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The mass of the asteroid (10) Hygiea derived from observations of (829) Academia

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Summary. A systematic search for new mass determinations of asteroids yielded only two promising candidates, (10) Hygiea and (15) Eunomia. Using a close approach between the asteroids Hygiea and (829) Academia of 0.006 AU at a relative velocity of 3.2 km s^{-1} in 1927, we determined a first value for the mass of the fourth largest asteroid Hygiea by analyzing the observations of Academia. Only 43 accurate observations of Academia were available. Some of these observations, in particular the earlier ones were re-reduced. We obtained a value of $4.7 \pm 2.3 \; 10^{-11} \; M_{\odot}$ for Hygiea which yields a density of 2.05 ± 1 g cm^{-3} assuming a diameter of 443 km. The large mean error obtained for Hygiea's mass shows the limitations to determine more asteroidal masses.

Key words: asteroids – celestial mechanics

1. Introduction

The mass of an asteroid is classically obtained by an analysis of its gravitational effects on the orbit of another asteroid. The determination of the three hitherto known masses of (1) Ceres, (2) Pallas, and (4) Vesta (Table 1) was based on mean motion resonance effects. A resonance between two asteroidal orbits results in successive close approaches between the two asteroids which enhances perturbational effects in longitude mainly. Hertz (1968) used the 5:4 resonance between the orbital periods of Vesta and (197) Arete in order to derive the first known asteroidal mass namely that of Vesta by analyzing the orbit of Arete. These two asteroids approach each other closely every 18 years because of this 5:4 resonance. Within 18 years, Vesta revolves five times about the Sun, while Arete completes 4 revolutions. Every 18 yr, these two asteroids approach each other closely with a minimum distance of 0.03 AU at a relative velocity of 2.1 km s^{-1}. Due to Jupiter perturbations, this 5:4 resonance between Vesta and Arete is only a temporary one. The minimum distance between these two asteroids will increase in the future to a value of 0.067 AU in the year 2012 according to our calculations. Hertz did not take into account the perturbations exerted by Arete on Vesta's orbit. This procedure is justified by the comparatively small diameter of Arete which is 12 times smaller than Vesta's diameter according to the TRIAD file (Bowell et al., 1979). Arete's mass is therefore presumably about 1000 times less than Vesta's mass.

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Including Arete's observations after 1968, Schubart (1977) improved Vesta's mass. It is Schubart's value which is given in Table 1.

In addition, Schubart (1974, 1975) determined the masses of Ceres and Pallas using a temporary 1:1 resonance which caused several close approaches between Ceres and Pallas in the 19th century. Between 1802 and 1848, these two largest asteroids approached each other within 0.3 AU at a relative velocity of 5 km s^{-1}. Since 1848 Ceres and Pallas drift slowly out of this 1:1 resonance. At present, their minimum distance is 1.3 AU, a value which appears to be too large for mass improvements.

A search for more resonances among asteroids carried out by the authors yielded only two more pairs of asteroids suitable for mass determinations: a 1:1 resonance between (15) Eunomia and (1284) Latvia and another 1:1 resonance between Eunomia and (1313) Berna. According to the TRIAD file Eunomia has a diameter of 261 km. Neither for Latvia nor for Berna values for diameters are given, which indicates that their perturbations on Eunomia's orbit can be neglected. Eunomia is the largest S-type asteroid. Since only a few observations of Berna and Latvia are available, a successful mass determination of Eunomia appears at present to be very difficult. Table 2 summarizes relevant data for the large resonant asteroids mentioned above.

Since the search for more resonant pairs of asteroids suitable for mass determinations yielded only one not very promising candidate, (15) Eunomia, we looked for single very close encounters between large and small asteroids at low relative velocities. Results about searches for such single close encounters between asteroids were published by Fayet (1949), Davis and Bender (1977), and Lazović and Kuzmanoski (1979). These results were mainly based on estimations using unperturbed two body approximations. After integrating numerically the orbits of about 2300 numbered asteroids over the period 1892–2015 we used the resulting data for a systematic search for single close encounters.

In order to be considered as a good candidate for a mass determination from a single close encounter with a smaller asteroid, we applied the following criteria:

– close approach within 0.01 AU;

– low velocity encounter less than 5 km s^{-1};

– sufficiently long time spans of observations of the smaller asteroid before and after a close encounter.

In case the minimum approach distance during an encounter is too larger or in case the relative encounter velocity is too large, the perturbations in the orbit of the smaller asteroid are to weak to be detected by analyzing the observations of the smaller asteroid. In

Table 1. Known masses and densities for asteroids. Values taken from Schubart and Matson (1979) and Bowell et al. (1979)

Name	Type	Mass (10^{-10} $M_\odot$)	Diameter (km)	Density (g cm^{-3})	Method
1 Ceres	*C*	5.9 ±0.33	1025	2.3±1.1	1:1 resonance with Pallas
2 Pallas	*U*	1.08±0.22	583	2.6±0.9	1:1 resonance with Ceres
4 Vesta	*V*	1.38±0.12	555	3.3±1.5	5:4 resonance with (197) Arete

Table 2. The 1:1 resonances between (15) Eunomia, (1284) Latvia and (1313) Berna. Only distances smaller than 0.1 AU are given

	Distance (AU)	Relative velocity (km s^{-1})	Year	Epoch (JD)
Eunomia	0.08	2.3	1938	2429200.5
–Latvia	0.08	2.3	1942	2430700.5
	0.06	2.3	1947	2432300.5
	0.04	2.0	1951	2433900.5
	0.04	1.9	1956	2435500.5
	0.03	1.8	1960	2437000.5
	0.02	1.6	1964	2438600.5
	0.05	1.4	1968	2440200.5
	0.07	1.4	1973	2441700.5
	0.09	1.5	1977	2443300.5
	0.08	1.2	1981	2444900.5
Eunomia	0.08	1.0	1954	2435100.5
–Berna	0.04	1.0	1955	2435200.5
	0.05	1.0	1955	2435300.5
	0.10	1.2	1955	2435400.5
	0.10	0.5	1959	2436600.5
	0.10	0.6	1960	2437000.5
	0.09	0.6	1960	2437100.5

addition, if the time span before or after the close encounter covered by observations is too short, the effect of the perturbations due to the single close encounter might not be recognizable in observations. In such cases, individual and systematic errors of observations are larger than perturbational effects.

The values of 0.01 AU for distances and of 5 km s^{-1} for encounter velocities are empirical values based on numerical experiments. In each individual case it is possible, as we will show later, to estimate the necessary precision of the observations of the smaller object in order to derive a significant value for the mass of the larger asteroid.

The most promising result of the search was the discovery of a very close encounter between the fourth largest asteroid (10) Hygiea and (829) Academia. In 1927, Hygiea approached Academia within 0.006 AU at a relative velocity of 3.2 km s^{-1}. Hygiea is a C-type asteroid with a diameter of 443 km according to the TRIAD file. Academia was discovered in 1914. The period covered by observations before the close encounter in 1927 was therefore 13 years.

2. The precision required for Academia's observations

Before the procedure of a mass determination was started, we estimated the necessary precision of Academia's observations. Assuming a density of 2.5 g cm^{-3} and a diameter of 443 km for Hygiea, we calculated the perturbations in mean longitude of Academia's orbit exerted by Hygiea over the period 1914–1978. Since Academia's diameter and subsequently its mass is some orders of magnitude less than Hygiea's mass, the perturbations of Academia exerted on Hygiea's orbit were neglected.

The estimation procedure was the following: Academia's orbit was integrated numerically backwards from 1978 to 1914 taking into account the planetary perturbations but not Hygiea's perturbations. This orbit is called the unperturbed orbit, unperturbed by Hygiea of course. A second run yielded the perturbed orbit of Academia, perturbed by Hygiea.

A comparison between corresponding mean longitudes of Academia's perturbed orbit L_p and unperturbed orbit L_u yields the precision estimation. We consider the difference $\Delta L(t) = L_u(t) - L_p(t)$, where t is the time. It is well known from perturbation theory that ΔL can be divided up in a linear and in a non-linear part.

The linear part of ΔL would disappear in an orbital improvement procedure by a suitable change of Academia's mean motion. The mean longitude is a linear function of the mean motion. It is therefore the non-linear part of ΔL which contains information about the necessary precision of observations. This non-linear part can disappear only by using a suitable value for Hygiea's mass. The linear part is represented by the function $\Delta L_l(t) = t^*(\Delta L(t=1914) - \Delta L(t=1979))/(1979-1914)$. The non-linear part is then obtained by $\Delta L_n(t) = \Delta L(t) - \Delta L_l(t)$.

The maximal value |MAX| of the non-linear part ΔL_n is the crucial value we are looking for. |MAX| should be larger than the estimated error of an observation. In the case of Academia, we found a value of |MAX| = 4″.9 which is large as compared to an estimated error of 1″ in Academia's "good" observations. Also varying Hygiea's mass within a reasonable interval yielded values of |MAX| larger than 4″. We, therefore, tried to determine Hygiea's mass.

3. Observations

According to the files of the Astronomisches Rechen-Institut, 72 observations of Academia are published. After a preliminary

Table 3. Observations of (829) Academia used for the mass determination

N	DATE (UT)	R. A.	Decl.	Equ.	Place	B
1	1914 2 28.83141	09 02 10.41	+23 53 48.9	1950.0	HEIDELBERG	
2	1916 9 2.93440	22 48 22.10	-11 13 29.4	1950.0	HEIDELBERG	S
3	1916 9 03.01089	22 48 17.39	-11 13 39.4	1950.0	HEIDELBERG	R
4	1916 9 17.83115	22 34 14.40	-11 30 10.0	1950.0	VIENNA	R
5	1916 9 22.84498	22 30 08.68	-11 30 40.9	1950.0	VIENNA	R
6	1916 9 22.87635	22 30 07.39	-11 30 40.1	1950.0	HEIDELBERG	R
7	1916 9 22.91346	22 30 05.54	-11 30 40.6	1950.0	VIENNA	R
8	1916 9 23.87536	22 29 22.29	-11 30 22.6	1950.0	HEIDELBERG	R
9	1916 9 24.87615	22 28 38.15	-11 30 02.0	1950.0	VIENNA	R
10	1916 9 26.88524	22 27 14.12	-11 28 48.7	1950.0	VIENNA	R
11	1916 9 27.86468	22 26 35.44	-11 28 04.3	1950.0	BERGEDORF	K
12	1916 9 27.94795	22 26 32.06	-11 27 57.3	1950.0	BERGEDORF	K
13	1916 10 01.82589	22 24 13.87	-11 23 37.4	1950.0	VIENNA	R
14	1916 10 17.80638	22 19 17.02	-10 44 31.5	1950.0	VIENNA	S
15	1916 10 19.79499	22 19 12.00	-10 37 18.6	1950.0	BERGEDORF	K
16	1916 10 22.77323	22 19 17.50	-10 25 38.9	1950.0	BERGEDORF	K
17	1916 10 30.76572	22 20 48.09	-09 48 58.9	1950.0	VIENNA	R
	1927 19 5	Close approach with (10)HYGIEA				
		Minimum distance 0.006 AU				
		Relative velocity 3.2 KM/S				
18	1929 11 22.80101	03 49 40.91	+33 40 15.6	1950.0	HEIDELBERG	
19	1929 12 5.77188	03 35 34.43	+33 04 16.3	1950.0	HEIDELBERG	S
20	1931 4 20.92772	11 25 33.88	+01 04 12.2	1950.0	HEIDELBERG	S
21	1953 7 6.94236	20 44 32.59	-29 25 03.1	1950.0	JOHANNESBURG	
22	1956 4 11.59806	12 21 14.36	-07 07 05.7	1950.0	NANKING	
23	1957 8 4.23227	18 45 41.08	-34 15 41.9	1950.0	LA PLATA	
24	1958 12 11.20485	03 23 52.00	+32 17 12.0	1950.0	FLAGSTAFF	
25	1960 3 25.15715	11 41 38.22	+00 26 16.3	1950.0	LA PLATA	
26	1960 3 25.18901	11 41 36.40	+00 26 22.2	1950.0	LA PLATA	
27	1961 6 12.96527	18 10 08.67	-36 31 33.0	1950.0	HARTBEESPOORT	
28	1966 9 19.00972	01 16 30.88	+12 57 49.6	1950.0	CRIMEA	
29	1966 9 21.07739	01 14 57.38	+12 59 37.0	1950.0	UCCLE	
30	1966 10 18.86525	00 48 43.37	+12 28 52.8	1950.0	CRIMEA	
31	1969 5 8.86249	16 01 28.32	-32 35 19.7	1950.0	HARTBEESPOORT	
32	1969 5 12.22688	15 58 06.01	-32 35 13.5	1950.0	CERRO EL ROBLE	
33	1973 5 3.87568	14 55 51.50	-27 46 35.2	1950.0	HARTBEESPOORT	
34	1974 8 22.94431	22 47 55.00	-12 18 16.9	1950.0	CRIMEA	
35	1977 4 21.85138	14 03 07.85	-21 43 23.9	1950.0	HARTBEESPOORT	
36	1978 7 10.70863	21 58 00.85	-21 36 42.4	1950.0	CANBERRA	
37	1978 7 10.72387	21 58 00.44	-21 36 44.7	1950.0	CANBERRA	
38	1978 7 25.63859	21 48 55.36	-22 13 16.0	1950.0	CANBERRA	
39	1978 7 25.65036	21 48 54.79	-22 13 17.5	1950.0	CANBERRA	
40	1978 7 28.66641	21 46 25.26	-22 21 08.5	1950.0	CANBERRA	
41	1978 7 28.67819	21 46 24.60	-22 21 10.9	1950.0	CANBERRA	
42	1978 7 30.69961	21 44 38.38	-22 26 20.1	1950.0	CANBERRA	
43	1978 7 30.71152	21 44 37.73	-22 26 21.7	1950.0	CANBERRA	

Remarks: K means observation was remeasured/reduced by L.Kohoutek
R " S.Roeser
S " L.Schmadel

orbital improvement without Hygiea's perturbations, we had to remove about 40% of this sample of 72 observations since the O – C exceeded 8″. This value of 8″ is a rather empirical limiting value. There remained 43 accurate observations which are shown in Table 3.

According to Table 3, 18 of the 43 observations lie before the close approach between Hygiea and Academia in 1927. Since the resulting mass and mean error depend very strongly on these early observations, the photographic plates taken in Heidelberg and in Bergedorf were remeasured by Schmadel and Kohoutek, respectively. In addition, the early Heidelberg plates were remeasured independently by West at ESO in Munich. A special enhancement technique developed at ESO for photographic plates was applied in order to determine the length of the tracks.

Many of the early observations of Academia, in particular the visual Vienna observations were reduced to the FK4 system by Röser. This reduction was carried out by reducing the given reference stars of each observation to the FK4 system with respect to position and proper motion.

The reference stars were identified with stars compiled in a special astrometric catalogue at the Astronomisches Rechen-Institut (Bastian and Lederle, 1985). This star catalogue contains stars of major astrometric catalogues like for instance AGK3, AGK3R, GC, and SAO reduced to the FK4 system. This reduction is based on the application of known systematic differences between a star catalogue and the FK4 (e.g. Schwan 1985).

After the reduction of the reference stars of an observation to the FK4 system, a new observed position was calculated using the given relative differences between the observation and the reference stars.

Table 4. Orbital elements of (10) Hygiea and (829) Academia at JD 2446000.5

Orbital element	Hygiea	Academia
Mean anomaly	72.47724	114.50017
Semimajor axis	3.13601 AU	2.57980 AU
Eccentricity	0.12034	0.09950
Argument of perihelion	317.00937	41.09434
Longitude of asc. node	283.07223	352.24601
Inclination	3.83985	8.31319

Table 5. Variation of elements Δe_i and results λ_i, $i=1,\ldots,7$ with mean errors. The λ_i are in units of the variations Δe_i

i	Element e_i	Δe_i	λ_i
1	$e \cos \tilde{\omega}$	10^{-5}	$+0.12 \pm 0.10$
2	$e \sin \tilde{\omega}$	10^{-5}	$+0.06 \pm 0.07$
3	$\sin i \cos \Omega$	10^{-5}	-0.01 ± 0.14
4	$\sin i \sin \Omega$	10^{-5}	0.00 ± 0.00
5	Mean anomaly (deg)	$-5\ 10^{-4}$	-0.64 ± 0.90
6	Semimajor axis (AU)	10^{-6}	-0.19 ± 0.10
7	Mass of Hygiea ($M_\odot$)	$5\ 10^{-11}$	$+0.93 \pm 0.45$

e, $\tilde{\omega}$, i, Ω denote eccentricity, longitude of perihelion, inclination, longitude of the ascending node respectively. $M_\odot$ means solar mass

4. Calculations

The numerical integration was based on the N-Body Program developed by Schubart and P. Stumpff (1966). This method is a purely predictor method of Adam's type. We included all the planets except Pluto. The orbits of the planets were not integrated numerically but their coordinates were taken from a magnetic tape provided by Newhall et al. (1983) from Jet Propulsion Laboratory known as the Development Ephemeris 102.

For the computation of the runs including Hygiea's mass, a different method was used since the program of Schubart and Stumpff does not change step size. Such a change is necessary during the close approach between Hygiea and Academia. The code DVDQ written by F. Krogh was applied which is based on a predictor-corrector method with variable order and variable step size. Again, the planetary coordinates were taken from the Development Ephemeris 102.

In a first step, Academia's and Hygiea's orbital elements taken from the Leningrad Ephemerides (1981) were improved. For this preliminary orbital improvement, no gravitational interaction between Hygiea and Academia was taken into account. The resulting orbital elements were published by Schmadel (1982). Table 4 shows the elements we used for the epoch JD 2446000.5.

In a second step, Academia's orbital elements and an assumed mass for Hygiea were simultaneously improved. In the following we will briefly describe the procedure: We denote by $e_1, \ldots. e_6$ the six orbital elements of Academia at an epoch t_0. The assumed mass of Hygiea is denoted by e_7. Integrating numerically Newton's equations of motion, we obtain for time t_i a calculated position $C(t_i)$ of Academia in right ascension α and declination δ. Denoting by $V(e_1, \ldots, e_7, t_i)$ the solution of the equations of motion at time t_i we can formally write

$$C(t_i) = V(e_1, \ldots, e_7, t_i).$$

At time t_i we assume to have an observation $\mathrm{O}(t_i)$ which is of the form

$$\mathrm{O}(t_i) = V(\bar{e}_1, \ldots, \bar{e}_7, t_i) + \varepsilon(t_i),$$

where $\bar{e}_1, \ldots, \bar{e}_6$ are the "true" orbital elements of Academia and where $\bar{e}_7$ is the "true" mass of Hygiea. $\varepsilon(t_i)$ represents the error in the observation $\mathrm{O}(t_i)$. The elements e_j and $\bar{e}_j, j = 1 \ldots 7$, are related by

$$\bar{e}_j = e_j + \lambda_j \cdot \Delta e_j, \qquad j = 1 \ldots 7.$$

The λ_j are unknowns in units of variations Δe_j. Developing $V(\ldots, e_j + \lambda_j \Delta e_j, \ldots)$ in a first order Taylor series, one obtains

$$\mathrm{O}(t_i) - \mathrm{C}(t_i) = \sum_{j=1}^{7} \lambda_j \Delta e_j \frac{\partial V(e_1, \ldots, e_7, t_i)}{\partial e_j} + \varepsilon(t_i).$$

A respective partial derivative is obtained numerically by calculating the difference between the variational orbit $V(e_1, \ldots, e_j + \Delta e_j, \ldots, t_i)$ and the reference orbit $V(e_1, \ldots, e_j, \ldots, e_7, t_i)$.

Assuming a Gaussian distribution of the errors $\varepsilon(t_i)$, the linear system of equations is solved by least squares fit.

In a third step, the improved elements $e_j + \lambda_j \Delta e_j$ are used in order to determine the new reference orbit. This new reference orbit again is improved by the same procedure.

This iterative method for mass determination is stopped if the new resulting improvements are smaller than their corresponding mean errors. Our procedure is based on an algorithm applied already earlier (see for instance Zech, 1968).

It is well known that the application of the least squares method classically requires in our case the inversion of a 7×7 matrix which might be troublesome in case the matrix is ill-conditioned. A symmetric matrix is ill-conditioned if the ratio R between the largest and the smallest eigenvalue exceeds a certain value. This value is an empirical value and is assumed to be of the order of 10^2 (Duma et al., 1980). In our cases, R never exceeded 10^2. The application of singular value methods for ill-conditioned matrices (e.g. Duma et al., 1980; Laubscher, 1976) did not affect our results.

Another difficulty may arise from large correlation coefficients. Particularly large correlation coefficients appear between the semimajor axis of Academia's orbit and the variable representing the improvement of Hygiea's mass. The correlation coefficients for our solution are given below.

5. The resulting mass for Academia

After an another improvement for Academia's orbit we applied the mass determination procedure described in Sect 3. For the variations of eccentricity, longitude of perihelion, longitude of ascending node and inclination, we used suitable combinations between these variables. Table 5 shows the variations Δe_i, $i = 1, \ldots, 7$, which we used and the results for the corresponding unknowns λ_i with mean errors.

As expected, the orbital elements of Academia vary only little except the semimajor axis.

Unfortunately, the resulting value for Hygiea's mass λ_7 has a mean error of about 50% though we have made a great effort improving the early observations. The number of only 43

Table 6. Correlation coefficients resulting from the least squares solution for Hygiea's mass

Element number							
1	2	3	4	5	6	7	
1.00	0.17	−0.03	−0.01	−0.75	−0.64	0.60	1
	1.00	0.01	0.01	0.50	−0.30	0.40	2
		1.00	−0.27	0.03	0.01	−0.02	3
			1.00	0.01	0.00	−0.01	4
				1.00	0.48	−0.34	5
					1.00	−0.94	6
						1.00	7

observations may be too small and the distribution of the observations along Academia's orbit may not be optimal. We did not make more detailed calculations about this latter point since it would not improve our result. Also our error estimation of Sect. 2 may have been too optimistic. On the other hand, the close approach between Academia and Hygiea was the very best event obtained in the systematic search for new mass determinations.

As an additional test run, we included in the mass determination procedure also the perturbations by Ceres, Pallas and Vesta due to their masses given in Table 1. We obtained almost the same value for Hygiea's mass.

Also a singular value analysis did not improve our result.

The correlation matrix (Table 6) shows a very strong correlation of −0.97 between the mass of Hygiea and Academia's semimajor axis. This might be another reason for the large mean error we obtained for Hygiea's mass.

Table 7. O−C for Academia before and after the mass determination of Hygiea

		Right ascension		Declination	
N	JD (UT)	DA1 (")	DA2(")	DD1(")	DD2(")
1	2420192.33141	0.18	-1.33	-1.09	-0.47
2	2421109.43439	2.33	2.20	2.55	2.49
3	2421109.51089	1.86	1.73	0.31	0.26
4	2421124.33115	-1.78	-1.91	-1.18	-1.23
5	2421129.34498	-1.67	-1.79	-0.63	-0.69
6	2421129.37635	0.83	0.71	-0.13	-0.19
7	2421129.41346	0.15	0.02	-1.14	-1.20
8	2421130.37536	2.79	2.67	1.42	1.36
9	2421131.37615	0.72	0.60	-1.65	-1.71
10	2421133.38524	0.12	0.00	0.79	0.73
11	2421134.36468	0.68	0.56	-0.63	-0.69
12	2421134.44795	0.30	0.18	2.06	2.00
13	2421138.32589	-1.08	-1.19	1.03	0.97
14	2421154.30638	-3.22	-3.33	-1.25	-1.31
15	2421156.29499	-2.10	-2.21	0.93	0.87
16	2421159.27323	-1.14	-1.24	-0.08	-0.14
17	2421167.26572	2.47	2.37	0.11	0.05
18	2425938.30101	-0.15	1.24	1.48	1.83
19	2425951.27187	-5.53	-4.19	2.43	2.83
20	2426452.42772	-3.65	-2.99	-1.64	-2.07
21	2434565.44236	-0.47	0.70	0.03	0.48
22	2435575.09806	1.20	1.20	-2.13	-2.15
23	2436054.73227	-0.79	0.20	-1.47	-1.38
24	2436548.70485	2.41	2.04	-4.81	-4.97
25	2437018.65715	-0.78	-1.06	-1.03	-0.86
26	2437018.68901	-0.74	-1.02	-1.13	-0.96
27	2437463.46527	-0.76	0.00	2.94	2.94
28	2439387.50972	1.93	1.74	-0.39	-0.50
29	2439389.57739	-0.03	-0.23	-1.02	-1.13
30	2439417.36525	2.83	2.65	0.84	0.69
31	2440350.36249	2.50	2.64	-0.08	-0.15
32	2440353.72688	-1.08	-0.94	-0.89	-0.96
33	2441806.37568	1.04	0.87	0.52	0.54
34	2442282.44431	3.33	3.24	-2.58	-2.63
35	2443255.35138	0.49	0.02	0.14	0.34
36	2443700.20863	0.06	-0.03	-0.23	-0.28
37	2443700.22387	-0.03	-0.12	-0.63	-0.68
38	2443715.13859	-0.83	-0.94	-0.32	-0.38
39	2443715.15036	-0.62	-0.73	0.03	-0.02
40	2443718.16641	-1.09	-1.19	-0.26	-0.31
41	2443718.17819	-1.50	-1.60	-0.86	-0.91
42	2443720.19961	-0.33	-0.44	-1.12	-1.18
43	2443720.21152	-0.16	-0.27	-0.98	-1.04

Meaning of symbols in Table 7 :
N : Running number of observations like in Table 3
DA1 : O-C in right ascension before mass determination
DA2 : O-C in right ascension after mass determination
DD1 : O-C in declination before mass determination
DD2 : O-C in declination after mass determination

Table 7 shows the shift in the O−C's for Academia's observations without and with Hygiea's perturbations. Due to the large correlation coefficient mentioned above and due to the large mean error for Hygiea's mass, this shift is not very striking.

6. Conclusions

Analyzing 43 available observations of Academia, we obtain a value of $4.7 \pm 2.3\ 10^{11}\ M_{\odot}$ for Hygiea's mass by a Gaussian least squares fit. This is the fourth determination of an asteroidal mass after Ceres, Pallas and Vesta. It is also the smallest asteroidal mass ever determined. The mass of Ceres is ten times, the masses of Pallas and Vesta five times larger.

The comparatively large error of about 50% for Hygiea's mass shows the limitations for future asteroidal mass determinations. According to our systematic search for new mass determinations, we doubt that in the near future better values for asteroidal masses can be obtained using the classical method of perturbations on the orbit of smaller asteroid.

In spite of the large mean error for Hygiea's mass and additional uncertainties in its diameter, we also calculate a value for the mean density, which of course is a more physically relevant quantity than mass. Assuming a diameter of 443 km, we obtain a mean density of 2.05 g cm^{-3} with an error of 50% only due to the error in the mass. This again shows the limitations of the application of classical perturbation methods in order to derive mean densities for more asteroids.

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