Mira stars – I:
R Ari, R Aur, X Aur, R Boo and S Boo

J. E. Isles and D. R. B. Saw

Some 43 000 visual observations of five Mira stars, each observed for more than 50 years, are analysed and the results are compared with catalogue data. A number of interesting correlations are found among features in their light curves. The method of analysis, to be used in future reports on Mira stars, is described in detail.

Report of the Variable Star Section *Director: D. R. B. Saw*

Introduction

Mira stars are radially pulsating red giants. Their changes in brightness generally exceed 2.5 magnitudes, and their periods range from 80 to 1000 days or more. The light changes of a Mira star are due to variations in its radius and temperature, and to the obscuring effect of molecular bands in the spectrum which strengthen when the star cools. Most Mira stars have spectra of type M with emission lines, but some are of types C (R, N) and S. The five stars discussed in this report have spectral classes ranging from M3e to M6.5e at maximum brightness, and from M6e to M9.5e at minimum, according to the catalogue[1].

Mira stars are sometimes referred to as Long Period Variables (LPVs). But many authors include among LPVs some semiregular variables of amplitude less than 2.5 magnitude. Strohmeier[2], indeed, draws a distinction between Mira stars (amplitude } 2.5 magnitude) and LPVs ({2.5 magnitude). To prevent any possible confusion, we avoid the term LPVs.

For many years, Mira stars were the variable star observer's staple fare. Their large amplitudes and relatively slow variations usually make it easy to define their light curves by visual observation. Variations from one cycle to the next in the shape of the curve, the magnitudes of maximum and minimum, and the cycle length, are a constant source of interest to the observer.

Until about 25 years ago, the Variable Star Section (VSS) programme consisted almost entirely of Mira stars. But by the early 1970s it had become clear that 31 Mira stars, whose variations were adequately recorded by other groups such as the American Association of Variable Star Observers (AAVSO), could profitably be dropped from the BAA VSS progamme. Those Mira stars which were retained were objects of particular interest by virtue of their brightness, variable periods or variable amplitudes.

This is the first in a projected series of reports covering the Section's observations of Mira stars, commencing with the stars which were dropped in 1974. The present report discusses the five stars listed in the title, which had been observed by the VSS for intervals ranging from 54 to 72 years. Future reports will use the same methods, described here once and for all.

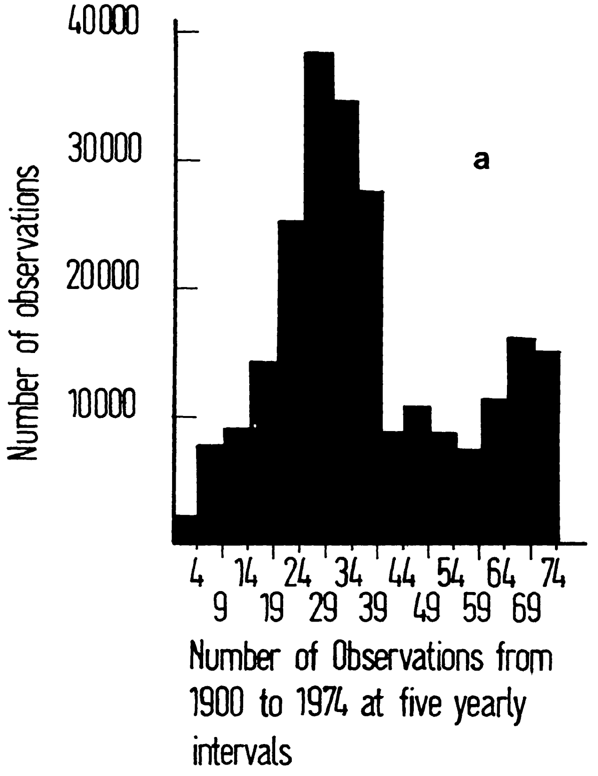
Observations

The Section's observations up to the year 1929 have been published in full in *Memoirs*[3], and those for 1930-34 on microfilm[4]. Annual reports in the *Journal*[5] have dealt with further observations up to 1938. It has not proved feasible so far to publish the individual light estimates from 1935 to date, which remain in the Section's files. Computer-plotted light curves are, however, now published by the Section in an annual booklet. Anyone requiring access to unpublished data is invited to contact the Director.

The following summary of numbers of estimates and principal observers covers all the 31 dropped stars, and will not be repeated in future reports. Altogether some 237 000 observations are available for the years 1900-74. The numbers available from 1900 to 1974 are shown in Figure 1(a) in five-year intervals, and in Figure 1(b) annually from 1939. The exponential growth in the Section's output was reversed after the 1920s, but observations are reasonably plentiful except in the later years of the war, and in the late 1950s. Unfortunately, most of the observations for 1947 and 1949 have long been missing from the Section's files. Despite efforts to recover these, only 238 observations of the 31 stars are now available for each of those years.

The *Memoirs* and *Journals* cited above give details of the observers during the years 1900-38. Since 1939, some 200 observers have made 84 279 usable observations of the 31 Mira stars. Eighty-three per cent of the observations were made by 30 observers, each of whom made over 400. Their individual totals in five-year intervals are given in Table 1. Six other observers (R. G. Chandra, E. H. Collinson, W. T. Gayfer, J. W. Macvey, R. A. H. Paterson, A. K. Porter) made between 150 and 350 observations.

The tremendous efforts of many observers, to which this series of reports does inadequate justice, may readily be appreciated from inspection of Table 1. We



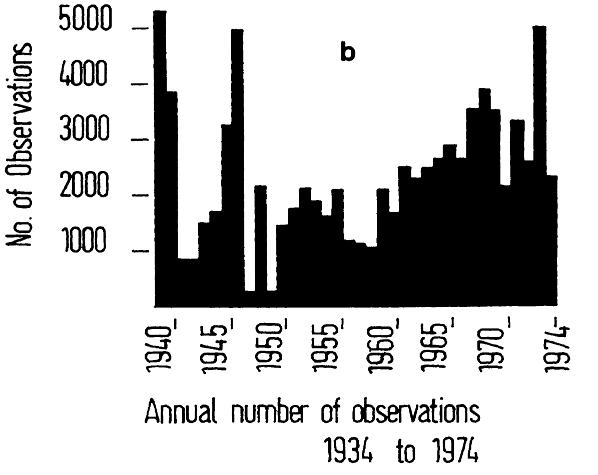


Figure 1. Time distribution of observations of 31 stars. (a) 1900-74 in 5-year intervals. (b) 1939-74, annual totals.

cannot avoid giving special mention to three observers. F. M. Holborn began observing in 1926 and continued until 1962. Both the profusion and the consistency of his estimates are astonishing. It was fortunate that, at the time when Holborn's observations were decreasing in number, another particularly reliable observer became active. C. R. Munford started in 1954 and remains today the Section's most consistent observer. Observers who make estimates of stars when they are morning objects provide a particularly important contribution, and among these O. J. Knox has been outstanding in recent years.

Table 1

Observers and numbers of observations: 31 stars, years 1939-1974

Observer	*1939-1942*	*1943-1947*	*1948-1952*	*1953-1957*	*1958-1962*	*1963-1967*	*1968-1972*	*1973-1974*	*Total*
A. F. Alexander	422	12	–	–	–	–	–	–	434
R. G. Andrews	–	452	1574	1186	430	–	–	–	3642
M. K. Bappu	669	142	–	–	–	–	–	–	811
J. R. Bazin	1821	3429	1338	1	–	–	–	–	6589
C. F. Butterworth	3256	–	–	–	–	–	–	–	3256
B. A. Carter	–	–	–	–	1252	3681	1315	–	6248
H. W. Cox	527	1600	1	–	–	–	–	–	2128
J. Friends	–	–	617	159	588	248	–	–	1612
M. J. Gainsford	–	–	–	–	–	544	1450	302	2296
R. F. Griffin	–	–	–	1325	–	–	–	–	1325
P. Harvey	613	–	–	–	–	–	–	–	613
G. P. Hawkins	–	–	–	–	–	13	446	146	605
A. Heath	–	689	–	–	–	–	–	–	689
F. M. Holborn	1084	2108	1987	2240	526	–	–	–	7945
R. H. Jones	–	–	–	–	47	1815	159	–	2021
N. F. H. Knight	575	474	–	–	–	–	–	–	1049
O. J. Knox	–	–	–	–	–	230	4281	1958	6469
R. J. Livesey	–	–	–	–	441	1114	1045	90	2690
R. S. Lomas	–	–	–	–	9	1437	178	–	1624
P. A. Moore	–	–	–	–	–	16	1059	239	1314
T. J. C. A. Moseley	–	–	–	–	–	55	554	–	609
C. R. Munford	–	–	–	150	1052	1175	987	287	3651
R. D. Pickard	–	–	–	–	–	33	339	100	472
F. A. Roper	–	–	452	1816	2054	226	–	–	4548
D. R. B. Saw	–	–	–	–	–	602	1210	143	1955
J. D. Shepherd	–	–	–	–	236	432	32	–	700
W. L. Shepherd	–	806	857	–	–	–	–	–	1663
R. C. Shinkfield	–	–	–	–	197	252	37	5	491
H. Wildey	712	1003	102	–	–	–	–	–	1817
P. J. Young	–	–	–	–	–	–	92	515	607
All above	9679	10715	6928	6877	6832	11873	13184	3785	69873
All observers	10828	11589	7635	7802	9523	14149	15491	7262	84279

Table 2

Stars and numbers of observations, years 1900-1974

Star	*1900-1909*	*1910-1919*	*1920-1929*	*1930-1939*	*1940-1949*	*1950-1959*	*1960-1969*	*1970-1974*	*Total*
R Ari	766	1058	2185	2025	487	334	912	701	8468
R Aur	570	1283	1927	1818	566	429	390	381	7364
X Aur	–	1368	2501	2671	835	818	340	552	9085
R Boo	690	1422	2265	2581	576	253	1338	795	9920
S Boo	–	–	2557	2891	997	279	747	306	7777
R Cam	–	–	2482	1863	797	803	477	–	6422
R Cas	385	1325	2426	2311	836	913	1089	465	9750
W Cas	–	–	2169	2352	790	185	1154	825	7475
S Cep	–	–	1483	2153	624	513	1023	377	6173
T Cep	1257	1774	2781	2647	961	1457	2610	1008	14495
U Cyg	–	–	2049	1658	774	698	1436	951	7566
S Del	–	–	–	–	–	–	428	245	673
R Dra	1093	1295	3092	2690	818	698	1238	825	11749
R Gem	111	940	2045	2642	896	573	994	471	8672
S Her	479	1156	2005	1785	382	405	772	24	6958
T Her	715	1141	2110	1508	522	431	1161	784	8372
U Her	–	–	1805	2036	393	350	957	330	5871
R Leo	1106	1513	2488	2400	693	1007	1687	1094	11988
R Lyn	–	–	1777	1600	656	609	834	277	5753
W Lyr	–	918	2565	2235	1004	659	762	605	8748
RY Oph	–	362	1625	1318	415	199	503	12	4434
R Peg	517	923	1586	1508	557	452	545	410	6498
X Peg	–	428	1453	1649	425	366	411	109	4841
R Per	–	–	1403	1667	601	489	805	294	5259
V Tau	–	–	1818	1697	544	254	482	33	4828
R Tri	–	–	–	–	–	–	743	429	1172
R UMa	854	1492	2666	2676	874	730	527	904	10723
S UMa	1119	1399	4280	4422	1088	1102	1892	1096	16398
S UMi	–	1248	2307	2003	472	360	1009	722	8121
S Vir	178	569	1418	1471	427	512	333	211	5119
R Vul	–	1217	2318	1861	484	285	400	139	6704
Total	9840	22831	63586	62138	19494	16163	27949	15375	237376

Table 2 presents a summary of the numbers of observations available for each star, in ten-year intervals. S Del and R Tri were only observed for a few years, but the other 29 stars were all observed for periods between 50 and 75 years.

The comparison star sequences used are based on those published by Harvard College and Leander McCormick Observatories and are very similar to those used by the AAVSO. In a few cases, minor adjustments have been made, based on visual estimates by members of the Section. Further details may be obtained from the Director.

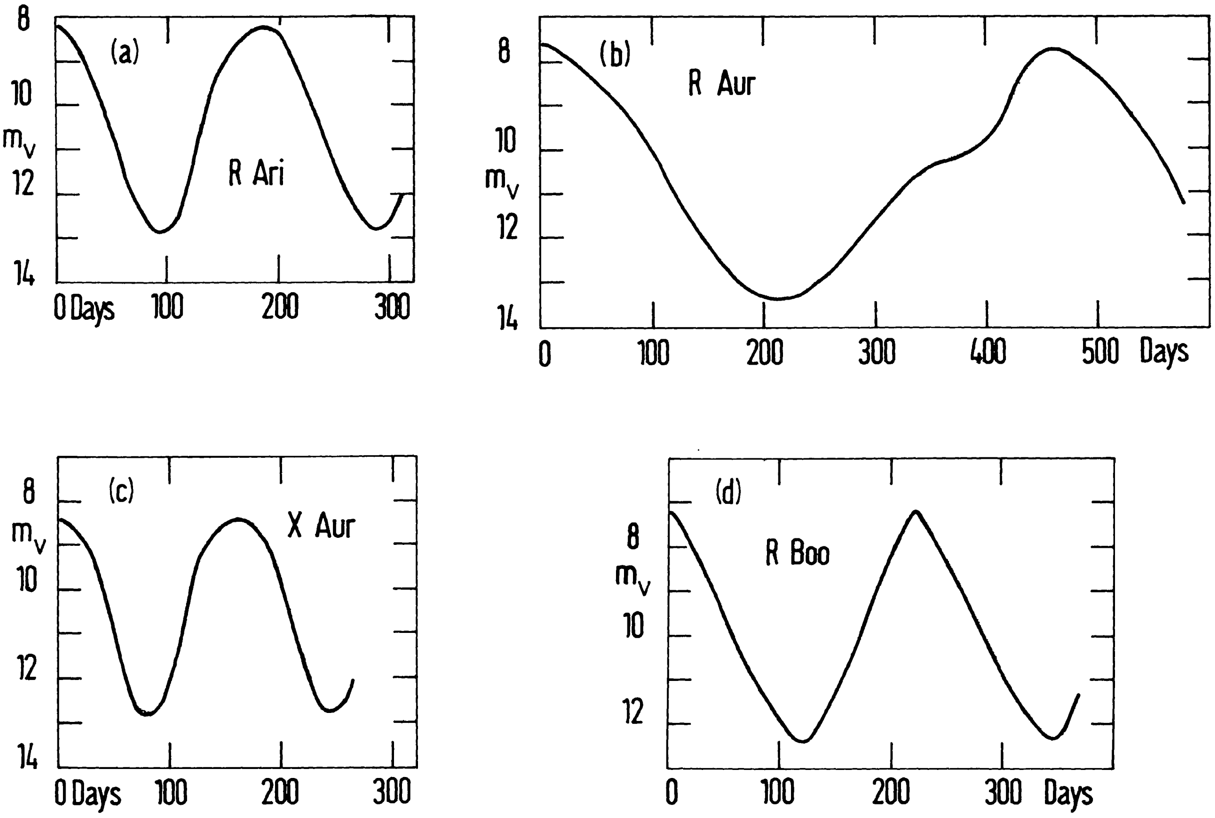
Dates and magnitudes of maxima and minima for most of the 31 stars have been published in Appendices to the *Memoirs*[6] and in the *Journal*[5,7] for the period up to 1938. These derived results are used in the present series of reports, except that, where there has since been significant revision to the comparison star sequences, appropriate adjustments have been made to the magnitudes.

For the period from 1939 to date, dates and magnitudes have been derived by the following procedure, which is in fact the same as that used up to 1938. The individual observations of each star were plotted, and the most probable curve drawn by hand. Dates of maxima and minima were then derived by Pogson's method of bisected chords. We believe the dates are in general correct to within three days, though sometimes there was difficulty when observations were sparse or where the rise and fall were very asymmetric. The dates are those when the star was most probably brightest and faintest. This procedure differs from that used by the AAVSO, in which the plot of observations is compared with a mean light curve on which dates of maximum and minimum have been marked. The latter method, we feel, incurs a risk of imposing a spurious regularity on the data.

As an example of the data extracted, Table 3 (p. 110) gives the dates and magnitudes of maxima and minima of R Aur in 1904-74. The cycle numbers are a continuation of the Harvard sequence, as used also by the AAVSO, for example in Campbell's study[8]. Owing to the pressure on *Journal* space, we do not give such lists for each star, but they are available from the writers and we hope it will prove possible to publish them separately in some form when analysis of all the Section's observations of Mira stars has been completed.

Light curves

A typical light curve was obtained for each star by plotting the three rises and three falls of well observed cycles that had rise and fall times and maximum and minimum magnitudes nearest to the mean values



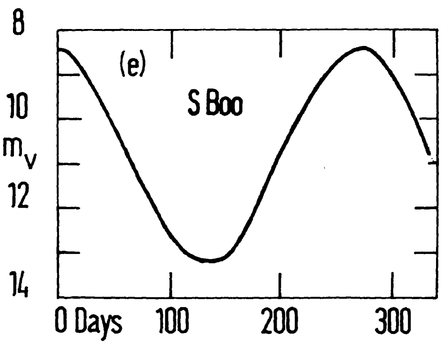


Figure 2. Typical light curves for (a) R Ari; (b) R Aur; (c) X Aur; (d) R Boo; (e) S Boo.

observed by the Section. These curves are shown in Figure 2.

R Aur is clearly in a different category from the other four stars, having a much longer period and also a pronounced hump on the rise, whose position in fact varies somewhat from cycle to cycle. The other four stars have roughly symmetrical curves, with the rise being slightly faster than the fade. R Boo has sharper maxima and minima than the other three.

Strohmeier[2] divides the light curves of Mira stars into three main types: (a) the rise is steeper than the decline, the maximum is consequently sharp and the minimum wide; (b) the form is essentially symmetrical; (c) the light curve shows a hump or stand-still on the rising branch, or has a double maximum. R Ari, X Aur, R Boo and S Boo are clearly of type b, and R Aur of type c.

The classification is evidently based on that of Ludendorff[9], which is quoted in greater detail by Hoffmeister *et al.*[10] This uses the letters α, β, γ rather than a, b, c, and also gives various subdivisions, including: (β_1) the maximum is sharper than the minimum (R Boo); (β_2) the maximum is as sharp or as flat as the minimum (S Boo); (β_3) the maximum is flatter than the minimum (R Ari, X Aur); (γ_1) waves on the ascending branch (R Aur). Ludendorff actually lists S Boo as intermediate between types β_1 and α_4, where α_4 signifies: rise markedly steeper than decline, minimum not so wide nor rise so steep as in other stars in class α. Our observations do not conform Ludendorff's classification for S Boo; in particular, the rise is not markedly steeper than the decline. Such classification of light curves may not help directly towards understanding the physical processes involved, but there is certainly a statistical relationship between the shape of the light curve and the period, as was shown by Campbell[11], and more recently by Tuchman[12].

Analysis of maxima and minima

In this section we describe the methods of analysis to be used in this series of reports. Our analysis uses mainly data such as those in Table 3 (the dates and magnitudes

Table 3
Maxima and minima of R Aur, 1904-1974

Seq.	*Maximum*		*Minimum*		*Seq.*	*Maximum*		*Minimum*	
No.	*JD 24...*	*Mag.*	*JD 24...*	*Mag.*	*No.*	*JD 24...*	*Mag.*	*JD 24...*	*Mag.*
		m		m			m		m
33	16564	7.8	...	...	62	29939:	8.7	...	...
34	17031	8.2	17201:	13.3:	63	30385:	7.0	...	...
35	17463	7.7	17686	13.2	64	30815	7.5	31028	13.5
36	17905	8.6	18112	13.4	65	31279	7.8	31507	13.6
37	18361	8.0	18576	13.3	66	31732	7.8	31930	13.6
38	18800	7.9	19010	12.8	67	32185	7.4	...	...
39	19242	8.0	19454	13.0	68	32641	6.7	32865	13.7
40	19692	8.3	19902	12.9	69	...	...	33335	13.7
41	20168	8.0	20387	12.8	70	33585	7.8	33792	13.6
42	20613	7.8	20839	13.3	71	34050	7.8	34260	13.4
43	21071	7.7	21284	12.9	72	34495	6.6	34704	13.7
44	21533	7.4	21756	13.1	73	34946	7.6	35167	13.4
45	21990	8.1	22209	13.4	74	35402	7.8	35617:	13.2:
46	22458	7.4	22703	13.5	75	35852	8.0	36075	13.2
47	22923	8.4	23154	13.4	76	36304	7.6	36518	13.3
48	23375	7.4	23634	13.6	77	36748	7.5	36967	13.2
49	23850	8.0	24092	13.2	78	37221	8.0	37420	13.2
50	24312	7.2	24587	13.6	79	37661	8.0	37869	13.1
51	24809	8.2	25023	13.3	80	38100:	7.8	...	...
52	25277	7.9	25495	13.3	81	...	...	...	...
53	25735	7.4	25966	13.6	82	39019	7.5	...	...
54	26203	7.8	26415	13.6	83	39466:	7.7:	...	...
55	26667	7.4	26899	13.6	84	39916	7.6	40138	13.4:
56	27125	7.6	27343	13.4	85	40379	7.9	40579	13.2
57	27586	6.8	27810	13.5	86	40804	7.6	41052	13.5
58	28040	7.7	28285	13.8	87	41274	8.3	41506	13.4
59	28509	7.2	28738:	13.6:	88	41738	7.8	41947	13.4
60	28973	7.7	29209	13.7	89	42182	7.5	42394	13.2
61	29435	7.4	29681	13.5					

of maxima and minima) as a convenient summary of the observations. We do not of course claim that they represent all the information contained in the original observations. Ten quantities of interest, X_1 to X_{10}, have been derived for each cycle. These are shown in Figure 3 and described in Table 4, which gives summary statistics for them.

Our adopted 'best estimate' for the mean period of a star is an appropriately weighted mean of the mean interval between the first and last observed maxima, and the mean interval between the first and last minima. For example, in the case of R Ari we have 25 783 days separating maxima 90 and 228, and 25 619 days separating minima 91 and 228, giving a total of 51 402 days corresponding to 275 cycles, that is a mean period of 186.92 days.

Another parameter of interest is the rise expressed as a fraction of the period. In R Ari, for example, the rise time is slightly less than the fall time on average, the rise taking 0.47 of the period length (the fraction being derived as rise time/(rise time + fall time), using the mean values from Table 4).

The mean periods and rise times, the extreme ranges, and the mean ranges observed by the Section are summarised in Table 5, which also gives the most recent catalogue data for comparison. The mean ranges are taken from the 1976 supplement[13], as the 1985 catalogue[1] gives only extreme ranges. Mean ranges are probably more useful, particularly because the extreme range of a variable star is likely to expand the longer it is observed.

We may now wish to look for correlations among these quantities. For example, does a long rise tend to lead to a bright maximum; does the amplitude vary systematically (maximum and minimum magnitudes negatively correlated); does the mean magnitude vary (maximum and minimum magnitudes positively correlated)? In order to test in a systematic way for the existence of these correlations, and others we might not have thought to look for, we have examined correlations between each pair of 20 quantities: X_1 to X_{10} considered above, and the ten corresponding quantities (X_{11} to X_{20}) for the following cycle.

This does not exhaust the possibilities, but it covers most of the obvious ones, and in fact it gives 145 different correlation coefficients for each star, which might be independently tested for significance. But even at the one per cent significance level, we should expect to get one or two spuriously 'significant' correlations emerging for each star even if none of the 145 are real. We therefore adopt a much stricter criterion as to what we accept as significant for a particular star.

For each correlation we can derive a P value, giving the probability of obtaining purely by chance a correlation (positive or negative) at least as extreme as the one observed. If none of these correlations is real, the 145 P values will be drawn from a uniform distribution with range from 0 to 1. The probability that the smallest will be less than P^* is 0.01, where

$$P^* = 1 - 0.99^{(1/145)} = 6.93 \times 10^{-5}.$$

In these reports we accept a correlation for a particular star as significant only if $P < P^*$; that is, only if the

Table 4
Summary statistics

	No.	*Mean*	*S.D.*	*Extremes*	
R Ari					
X_1 Period (max. to max.)	90	186.59	7.68	163	205
X_2 Period (min. to min.)	51	185.96	8.17	164	200
X_3 Rise time	77	87.21	6.48	74	101
X_4 Fall time	77	99.81	8.32	71	113
X_5 Maximum magnitude	114	8.18	0.35	7.6	9.4
X_6 Minimum magnitude	88	12.83	0.29	12.0	13.7
X_7 Amplitude (rise)	70	4.64	0.35	3.7	5.4
X_8 Amplitude (fall)	70	4.67	0.49	3.0	5.8
X_9 Mid-range (rise)	70	10.51	0.28	9.95	11.35
X_{10} Mid-range (fall)	70	10.49	0.22	10.05	11.1
R Aur					
X_1 Period (max. to max.)	52	456.83	14.60	425	504
X_2 Period (min. to min.)	45	458.71	17.26	423	495
X_3 Rise time	47	236.81	12.95	216	266
X_4 Fall time	48	221.15	17.25	170	275
X_5 Maximum magnitude	55	7.71	0.42	6.6	8.7
X_6 Minimum magnitude	49	13.37	0.24	12.8	13.8
X_7 Amplitude (rise)	47	5.61	0.50	4.6	6.8
X_8 Amplitude (fall)	48	5.65	0.56	4.6	7.1
X_9 Mid-range (rise)	47	10.56	0.22	10.0	11.1
X_{10} Mid-range (fall)	48	10.54	0.18	10.15	11.0
X Aur					
X_1 Period (max. to max.)	114	163.56	7.96	141	182
X_2 Period (min. to min.)	106	163.75	8.15	132	186
X_3 Rise time	112	80.57	8.03	55	105
X_4 Fall time	112	82.79	8.10	64	107
X_5 Maximum magnitude	123	8.44	0.24	7.9	9.3
X_6 Minimum magnitude	116	12.83	0.34	11.9	13.5
X_7 Amplitude (rise)	109	4.38	0.37	3.0	5.0
X_8 Amplitude (fall)	109	4.40	0.45	3.1	5.4
X_9 Mid-range (rise)	109	10.63	0.23	9.9	11.2
X_{10} Mid-range (fall)	109	10.64	0.19	10.15	11.05
R Boo					
X_1 Period (max. to max.)	91	223.74	6.94	210	240
X_2 Period (min. to min.)	93	223.47	7.76	194	244
X_3 Rise time	92	102.50	8.25	81	129
X_4 Fall time	92	121.43	8.01	100	144
X_5 Maximum magnitude	100	7.16	0.38	6.3	8.0
X_6 Minimum magnitude	102	12.33	0.26	11.4	12.9
X_7 Amplitude (rise)	92	5.19	0.35	4.3	5.8
X_8 Amplitude (fall)	92	5.18	0.48	4.2	6.2
X_9 Mid-range (rise)	92	9.74	0.28	9.1	10.4
X_{10} Mid-range (fall)	92	9.74	0.23	9.2	10.35
S Boo					
X_1 Period (max. to max.)	63	271.19	8.39	250	301
X_2 Period (min. to min.)	61	270.72	8.85	250	292
X_3 Rise time	63	130.38	9.82	92	148
X_4 Fall time	63	140.86	10.12	122	176
X_5 Maximum magnitude	67	8.40	0.24	8.0	9.2
X_6 Minimum magnitude	67	13.30	0.25	12.6	13.8
X_7 Amplitude (rise)	63	4.89	0.30	4.2	5.5
X_8 Amplitude (fall)	63	4.91	0.34	4.0	5.7
X_9 Mid-range (rise)	63	10.85	0.20	10.4	11.4
X_{10} Mid-range (fall)	63	10.84	0.16	10.4	11.2

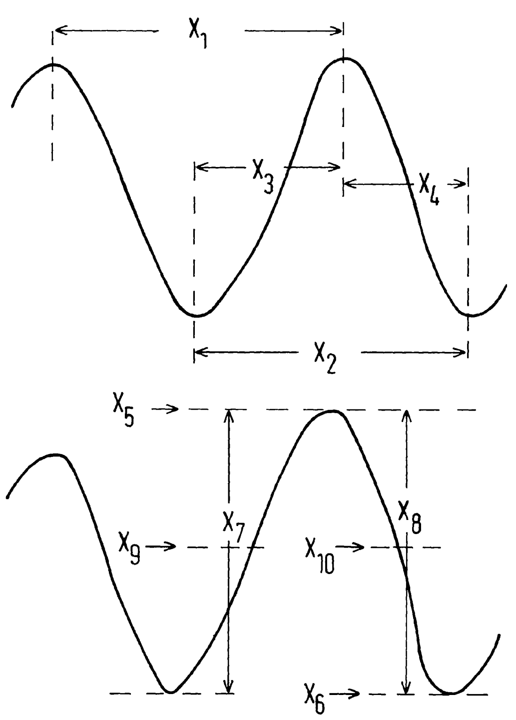


Figure 3. Imaginary light curves showing (a) the time quantities X_1 to X_4 and (b) the magnitude quantities X_5 to X_{10}.

Table 5
Results compared with the catalogue

Star	*Period*	*(M-m)/P*	*Extreme range*	*Mean range*	*Source*
R Ari	186.92	0.47	7.6 – 13.7	8.18 – 12.83	VSS 1903-1974
	186.78	0.45	7.4 – 13.7	8.2 – 13.2	GCVS*
R Aur	457.76	0.52	6.6 – 13.8	7.71 – 13.37	VSS 1904-1974
	457.51v	0.51	6.7 – 13.9	7.7 – 13.3	GCVS*
X Aur	163.90	0.49	7.9 – 13.5	8.44 – 12.83	VSS 1910-1974
	163.79	0.50	8.0 – 13.6	8.6 – 12.7	GCVS*
R Boo	223.51	0.46	6.3 – 12.9	7.16 – 12.33	VSS 1906-1974
	223.40	0.46	6.2 – 13.1	7.2 – 12.3	GCVS*
S Boo	271.08	0.48	8.0 – 13.8	8.40 – 13.30	VSS 1921–1974
	270.73	0.44	7.8 – 13.8	8.4 – 13.3	GCVS*

*1976 supplement[13] for mean range, 1985[1] for other details.

apparent correlation is more marked than the most marked correlation we would get purely by chance 99% of the time among 145 possible correlations.

To take the case of R Ari again, as many as 18 of the 145 correlations are 'significant' in this sense. Eleven of these are, however, trivial. For example, the cycle length between minima X_2 is positively correlated with the fall time X_4 ($r = +0.694$); but as X_2 is the sum of X_4 and the previous rise time we should expect X_2 and X_4 to show a positive correlation ($r \times +0.7$) if fall time and rise time vary independently. We do not consider further such trivial correlations.

As it happens, none of the remaining correlations tested among time quantities (cycle lengths, rise times, fall times) is significant in the case of R Ari. There are, however, two significant correlations among magnitude quantities, and five which interrelate times and magnitudes. Details are given in Table 6, where these two groups of correlations are listed in order of strength. The first one is a positive correlation between X_8 and X_{15}, which we describe as "a large amplitude fall tends to be followed by a faint maximum". It is of course equally true that "a small amplitude fall tends to be followed by a bright maximum", and similarly in other cases. Our apparent emphasis on faintness in Table 6 merely reflects the fact that magnitudes increase numerically with faintness.

It is clear from these correlations that these Mira stars do not produce bright or faint maxima, long or short cycles, and so on, completely at random. It is rather the case that in 'deciding' whether to produce them they 'remember' what they have done over at least the last cycle, and modify what comes in accordance with certain statistical laws which we may attempt to infer. To this end we intend in a later report to discuss the distribution across stars of the correlation coefficients.

It is of particular interest to establish whether the periods of Mira stars vary. Certainly there are variations from one cycle to the next in the intervals between consecutive maxima or minima; but it has long been known that in most stars these can be explained as random variations about the mean period. This was first clearly demonstrated for Mira Ceti and Chi Cygni by Eddington and Plakidis[14], and for larger numbers of stars by Sterne[15]. Sterne used a simple Chi-square test on the numbers of cycles greater and smaller than the mean in groups of about 10 consecutive cycles to test for period change, and this method has since been widely used.

A positive correlation between X_1 and X_{11} (consecutive intervals from maximum to maximum), or between X_2 and X_{12} (the same thing for minima) would be a more sensitive indicator than Sterne's test of a systematic change in the period, but even this is not a very sensitive test. If a particular observed maximum (or minimum) is late, because of observational error or a minor fluctuation in the light curve, this will cause the preceding cycle to be longer, and the following cycle to be shorter; and conversely. Therefore, we may expect consecutive cycle lengths to be negatively correlated. This effect could swamp any positive correlation arising from change in the mean period.

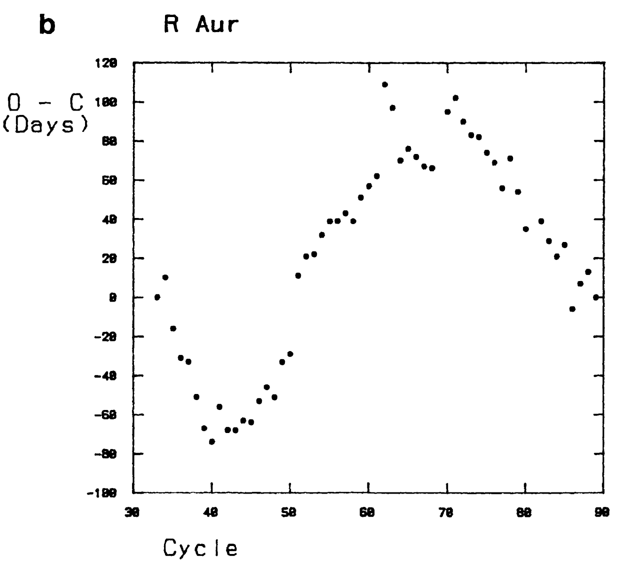
We therefore use a different test for period variation. From the date of the first observed maximum and the mean period from maxima (for example, for R Ari, 25 783/138 = 186^d.83) we find the expected, or calculated (C), dates of maxima. These are compared with the observed dates (O). Plots of the residuals (O-C) are given in Figure 4. Because the calculated dates are derived from the first and last observed maxima, the O-C curves start and finish at zero. Between those points

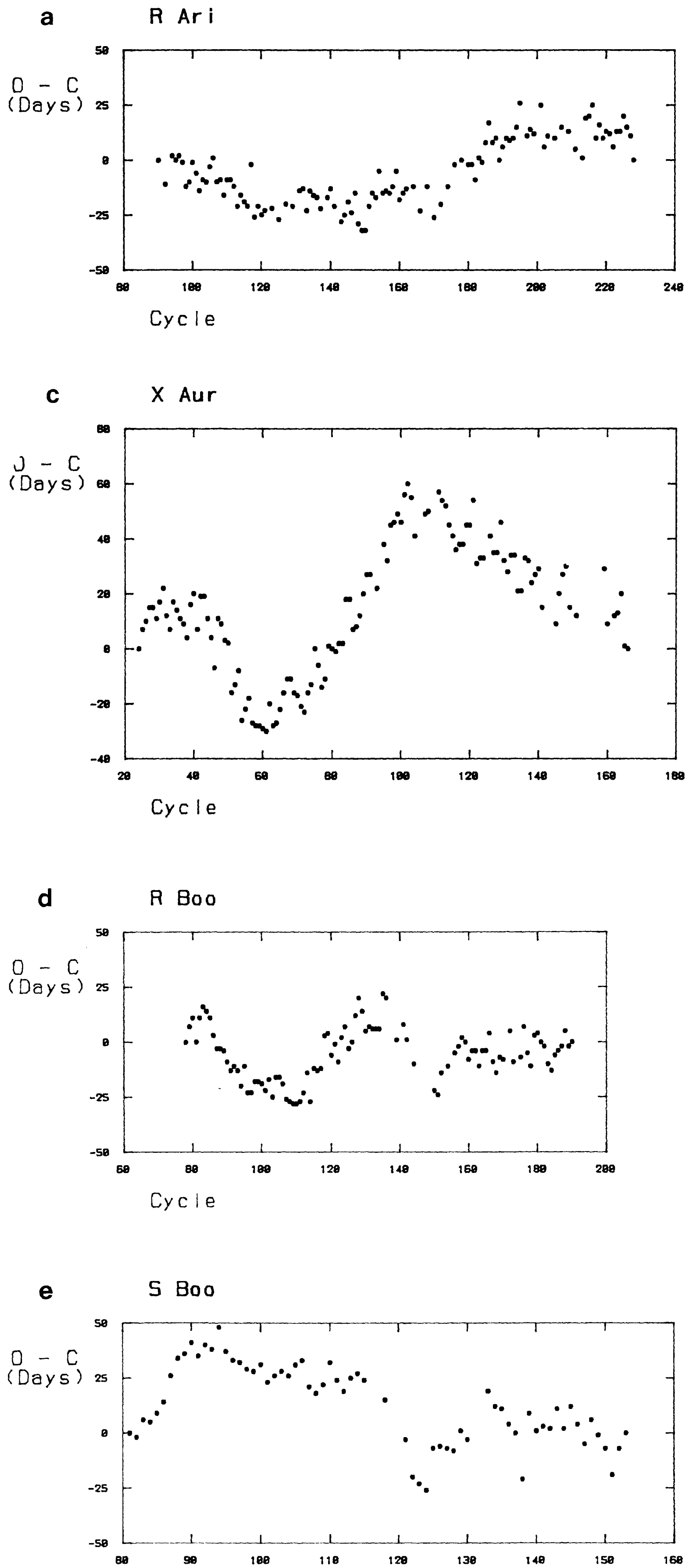
Table 6
Significant non-trivial correlations

	r	*P*	*Description*
R Ari			
X_8,X_{15}	+0.555	1×10^{-5}	A large amp. fall => a faint max.
X_6,X_{15}	+0.464	4×10^{-5}	A faint min. => a faint max.
X_2,X_6	+0.613	4×10^{-6}	A long cycle => a faint min.
X_8,X_{11}	+0.544	2×10^{-5}	A large amp. fall => a long cycle
X_9,X_{11}	−0.510	5×10^{-5}	A faint rise => a short cycle
X_5,X_{11}	−0.448	8×10^{-6}	A faint max. => a short cycle
X_1,X_5	+0.407	6×10^{-5}	A long cycle => a faint max.
R Aur			
X_6,X_7	+0.609	6×10^{-6}	A faint rise => a faint min.
X_6,X_{16}	+0.591	1×10^{-5}	A faint min. => a faint min.
X Aur			
X_8,X_{20}	+0.443	4×10^{-6}	A large amp. fall => a faint fall
X_1,X_9	+0.397	2×10^{-5}	A long cycle tends to include a faint rise
R Boo			
X_8,X_{20}	+0.543	8×10^{-8}	A large amp. fall => a faint fall
X_8,X_{15}	+0.498	6×10^{-7}	A large amp. fall => a faint max.
X_6,X_{15}	+0.496	3×10^{-7}	A faint min. => a faint max.
X_6,X_{20}	+0.466	4×10^{-6}	A faint min. => a faint fall
X_7,X_9	−0.401	6×10^{-5}	A faint rise tends to have a large amp.

The symbol => signifies 'tends to be followed by'.

Figure 4. (opposite and below) O-C plots for (a) R Ari; (b) R Aur; (c) X Aur; (d) R Boo; (e) S Boo.




a
R Ari
O - C
(Days)
50
25
0
-25
-50
80
100
120
140
160
180
200
220
240
Cycle
c
X Aur
O - C
(Days)
80
60
40
20
0
-20
-40
20
40
60
80
100
120
140
160
180
Cycle
d
R Boo
O - C
(Days)
50
25
0
-25
-50
60
80
100
120
140
160
180
200
Cycle
e
S Boo
O - C
(Days)
50
25
0
-25
-50
80
90
100
110
120
130
140
150
160

they oscillate about zero, the extreme residual in the case of R Ari for example being -32^d for cycles 149 and 150. This is nearly a fifth of a cycle, and is undoubtedly a real deviation; but could it merely be due to the accumulation of random deviations in the individual cycles up to that point?

The problem is closely analogous to the analysis of cusums, which are often used in industrial quality control. The O-C value is in fact the cusum (cumulative sum) of the deviations from the mean, over all cycle lengths since the first maximum. We test the significance of the greatest positive or negative O-C value, using a 'span test', described in the Appendix. Essentially, this compares the observed greatest |O-C| value with what we might expect to get by chance if the mean period does not vary, and individual cycles are independent and normally distributed about the mean, with a standard deviation estimated from the data.

For R Ari we find that a maximum |O-C| of 32^d is not significant; only a deviation greater than 117^d would be evidence at the 5% significance level of change in the mean period.

Results of analysis

R Ari was discovered by Argelander in 1857. The catalogue[1] remarks that the star shows OH emission, presumably from infrared-pumped masering clouds produced by its stellar wind. BAA observations started in 1903.

We find a longer rise time than the catalogue, and a brighter mean minimum magnitude (Table 5). A comparison of our minimum magnitudes with those observed by the AAVSO as published by Campbell[8] shows that they are in good agreement. For cycles 124 to 177 the mean is $12^d.93$ (AAVSO), or $12^d.87$ (BAA). We do not know the source of the catalogue value of $13^d.2$.

The correlations in Table 6 indicate that there is some tendency for alternating behaviour from cycle to cycle. A bright maximum will tend to be followed by a long cycle, which in turn tends to be followed by a faint maximum. There does, in fact, appear to be a weak negative correlation between consecutive maximum magnitudes ($r = -0.271$, $P = 0.01$).

Figure 4(a) shows the varation in O-C. The span test does not reveal evidence of variation in the mean period.

R Aur was discovered by Argelander in 1862, and BAA observations began in 1904. Our results are in close agreement with the main catalogue details, but we find no real evidence from BAA data that the period varies as claimed in the catalogue's remarks. These quote a study by Nudzhenko[16], who applied Sterne's test to some 60 years' dates of maxima. The catalogue also mentions a possible variation in the period with a cycle of 64.5 years. The O-C plot (Figure 4(b)) shows a maximum |O-C| of 109^d, less than the critical value of 136^d for significance at the 5% level. An inspection of the O-C plot might suggest at least two period changes, around maxima Nos. 40 and 70; but in fact the variations in O-C are quite consistent with the accumulation of random deviations in individual cycle lengths. Their standard deviation of 15^d is relatively large, but not large in proportion to the period of R Aur: 3% compared with 3 to 5% for the other four stars discussed in this report.

We have verified this result in two ways. First, Figure 5 shows the observed individual cycle lengths. Although there is an apparent tendency for the period to be longer between about cycles 43 and 62, consecutive cycle lengths do not show a significant positive correlation, whether we consider intervals between maxima ($r = +0.019$, $P = 0.9$) or those between minima (which are negatively correlated! $r = -0.378$, $P = 0.01$).

Secondly, by a Monte Carlo method we have produced 10 simulated O-C diagrams for an imaginary star with a constant period, and cycle length standard deviation of 15^d, the same value as for R Aur. These do not look any different in character from the diagram for R Aur. One of the more 'interesting' simulations is shown in Figure 6, for comparison with Figure 4(b).

Even if the period of R Aur does vary, it seems premature to suggest a cycle of 64.5 years, when the star has been observed for less than two of these supposed cycles.

The correlations listed in Table 6 show that there is a tendency for a faint minimum, or a faint rise to maximum, to be followed by another minimum fainter than average.

X Aur was discovered by Anderson in 1900, and BAA observations began in 1910. Our mean range is 0.3 magnitude greater than that given in the catalogue. From a comparison of our results with those given by Campbell[8], who is the source of the mean range given in the catalogue, it appears that the BAA and AAVSO results on individual maxima and minima agree closely. The discrepancy arises from the fact that Campbell's study covers only 29 years. Otherwise we agree closely with the catalogue.

The first correlation in Table 6 indicates that the star has a memory of at least one cycle. The second may be connected with the fact that cycles including faint minima appear to show a weak tendency to be longer (X_6, X_{11}: $r = +0.257$, $P = 0.008$).

Figure 4(c) shows the variation in O-C. The span test does not reveal evidence of variation in the mean period.

R Boo was discovered by Argelander in 1858, and BAA observations began in 1906. Our results are in good agreement with the catalogue. The minimum magnitude of 13.1, 0.2 magnitude below our faintest, appears to be confirmed by AAVSO data[17], which show a minimum of $13^m.1$ in 1975 after our observations had been discontinued. A less well observed AAVSO mini-

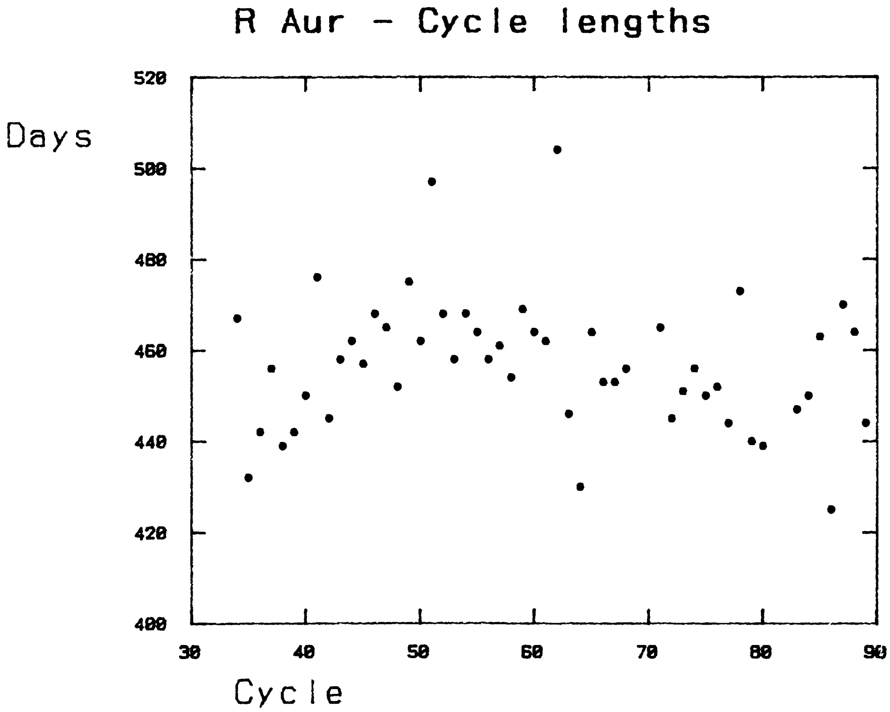


Figure 5. R Aur: cycle lengths, from consecutive observed maxima.

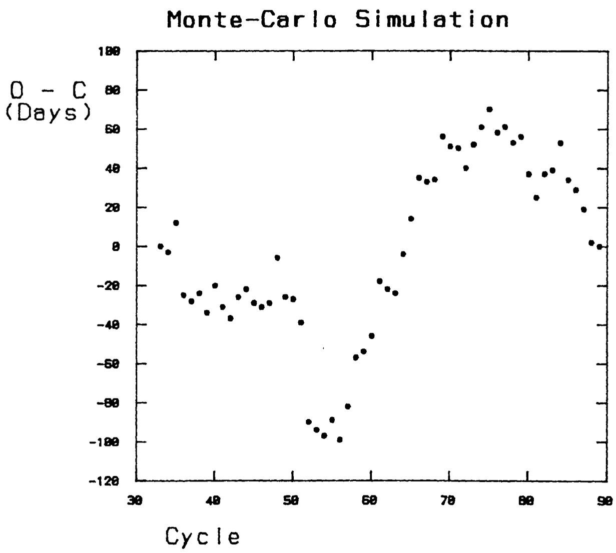


Figure 6. Monte Carlo simulation of the O-C diagram for a star with constant period, and cycle length standard deviation of 15^d, the same value as for R Aur. Compare Figure 4(b).

mum[18] of 13^m.1 in 1962 was however about 12^m.4 according to our (also fragmentary) observations.

The correlations indicate a memory of at least one cycle. Fall amplitudes show a weak tendency to be alternately large and small (X_8,X_{18}: $r=-0.340$, $P=0.002$), but for rise amplitudes and for mid-ranges on both fall and rise any tendency to alternate is insignificant or apparently absent.

Figure 4(d) shows the variation in O-C. The span test does not reveal evidence of variation in the mean period.

S Boo was discovered by Argelander in 1860, and BAA observations began in 1921. We find that the rise time is 0.48 of the period, rather than 0.44 as given in the catalogue; otherwise there is close agreement. The extreme maximum given of 7^m.8, 0.2 magnitude brighter than ours, is confirmed by two maxima of this brightness observed after our series ends by the AAVSO[17] in 1975 and 1976, and one by the Association Française des Observateurs d'Etoiles Variables (AFOEV)[19] in 1977. Hopp[20] independently reports 7^m.7 for the first of these.

There are no significant non-trivial correlations. Figure 4(e) shows the variation in O-C. The span test does not reveal evidence of variation in the mean period.

Conclusions

It is early to draw general conclusions about Mira stars from so small a sample as five stars. We shall therefore content ourselves with the following observations.

1) We find good agreement with the extreme ranges and mean periods given in the catalogue.

2) We disagree in a number of cases with the catalogue data on rise times $(M\text{-}m)/P$, and on mean ranges. These are mainly based on Campbell's discussion of observations over only 29 years, less than half the timespan of the present report. (In the case of the mean magnitude of R Ari at minimum, however, we agree with Campbell against the catalogue.) We also find no evidence that the period of R Aur varies, as the catalogue asserts.

3) There are numerous correlations indicating that the stars do not vary at random but have a memory of up to at least one cycle in most cases. Sometimes there is a tendency for the new cycle to resemble the preced-

ing one in some features; sometimes there is a tendency to alternate. But where the same quantities have been found to be correlated in more than one star, the correlations have so far always been in the same sense. This may indicate that it is to some extent a matter of chance which of these correlations turn out to be statistically significant in the data we have for each star. Such a conclusion would be consistent with the investigation by Harrington[21] of the single correlation 'a long cycle tends to be followed by a faint maximum', one we find to be significant for R Ari. Harrington considered this might be due to the outward propagation of a shock wave taking more or less time to reach the surface depending on the energy it carries. Of 165 Mira and semiregular variables he analysed, only 41% showed a correlation significant at the 5% level; but the distribution of correlation coefficients was consistent with statistical fluctuations about the mean, and no difference was discernible between the 'significant' stars and the rest, whether one considered period, spectral type or space velocity. It may really be the case that the light curves of all Mira stars are governed by similar statistical laws. Analysis of further stars will help us to infer what these laws might be.

4) Although it would have been possible to gain a general idea of the ranges and periods of Mira stars from a relatively short run of observations, more detailed analysis such as we have attempted requires very long runs. We are greatly indebted to past observers whose efforts have provided these, and we would encourage present observers to continue to monitor the Mira stars still on the programme.

Acknowledgements

We acknowledge with gratitude the immense help given by Herbert Joy, a former observer, who single-handed has logged in chronological order over 90 000 observations of the 31 stars made between 1935 and 1974. We are also grateful to Storm Dunlop for comments on a draft of this report, and to both Storm Dunlop and Melvyn Taylor for help with the illustrations.

References

1 Kholopov, P. N. (ed.), *General Catalogue of Variable Stars*, Fourth Edition, **1**, Moscow, 1985.
2 Strohmeier, W., *Variable Stars*, Oxford, 1972.
3 *Mem. Brit. astron. Assoc.*, **15, 18, 22, 25, 28, 31** (1906-34).
4 Lindley, W. M., *Mem. Brit. astron. Assoc.*, **38**, 1 (1958).
5 *J. Brit. astron. Assoc.*, **71**, 18, 192; **72**, 122; **73**, 97, 152, 157; **74**, 185, 190 (1960-64).
6 *App. Mem. Brit. astron. Assoc.*, **15, 18, 22, 25** (1913-24).
7 *J. Brit. astron. Assoc.*, **31**, 257; **32**, 296; **33**. 316; **34**, 97, 135, 175, 224; **35**, 187, 227; **36**, 226, 277; **37**, 256, 302; **38**, 209; **39**, 179, 285, 360; **40**, 388; **41**, 61, 408; **42**, 18, 284; **43**; 187; **44**, 213, 383; **50**, 97, 140, 177 (1921-40).
8 Campbell, L., *Studies of Long Period Variables*, Cambridge, Mass., 1955.
9 Ludendorff, H., *Handbuch der Astrophysik*, **6**, 99 (1928).
10 Hoffmeister, C., *et al.*, *Variable Stars*, Berlin, 1985.
11 Campbell, L., *Harvard Repr.* No. 21 (1925).
12 Tuchman, Y., *Mon. Not. R. astron. Soc.*, **208**, 215 (1984).
13 Kukarkin, B. V., *et al.*, *Third Supplement to the General Catalogue of Variable Stars*, Moscow, 1976.
14 Eddington, A. S., and Plakidis, S., *Mon. Not. R. astron. Soc.*, **90**, 65 (1929).
15 Sterne, T. E., and Campbell, L., *Harvard Ann.*, **105**, 459 (1936).
16 Nudzhenko, A. G., *Peremennye Zvezdy*, **19**, 381 (1974).
17 *AAVSO Report* 38 (1983).
18 Mattei, J. A., *et al.*, *Sky & Telesc.*, **60**, 180 (1980).
19 *Bull. AFOEV*, **11**, 47 (1977).
20 Hopp, U., *Inf. Bull. Var. Stars*, 1437 (1978).
21 Harrington, J. P., *Astron. J.*, **70**, 569 (1965).
22 Williamson, R. J., *The Statistician*, **34**, 345 (1985).

Addresses: J. E. Isles, P. O. Box 6322, Limassol, Cyprus
D. R. B. Saw, Upanova, 18 Dollicott, Haddenham, Aylesbury, HP17 8JG

Appendix

The span test for period change

If we have observed m individual cycle lengths between maxima, $P_1,\ldots,P_m$, and if $\bar{P}$ is the mean period between the first and last maxima, we calculate

$$s = \sqrt{\frac{\sum_{i=1}^{m} (P_i - \bar{P})^2}{m-1}}$$

The test statistic is then (maximum |O-C|)/s. Critical values of this statistic are tabulated by Williamson[22] for various values of n, which in our context is the number of cycles between the first and last observed maxima. These tables are for the case $m=n$. Where $m\{n$, that is where some maxima have been missed, the critical value may be greater (because s, which is an estimate of the standard deviation of the cycle length, will have greater variance) or smaller (because the greatest O-C value may be among those that were unobserved). But generally few maxima have been missed, so the true critical value should be close to the tabulated value.

The use of an alternative estimator of the standard deviation is often advocated. This is

$$r = \sqrt{\frac{\sum_{i=1}^{k-1} (P_{i+1} - P_i)^2}{2\,(k-1)}}$$

where the summation is over k consecutive pairs of cycle lengths. Where the mean period does vary, r will tend to overestimate the true standard deviation less than will s, provided the dates of maxima can be observed without error. In practice, however, when the mean period is constant, consecutive cycle lengths tend to be negatively correlated because of observational error, with the result that r is greater than s. We have used this alternative formula, in conjunction with critical values also tabulated by Williamson[22], only as a check on the main calculation using s.

We have also checked the results for maxima by a similar analysis of minima. The data are, of course, not independent. Generally minima are less well observed, and more of them are missed. All quoted results are therefore for maxima.

In investigating 31 Mira stars for possible period change, we may expect to find one or two results of apparent significance at the 5% level, even if none of the periods really vary. We shall therefore consider the question of how many of these stars have genuinely variable periods in a later report.