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ON THE MAXIMUM MASS OF A UNIFORMLY ROTATING NEUTRON STAR

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ABSTRACT

The effect of rapid uniform rotation on the upper mass limit for neutron stars is studied under the assumption that the equation of state is subject to only a minimal set of physical constraints. If only the energy condition $\epsilon \geq 0$, the microstability constraint $dp/d\epsilon \geq 0$, and the causality constraint $dp/d\epsilon \leq 1$ are imposed on the density ϵ and the pressure p above a fiducial value ϵ_0 of ϵ, the maximum mass of a uniformly rotating neutron star is approximately equal to $8.4(\epsilon_0/10^{14}\ \mathrm{g\ cm^{-3}})^{-1/2}\ M_\odot$. This amounts to an increase of $\sim$24%–25% over the corresponding nonrotating maximum mass, and it is to be compared with the value $\sim 3.2\ M_\odot$ of the maximum mass of uniformly rotating configurations associated with the stiffest of the equations of state proposed for actual neutron-star matter. Rotation increases the upper limit on baryon mass by only $\sim$20%, while the limiting moment of inertia is about twice its value for the nonrotating case. Also found are upper limits on the frequency of rotation, the frequency of frame dragging, and the maximum polar and equatorial redshifts. It is estimated that discarding the causality constraint would allow the upper mass limit to increase to $\sim 14(\epsilon_0/10^{14}\ \mathrm{g\ cm^{-3}})^{-1/2}\ M_\odot$. Rotating configurations near the upper mass limits found here probably exhibit a rotationally induced, bar-mode instability driven by gravitational radiation.

Subject headings: stars: neutral — stars: rotation

I. INTRODUCTION

The problem of determining the upper limit on the mass of a neutron star has received considerable attention. Knowledge of this limit is important in a variety of astrophysical contexts. For example, it is central to arguments that the compact objects in certain binary systems are so massive that they must be black holes rather than neutron stars (see, e.g., the recent argument from McClintock and Remillard [1986] involving the binary A0620 − 00).

In essence, the upper mass limits for *nonrotating* neutron stars were determined some time ago. This is so not only for the case in which one assumes that the equation of state is one of those proposed for "realistic" neutron-star matter (e.g., Arnett and Bowers 1977), but also for the more general case in which one assumes that the equation of state is subject to only a minimal set of physical constraints at high densities approximately equal to or greater than nuclear densities (Hartle 1978 and references cited therein). The latter case is the only one that really yields firm upper limits, given the incomplete state of understanding of neutron-star matter at high densities.

It is clear that the upper mass limits for *rotating* neutron stars will be larger than those for nonrotating configurations. But how much larger? While intrinsically interesting, this question takes on added importance in the context of the discovery of the fast pulsar (Backer *et al.* 1982), whose rotational period is $\sim$1.56 ms. In certain theories of the origin of fast pulsars (see, e.g., Papaloizou and Pringle 1978; Alpar *et al.* 1982, Fabian *et al.* 1983), they are spun up via accretion in binary systems. This suggests the possibility that a significant fraction of the neutron stars possessing weak magnetic fields and residing in binary systems are rotating rapidly, in the sense that their rotational energies are of the order of at least a 10th or so of their gravitational energies.

The question of the effect of rotation on the maximum has recently been answered by Friedman, Ipser, and Parker (1986; henceforth Paper I) for the former case above: they found that neutron stars in rapidly uniform rotation—attention is restricted to uniform rotation since it is thought that neutron-star matter does not permit significant differential rotation—can have masses ranging up to $\sim 3.2\ M_\odot$ for the proposed equations of state. This corresponds to increases of up to $\sim$20% over the corresponding nonrotating values.

Our purpose in this paper is to investigate the upper mass limits for the more general latter case above, in which the equation of state is subjected to a minimal set of physical constraints. Our investigation begins with a discussion of the constraints and then proceeds to the presentation of numerical results describing the structural features and mass limits for the corresponding models. Special attention is paid to the stability of the models.

II. THE CALCULATIONS

a) The Limiting Equation of State for Neutron Stars

The physical constraints that we shall impose on the equation of state for cold neutron-star matter have been discussed before (Hartle 1978): (i) the total mass-energy density $\epsilon \geq 0$ and is related to the isotropic pressure p by equation of state $p = p(\epsilon)$, (ii) the equation of state reduces to the presumed known form below a prescribed fiducial density $\epsilon = \epsilon_0$, (iii) the equation of state satisfies the microstability constraint

$$dp/d\epsilon \geq 0\ , \tag{1}$$

and (iv) it satisfies the causality constraint

$$dp/d\epsilon \leq 1 \tag{2}$$

(in units with $G = c = 1$).

TABLE 1

Properties of Nonrotating Models with $\epsilon_0 = 10^{14}$ g cm^{-3}

ϵ_c	$M/M_\odot$	$M_0/M_\odot$	R	I	β
1.1	2.24	2.44	23.7	0.92	0.722
1.2	3.57	4.08	26.5	1.92	0.603
1.3	4.51	5.32	27.9	3.40	0.526
1.5	5.58	6.87	29.1	5.20	0.436
1.7	6.08	7.64	29.4	6.16	0.393
2.1	6.57	8.44	29.3	6.85	0.342
2.5	6.71	8.69	29.0	6.94	0.320
2.9	6.75	8.76	28.6	6.85	0.306
3.3	6.74	8.74	28.2	6.66	0.297
4.4	6.64	8.54	27.4	6.23	0.287

Note.—For each model the following quantities are exhibited: ϵ_c, the central mass-energy density in units of 10^{14} g cm^{-3}; $M/M_\odot$, the total gravitational mass in solar units; $M_0/M_\odot$, the rest, or baryon, mass in solar units; R, the proper circumference radius ≡ (proper distance around equator)/2π, in units of kilometers; I, the moment of inertia ≡ (angular momentum)/(angular velocity), in units of 10^{46} g cm^2; and β, the relativity parameter ≡ [(polar redshift) + 1]$^{-2}$. The definitions of these quantities are discussed in more detail in Paper I.

TABLE 3

Properties of Nonrotating Models with $\epsilon_0 = 3 \times 10^{14}$ g cm^{-3}

ϵ_c	$M/M_\odot$	$M_0/M_\odot$	R	I	β
3.1	0.54	0.57	12.8	0.04	0.874
3.2	0.93	1.00	13.5	0.11	0.796
3.4	1.60	1.78	14.9	0.26	0.682
3.6	2.10	2.41	15.7	0.42	0.606
4.0	2.76	3.29	16.6	0.66	0.510
5.0	3.48	4.38	17.2	0.95	0.401
6.0	3.75	4.81	17.1	1.03	0.355
7.0	3.85	4.99	17.0	1.04	0.330
9.0	3.90	5.08	16.6	1.00	0.306
10.0	3.89	5.06	16.4	0.97	0.300

Note.—The notation is the same as that of Table 1.

Actually, the causality constraint (2) is not above suspicion. The quantity $(dp/d\epsilon)^{1/2}$ is the phase velocity of hydrodynamic waves in a neutron star, and it would also be the physical group velocity if the medium were not dispersive. In such a situation, constraint (2) would clearly be appropriate. It is unfortunate that neutron stars are in fact thought to be dispersive, and the resulting effect on the group velocity is not well understood (see, e.g., Caporaso and Brecher 1983; Glass 1983). While we shall retain constraint (2), it must be remembered that discarding it would increase our upper mass limits by significant amounts. We shall give rough estimates of those amounts below.

Our numerical studies support the conclusion that the upper mass limit increases with increasing stiffness of the equation of state (i.e., increasing dp/de). Indeed, although a rigorous proof is lacking, our investigations and those described by Hartle leave little doubt that, given the constraints (i)–(iv) above, the maximum mass occurs when $dp/d\epsilon = 1$ for $\epsilon \geq \epsilon_0$. This corresponds to the CL (causal limit) equation of state

$$p = \epsilon + \text{constant for } \epsilon \geq \epsilon_0 \,, \tag{3}$$

where the constant is chosen to guarantee continuity of the pressure at $\epsilon = \epsilon_0$.

The appropriate value of ϵ_0 is somewhat arbitrary, but it is of the order of nuclear densities. We shall present results for the values $\epsilon_0 = 10^{14}$ g cm^{-3} and 3×10^{14} g cm^{-3}. The precise form of the equation of state at $\epsilon < \epsilon_0$ is also uncertain, but it turns out that this is unimportant, since almost all of the mass of a configuration is contained within the stiff core at densities $\epsilon \geq \epsilon_0$. For definiteness we shall employ either the Baym, Pethick, and Sutherland (1971, hereafter BPS) or Negele and Vautherin (1973, hereafter NV) equation of state at $\epsilon < \epsilon_0$.

b) Numerical Results

i) *Upper Mass Limits*

Tables 1–4 and Figure 1 exhibit our results for the CL equation of state and the two values $\epsilon_0 = 10^{14}$ g cm^{-3} and $\epsilon_0 = 3 \times 10^{14}$ g cm^{-3}. The rotating models shown are maximally rotating in the sense described in Paper I: They lie at the termination points of the uniformly rotating equilibrium sequences obtained by fixing the relativity parameter $\beta \equiv [(\text{polar redshift}) + 1]^{-2}$ and by varying the uniform angular velocity Ω. As Ω is increased to its value at the termination point along a sequence, the velocity of rotation becomes equal to the Keplerian velocity of geodesic circular orbits, and equilibrium configurations cease to exist, since further increase in Ω would result in matter being shed from the equatorial regions. Rotation permits larger amounts of matter to be supported in equilibrium, and the model with maximum gravitational mass and baryon mass, for a given equation of state, lies at a termination point.

Tables 1 and 2 exhibit the properties of nonrotating and maximally rotating models for $\epsilon_0 = 10^{14}$ g cm^{-3}, with the BPS

TABLE 2

Properties of Maximally Rotating Models with $\epsilon_0 = 10^{14}$ g cm^{-3}

Ω	ϵ_c	$M/M_\odot$	$M_0/M_\odot$	R	ω_c/Ω	T/W	v_{eq}/c	I	cJ/GM^2	β	z_P	z_B	z_F
0.309	1.08	2.82	3.08	30.7	0.38	0.147	0.35	1.96	0.87	0.640	0.25	0.75	−0.21
0.330	1.12	3.93	4.41	33.7	0.48	0.159	0.41	3.48	0.85	0.540	0.36	1.05	−0.24
0.351	1.19	5.30	6.13	35.3	0.59	0.172	0.45	5.81	0.89	0.436	0.51	1.50	−0.26
0.375	1.30	6.59	7.96	36.5	0.70	0.174	0.49	8.41	0.82	0.330	0.74	2.31	−0.27
0.378	1.33	7.02	8.44	38.4	0.71	0.186	0.51	9.23	0.79	0.320	0.77	2.39	−0.28
0.381	1.44	7.80	9.46	36.8	0.74	0.186	0.52	10.9	0.78	0.285	0.87	2.76	−0.28
0.384	1.87	8.20	10.25	34.6	0.81	0.155	0.49	11.1	0.72	0.240	1.04	3.42	−0.19
0.387	2.03	8.34	10.45	34.1	0.83	0.153	0.48	11.2	0.71	0.230	1.09	3.67	−0.18
0.396	2.15	8.33	10.42	34.0	0.84	0.153	0.53	11.2	0.73	0.220	1.13	3.67	−0.23

Note.—For each model the following quantities, in addition to those of Table 1, are exhibited: Ω, the uniform angular velocity of rotation in units of 10^4 s^{-1}; ω_c/Ω, the percentage of central dragging of inertial frames; T/W, the ratio of rotational energy to gravitational energy; v_{eq}/c, the velocity of a comoving observer at the equator relative to the locally nonrotating observer, in units of the speed of light; cJ/GM^2, the dimensionless ratio of angular momentum J to M^2; z_P, the polar redshift; z_B, the equatorial redshift in the backward direction; and z_F, the equatorial redshift in the forward direction. The definitions of these quantities are discussed in more detail in Paper I.

TABLE 4

PROPERTIES OF MAXIMALLY ROTATING MODELS WITH $\epsilon_0 = 3 \times 10^{14}$ g cm^{-3}

Ω	ϵ_c	$M/M_\odot$	$M_0/M_\odot$	R	ω_c/Ω	T/W	v_{eq}/c	I	cJ/GM^2	β	z_P	z_B	z_F
0.588........	3.40	2.46	2.76	21.1	0.51	0.180	0.42	0.79	0.69	0.510	0.40	1.19	−0.28
0.615........	3.69	3.25	3.77	21.2	0.63	0.181	0.44	1.25	0.62	0.401	0.58	1.69	−0.26
0.659........	4.23	4.03	4.91	21.4	0.73	0.179	0.47	1.75	0.54	0.299	0.83	2.57	−0.27
0.665........	4.37	4.17	5.06	21.4	0.74	0.183	0.48	1.88	0.55	0.280	0.89	2.79	−0.28
0.675........	4.50	4.32	5.27	21.4	0.76	0.180	0.48	1.93	0.53	0.270	0.93	3.01	−0.27
0.681........	4.54	4.38	5.36	21.5	0.77	0.181	0.49	1.96	0.53	0.265	0.94	3.08	−0.28
0.713........	5.40	4.72	5.85	21.1	0.82	0.180	0.50	2.13	0.51	0.220	1.13	4.02	−0.28
0.725........	5.57	4.76	5.95	21.0	0.83	0.180	0.51	2.16	0.50	0.210	1.18	4.39	−0.28
0.737........	5.93	4.81	6.03	20.9	0.84	0.180	0.51	2.17	0.50	0.200	1.24	4.75	−0.29
0.758........	6.68	4.87	6.11	20.7	0.86	0.180	0.52	2.16	0.50	0.185	1.33	5.36	−0.30
0.761........	6.72	4.86	6.11	20.6	0.88	0.180	0.53	2.15	0.50	0.183	1.34	5.46	−0.30

NOTE.—The notation is the same as that of Table 2.

equation of state used below ϵ_0, and Tables 3 and 4 exhibit the models for $\epsilon_0 = 3 \times 10^{14}$ g cm^{-3}, with the NV equation of state used below ϵ_0. Figure 1 exhibits the plots of mass versus radius for the models. We note that for the higher value of ϵ_0, the calculated properties of models based on the NV equation of state agree to within 1% with those of models based on the BPS equation of state.

The upper mass limits shown for the nonrotating configurations obey the known empirical formula (cf. Hartle 1978)

$$M^{\max}_{(\text{nonrot})} \approx 6.8\left(\frac{\epsilon_0}{10^{14}\ \text{g cm}^{-3}}\right)^{-1/2} M_\odot\ . \tag{4}$$

It is evident that the upper mass limits for uniformly rotating configurations also scale approximately like $\epsilon_0^{-1/2}$. In numerical terms, the effect of uniform rotation for $\epsilon_0 = 10^{14}$ g cm^{-3} is to increase the upper mass limit to 8.34 $M_\odot$, representing an increase of ~24% over the nonrotating value. And the effect for $\epsilon_0 = 3 \times 10^{14}$ g cm^{-3} is to increase the upper mass limit to 4.87 $M_\odot$, representing an increase of ~25%. These results are to be compared with the percentage increases ~20% for the stiffest of the proposed equations of state studied in Paper I. The maximum mass in Table 2, 8.34 $M_\odot$, is much greater than the largest value in Paper I, 3.18 $M_\odot$. Even so, given the trend evident in Table 2 of Paper I, involving the way in which the percentage increase grows with maximum mass, one might have expected increases larger than 25% in the present case.

ii) *Implications for the Stability of Configurations*

Nonrotating models obeying a fixed equation of state become dynamically unstable to gravitational collapse at the mass peak along a sequence parameterized by central density ϵ_c (cf. the solid curves in Fig. 1). As is discussed briefly in Paper I and proved in detail by Friedman, Ipser, and Sorkin (1986), a similar result holds for the two-parameter family of uniformly rotating models obeying a fixed equation of state (the models filling the regions between solid and dashed curves in Fig. 1): models become secularly unstable to axisymmetric collapse at the mass peak along a sequence of fixed angular momentum. The instability is secular because the corresponding fluid motion generally involves the redistribution of angular

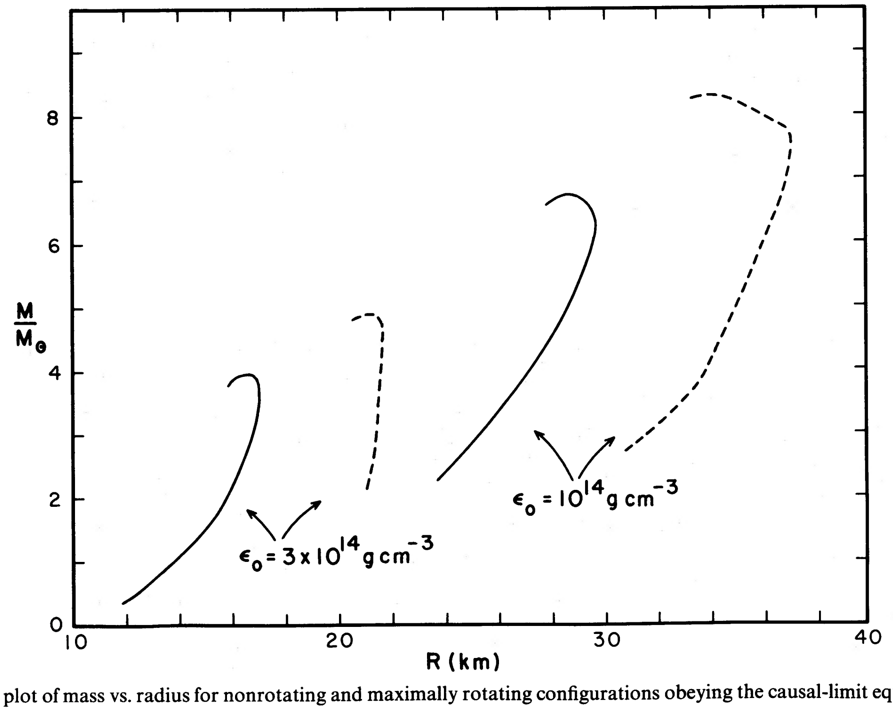

FIG. 1.—A plot of mass vs. radius for nonrotating and maximally rotating configurations obeying the causal-limit equation of state

momentum of fluid elements, a process whose time scale typically exceeds the dynamical time scale. The line of onset of secular instability runs from the mass peak of a spherical sequence in Figure 1 to the mass peak of the corresponding termination sequence. A consequence, evident from the tables, is that the maximum-mass model, in addition to possessing the largest baryon mass, also possesses the largest values of angular velocity Ω, central frame-dragging angular velocity ω_c, and polar and backward redshifts Z_p and Z_B among all models secularly stable against collapse. To within the numerical accuracy, it also appears to possess the maximum moment of inertia I, which is not true for the proposed equations of state studied in Paper I. The maximum values of I in Tables 2 and 4 are one order of magnitude larger than those of Paper I for maximally rotating configurations.

Another feature absent from the models of Paper I but present in the CL models of Tables 2 and 4 is ratios T/W of rotational energy to gravitational energy in excess of 0.14. This value is approximately that at which Newtonian polytropes and homogeneous Maclaurin spheroids succumb to a nonaxisymmetric bar-mode (spherical-harmonic index $m = 2$) secular instability driven by gravitational radiation (cf. Imamura, Durisen, and Friedman 1985; Managan 1985). Hence rapidly rotating CL models, including the maximum-mass models, quite likely have unstable bar modes. The growth times should be significantly shorter than those for the higher m nonaxisymmetric modes that are also expected to be unstable (cf. Paper I). Unstable models would spin down via gravitational radiation emission and evolve toward the line of onset of the axisymmetric collapse instability. This has the effect of limiting the mass of a completely stable model to something less than the upper mass limit. Further work is required before the magnitude of this effect can be assessed.

Despite their relatively large values of T/W, none of the models had ergospheres, regions where the asymptotically timelike Killing vector becomes spacelike. Apparently, uniformly rotating models can have ergospheres only if their equations of state are stiff enough to violate causality.

iii) *Effect of Relaxing the Causality Constraint*

If the causality constraint (2) is discarded, the upper mass limits will be increased significantly. In such a case it turns out that one ends up considering models with homogeneous cores, and it is known (Hartle 1978) that in the nonrotating limit the maximum mass is increased to the value

$$M^{\max}_{(\mathrm{nonrot})} \approx 11.4\left(\frac{\epsilon_0}{10^{14}\ \mathrm{g\ cm}^{-3}}\right)^{-1/2} M_\odot \,. \qquad (5)$$

We have not determined the corresponding limit $M^{\max}_{(\mathrm{rot})}$ for uniformly rotating configurations, but two remarks are appropriate: first, Table 3 of Butterworth and Ipser (1976) demonstrates that uniformly rotating homogeneous configurations can have masses that exceed $13(\epsilon_0/10^{14}\ \mathrm{g\ cm}^{-3})^{-1/2}\ M_\odot$, so the upper mass limit in the absence of the causality constraint is at least this large; and second, estimating that uniform rotation increases the upper mass limit by $\sim 25\%$ for configurations with homogeneous cores, we would guess

$$M^{\max}_{(\mathrm{rot})} \approx 14.3\left(\frac{\epsilon_0}{10^{14}\ \mathrm{g\ cm}^{-3}}\right)^{-1/2} M_\odot \qquad (6)$$

if the causality constraint (2) is discarded. In other words, we would guess that $M^{\max}_{(\mathrm{rot})}$ increases by a factor ~ 1.7 as $dp/d\epsilon$ increases from 1 to ∞ at high densities. Caporaso and Brecher (1983) and Glass (1983) have discussed whether degrees of stiffness in this range are actually physically reasonable.

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