

ATMOSPHERIC EXTINCTION EFFECTS ON STELLAR ALIGNMENTS

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1. Introduction

The extinction angle is the lowest angle on the horizon at which a star is visible. In other words, if the star were any lower on the horizon than the extinction angle, then the increased amount of atmospheric scattering and haze would render the star invisible. It should be noted that the value of the extinction angle can vary from place to place and from night to night. In this paper, the extinction angle will be measured from the zenith and will be denoted by z .

The determination of extinction angles is an important problem for archaeoastronomy, because z is usually needed for calculations regarding the azimuth of proposed stellar alignments: errors in the value of z will translate into errors in azimuth of the first or last sighting of a star. In many practical cases, the error in azimuth is greatly magnified from the error in z by the low angle at which stars may rise relative to the horizon. Hence, a thorough understanding of extinction angles is required to discover and assess any stellar alignment.

There have been many studies of models of extinction as a function of zenith distance, which is an important quantity in modern astrophysical photometry. There is a survey of much of this earlier work in Schoenberg (1929), including a reference to an interesting study by Müller (1891) of extinction at all zenith distances up to 89° from both low and high altitude observing sites.

Unfortunately, to the best of my knowledge, no accurate model of extinction angles as defined in this paper has ever been proposed; and worse, no actual observations of such extinction angles have ever appeared in the literature. In this paper, I seek to fill these gaps with a three-part approach. In Section 2, I will present numerous observations of extinction angles for objects ranging between -2.7 and 5.4 magnitudes (m). In Section 3, I advance a quantitative and modern astronomical model for predicting z . In Section 4, I analyse the statistical errors associated with the azimuth of a star's first appearance. The results from these sections are then applied to a wide variety of claimed stellar alignments.

2. Observations

I have made a total of 60 observations of extinction angles from Cerro Tololo Inter-American Observatory (CTIO) near La Serena, Chile and from my home in Bowie, Maryland. The clarity of the atmosphere at these sites can be characterized by the visual extinction coefficient (k_v), which is defined as the magnitudes of extinction suffered by starlight in the direction of the zenith. The value of k_v at CTIO was measured photometrically to vary between 0.13 and

0.20 on the 6 nights from which data were obtained. The faintest star visible to my unaided eyes at zenith (m_z) was 6.0 on all nights. It should be remembered that CTIO is sited on a mountain top in the world's driest desert and was specifically chosen because it has the best observing conditions obtainable in the world. In contrast, Bowie, Maryland is near sea level, in a humid climate and 15 miles from the centre of Washington, D.C. For the nights of observation, k_v was estimated to be 0.3, while m_z was roughly 4.1.

An observation consisted of my watching a star rise or set, and noting the time of its appearance or disappearance. This time was then converted into sidereal time (T_s), from which the true zenith angle (Z_T) of the star can be deduced from

$$\cos Z_T = \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos (T_s - \alpha).$$

(1)

In this equation, λ is the observer's latitude and α and δ are the star's right ascension and declination, respectively. The true zenith angle must then be corrected for refraction to find the apparent zenith angle. Refraction tables can be found in Thom (1967) or Hawkins (1963). These corrections depend weakly on atmospheric pressure and temperature; however, these effects are always insignificant.

All observations were made with my own unaided eyes, which were always fully dark adapted and are considered to be 'average'. The horizon was always below the extinction angle. Absolute times were known to within 5 seconds by comparison with WWV time signals, and the star was always positively identified by the comparison of surrounding stars with a star chart. The data were compared with model predictions only long after the time of observation.

The individual data are presented in Figures 1 and 2 for CTIO and Bowie, respectively, observations made on separate nights being indicated by different symbols. No significant differences are noted between extinction angles for rising and setting stars. Note that Jupiter ($m = -2.7$) was visible for zenith distances greater than 90° , because at CTIO the apparent western horizon lies $91^\circ.5$ from the zenith.

One item which impressed me was that it was difficult to assign a unique time to the appearance or disappearance of a star: usually there was an interval of

TABLE 1. Extinction coefficients.

k_v	Description ^a
0.10	Best night on dry mountain top
0.15	Average night on dry mountain top
0.20	Poor night on dry mountain top
	Best night at dry sea level site
0.25	Average night at dry sea level site
	Best night at humid sea level site
0.30	Average night at humid sea level site
0.40	Average night at poor site with much dust or humidity
0.50	Very poor night

^aThese descriptions are provided so that estimates of the *average* k_v can be deduced in the absence of better data. Variations around the stated mean will occur at different sites, times of day, and times of year. So for example, a "dry sea level site" may have quite clear skies or it may have morning hazes or it may have substantial amounts of windborn dust. The user should also guard against known climatic changes.

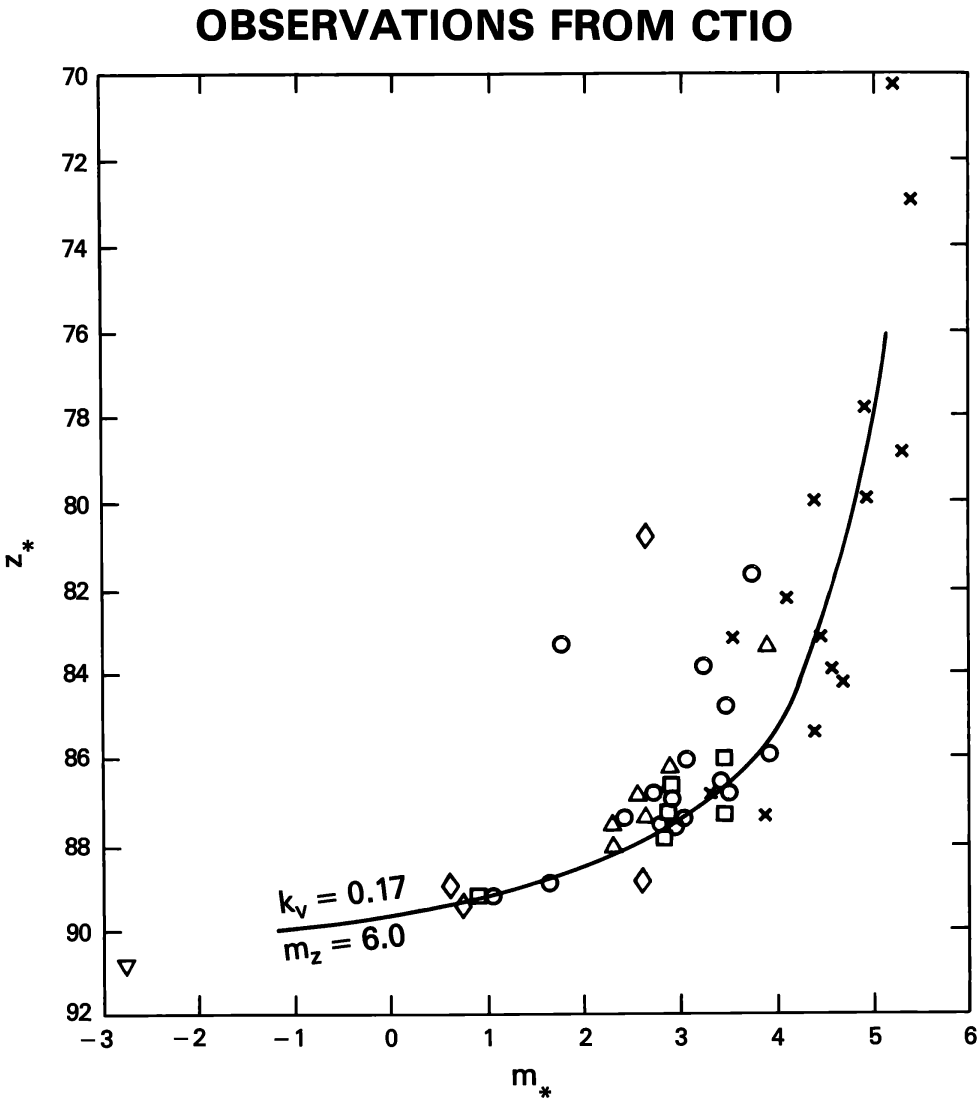


FIG. 1. Observations from CTIO. Individual points represent individual observations, while separate symbols signify different nights. The crosses are for observations of rising stars, while the other symbols are for setting stars. The photoelectrically observed mean k_v is 0.17 for these nights. The model prediction for $k_v = 0.17$ and $m_z = 6.0$ is shown, and the agreement between observed and predicted values is satisfactory. The wide scatter about the predicted value illustrates the point that the azimuth of a star's first appearance can vary quite widely from night to night.

time during which I was uncertain whether the star was visible. For bright stars this duration is typically 1 minute, for stars with $m = 3$ the uncertainty may be 3 or 4 minutes, while faint stars with $m = 5.4$ will have a time of uncertainty of 20 minutes or more. This observational uncertainty is likely to be the primary cause of the scatter about the model predictions.

3. Model

An accurate model for predicting z must quantitatively account for sky

OBSERVATIONS FROM BOWIE

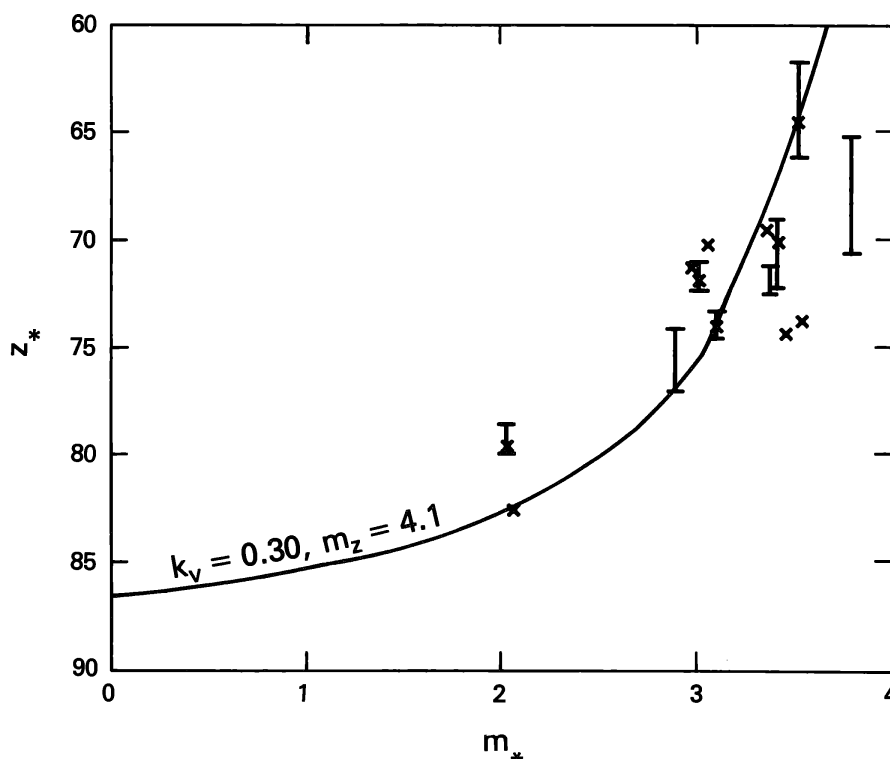


FIG. 2. Observations from Bowie. For Bowie, a mean k_v of 0.30 was estimated while $m_z = 4.1$. The model prediction for these parameters is shown by the line, which adequately describes the data. All observations are for rising stars, since the western sky had significant light from Washington, D.C. The vertical bars correspond to the time interval during which it was uncertain whether the star was visible.

conditions, the complicated physiological sensitivity of the eye, and the variation of path length within the atmosphere. The model presented here will determine the extinction angle z as a function of stellar magnitude m , zenith limiting magnitude m_z , and the visual extinction coefficient k_v .

The sky conditions will be characterized by two parameters, m_z and k_v . The value for m_z may be found by direct observation. For normal dark skies, m_z will be 6.0. However, moonlight, city lights, haze, or poor eyesight may decrease this value. The only accurate method for measuring k_v is by photometry. Unfortunately, all photometrists know by experience that k_v can vary quite widely with altitude and azimuth as well as from hour to hour within a given night, and so the best we can do is to use an average value to represent the sky conditions. Even without photometry, a fairly good value of k_v may be estimated by an experienced photometrist, and reasonably accurate average values can be estimated for locations at which only altitude and humidity are known. Table 1 contains a guide for estimating k_v . For example, a site in England is likely to have $k_v = 0.30$ on average and certainly no better than 0.25 on any night.

The magnitude of the faintest visible star will depend on the brightness of the background sky. For a set of 'standard' observing conditions ($k_v = 0.20$ and m_z

EXTINCTION ANGLE, $m_z = 6.0$

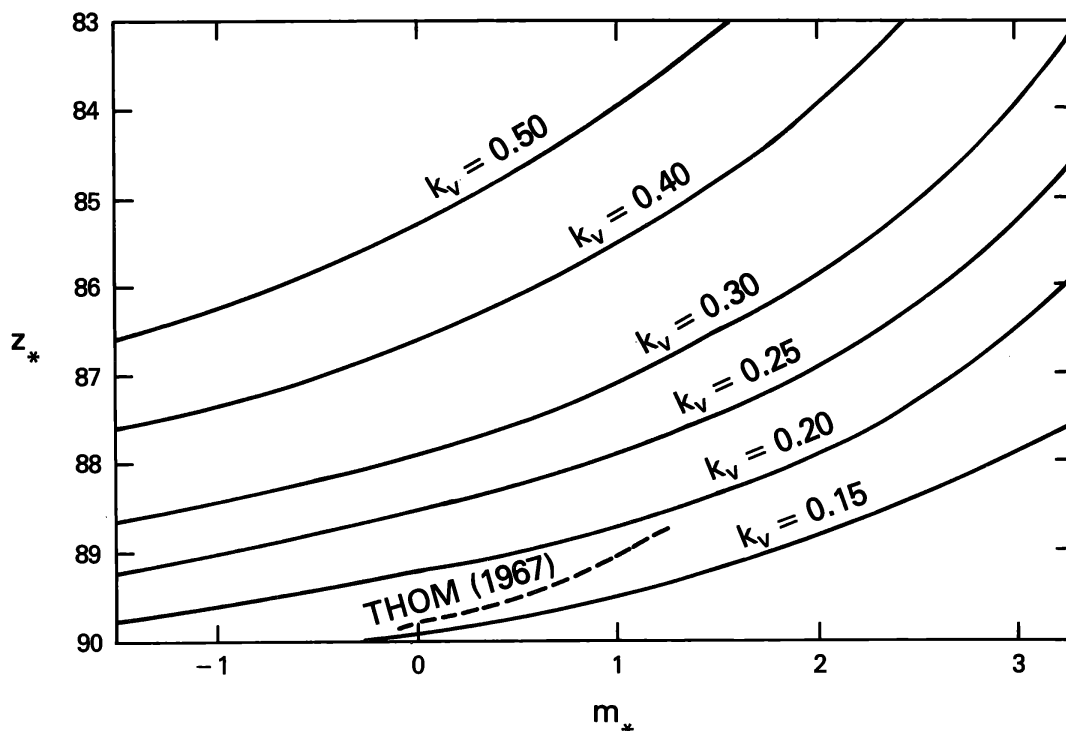


FIG. 3. Model extinction angles. For parameters of archaeoastronomical interest ($m < 3.0$ and $m_z = 6.0$), this figure presents the model extinction angle law for various values of k_v . The dashed line shows the extinction angle law used by Thom (1967).

= 6.0), the zenith sky brightness will be $118 \text{ } m\mu L$, as deduced from Equations (4) and (5). The sky brightness near the horizon will be roughly twice that at zenith for the skies at CTIO and at the Kitt Peak National Observatory. (The results are very insensitive to this ratio.) The sky brightness near the horizon for nonstandard conditions can be modelled as

$$b = \beta + (f/f_s) \cdot 236 \text{ } m\mu L, \quad (2)$$

$$f = 1 - 10^{k_v/2.5}, \quad f/f_s \cong k_v/0.20.$$

Here, f represents the relative amount of scattered light illuminating the sky, f_s is the value of f under standard conditions, and β is the background brightness caused by moonlight, etc. The value of β can be found by applying Equations (2) and (4)-(6) to the observation of m_z :

$$\beta = (10^{(m_z + 16.57 + k_v + 2.5 \log C)/-5} - 1)^{2/K} - (f/f_s) 236 \text{ } m\mu L. \quad (3)$$

From Equations (2) and (3), the value of the sky brightness on the horizon may be estimated.

For this sky brightness, the faintest intensity of a point source is given by the

formula of Hecht (1947):

$$\begin{aligned}
 i &= C(1 + \sqrt{Kb})^2 \\
 K &= 10^{-1.90} \text{ and } \log C = -9.80 \text{ for } \log b < 3.17 \\
 K &= 10^{-5.90} \text{ and } \log C = -8.35 \text{ for } \log b > 3.17,
 \end{aligned}
 \tag{4}$$

where b is in $m\mu L$ and i is in footcandles. This intensity can be related to visual magnitude with the relation

$$m = -16.57 - 2.5 \log i - k_v X.
 \tag{5}$$

The constant -16.57 is based on data in Allen (1973). Here, X is the number of airmasses (i.e., that quantity of atmosphere seen by an observer looking towards the zenith) through which the line of sight must pass. Rozenberg (1966) presents a formula for X which is accurate to roughly 5% all the way to the horizon:

$$X = 1/(\cos Z - 0.025 \exp(-11 \cos Z)),
 \tag{6}$$

where Z is the angle from the zenith.

The Equations (2)-(6) can be used in a simple computer program involving roughly 20 lines of instruction. This program will predict the extinction angle z for inputs of m , m_z , and k_v . The results of the model for practical situations are presented in Figure 3; in addition, Figures 1 and 2 show the predicted extinction angles for the appropriate conditions. The observations are well described by the model.

4. Error Analysis

The existence of a quantative extinction angle model allows a quantative error analysis. The importance of an error analysis lies in the fact that any error in extinction angle is translated into error in the azimuth of first sighting of a rising star. Hence, for the first time, we can evaluate the significance of a proposed alignment pointing to the first appearance of a star.

There are a number of effects that will cause significant night-to-night variation in the extinction angle. These effects include (1) the uncertainty in time of the observer over whether a star is visible, (2) the night-to-night variation in k_v , and (3) the night-to-night variation in m_z .

The uncertainty in time of disappearance or reappearance (Δt) will translate into an uncertainty in the value of z :

$$\begin{aligned}
 \sigma_{\Delta t} &= \cos a \cdot \cos^{-1} (\sin^2 \delta + \cos^2 \delta \cos(\Delta t \cdot 0^\circ.25)) \\
 \sigma_{\Delta t} &\cong \cos a \cdot \cos \delta \cdot \Delta t \cdot 0^\circ.25.
 \end{aligned}
 \tag{7}$$

In Equation (7), Δt is measured in minutes and a is the angle from the horizontal at which the star rises. This angle is given by

$$\cos a = (\sin \lambda - \sin \delta \cos z)/(\cos \delta \sin z).
 \tag{8}$$

For bright stars in practical situations, $\sigma_{\Delta t}$ will be less than a degree.

The uncertainties in z caused by variation in k_v (σ_k) are harder to quantify. However, the experience of photometrists suggests that one sigma variations of 20% in k_v are reasonable. This being so, figures such as Figure 3 can be used to evaluate σ_k . For example, in a site with a mean k_v of 0.30, it is reasonable to expect the k_v of any specific night to be between 0.24 and 0.36. For a first magnitude star, Figure 3 then shows z to vary from 88.0 to 86.0, or $\sigma_k \leq 1^\circ.0$.

As long as $m_z \gg m$, the error in z caused by changes in m_z will be small. Hence, in practical situations (with $m_z \cong 6$ and $m \cong 1$), σ_m will be insignificant.

The total uncertainty in z (σ_z) will be the usual sum in quadrature of the component uncertainties,

$$\sigma_z^2 = \sigma_{\Delta t}^2 + \sigma_k^2 + \sigma_m^2. \tag{9}$$

This can then be related to the uncertainty in azimuth (σ_A) of the star's first appearance,

$$\sigma_A = \sigma_z \cdot \cot a. \tag{10}$$

There are two distinct ranges of azimuth (one for rising and one for setting), each $2\sigma_A$ in size, for which there is a 68% probability of including the rising or setting azimuth of a given star. Hence the probability (p) of a random alignment pointing at the horizon phenomena of a particular star at the one sigma level of significance is $\sigma_A/90^\circ$.

Let us see what the implications of this probability are for even the best of cases. Let us assume that (1) the date of the site is known exactly, (2) only one sight line is available at the site, (3) only one star could be considered a possible candidate for alignments, and (4) only one site was examined for possible alignments. Now suppose that the one possible alignment does point to within the one sigma range in azimuth for the star's horizon phenomena. In practical cases, σ_A will be larger than a degree. This will make $p > 1.1\%$, which is much worse than the standard three sigma ($p < 0.27\%$) requirement for accepting a hypothesis. In other words, *except at low terrestrial latitudes, no single stellar alignment can be considered statistically significant.*

In the real world, the probabilities for accepting the hypothesis will be much worse than the above case. Let us look at a more realistic situation. The uncertainty in azimuth (σ) will be caused by uncertainty in the extinction angle (σ_A), the uncertainty in the date of construction (σ_D), and the uncertainty in measuring the alignment's azimuth (σ_E):

$$\sigma^2 = \sigma_A^2 + \sigma_D^2 + \sigma_E^2. \tag{11}$$

The total number of alignments (N) possible at the many sites is always quite large. The number of stars for which alignments would be allowed (s) is typically of order 10. The probability (π) of getting a total of A or more successful alignments on stars is then

$$\pi = \sum_{j=A}^N P(j), \tag{12}$$

where $P(A) = \rho^A (1 - \rho)^{N-A} N! / A!(N - A)!$ (13)

and $\rho = 1 - \exp (-s\sigma/90^\circ)$. (14)

Equation (14) is used to find the probability that any given alignment points at a significant azimuth and this result is used in Equation (13) to find the probability that exactly A alignments point to significant azimuths. It is implicitly assumed that the significant azimuths are randomly scattered over half the horizon and that another significant azimuth exists for the same star on the opposite horizon. For a sample case ($s=10$, $\sigma = 2^\circ$, $N = 50$), we would need $A \geq 19$ for $\pi < 0.27\%$. It must be stressed that π is the probability that the observed number of alignments is due to chance alone. It is possible for a civilization to have incorporated real alignments into a structure, yet for π to be near unity. However, if no ethnographic evidence is available and π is not small, then it would be extremely risky to claim that any alignments are intentional.

There is only one way to avoid these strict but necessary requirements. This can only be by having additional information from the outside in the form of ethnoastronomy. This extra information may reduce S and N . Without ethnoastronomical information, π will be too large in all practical cases for the hypothesis to be accepted. Hence, I conclude that, other than at low terrestrial latitudes, the azimuths of rising and setting stars cannot be used in archaeoastronomical applications except with *extreme caution*.

5. Claimed Alignments

Most of the claimed stellar alignments use the extinction angle law of Thom (1967). Thom took his extinction angle law from Neugebauer (1929), and then used his data to ‘prove’ the validity of the law. Unfortunately, Thom has two procedural difficulties.

The first difficulty arises when Thom uses Neugebauer’s law to demonstrate the validity of his alignments – and then uses his alignments to prove the validity of Neugebauer’s law. This is a classic example of circular reasoning.

The second difficulty arises because Thom’s “definition [of the extinction angle] excludes anything but the clearest conditions” (Thom, 1967). A comparison with the model predictions (Figure 3) shows that Neugebauer’s law is similar to my model with $k_v = 0.17$. This k_v is equal to the k_v at CTIO, which is one of the very best sites in the world. No one claims that the observing quality in England, the Great Plains, or the Yucatan rivals the best in the world. By taking the proper k_v into account, the true extinction angle will be different by several degrees from that calculated using Neugebauer’s law. Especially at high latitudes, this translates into a systematic error in the azimuth of a star’s first appearance.

Thom (1967) proposes 67 stellar alignments for stone circles in England and Scotland. In 32 cases, the apparent horizon is higher than the model extinction angle, for $k_v = 0.30$. The remaining 35 alignments (i.e. the majority) are systematically offset by $3^\circ.6$ on average. The details are given in Table 2, where the third column gives the difference in azimuth between the predictions from Thom’s model and those from my model for $k_v = 0.30$. Since it is implausible

TABLE 2. Evaluation of claimed alignments.

Site	Star	Azimuth Error	σ_A	Reference
A 2/14	β Gem	$-2^{\circ}.2$	$2^{\circ}.4$	Thom (1967)
	α Aur	$-2^{\circ}.2$	$2^{\circ}.7$	
A 3/4	α Gem	$4^{\circ}.0$	$2^{\circ}.9$	
		$5^{\circ}.6$	$2^{\circ}.9$	
		$3^{\circ}.5$	$2^{\circ}.9$	
A 6/4	α Vir	$1^{\circ}.3$	$1^{\circ}.7$	
A 10/3	β Ori	$-0^{\circ}.6$	$2^{\circ}.0$	
B 3/3	γ Ori	$-2^{\circ}.7$	$1^{\circ}.8$	
D 1/7	α Vir	$-0^{\circ}.7$	$1^{\circ}.7$	
G 1/4	α Cyg	$2^{\circ}.5$	$6^{\circ}.2$	
G 4/14	α Aql	$2^{\circ}.6$	$1^{\circ}.7$	
H 1/1	α Aur	$1^{\circ}.6$	$2^{\circ}.8$	
		$1^{\circ}.4$	$2^{\circ}.7$	
	α Aql	$2^{\circ}.6$	$1^{\circ}.7$	
H 1/3	α Sco	$-1^{\circ}.7$	$1^{\circ}.8$	
H 1/12	α Aql	$2^{\circ}.6$	$1^{\circ}.7$	
H 3/12	α Aql	$-2^{\circ}.6$	$1^{\circ}.7$	
	α CMi	$-2^{\circ}.6$	$1^{\circ}.6$	
L 1/6	α Sco	$0^{\circ}.3$	$1^{\circ}.8$	
L 1/10	α Cyg	$-8^{\circ}.8$	$6^{\circ}.2$	
L 6/1	α Aur	$-3^{\circ}.8$	$2^{\circ}.8$	
M 1/4	α Aur	$-4^{\circ}.4$	$2^{\circ}.8$	
M 1/5	α Gem	$-6^{\circ}.5$	$2^{\circ}.9$	
M 1/9	α Aur	$-0^{\circ}.3$	$2^{\circ}.7$	
M 8/2	β Gem	$-1^{\circ}.2$	$2^{\circ}.4$	
N 1/8	α Gem	$-9^{\circ}.1$	$2^{\circ}.9$	
P 1/2	α Aur	$4^{\circ}.4$	$2^{\circ}.7$	
S 1/5	α Gem	$-4^{\circ}.3$	$2^{\circ}.9$	
S 1/9	α Cyg	$1^{\circ}.5$	$6^{\circ}.2$	
S 5/3	α Cyg	$-8^{\circ}.8$	$6^{\circ}.2$	
S 5/4	α Aur	$4^{\circ}.4$	$2^{\circ}.7$	
S 6/1	α Aur	$4^{\circ}.4$	$2^{\circ}.7$	
W 2/1	α Cyg	$8^{\circ}.8$	$6^{\circ}.2$	
W 5/1	α Cyg	$8^{\circ}.8$	$6^{\circ}.2$	
W 11/4	α Vir	$-1^{\circ}.9$	$1^{\circ}.7$	
Stonehenge	α Leo	$2^{\circ}.9$	$2^{\circ}.1$	Robinson (1983)
Moose Mountain	α Tau	$1^{\circ}.5$	$1^{\circ}.6$	Eddy (1977)
	β Ori	$1^{\circ}.4$	$1^{\circ}.2$	
	α CMa	$1^{\circ}.0$	$1^{\circ}.1$	
	α Aur	$3^{\circ}.6$	$20^{\circ}.5$	
	α PsA	$2^{\circ}.3$	$2^{\circ}.3$	Robinson (1980)
Fort Smith	α Cyg	$3^{\circ}.5$	$3^{\circ}.5$	Robinson (1981)
	β Per	$2^{\circ}.1$	$2^{\circ}.5$	
	α Leo	$1^{\circ}.4$	$1^{\circ}.5$	
Bighorn Mountain	α Tau	$1^{\circ}.3$	$1^{\circ}.3$	Eddy (1979)
	β Ori	$1^{\circ}.2$	$1^{\circ}.0$	
	α PsA	$1^{\circ}.7$	$1^{\circ}.8$	Robinson (1980)
Chichén Itzá	α Eri	$-0^{\circ}.8$	$0^{\circ}.6$	Aveni <i>et al.</i> (1975)
	α PsA	$-0^{\circ}.9$	$0^{\circ}.6$	
	α Gem	$0^{\circ}.9$	$0^{\circ}.7$	
	β Gem	$0^{\circ}.9$	$0^{\circ}.6$	
	α Car	$1^{\circ}.1$	$0^{\circ}.7$	
Monte Albán	α Aur	$0^{\circ}.7$	$0^{\circ}.4$	Aveni and Linsley (1972)
	α Cen	$0^{\circ}.9$	$0^{\circ}.5$	
Tenochtitlán	η Tau	$0^{\circ}.6$	$1^{\circ}.0$	Aveni and Gibbs (1976)

that $k_v \sim 0.17$ in ancient Britain, I take these values to be systematic errors in the Thom's alignments. With the majority of Thom's stellar alignments offset by several degrees, it is doubtful whether the remaining minority are significant.

Similarly, many other claimed stellar alignments systematically do not point

at the star's appearance or disappearance. Once again, details for affected alignments are summarized in Table 2. The value of k_v used was 0.30, except for the Great Plains sites where k_v is taken to be 0.25. It must be noted that the azimuth errors of the Mesoamerican alignments are relatively small because of the low latitude. Also, the azimuth of the Pleiades rising from Tenochtitlán was calculated assuming a point source with $m = 2.0$ although this cannot be accurate because Equation (4) is not valid for extended sources.

Let us now calculate the significance of the various claimed stellar alignments (while ignoring the above conclusion that most of them have systematic offsets of several degrees). Thom uses alignments on stars as faint as 1.64, of which 20 are visible in the skies of northern England (hence $s = 20$). Thom's Table 8.1 gives 67 stellar alignments out of a list of 270 sightlines for 145 sites. This gives $N = 270$ and $A = 67$, although a perusal of Thom's diagrams reveals that N should be possibly two or three times larger completely to satisfy his selection criteria. With the claimed inaccuracy of alignments, the large σ_A (see Table 2), and the typical uncertainty in construction dates of a millenium, an average value of $\sigma = 3^\circ$ is reasonable. The calculated π is 100.00%.

The similar calculations can be made for the three medicine wheels which appear in Table 2. Eddy uses alignments only on stars brighter than 0.85 (hence $s = 10$), while Robinson is willing to use stars as faint as 2.12 (so $s = 46$). Both authors state that σ_E is 2° , which when combined with σ_A and the very large σ_D suggests that σ is larger than 3° . The values of N are 42, 30, and 6 and the values of A are 6, 5, and 6, which lead to calculated values of π being 100.00%, 100.00%, and 6.9% for Big Horn, Moose Mountain, and Fort Smith, respectively. In other words, there is no statistical evidence for stellar alignments from these sites.

Haack (1984) proposed a specific mechanism for how the ancient Egyptians could have cardinally aligned their pyramids. His idea provides a plausible explanation for the shifts in alignment as a function of time. The proposal is that the Egyptians made observations of equatorial stars near the horizon, and he explicitly assumes Thom's extinction angle law. With the results from this paper, his specific dating and star identifications may need to be changed. In addition, the relatively large value of σ_A suggests that it may be difficult to achieve the needed accuracy with this technique.

In summary, I have constructed a model for the extinction angle, and have confirmed the validity of the model by observation. The model is then used to test the accuracy and significance of claimed stellar alignments both in the general case and in a variety of particular cases. I find that uncertainties in the extinction angle lead to relatively large uncertainties in the azimuth of a star's first or last appearance, and this seriously compromises the statistical significance of any claimed stellar alignment. I stress a number of caveats to the above conclusions: (1) the extinction angle is irrelevant for those cases (e.g., Sun, Moon, and Venus) for which it can be shown that the extinction angle is greater than the zenith distance of the apparent horizon; (2) at low terrestrial latitudes, the uncertainties in the azimuth of a star's first or last appearance can become quite small; and (3) the observing conditions at ancient sites can only be estimated, albeit intelligently.

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