences are then interpolated at the usual dates at ten-day intervals (MJD ending by 9.0), with 3rd order Legendre polynomials and the origin is adjusted to conform with the IAU recommendation. $\text{TT}(\text{BIPM87}) - \text{EAL}$ is thus obtained and transformed into $\text{TT}(\text{BIPM87}) - \text{TAI}$.

3.3. Results

Figure 3 shows $\text{TT}(\text{BIPM87}) - \text{TAI}$, and also $\text{TT}(\text{BIPM87}) - \text{TA}(\text{NRC})$ and $\text{TT}(\text{BIPM87}) - \text{TA}(\text{PTB})$. All these time scales have the same relativistic definition as TAI (Guinot, 1986). They can also be considered as proper time of clocks at the geocenter, the time unit being chosen so that it agrees with the second at sea level.

4. Discussion and conclusion

Figure 3 confirms the outstanding agreement of the standards NRC-CsV and PTB-CS1, except for a short interval of troubles of NRC-CsV. During the intervals where the two standards operated normally, TT(BIPM87) is, of course, close to the mean of their readings (plus constant), and we cannot say which of the three time scales is the best.

However, it would not be safe to use the data of a single standard in real time, before time comparisons with other standards and time scales confirm that it was operating well.

Since 1976.0, the following approximations to TT can be recommended:

1976.0–1987.0: TT(BIPM87)

1976.0–1984.0: TT(NRC) = TA(NRC) + 32.1840306 s

1985.4–1987.0: TT(NRC) = TA(NRC) + 32.1840266 s

1978.5–1981.0: TT(PTB) = PTB-CS1 + 32.1836476 s

1981.0–1987.0: TT(PTB) = PTB-CS1 + 32.1840090 s

1980.0–1987.0: TT(PTB) = TA(PTB) + 32.1836456 s.

The time scales TA(NRC), TA(PTB), PTB-CS1, are available through the differences “TAI-time scale” in annual reports and circulars on time issued by BIPM (by BIH, prior to 1988.0). These publications contain also the data for linking these scales to the UTC(lab) of many laboratories.

TT(BIPM87) is available at BIPM on request.

All the above data can be made accessible by various means, such as files in General Electric Mark 3 System, and direct access by telephone.

For use after 1987.0, until a revision of TT(BIPM87) is available, it can be extrapolated by

$$TT(BIPM87) = TAI + (32\,184\,004.7 + 0.2 \cos 2\pi t) \mu\text{s},$$

where t is in years since 1987.0.

The subsequent realizations of TT by BIPM will be made available, under the same conditions as TT(BIPM87), without prior notice.

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