DYNAMICAL FRICTION AND FORMATION OF STRUCTURES: SOME COSMOLOGICAL AND COSMOGONICAL ASPECTS

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Abstract. The influence of dynamical friction on peculiar velocities in a two-component medium is considered. One of the components is assumed to consist of heavy particles with small gravitational interactions and the other is assumed uniform and its particles of much less weight. Here the solutions for the stationary as well as for the cosmologically expanding medium are obtained. These solutions illustrate the possibility for clusters of galaxies to have super-low velocities.

It is emphasized that the presence of two components leads to the formation of individual strings and general string structure in one of the component. A string-like structure of the Universe becomes inevitable in the case when an initial perturbations spectrum has a specific scale, and falls down abruptly for larger and smaller distances.

Such a picture of the formation of large-scale structure of the Universe is similar to the formation of particle tracks in the Wilson chamber, where tracks are the observational clusters of galaxies, and 'particles' are invisible perturbations in the second (neutrino?) component.

1. Introduction

In a whole series of astrophysical objects we deal with the situation when it is necessary to consider the gravitational interaction in physically different particles in a two-component medium. For example, there are the system of globular clusters and the system of field stars in the Galaxy. Average masses of 'particles' in these systems differ by a factor 10^5–10^6. In the last decade it became clear that there is another kind of 'heavy particles' in the Galaxy: namely Giant Molecular Clouds (GMC) (cf. Solomon and Sanders, 1979). An average mass of GMC is 10^5–10^6 $M_\odot$. There exist an analogous situation in the clusters of galaxies where the galaxies themselves can be regarded as 'heavy particles', the second component being the intergalactic gas and low-mass galaxies.

An entirely new situation has arisen in cosmology owing the arguments for non-zero rest mass (energy) of neutrino (Lubimov *et al.*, 1980a, b) and advances of particle physics. The current model of the Universe involves two components, an ordinary matter and background particles (for instance, massive neutrino; see Zel'dovich and Sunyaev, 1982).

Therefore, an interest in investigations of gravitational interaction in the multi-component media has increased. A problem of gravitational instability of such medium was concerned by Grishchuk and Zel'dovich (1981) who gave the general solution for a linear case in the rest environment.

However, a linear problem disregards a series of new interesting effects of the multi-component structure. These effects may arise at an essentially nonlinear stage. For example, let there be a two-component medium, in one of which the gravitational instability has developed leading to formation of gravitationally-bounded objects which we shale refer to as heavy particles. But perturbations of the other component do not increase to a first approximation, because of the high velocity of sound. Let us assume the temperature of this component decreases abruptly, so that the sound velocity becomes comparable to, or less than, the dispersion of heavy particles velocities in the first component. In this case, firstly, shock waves may arise, and, secondly, heavy particles will create axial perturbations, leading to an axial accretion pattern. Therefore, strings or filaments – i.e., gravitationally-bound objects – will appear in the accretion wake of heavy bodies. On the other hand, dynamical friction will reduce the velocities of heavy particles.

In the present paper we consider both effects in the case when self-gravitation of massive particles can be disregarded. It is a good approximation for a whole series of the real astrophysical situations – for example when considering the dynamical evolution of globular clusters in the Galaxy.

Note that the effects of dynamical friction on the motion of individual globular clusters was pointed out by Ostriker and Tremaine (1975), Tremaine and Ostriker (1975), Surdin and Charikov (1977); effect of dynamical friction on the motion of GMC was mentioned by Surdin (1980).

The evolution of a system of GMC due to dynamical friction was considered by Lupinov (1982), and the more detailed theory involving cloud collisions was developed by Silchenko and Lipunov (1985).

Effect of dynamical friction in two-component medium on the velocities of heavy particles and formation of a structure was discussed by Hadrava and Kareš (1984).

In the present paper the effect of dynamical friction on the velocity distribution of heavy particles will be considered.

2. The Dynamical Friction

For the dynamical friction force applied to a heavy particle we use the classical formula derived by Chandrasekhar (1943), of the form

$$F = \pi R_G^2 \Omega \rho v^2 \Lambda \,, \tag{1}$$

where $R_G = 2GM/v^2$ is the gravitational capture radius; M, the mass of heavy particle; V, its velocity; and $\Omega\rho$, the density of background particles of the second component; Λ, logarithmic factor is supposed to be constant: $\Lambda \cong \text{const} \approx 1$. A similar expression can be derived for a heavy body moving in the gas (Spiegel, 1975), if the velocities of massive particles are assumed to be much larger than the dispersion of velocities of light particles or the sound velocity in the gas.

In what follows we shall apply this expression for the case when both components are unstationary, for example, when considering the Hubble's expansion. This formula is applicable if a time-scale of a stationary accretion t_G is much less than the expansion time, t_{exp}: i.e.,

$$t_G \approx R_G/v = 2GM/v^3 \ll t_{\text{exp}} \,. \tag{2}$$

In the case of Hubble's expansion $t_{\exp} \sim 1/H$ and we obtain the following condition ($H = 100$ km s^{-1} Mpc^{-1}):

$$M_{15} V_8^{-3} \ll 1 , \tag{3}$$

where $M_{15} = M/10^{15} M_{\odot}$, $V_8 = V/10^8$ cm s^{-1}.

3. The Evolution of the Distribution Function in a Stationary Two-Component Medium

Let us consider the background medium to be homogeneous, and the other component of heavy particles – sufficiently rarefied and keeping homogeneity all the time. The kinetic equation is

$$f_t + v_i f_{r_i} - \left(F_{i\partial} + \frac{\partial \varphi}{\partial r_i} \right) f_{V_i} = 0 , \tag{4}$$

where

$$F_{i\partial} = \pi G^2 \rho_{cr} \Omega M V_i / V^3 , \tag{5}$$

ρ_{cr} being the critical density of matter, $\Omega = \rho/\rho_{cr}$. If inhomogeneities in the background matter due to presence of heavy particles are disregarded, Equation (4) takes the simplest form

$$\frac{\partial f}{\partial t} - F_{i\partial} \frac{\partial f}{\partial V_i} = 0 . \tag{6}$$

The solution of this equation can be written as

$$f = f(C) \tag{7}$$

where C is the characteristic constant of Equation (6) (see Zel'dovich and Mishkis, 1973)

$$C = \frac{v^3}{3} + \pi G^2 M \Omega \rho_{cr} t . \tag{8}$$

For the initial velocity distributions we may choose:
(1) δ-function

$$f_{10} = C_{10} \delta(v - w_0) ; \tag{9}$$

or (2) Maxwell's function

$$F_{20} = C_{20} v^2 \exp(v^2/\theta) , \tag{10}$$

where w_0 is the initial velocity spectrum; θ, the initial r.m.s. velocity. Using Equations (9)–(10) one may derive

$$f_1 = C_{10} \delta([v^3 + 3\pi G^2 M \Omega \rho_{cr}(t - t_1)]^{1/3} - w_o) , \tag{11}$$

$$f_2 = C_{20}(v^3 + 3\pi G^2 \Omega \rho_{cr} M(t - t_1)^{2/3} \times$$
$$\times \exp\left(- \frac{1}{\theta} [v^3 + 3\pi G^2 \Omega \rho_{cr} M(t - t_1)]^{2/3}\right); \tag{12}$$

where t_1 is the initial time moment, C_{10} and C_{20}, some normalization factors.

With the use of Equations (11)–(12) it easy to get expression for the mean-square velocity of the heavy component

$$\langle v^2 \rangle = \frac{\int v^2 f_{1,2}(v)\,\mathrm{d}v}{\int f_{1,2}(v)\,\mathrm{d}v} . \tag{13}$$

For the f_1-distribution a simple integration yields

$$\sqrt{\langle v^2 \rangle_1} = (w_0^3 - 3\pi G^2 M \Omega \rho_{cr}(t - t_1))^{1/3} . \tag{14}$$

In the second case (f_2-distribution) the saddle-point method (cf. Svechnikov and Tikhonov, 1979) yields

$$\sqrt{\langle v^2 \rangle_2} = \left[\left(\frac{3\theta}{2}\right)^{3/2} - 3\pi G^2 M \Omega \rho_{cr}(t - t_1)\right]^{1/3} . \tag{15}$$

4. Decrease of Peculiar Velocities in Nonstationary Medium with Cosmological Expansion

Let us estimate an efficiency of dynamical friction in braking heavy particles which have peculiar velocities V. The braking time due to dynamical friction is given by

$$t_G = \frac{MV^2}{2F_\partial V} = \frac{V^3}{\pi G^2 M \Omega \rho_{cr} \Lambda (1 + Z)^3} , \tag{16}$$

where $\Omega \rho_{cr}$ is the density of background component matter. Hereafter, the total density in the Universe is supposed to be equal to the critical density $\rho_{c_r}(\Omega = 1)$, given by

$$\rho_{cr} = 3H^2/8\pi G . \tag{17}$$

In this case the cosmological time (cf. Zel'dovich and Novikov, 1975) becomes

$$t_H = 2/3H_0(1 + Z)^{3/2} ; \tag{18}$$

and for $\rho = \rho_0(1 + Z)^3$ we have

$$\frac{t_G}{t_H} \cong 10 \times (1 + Z)^{-3/2} V_8^3 M_{15}^{-1} H_{100}^{-1} ; \tag{19}$$

$H_{100} = H/100$ km s^{-1} Mpc^{-1}. This estimate indicates that even for $V_8 \sim 1$, $Z \sim 5 - 10$, $\Omega \cong 1$ dynamical friction appears to be highly effective.

Thus, let us consider the case of cosmological expansion. Here we se the Newton's approximation in the Friedmann Universe with $\Omega = 1$. Therefore, in (1′) ρ depends om the time t as

$$\rho = 1/6\pi G t^2 . \tag{20}$$

The mean expansion of the Universe can be expressed in the explicit form (Zel'dovich and Novikov, 1975) as

$$\bar{U}(\bar{r}, t) = H(t)\bar{r} + \bar{V}(\bar{r}, t) , \tag{21}$$

$$\Phi(\bar{r}, t) = \tfrac{2}{3}\pi G\rho(t)\bar{r}^2 + \varphi(\bar{r}, t) ; \tag{22}$$

where $\bar{U}$ is the velocity; $\bar{V}$, the peculiar velocity; H, the Hubble's constant. Note that it is just the value of V that is involved in (1). It is appropriate to solve the problem in co-moving coordinate system after the change of the variables (Shandarin, 1980)

$$\begin{aligned} r_i' &= \frac{1}{a}\frac{r_i}{r_0} , \qquad \mathrm{d}t' = \frac{1}{a^2}\frac{\mathrm{d}t}{t_0} , \\ \rho' &= a^3 \frac{\rho}{\rho_0} , \qquad V_i' = a\frac{V_i}{V_0} , \qquad \varphi' = a^2 \frac{\varphi}{\varphi_0} ; \end{aligned} \tag{23}$$

where r_0 is an arbitrary distance scale, $a \sim (1 + Z)^{-1}$, the factor describing an increase of linear scale in the course of expansion,

$$t_0 = \frac{2}{H\sqrt{\Omega}} , \qquad V_0 = r_0/t_0 ,$$

$$\rho_0 = \frac{3\Omega H_0^2}{8\pi G} , \qquad \varphi_0 = V_0^2 ; \qquad a' = \frac{1}{t'^2} . \tag{24}$$

After this transition the kinetic equation can be rewritten as

$$f_{t'} + V_i' f_{r_i'} - \left(F_{\partial_i}' + \frac{\partial\varphi'}{\partial r_i'}\right) f_{V_i'} = 0 . \tag{25}$$

Disregarding perturbations of the potential from (25) one can obtain

$$f_{t_i'} - F_{\partial i}' f_{V_i'} = 0 , \tag{26}$$

where

$$F_{\partial i}' = \frac{3GM\Omega}{2V_0^3 t_0 V'^2 t_1'^2} . \tag{27}$$

This equation is similar to Equation (6).

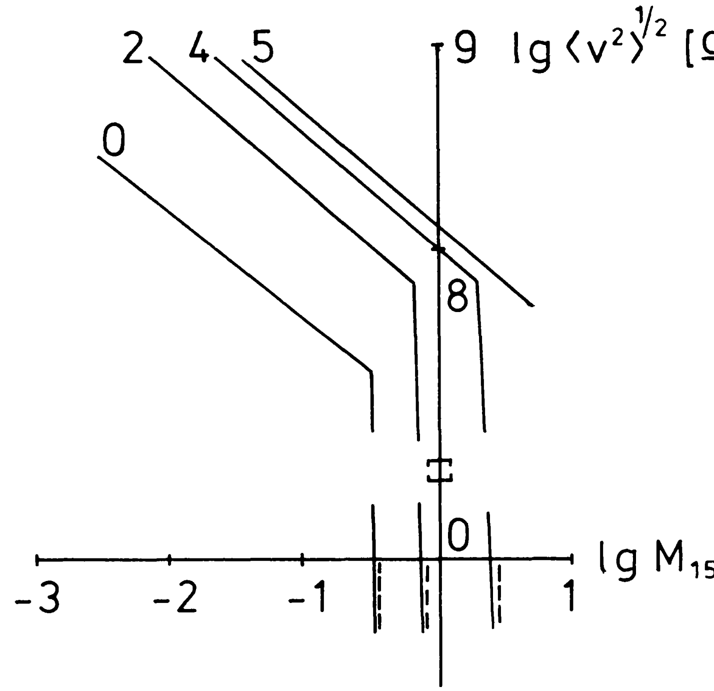

Fig. 1. Evolution of the r.m.s. velocity $(\langle v^2 \rangle)^{1/2}$ in unstationary model with $\Omega = 1$. The initial distribution is described by Maxwell function. The indices near the curves indicates the time moment (Z).

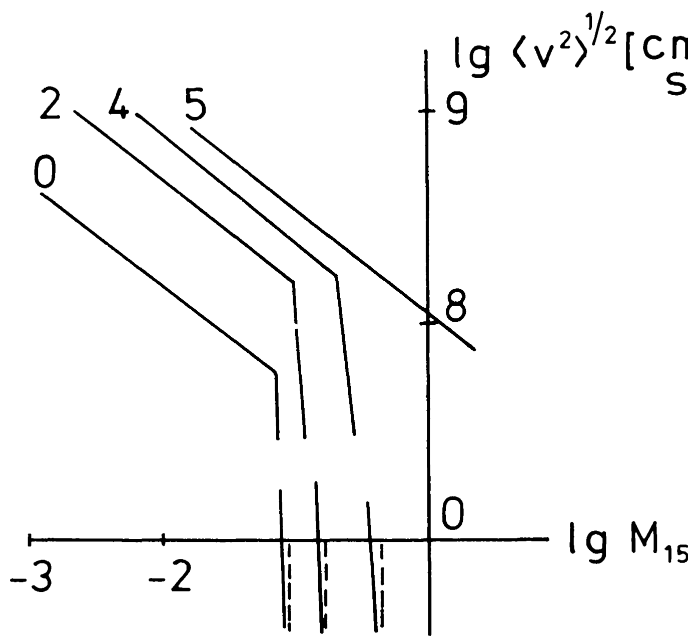

Fig. 2. The same as in Figure 1 for velocities distributed in agreement with δ-function at the moment $T_1 = 0 (Z = 5)$.

Choosing the same δ- and Maxwell's functions as the initial distributions, after identical consideration we obtain

$$\sqrt{\langle v^2 \rangle_1} = \left(\frac{t_1}{t}\right)^{1/3} \delta\left(w_0^3 - \frac{2GM\Omega}{t_1^2}\,(t - t_1)\right)^{1/3}, \tag{28}$$

$$\sqrt{\langle v^2 \rangle_2} = \left(\frac{t_1}{t}\right)^{1/3} \left[(\tfrac{3}{2}\theta)^{3/2} - \frac{2GM\Omega}{t_1^2} (t - t_1) \right]^{1/3} . \tag{29}$$

To illustrate the effects of dynamical friction, let us impose the initial condition

$$w_0 = 10^8 M_{15}^{-1/2} \quad \mathrm{cm\ s^{-1}} ; \tag{30}$$

and also $\theta = w_0^2$. This initial velocity spectrum corresponds to the case of constant 'temperature', $T = M\theta$, of the heavy component at zero time moment. We remind the reader that $\Omega = 1$, $H = 100$ km s^{-1} Mpc^{-1}. One can see from Figures (1)–(4) that the dynamical friction effect must be taken into account. This effect brings heavy particles to a stop and reduces the velocitys of light particles. This fact must be taken into account when analysing observational data on peculiar velocities of galaxy clusters. It is known (cf. Zel'dovich and Sunyaev, 1980, 1982) that measuring these velocities provides an important information concerning the spectrum of large-scale parturbations of the matter density in the early Universe on distance-scale of 100 Mpc. It was inferred in those works that if peculiar velocities of galaxy clusters depend on small-amplitude density perturbation on large scale, then observations of microwave beckground radiation in the direction of remote clusters yields the new opportunity for determination of cosmological parameter Ω.

Moreover, the smalness of peculiar velocities argues for low value of ρ – i.e., for the 'open' Universe. But the dynamical friction effect considered above indicates that the velocities can be small even if $\Omega = 1$. Thus, the value of $\langle v^2 \rangle$ does not allow us to infer a mean density in the Universe and, consequently, a character of its curvature.

Let us estimate now the effect of the dynamical friction on the peculiar velocities of galaxies. We will use the formulae for velocities derived in Zel'dovich and Sunyaev (1980). These expressions are the solutions of evolution for small density perturbation in a linear case: i.e.,

$$\overline{w}_0 = A \frac{H_0 \overline{k}}{k^2 (1 + Z)^{1/2}} , \tag{31}$$

where $\overline{k}$ is the wave vector; Z, the redshift; and A, the normalising constant. Fluctuation of the matter density is expressed as

$$\delta = \frac{\delta\rho}{\rho} = A(1 + Z) . \tag{32}$$

The initial velocity w_0 is chosen to be equal to $\sim 2 \times 10^8$ cm s^{-1}. For the moment $Z = 0$ the value 0.4×10^8 is obtained. In Figures 1 and 2 the result inferred from (28, 29) for $\Omega_1 = 1$ is shown. $\Omega_1 = 1$ corresponds to the case when some heavy particles of 'ordinary' matter are braking by invisible (neutrino?) background. Figures 3 and 4 shows the results of calculation for the case $\Omega_2 = 0.03$ when some hypothetical objects (which represent the main deposit to the mass of the Universe) are braking the 'ordinary' matter. This case differs considerably from the previous one; the deceleration being less

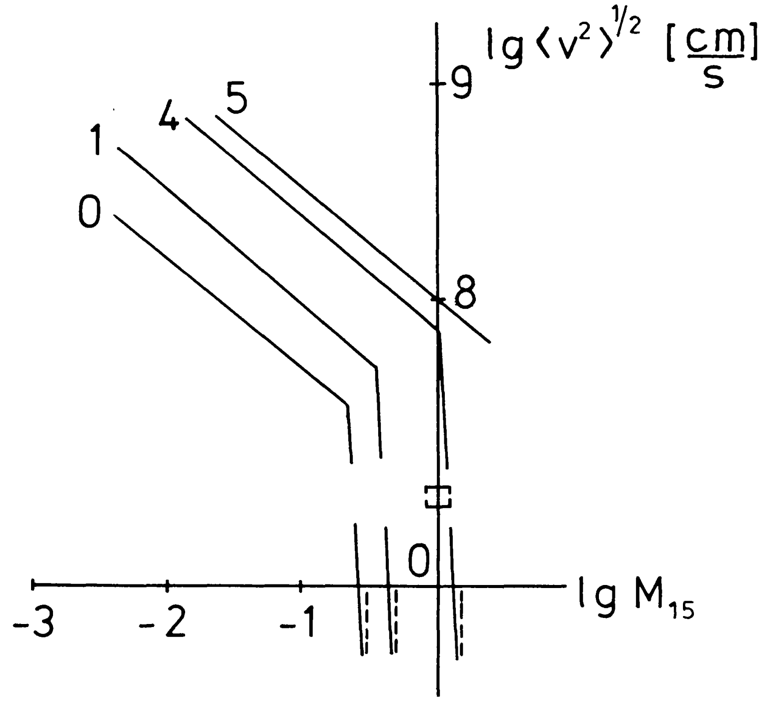

Fig. 3. The same as in Figure 2 in a model with $\Omega_2 = 0.03$ ($\Omega\rho_{cr}$ is the density of background matter, the full density is proposed to be equal $\Omega = 1$, see the text).

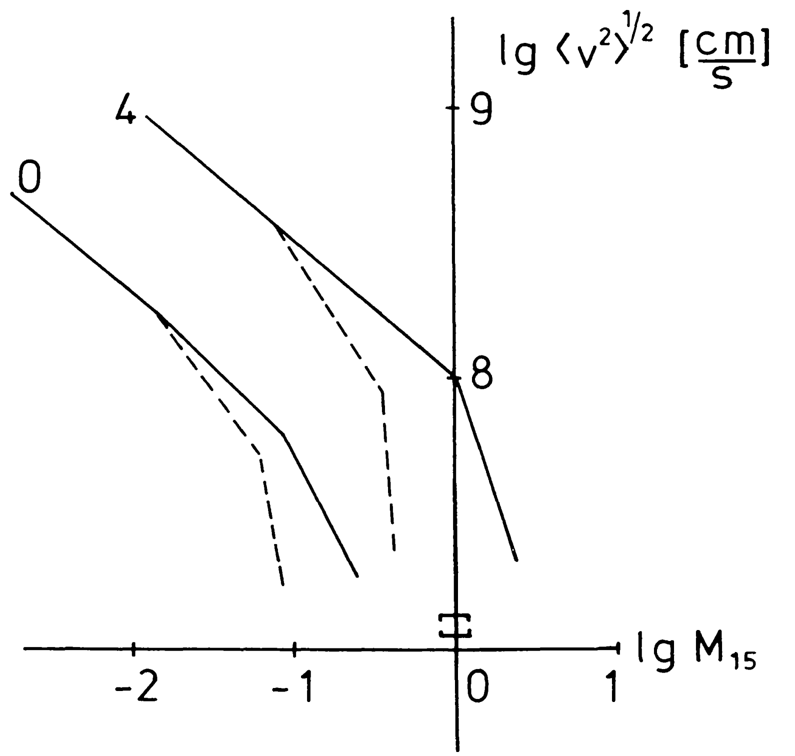

Fig. 4. Evolution of the value $(\langle v^2 \rangle)^{1/2}$ with the Maxwell initial velocity distribution for the model with $\Omega = 0.03$ in comparison with the $\Omega_1 = 1$-model (dotted line).

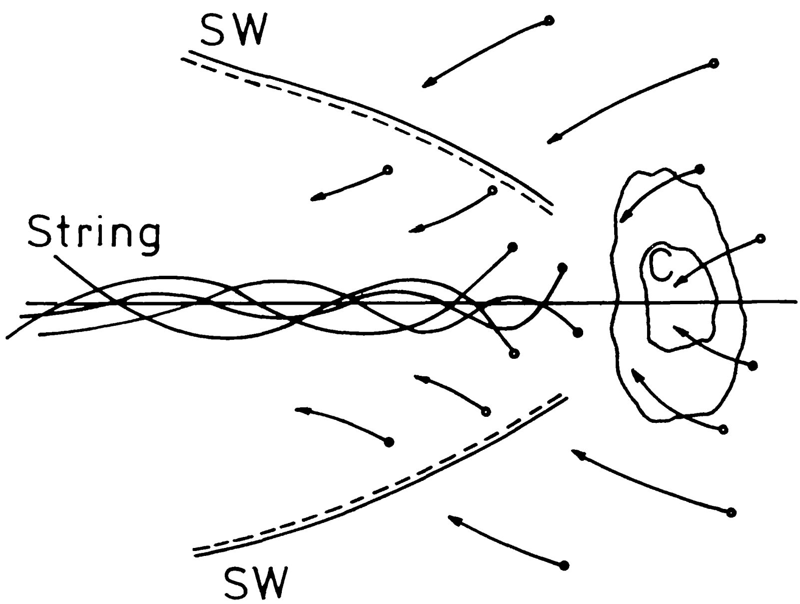

Fig. 5. Formation of the string-like structure via cylindrical accretion of lowmass marticles onto the heavy body C. Under certain conditions (see the Introduction) the shock wave SW can appear in this case.

in the second case. Thus, the following important conclusion can be inferred. The data about $\langle v^2 \rangle^{1/2}$ for the galaxy clusters obtained from observations are underestimated in comparison with those resulting from the theory of perturbations on the evolution caused dynamical friction. Using Equations (29)–(31) we find it easy to obtain the supposed values of $\sqrt{\langle v^2 \rangle}$.

5. String-like Structure in Two-Component Medium

Here we consider qualitatively effects connected with the formation of structure in two-component medium. Recall the example given in the introduction. One of the component consist of heavy, gravitationally-bound particles (appreciably nonlinear stage); while another component is homogeneous. If heavy particles move with sufficiently high velocities, then perturbation in the second component will develop. They will lead to the formation of strings stretched along the direction of heavy particle motion. The formation of this structure can be understood in terms of axial or cylindrical accretion (see Figure 5). A specific length l of a string is estimated as the distance passed by a particle until the stop is given by

$$l \approx Vt_G = V^4/\pi G^2 M \Omega \rho_{cr} \Lambda . \tag{33}$$

The formation of separate strings is expected in the following astrophysical situations:

(a) In the galactic medium during the pass of a globular cluster or GMC. The length of a string may reach some kiloparsecs (it is worth-while to mention that, in this case, the above-mentioned objects may lead to formation of chopped spiral arms);

(b) In the case of ‘nomadic’ mass represented by ‘black holes’ in the Galaxy (see Silchenko and Lipunov, 1985);

(c) During formation of clusters of galaxies. Imagine a super-massive object (neutrino complex?) moving in the homogeneous gas (such a gas must exist at the recombination moment). Bearing in mind what was said above, it is asserted that the length of a string is equal to

$$l \approx 10\ \mathrm{Mpc} \times V_8 (t_G / 10^{10}\ \mathrm{yr})\,. \tag{34}$$

Formation of the structure in this case is similar to formation of particles tracks in Wilson chamber.

Such a scheme for the formation of large-scale structure of the Universe seems quite real if an initial perturbations spectrum is assumed to have some specific distance-scale, and to fall down for larger and smaller distances. Recently, Kofman and Linde (1985) indicated that such a spectrum could be formed in the Universe expanding in accordance with the inflation picture. (In the outmost case such spectrum has a δ-function appearence.) For this spectrum a string-like structure of the Universe appears in a quite evident manner. In reality, at first the structure in the 'invisible' component is formed. At the moment of recombination, perturbations in the 'neutrino' (or some other) component can be small. Of importance is, however, the fact that these perturbations have a certain distance-scale; and every perturbation is a rudimentary protocluster. It is easy to infer that, just after the moment of recombination, peculiar velocities of such protoclusters exceed the sound velocity. Two consequences are as follows: (1) weak shock-waves are formed; (2) in the baryon matter string-like perturbations develop.

Is not the Universe a gigantic Wilson chamber?

References

Chandrasekhar, S.: 1943, *Astrophys. J.* **97**, 255.

Grishchuk, L. P. and Zel'dovich, Ya. B.: 1981, *Astron. Zh.* **58**, 472.

Hadrava, P. and Kares, V.: 1984, *Bull. Astron. Inst. Czechosl.* **35**(**6**), 343.

Kofman, D. and Linde, A.: 1985, Preprint, Acad. Science Estonian SSR A-6, Tartu.

Lipunov, V. M.: 1982, *Astron. Zh.* **59**, 286.

Lubimov, V. A. *et al.*: 1980a, Preprint, *ITEP* **62**.

Lubimov, V. A. *et al.*: 1980b, *J. Phys.* **32**, 301.

Ostriker, J. P. and Tremaine, S.: 1975, *Astrophys. J.* **202**, L113.

Shandarin, S. Ph.: 1980, *Astrofizika* **10**, 769.

Silchenko, O. K. and Lipunov, V. M.: 1985, *Astrophys. Space Sci.* **117**, 293.

Solomon, P. M. and Sanders, D. B.: 1979, in P. M. Solomon and P. M. Edmunds (eds.), *Giant Molecular Clouds in the Galaxy*. Pergamon Press, London.

Spiegel, E. A.: 1970, in H. J. Habing (ed.), *Interstellar Gas Dynamics*. D. Reidel Publ. Co., Dordrecht, Holland.

Surdin, V. G. and Charikov, A. V.: 1977, *Astron. Zh.* **54**, 24.

Surdin, V. G.: 1980, *Astron. Tsirc.* **113**, 3.

Sveshnikov, A. G. and Tikhonov, A. N.: 1979, *Teorya Funkcii Kompleksnoy Peremennoy*, Nauka, Moscow.

Tremaine, S., Ostriker, J. P., and Spitzer, L. Jr.: 1975, *Astrophys. J.* **196**, 407.

Zel'dovich, Ya. B. and Myshkis, A. D.: *Elementy Matematicherkoy Fyziki*, Nauka, Moscow.

Zel'dovich, Ya. B. and Novikov, I. D.: 1975, *Stroenie i evolutsiya Vselennoy*, Nauka, Moscow.

Zel'dovich, Ya. B. and Sunyaev, R. A.: 1980, *Pis'ma v Astron. Zh.* **6**, 737.