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Review of galactic constants

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Summary. Whereas the 1964 IAU standard values of the constants $R_0=10$ kpc, $\theta_0=250\,\mathrm{km\,s^{-1}}$, $A=15\,\mathrm{km\,s^{-1}\,kpc^{-1}}$, and $B=-10\,\mathrm{km\,s^{-1}\,kpc^{-1}}$, differ from those currently considered best, the errors of current determinations are still rather large. Although the means of recent determinations lead to a fairly consistent set of values for the four main constants, the agreement may be largely coincidental. However, a case can be made for agreeing on the pair of standard values $R_0=8.5$ kpc and $\theta_0=220\,\mathrm{km\,s^{-1}}$. On the basis of this review these were adopted by resolution of IAU Commission 33 on 1985 November 21 at Delhi.

1 Introduction and definitions

At the General Assembly of the IAU in Patras, Commission 33 set up a working group to consider the current values of the galactic constants and publish a review. This document by the Chairman and Vice-Chairman of the working party owes much to the submissions of others detailed in the acknowledgments. A summary of recommendations by Commission 33 at Delhi that resulted from the report is published in *Highlights of Astronomy* and the recommendations themselves in Transactions of the IAU. These include the recommendation that where standardization on a common set of galactic parameters is desirable the values

$R_0=8.5$ kpc

$\theta_0=220\,\mathrm{km\,s^{-1}}$

should be used; these imply $A-B=25.9\,\mathrm{km\,s^{-1}\,kpc^{-1}}$.

This review first gives definitions of the quantities, followed by comments concerning the presuppositions implied by the definitions. These comments are aimed at extending the definitions to a galaxy which shows departures from axial symmetry, etc.

Secondly, different determinations are listed and their inconsistencies and inadequacies are briefly discussed.

1.1 R_0

Definition: R_0 is the distance of the Sun from the Galactic Centre.
Comment: Currently it is *assumed* that the Galactic Centre coincides sufficiently well with the Galaxy's barycentre that a distinction between the point of greatest star density (or any other central singularity) and the barycentre (centre of mass) is not necessary. It is also *assumed* that to sufficient accuracy for the internal dynamics of the Galaxy the Galactic Centre defines an inertial coordinate system. These assumptions could prove to be untrue if for instance the centre of the distribution of mysterious mass in the heavy halo were displaced from the mass centre of the visible Galaxy.

1.2 CIRCULAR VELOCITY

Definition: $\theta(R)$ is the velocity of an object moving in a circle of radius R about the Galactic Centre with centrifugal force balancing the Galaxy's gravity. The circle should be taken in the galactic plane.
Comment: The definition assumes an idealized axially symmetrical galaxy. It follows that

$$\theta^2 = -\mathbf{R}\cdot\mathbf{K} = -RK_R \tag{1}$$

where $\mathbf{K}$ is the acceleration due to the Galaxy's gravity. For a galaxy lacking axial symmetry, $\theta(R)$ may be defined by averaging equation (1) around a circle and over a small range of radii to obtain a regional average. Thus, only the axially symmetrical Fourier component of K_R is needed in the definition of θ. The circular velocity at the Sun $\theta_0 = \theta(R_0)$ is also commonly called V_c. The tangential velocity increases outwards across a spiral arm. This has been known in our Galaxy for some time (e.g. Georgelin 1967), but is best illustrated in external galaxies.

1.3 OORT'S CONSTANTS

These are defined in terms of the global rotation law $\theta(R)$:

$$A = -\tfrac{1}{2}R\, d/dR\,(\theta/R) = -\tfrac{1}{2}R\, d\omega/dR,$$

$$B = -\tfrac{1}{2}R^{-1}\, d/dR\,(R\theta) = -\tfrac{1}{2}R^{-1}\, dh/dR.$$

These constants are normally evaluated at $R = R_0$, but we notice that $A(R)$ and $B(R)$ may be defined (though not easily measured) as functions of R by the same formulae evaluated at other radii. $A(R)$ is the rate of shearing of circular motion; $2B(R)$ is the vorticity of a fluid taking part in the circular motion. The angular velocity of galactic rotation is

$$\omega(R) = \theta(R)/R = A(R) - B(R),$$

or at the Sun

$$\omega_0 = \omega(R_0) = A - B.$$

The specific angular momentum of the circular motion at R is

$$h(R) = R\theta(R).$$

The epicyclic frequency $\varkappa(R)$ is defined so that the period of radial motion in the near-circular galactic orbit of a star is $2\pi/\varkappa$.

$$\varkappa^2 = 4\,|B(R)|\,\omega(R) = 4B(R)[B(R) - A(R)].$$

Comment: In a non-axially symmetrical model, values of Oort's may be determined by taking angular averages around the Galaxy. In practice, the presence of spiral arms can give a wavy local-velocity curve with substantial local gradients which corrupt locally measured values. Fricke (1967, 1977) early separated his investigations into those on very nearby stars and those on more distant ones. Clube (1972, 1973), by exploring the full rate of deformation tensor, has shown that observed local proper motions are not adequately described by the Oort–Lindblad differential-rotation model. He finds that a significant radial compression rate is also present. Finally he points out that many of the results obtained from proper motions within 300 pc or so are seriously affected by the recent star formation that gave rise to Gould's belt of bright stars which have their own systematic motions superposed on normal galactic rotation.

Both the radial compression rate and the sense of the deviation of the velocity ellipsoid from the Galactic Centre can be understood, following Kalnajs (unpublished work), in terms of the density-wave picture of spiral structure. We assume that the displacement velocities of stars are on average those of their epicentres. A radial compression displaces stars of negative V (laggards) whose epicentres lie nearer the Galactic Centre towards smaller U, and stars of positive V whose epicentres are outside the solar circle towards larger U. (In this convention, U and V are the velocity components measured inwards and in the direction of rotation, respectively.) The tilt of the velocity ellipsoid results. An increasing density in the solar neighbourhood would result from us starting to catch up with a spiral density-wave enhancement (or from such an enhancement catching up on us). A more detailed treatment of spiral density-wave effects on the velocity ellipsoid has been given by Mayor (1970).

With the above complications in mind, it is important to state the range of distances of the stars used in any determination of the Oort constants. The relation between local and global values of the constants will be discussed later in this review.

1.4 THE SOLAR MOTION

The solar motion has been determined by numerous authors with respect to a great range of stellar and interstellar constituents of the Galaxy, and there is an extensive literature on the subject. In the present context, we are most concerned with the velocity of the Sun with respect to the Local Standard of Rest (LSR) which by definition moves around the Galaxy with the circular velocity θ_0.

For over 30 years the so-called 'Standard Solar Motion' has been used, for the solar motion with respect to the LSR. This is a velocity of 20 km s^{-1}, in the direction $\alpha = 18^h$, $\delta = +30°$ (1900).★ There is no clear evidence that this value should be changed, although several authors have discussed the possibility that an additional outward component of 5–7 km s^{-1} should be allowed for (e.g. Kerr 1962; Yuan 1983; Shuter 1982). The precise determination of the Local Standard of Rest is a complex subject, involving possible spiral-arm effects on the local kinematics, local peculiar motions, and non-uniformities of the distribution of the constituent under investigation (see Mihalas & Routly, 1968).

1.5 K_z

Definition: K_z is the component of the Galaxy's gravitational acceleration towards the galactic plane.

Comment: If the Galaxy is assumed to be symmetrical and approximately plane stratified then K_z at any point is proportional to the total surface density of mass closer to the plane than that point. Thus under these assumptions K_z increases monotonically with $|z|$. However, the plane-stratified

★In galactic Cartesian components $X = r\cos l\cos b$, etc. $\dot{X} = U = 10.0$, $\dot{Y} = V = 15.4$, $\dot{Z} = W = 7.8$ km s^{-1}.

approximation breaks down above a few kpc from the plane and $K_z \to 0$ at ∞. For a steady-state axially symmetrical stellar system Jeans's stellar hydrodynamical equations give

$$-K_z = \frac{1}{R}\langle v_R v_z\rangle + \frac{1}{n}\frac{\partial}{\partial R}\langle n v_R v_z\rangle + \langle v_z^2\rangle \frac{\partial \ln \langle n v_z^2\rangle}{\partial z}$$

where v_R and v_z are velocity components and $n(\mathbf{r})$ is the number density of the type of star used. All analyses to date have assumed that the $\langle v_R v_z\rangle$ terms, which are zero on the plane of symmetry, are also small elsewhere.

Observational determinations of $K_z(z)$ require the isolation of a suitable tracer stellar population whose number and velocities can be determined as functions of height. Most analyses have modelled the distribution as isothermal or as a sum of isothermals. All determinations that are not further constrained give an unphysical force law with a maximum in K_z only a few hundred pc above the plane and a negative mass density is implied. If, however, the implied density is *forced* to be positive, one can show that the data are not inconsistent with the theoretical form that arises from the density of a sum of several isothermal components with K_z monotonic in z to much greater heights (Oort 1960; Bahcall 1984).

1.6 ϱ_0

Definition: ϱ_0 is the mass density on the galactic plane in the neighbourhood of the Sun.
Comment: Since only a fraction of the material has been seen, ϱ_0 can only be determined from Poisson's equation

$$4\pi G\varrho_0 = -\nabla^2\psi.$$

For axial symmetry this reduces to

$$4\pi G\varrho_0 = -\frac{\partial^2\psi}{\partial z^2} + \frac{2V_c}{R}\frac{\partial V_c}{\partial R} = \frac{\partial K_z}{\partial z} - 2(A^2 - B^2).$$

The $A^2 - B^2$ term is zero for V_c=constant and is ~3 per cent of the total for the values of A and B quoted above. The $\partial K_z/\partial z$ term involves the second derivative of the stellar space density, consequently both ϱ_0 and K_z are critically dependent on the distances of the stars used. Systematic errors in these dominate the problems of all determinations to date. A 0.6 mag error in absolute magnitude gives ϱ_0 errors of 40 per cent. The total mass below 500 pc can be determined somewhat more accurately.

2 R_0 the distance to the Galactic Centre

Three direct methods have been much exploited:

(i) Shapley's method of finding the centre of the system of globular clusters. This method is limited by the relatively small number of globular clusters but is helped by their concentration towards the Galactic Centre. The most concentrated component consists of less metal-poor clusters with no RR Lyrae stars so a good distance estimator to these is required. Differences between investigators depend importantly on distance estimates to these metal-richer clusters.

(ii) Baade's method of finding holes in the galactic obscuration and determining the peak in the number of RR Lyrae stars as a function of magnitude. The chief difficulties arise from the large and variable extinction. It is assumed that the RR Lyraes seen near the centre have the same magnitude as calibration stars.

(iii) Mira variables in galactic windows. This method depends on the period–luminosity relationship for the Mira variables used to determine the magnitudes and the strong peak in the density of Mira variables seen towards the Galactic Centre. Use of infrared magnitudes and colours considerably alleviates the extinction problems of Baade's method.

2.1 SHAPLEY'S METHOD

Harris (1976) collected data on 111 globular clusters and determined their distances using a horizontal branch $M_V=0.6$. Using 106 clusters within 40 kpc of the centre he obtained

$$\langle X\rangle=7.28\pm0.57\,\text{kpc}; \qquad \langle Y\rangle=0.40\pm0.58\,\text{kpc}; \qquad \langle Z\rangle=0.26\pm0.61\,\text{kpc}.$$

After allowance for observational selection favouring nearby clusters above those beyond the centre and close to the galactic plane, he deduced $R_0=8.5\pm1.6$. The distribution of the globular clusters in X, Y and Z (the distance components from the Sun) is very non-Gaussian and a few distant globulars can seriously affect the mean. This is the reasoning behind Harris's limitation to galactocentric distances less than 40 kpc. However, as Frenk & White (1982) pointed out, the median gives a much better estimate for a centrally concentrated distribution with a long tail even when points in the tail can seriously affect the mean. The median X of Harris's data is only 5.8 kpc. This will be affected by the observational bias but even if we restrict consideration to those of his clusters with $|Z|>2$ kpc, this median only increases to 6.0 kpc and the median X of the 38 clusters with $|Z|>4$ kpc is only 4.8 kpc. Frenk & White find medians of 5.63 for their metal-poor sample, 8.56 for their metal-richer subset and 6.21 for the whole set. After discussion of the observational selection they deduce $R_0=6.8\pm0.8$ kpc on the scale on which the low-metallicity clusters have $M_V(\text{HB})=0.6$. They suggest that $M_V(\text{HB})$ for the higher metallicity clusters must be shifted by half a magnitude to 1.1 in order to bring their two groups into agreement. Such a shift is about twice that given by Sandage's empirical relationship $\Delta M_V(\text{HB})=0.35\,\Delta(\text{Fe/H})$. Harris in a re-discussion of his data uses $M_V(\text{HB})=0.9$ for the metal-rich clusters but, again using means, deduces $R_0=8.5\pm1.0$.

In summary, means are a less accurate method than medians for this sort of data and the overall result from medians is best described as $R_0=7\pm1$ kpc on the basis of $M_V(\text{HB})=0.6$.

2.2 BAADE'S METHOD

The results of the massive Palomar–Groningen survey of RR Lyrae stars carried out by Plaut have been given their definitive reduction by Oort & Plaut (1975). Originally it had been hoped to measure the extinction both from the reddenings and from counts of background galaxies, but the results from galaxy counts were too inconsistent. The major problem of measuring the extinction accurately can only be alleviated by working further into the infrared. In this very crowded region it is likely that this must await the development of infrared panoramic detectors.

Table 1. Weighted average 8.7 kpc (assumes $M_{pg}=0.70$, $\langle M_V\rangle=0.63$ for RR Lyrae stars).

b^0	R_0/kpc	$\langle A_{pg}\rangle$	Wt
−3.9	9.5	1.80	1
−8	8.41	0.73	1
−12	8.41	0.49	2/3
+14	8.81	0.85	2/3
+29	7.3	0.6	1/5

The apparent correlation between the R_0 deduced and the extinction found is solely due to the high values for Baade's field $b=-3.9$.* However, this serves to illustrate that correct allowance for extinction is the primary difficulty with this method.

In both the above methods the difficulty in calibration of the absolute magnitude of the RR Lyrae stars remains. Clube & Jones 1971 and Clube & Dawe 1980 have emphasized that their statistical parallax disagrees with their secular parallax.

Blanco & Blanco (1986) have recently reported a new study of RR Lyraes in Baade's Window. They point out that the photometric sequence used by Oort & Plaut is now considered to have unusually large errors, and say that their new study does what Oort & Plaut would do if they were repeating their project now. Blanco & Blanco present two alternative results. First, a conservative assumption about RR Lyrae absolute magnitudes yields $R_0=7.95\pm0.69$ kpc. In the second case, they allow for a relationship between the luminosity and metallicity of RR Lyraes, which some recent evidence seems to imply. In that case, $R_0=6.94\pm0.58$ kpc. They conclude that 'a reasonable 1984 value for R_0 as determined from RR Lyrae data may be about 7.3 ± 0.5 kpc', taking into account the uncertainties and recent trends.

2.3 MIRA VARIABLES

Glass & Feast (1982) find a clear peak in the number density of these variables in Baade's Galactic Centre windows, Sgr I and NGC 6522. Their results are from infrared photometry, so are relatively insensitive to absorption as Table 2 shows. However, the result is sensitive to the calibration of the zero-point of the period–luminosity relationship for Miras. This relationship is best determined in the Large Magellanic Cloud. At the time their paper was written, they used a true modulus of $m-M=18.69$. More recent work by Feast (1984) reduces this to $m-M=18.50$. Taking this into account in the last column, their table reads as seen below.

Table 2.

$\langle A_V \rangle$	$\langle A_B \rangle$	Galactic calibration R_0	Old LMC calibration R_0	New LMC calibration R_0
2.03	2.71	7.9	8.8	8.1
1.37	1.83	8.4	9.3	8.5
1.5	2.00	8.3	9.2	8.4

Thus, current values from this method give $R_0=8.3\pm0.6$ kpc.

2.4 INDIRECT METHODS

The major difficulties with these is their assumption of a kinematic model of the Galaxy to relate the observed distances to R_0. In any particular region streaming motions associated with spiral arms can upset the velocities, seriously change a locally determined Oort constant A and so upset the kinematical model. However, Byl & Ovenden (see also Ovenden, Pryce & Shuter 1983) show that a more global fitting of OB star data is far less upset by local streaming.

Balona & Feast (1974) $R_0=9.0$ $7.7<R_0<10.9$
Byl & Ovenden (1978) $R_0=10.4\pm0.7$.

*Determinations of extinction in this field using infrared photometry of 22 Mira stars yields $\langle A_V \rangle=2.03$, $\langle A_{pg} \rangle 2.7$, see Glass & Feast (1982). Adopting such a value would reduce R_0 determined from this field to 6.2 kpc. van den Bergh's work in the visible agrees with the Oort & Plaut extinction.

The determination depends on the OB star calibration. A new method of ingenious direct determination of distances is the subject of a recent Leiden Thesis by J. Herman; his distances to OH masers associated with infrared variables are consistent with an R_0 of 9.2 kpc.

We shall leave determinations of R_0 from radio astronomical measurements of AR_0 and optical determinations of A until later.

Table 3 summarizes the results published in the last decade. In this paper, we have taken the straight means of the published determinations of each quantity, without attempting any weighting, because we believe that systematic errors are more important than the formal errors. In each case, we have added the standard deviation (S) of the individual determinations about their mean, as an indication of the spread of these values. We have also added parenthetically the standard error of the mean ($S/\sqrt{n}$), a number which should not receive too much attention because of the large systematic uncertainties. The errors quoted for individual determinations are the values quoted by the authors.

Table 3. R_0.

			(kpc)
1974	Balona & Feast	Kinematic	9.0±1.6
1974	van den Bergh & Herbst	Galactic bulge turn-off point	9.2±2.2
1974	Cruz-Gonzalez	Stars 25 pc	8.9±0.5
1974	Rybicki *et al.*	Galactic disc modelling	9.0
1975	Oort & Plaut	RR Lyraes in windows	8.7±0.6
1976	Harris	Globular cluster system	8.5±1.0
1976	Crampton *et al.*	Kinematic	8.4±1.0
1977	Belikov & Syrovoi	Globular cluster distances	8.5
1978	Byl & Ovenden	Kinematic	10.4±0.7
1978	de Vaucouleurs & Buta	Globular cluster system	7.0±0.7
1978	Sasaki & Ishizawa	Symmetry of distribution	9.4±1.2
1978	Clube & Watson	RR Lyraes	6.7
1980	Quiroga	H II regions	8.4
1980a	Surdin	Symmetry of metallicity (I)	10.3±0.6
1980b	Surdin	Symmetry of metallicity (II)	9.9±0.3
1980	Clube & Dawe	RR Lyraes	7.0
1981	Genzel *et al.*	H II regions	10.5±4.0
1982	Frenk & White	Globular cluster system	6.8±0.8
1982	Glass & Feast, revised	Miras in windows	8.3±0.6
1983	Knapp	H II regions	8.0±2.0
1983	Ostriker & Caldwell	Model solution	8.2
1985	Rohlfs (private communication)	H I	7.9±0.1
1983	Herman	OH masers	9.2
1986	Blanco & Blanco	RR Lyraes	7.3±0.5
1986	Walker & Mack	RR Lyraes	8.1±0.4
Straight mean and the standard deviation (standard error of the mean=0.22)			8.54±1.1

3 θ_0, circular velocity

Three recent compilations of determinations of θ_0 have been given by Knapp (1983), Einasto, Haud & Joeveer (1979), and de Vaucouleurs (1983). A wide variety of methods has been used, most of which are indirect to a greater or lesser degree. A common approach is to attempt to measure the motion of the LSR with respect to some group of objects whose motions are sufficiently different to that of the LSR that their average can be taken as zero or some other known value. This method has been especially applied to the globular cluster system and to generalized halo objects. If the method is used in relation to the Local Group of galaxies (e.g. Lynden-Bell & Lin 1977), implausibly high values of θ_0 are obtained, but these have very large probable errors.

Some other determinations have made use of an apparently good relation between the velocity dispersion and the rotational velocity which has been demonstrated for a number of other galaxies, for example in the globular cluster system of M31. This relationship has then been applied to the spheroid of our Galaxy, assuming the motions are isotropic (Einasto *et al.* 1979; Lynden-Bell & Frenk 1981).

Table 4 presents results of the determinations that have been made in the last decade. As before, we take a straight mean, because of the importance of systematic errors. We see that many of the values are in the region 220–230 km s^{-1}, but caution is needed in interpreting this result, because of the possibility that the Galaxy may not be axisymmetric.

We know for example that the extreme radial velocities observed for the H I emission in the outer parts of the galactic disc are different on the north and south sides by about 15 km s^{-1} (Jackson & Kerr 1981). This is directly significant because of the most-quoted recent determinations of θ_0 is that of Knapp, Tremaine & Gunn (1978), who obtained a value of 220 km s^{-1} from a study of H I at the distant edges of the galactic disc in the first and second quadrants. A corresponding study by Jackson (1985) for the third and fourth quadrants implied that a value of θ_0=250 km s^{-1} was a better interpretation in that case. It is not yet known whether the asymmetry in the outer parts is kinematic or spatial, but observational programs at present in progress should enable this to be determined. If it is kinematic, this could imply that galactic orbits in the disc are somewhat elliptical, rather than circular. If this description should apply in the solar vicinity, the quantity we are measuring will clearly be affected. As well as such a large-scale effect, there could also be a localized effect through the presence of a tangential

Table 4.

			(km s^{-1})
1974	Cruz-Gonzalez	Stars<25 pc	228±24
1974	Mathewson *et al.*	Magellanic Stream data	220
1974	Isobe	High-velocity stars	275
1975	Woltjer	Globular cluster motions	200–225
1978	Knapp *et al.*	H I at edges of disc	220
1978	Hartwick & Sargent	Dwarf spheroidals	220
1979	Einasto	Best fit of galactic constants	225±10
1979	Einasto *et al.*	Halo globular clusters	212
1979	Einasto *et al.*	Disc population objects	230
1979	Einasto *et al.*	Oort limiting velocity	225
1979	Einasto *et al.*	Companions of Galaxy	220
1980	Clube & Dawe	RR Lyraes	250
1980	Frenk & White	Globular clusters	200–225
1981	Lynden-Bell & Frenk	Globular cluster motions	212±16
1981	Shuter	H I and CO	184±9
1985	Jackson	H I at edges of disc	250
1983	Ostriker & Caldwell	Model solution	238
1984	Pier	Halo field stars	⩾212
1985	Rohlfs (private communication)	H I	184
1985	Alvarez	CO	214
Straight mean and the standard deviation (standard error of the mean=4.9)			222.2±20

velocity component arising from a spiral density wave, which could not be separated from the real circular velocity.

4 Oort's constants, *A* and *B*

The definitions of Oort's constants A and B have been given earlier. These constants can be determined from local kinematics, and should be regarded as local quantities. They are normally derived from the double sine wave of a velocity component with longitude; for radial velocities, the double sine wave is proportional to A; for proper motions its mean is offset by B (see Mihalas & Routly 1968). Thus A can be determined from either radial velocities or proper motions, and B from proper motions only.

To use proper motion data it is necessary to use stars distant enough that their random motions do not dominate their proper motions and to have sufficient sky coverage to allow the separation of the double sine curve of galactic differential rotation from the single sine curve due to the solar motion. The latter gives proper motions that decrease inversely with distance. For incomplete sky coverage approximate knowledge of distances is helpful in decoupling these effects.

Tables 5 and 6 give determinations from proper motions and radial velocities respectively. It is noticeable that $A-B$ is better determined from proper motions than either A or B. Perhaps this is because $A-B$ is the maximum proper motion due to galactic rotation achieved at longitudes=90° and 270°.

As mentioned earlier, we have not weighted the different measurements according to their formal errors because systematic errors dominate in these investigations. With the exception of the work by Tsioumis & Fricke, the results are dominated by stars at less than 0.8 kpc. Thus these are very locally determined values.

Table 5. A and B from proper motions.

	Author		$A/\mathrm{km\,s^{-1}\,kpc^{-1}}$	$B/\mathrm{km\,s^{-1}\,kpc^{-1}}$	$A-B/\mathrm{km\,s^{-1}\,kpc^{-1}}$
1928	Oort		(19)	(−24)	
1951	Morgan & Oort		20.0	−7.0	+27.0
1967	Fricke	FK4	14.2±1.9	−11.8±1.9	+26.0
	Fricke	N30	17.1±1.9	−10.0±1.9	+27.1
1968	Dieckvoss		14.8±0.7	−11.3±0.6	+26.1
1968	Filin		12.1	−12.0	+24.1
1971	Vasilevskis & Klemola	AGK3 rel galaxies	8.9±3.6	−12.5±4.0	+21.4
1974	Cruz-Gonzalez	Nearby stars	–	−10.7±2.3	–
1975	Fricke & Tsioumis	FK4	15.6±0.7	−11.4±2.8	+27.0
1977	Asteriadis	AGK3	16.1±1.9	−9.0±1.9	+25.1
1977	Fricke	4 catalogues	15.6±2.8	−10.9±2.8	+26.5
1977	du Mont	FK4	16.1±3.8	−12.7±2.8	+28.8
1977	Corbin	AGK3R	13.5±2.3	−12.1±2.2	+25.6
1978	du Mont	AGK3	13.7	−15.2	+28.9
1978	Dieckvoss	AGK3	13.4±0.7	−14.8±1.8	+28.2
1979	Tsioumis & Fricke	(stars 0.6–3 kpc)	13.0±3.8	−13.3±2.9	+26.3
1982	Dieckvoss	AGK3 1 kpc A–K stars	11.6	−18	+29.6
Straight mean±S.D.			14.4±2.7	−12.0±2.8	26.4±1.9
Standard error of the mean			±0.7	±0.7	±0.5
1964 IAU values			15	−10	25

Table 6. *A* from radial velocities.

			A/km s^{-1} kpc^{-1}
1961	Johnson & Svolopoulos	Galactic clusters	15.0±3.0
1963	Kraft & Schmidt	Cepheids	15.0±2.0
1965	Feast & Shuttleworth	OB stars	14.3±0.8
1968	Crampton	Be stars 1.5 kpc	11.8±1.9
1968	Crampton	Cepheids	13.5±1.9
1968	Petri & Petrie	OB stars, H	15.1±0.4
1968	Miller	H II regions	15.7±0.5
1968	Crampton & Fernie	Cepheids	12.5±2.0
1970	Humphreys	Supergiants	14.0
1970	Georgelin & Georgelin	H II regions	14.2±0.5
1970	Crézé	Recalibration of older investigations	15.5±1.5
1974	Balona & Feast	OB stars (H width recalibrated)	16.8±0.6
1975	Fricke & Tsioumis	FK4 stars	15.6±0.7
1976a, b	Barkhatova & Gerasimenko	Open clusters	15.0
1978	Cruz-Gonzalez & Arellano Ferro	O stars	13.0
1980	Clube & Dawe	RR Lyraes	16.0
1980	Blitz *et al.*	CO	13.3
1983	Blitz & Fich	CO	12.5 or 15.0
1985	Rohlfs (private communication)	H I	16.0±0.13
Straight mean and the standard deviation (standard error of the mean=±0.3).			14.5±1.3

5 AR_0

The product AR_0 can be determined directly from measurements of tangential-point velocities, the maximum velocities observed on a line-of-sight in the first or fourth galactic quadrant (see Mihalas & Routly 1968). The method has usually been applied to radio astronomical survey data, but it can also in principle be used with observations of distant stars. The maximum velocity is given by

$$V_\mathrm{M}=\theta(R_\mathrm{M})-\theta_0 \sin l,$$

where

$$R_\mathrm{M}=R_0 \sin l.$$

Differentiation gives

$$\frac{dV_\mathrm{M}}{d(\sin l)}=R_0\frac{d\theta(R)}{dR}-\theta_0.$$

Since A can be expressed as

$$A=\frac{1}{2}\left[\frac{\theta_0}{R_0}-\left(\frac{d\theta}{dR}\right)_{R_0}\right]$$

this leads to

$$AR_0=-\frac{1}{2}\left[\frac{dV_\mathrm{M}}{d(\sin l)}\right]_{R_0},$$

so that AR_0 can be determined directly from a plot of the tangential point velocities against $\sin l$.

This expression has been derived on the basis of circular motion. In fact; (i) the gas clouds making up the source region have a substantial velocity dispersion which may or may not be the same everywhere; (ii) there may be streaming motions related to the spiral structure which would mean that the maximum velocity gas in certain directions would be closer or further than the tangential point; (iii) there may be a lack of gas in parts of the tangential-point region. Some of these effects can be reduced through studying the whole run of V_M against $\sin l$ but they still leave uncertainties.

The earlier determinations used an alternative formula,

$$V_M = 2AR_0(1-|\sin l|)\sin l.$$

This is the first term of a Taylor series expansion, and is only accurate for R close to R_0 (i.e. $|l|$ close to 90°). Substantial errors occur when the method is used over a wider longitude range.

Table 7 summarizes the main results obtained to date. A straight mean of the whole set is 124 km s^{-1}, while a mean of the 'modern' results is 110 km s^{-1}. We may note that the product of the mean values for A and R_0 comes to 123.7 km s^{-1}.

Table 7. AR_0.

			(km s^{-1})
1957	Schmidt	H I, 1st quad	156
1958	Feast & Thackeray	OB, 4th	135
1959	Kerr *et al.*	H I, 4th	130
1974	Lohmann	H I, 1st	142.1
1974	Cruz-Gonzalez	stars	133±8
1979	Gunn *et al.*	H I, 1 and 4	110
1979	Einasto (Haud)	?	120
1983	Shuter	H I, 1 and 4	110
1983	Knapp (McCutcheon)	CO, 4	103±5
1985	Rohlfs (private communication)	H I, 1 and 4	106.5±2.5
Straight mean and the standard deviation (standard error of the mean=5.5)			124.5±17

6 Velocity-ellipsoid ratio

The shape of the velocity ellipsoid for particular groups of stars has long been used to provide additional information on the values of Oort constants. The velocity dispersions in u and v for some class of stars is related to A and B by the expression

$$\left(\frac{\sigma_V}{\sigma_u}\right)^2 = \frac{B}{B-A}.$$

Table 8 presents the values obtained by a number of authors for this ratio. This list is a representative one, and not as comprehensive as the others in this report.

It is noticeable that the two investigations based on different sets of giant stars give values close to 1/2. This is the value expected for a flat velocity curve $\theta(R)$=constant. Giants are seen at great distances so it might be thought that we sample conditions over a wider region by observing them. However, this is hardly true because the fainter giants studied by Woolley *et al.* were typically 300 pc distant while a typical epicyclic orbit of a dwarf will make it range over about 1 kpc.

7 Combinations of constants

A check on the reasonableness of the system can be obtained by looking at combinations of the

Table 8. Velocity-ellipsoid ratio squared.

			$(\sigma_v/\sigma u)^2 = B/B-A$
1965	Delhaye	Giants G–M	0.49
		Dwarfs F–M	0.36
1974	Wielen	317 dwarfs K–M	0.37
1974	Mayor	362 older F stars (10^9 yr)	0.41
1975	Erickson	870 Gliese stars	0.42
1977	Woolley *et al.*	588 G–K giants 300 pc	0.50
		760 dwarfs	0.38
1983	Oblak	230 (older F stars 10^9 yr)	0.40
Straight mean and the standard deviation (standard error of the mean $=\pm0.02$)			0.42 ± 0.06
1964 IAU values			0.40

constants. First, the product AR_0 can be separately determined, as was discussed earlier, and secondly, the set should obey the relationship $\theta_0=R_0(A-B)$.

7.1 AR_0

The direct determinations of the product AR_0 in recent years led to a mean value of $110\,\mathrm{km\,s^{-1}}$. The product of the mean values derived individually for A and R_0 is $123.7\,\mathrm{km\,s^{-1}}$. We notice that the product is derived from velocity measurements made over large distances, and therefore involves a global value of A, whereas the determinations of A individually all refer to a much more local situation.

7.2 $R_0(A-B)$

The product of the mean values derived earlier for R_0, A, and B is

$$R_0(A-B)=8.54\ (14.45+12.0)$$
$$=225.9\,\mathrm{km\,s^{-1}}.$$

This value is in surprisingly close agreement with the mean of the listed determinations of θ_0 which is $222.2\,\mathrm{km\,s^{-1}}$. Considering all the uncertainties involved, this agreement must be viewed as rather coincidental.

7.3 $B/(B-A)$

The table of determinations of the square of the velocity-ellipsoid ratio led to a mean value for $B/(B-A)$ of 0.42. The corresponding value derived from the mean values of B (-12.0) and A (14.45), that have been obtained in this paper, is 0.45. The difference between these two values is well within the range of uncertainty, because $B/(B-A)$ is not an accurately determined quantity, as the apparent velocity-ellipsoid ratio is strongly influenced by star streaming and other aspects of non-uniformity.

7.4 COMMENTS

The first two of these comparisons give a certain amount of confidence that the set of values coming out of this study is meaningful as referred to the real Galaxy, but at the same time the AR_0

comparison leaves open the possibility that there may be a problem of interpretation arising from the presence of non-circular motions, or some other cause. The approximate agreement of the values also suggests that we are not suffering too much from the fact that A and B can only be determined over fairly short distances.

8 Some other quantities

8.1 K_z AND ϱ_0

The quantities K_z and ϱ_0, the vertical acceleration and the local mass density, have been defined earlier. Table 9 gives a list of determinations of ϱ_0. It is clear that there is still considerable uncertainty in these two quantities. These results give a value K_z at $z=500$ pc of $-6(\pm 3)\times 10^{-11}\,\mathrm{s}^{-2}$, and a local surface mass density of $\mu(R_0)=75\pm 30\,M_\odot\,\mathrm{pc}^{-2}$.

Table 9. K_z and ϱ_0.

Authors	Types of stellar tracer		$\varrho_0\,M_\odot\,\mathrm{pc}^{-3}$
Oort	1932	gK	0.09
Nahon	1957	gK	0.23
Woolley	1957	A	0.18
Hill	1960	gK	0.13
Oort	1960	gK	0.15
Yasuda	1961	high-velocity stars	0.15
Eelsalu	1961	gK, dF	0.08
Jones	1962	AO	0.14
Stothers & Tech	1964	OB	0.13
Woolley & Stewart	1967	A	0.11
Perry	1969	A	no solution
Lacarrieu	1971	gK	0.16
Joeveer	1972	B	0.09
Gould & Vandervoort	1972	A	0.19–0.28
Jones	1972	gM	0.21
Hill *et al.*	1979	A, dF	0.14
House & Kilkenny	1980	OB	0.13
Bahcall	1984	gK, dF	0.12–0.25

8.2 ROTATION CURVE

We have discussed at length the circular velocity of rotation at the Sun, θ_0. The whole rotation curve as a function of distance from the centre has of course also been studied by many authors. The main conclusion to be drawn at the present time is that the curve is nearly flat over a large range of R (~4–18 kpc), or it may be rising in the outer parts of the Galaxy. A flat or rising curve is consistent with the results obtained in recent years for a substantial number of external galaxies.

For a flat rotation curve, we would expect to have $A=-B$. This possibility is within the errors of the mean values we have obtained for the two quantities, but in making comparisons we should remember again that the values for A and B come from local determinations, whereas statements about a flat rotation curve really refer to a mean curve, with local irregularities smoothed out.

It is interesting to consider the effect that a change in R_0 or θ_0 would have on the rotation curve and other derived quantities. First, when a radial-velocity measurement is used to derive a kinematic distance, through the assumption of some rotation curve, such a distance will scale directly with R_0. The kinematic distance is also independent of θ_0 since, in deriving a rotation curve, a value of θ_0 is assumed, and this drops out again when the rotation curve is used to derive a kinematic distance.

Further, in the outer part of the Galaxy, where the direct kinematic approach cannot be used, a rotation curve is derived by measuring the radial velocities, V_r, and estimating the distances, r, for a series of individual objects. Under such circumstances, the rotation curve velocities will be increased if we assume a lower value for R_0, and will be decreased for a lower value of θ_0. The exact relation is

$$\theta=\left[1+\frac{r}{R_0}\left(\frac{r}{R_0}-2\cos l\right)\right]^{1/2}\left(\frac{V_r}{\sin l}+\theta_0\right).$$

8.3 SOLAR MOTION

When this study began, we had intended to include an overall study of solar-motion determinations and their limitations. It soon became clear that this would amount to another very lengthy study which should be carried out in a different context.

9 Conclusions

Published determinations of the four main galactic constants have been studied. Straight, unweighted means have been preferred because systematic uncertainties are more important than internal errors. The values obtained are shown in Table 10.

Table 10.

$$R_0=8.5\pm1.1\ \mathrm{kpc}$$
$$\theta_0=222\pm20\ \mathrm{km\,s^{-1}}$$
$$A=14.4\pm1.2\ \mathrm{km\,s^{-1}\,kpc^{-1}}$$
$$B=-12.0\pm2.8\ \mathrm{km\,s^{-1}\,kpc^{-1}}$$
$$A-B=26.4\pm1.9\ \mathrm{km\,s^{-1}\,kpc^{-1}}$$

The values for R_0 and θ_0 (Table 10) are substantially different from the IAU standard values, but we should note that the spread of the individual determinations is quite large, especially for R_0. Particular sources of uncertainty have been mentioned in the earlier discussions.

At the same time, we have seen that the two combinations of constants, AR_0 and $R_0(A-B)$, give results which are reasonably close to the values directly determined, suggesting that the derived numbers comprise a fairly consistent set. In view of the uncertainties that are present, this agreement may be largely coincidental. We cannot claim that precise values have been derived for the four constants, but there is a good argument for agreeing on new standard values for R_0 and θ_0 as a matter of practical convenience, in order to give a good basis for comparisons between the work of different people. A suitable choice of rounded-off values would be as follows

$$R_0=8.5\ \mathrm{kpc}$$

$$\theta_0=220\ \mathrm{km\,s^{-1}}.$$

In the case of the constants A and B, we are of course interested to know their best current values, but there is no real need to recommend or adopt standard values for them, as they do not enter into the comparisons of maps, velocity curves, etc., which are so greatly aided by having standardized values of R_0 and θ_0. We note that rounded-off values of our derived means are $A=14$ and $B=-12\ \mathrm{km\,s^{-1}\,kpc^{-1}}$; assumption of a flat rotation curve would suggest a preference for $A=13$ and $B=-13$, although these may not be the best values of these constants for the local region.

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