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ALPL-1 Newtonian Ephemeris of the Planetary System Spanning 4000 Years

by

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ABSTRACT

Numerical integration of the equations of motion of the Solar System Planets (except Mercury) is described. The continuous ephemeris obtained in result, spanning the time interval from 1918 BC to 2092 AD is presented.

1. Introduction

For the last several years, a few numerical integrations of Planetary System spanning long intervals of time have been performed. For example: the system of five outer planets was integrated over the time period of one million years (Cohen *et al.*, 1973), at JPL the simultaneous integration of nine planets and Moon spanning forty-four centuries was performed, obtaining the DE 102 ephemeris (Newhall *et al.*, 1983).

Such ephemerides have several important applications, for example in:
- Planetary System evolutional investigations,
- comparison with analytical theories of planetary motion,
- calculation of ephemerides and evolutional investigation of the Solar System small bodies,
- linking historical observations with the present ones.

Up to present our observatory has not had at its disposal a long time interval planetary ephemeris, giving the positions of the perturbative bodies necessary for evolutional investigations of comets, minor planets and meteors. In our actual studies in this field such data were required so we have attempted calculations of a suitable planetary ephemeris.

Before such calculations can be performed one has to solve a few important problems:

(a) choice of the force model,
(b) selection of the initial set of data,
(c) choice of the integration method,
(d) arrangement of the output data,
(e) estimation of the precision and accuracy of the integration.

After examination of all the above problems in connection with our present numerical possibilities we decided to carry out numerical integration of the Planetary System, spanning over 4000 years and producing the ALPL-1 ephemeris.

2. Description of the numerical integration

In the calculation of the ALPL-1 ephemeris we have accepted the Newtonian force model in a barycentric reference frame. The initial data set including the components of the position and velocity vectors, masses of the bodies for which the integration was performed was taken from the DE 102 ephemeris, for the epoch JED = 2440000.5 (except Saturn for which a slightly different value of its mass was assumed). As alternative we considered initial data sets obtained by Oesterwinter and Cohen (1972) or by Bretagnon (1982). But it seems to us that JPL DE 102 initial data set is the best one for cur purposes.

The integration was performed for the system of nine point-masses: Sun-Mercury, Venus, Earth-Moon, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. The restriction to the Newtonian model and the treatment of pairs: Sun-Mercury and Earth-Moon as single point-masses has followed from the necessity of minimization of the computer time and the amount of output information. For example in our case the integration of Mercury as a single body would increase the computer time by a factor 8, and increase the amount of output information by a factor 4.

The barycentric equatorial reference frame of the ALPL-1 ephemeris is close to the FK4 equator and equinox. The alignment of ALPL-1 with FK4 system may be carried out as follows:

$$\boldsymbol{r}_{\mathrm{FK4}} = \hat{R}_x(-0.''00029)\ \hat{R}_y(-0.''11718)\ \hat{R}_z(0.''11747)\boldsymbol{r}_{\mathrm{ALPL-1}}, \qquad (1)$$

where $\hat{R}(\alpha)$ indicates a rotation about the designated axis through the angle α.

The above transformation is based on the data from (Newhall *et al.*, 1983) taking into account the difference between reference frames of DE 102 and ALPL-1 ephemerides. Due to the global accuracy of the ALPL-1 ephemeris this may be neglected in many applications. The initial data in question are shown in Table 1.

Table 1

The reciprocal masses (taking the mass of the Sun as a unit) and initial data (in AU and AU/day) prepared for the ALPL-1 ephemeris

BODY	1/M	$X/\dot{X}$	$Y/\dot{Y}$	$Z/\dot{Z}$
VENUS	408523.5	.517063699599794E+0	.474368406692689 7E+0	.181328757634923 2E+0
		-.143007488698945 9E-1	.126946712356084 0E-1	.662373705902960 0E-2
EMBARY	328900.53	-.461685938478011 4E+0	-.826524664253153 3E+0	-.358458920395069 8E+0
		150002313384379 0E-1	-.730678860398821 5E-2	-.316865820390039 8E-2
MARS	3098710	383539031532443 3E+0	.135572087176038 9E+1	.612095248733994 2E+0
		-.130063496407967 2E-1	.409485264629121 1E-2	.222858971632560 1E-2
JUPITER	1047.355	-.499927739772123 4E+1	.182555333500658 7E+1	.905396320094241 2E+0
		-.292698035322549 9E-2	-.612595533084910 0E-2	-.255649549247339 8E-2
SATURN	3498.0	.897134714786358 2E+1	.264423133376608 1E+1	.705568491505007 2E+0
		-.191602490651866 8E-2	.489817156513222 8E-2	.210816545143660 1E-2
URANUS	22869	-.182758923858822 3E+2	.549967601447862 4E+0	.499217020182273 2E+0
		-.182867683133887 5E-3	-.376863103993199 1E-2	-.164870555820139 8E-2
NEPTUNE	19314	-.174019702933704 0E+2	-.231298114927761 7E+2	-.903683140163323 4E+1
		.255075046060755 1E-2	-.162742744999269 1E-2	-.730856830362398 1E-3
PLUTO	3000000	-.305381540905185 2E+2	.727117433360753 4E+0	.948471321802074 3E+1
		.170171793679402 9E-3	-.314659947081882 2E-2	-.104547144439239 8E-2
SMBARY	1/(1+M)*)	,391888464111161 6E-2	-.132474891045995 3E-2	-.622818293274268 7E-3
		,321185656014733 5E-5	.468862084036321 7E-5	.194121246392546 4E-5

*) M = 1/6023600

Numerical integration was performed by the Taylor series method formulated for the barycentric Newtonian N-body problem. This method is based on finding local solutions of differential equations of motion in the form of Taylor series with respect to time:

$$\boldsymbol{r}_i(t_0+h) = \sum_{j=0}^{n} \frac{\boldsymbol{r}_i^{(j)}(t_0)}{j!} h^j, \tag{2}$$

where:

$$i = 1, 2, \ldots, N, \quad \boldsymbol{r}_i^{(j)}(t_0) = \left.\frac{\partial^j \boldsymbol{r}_i}{\partial t^j}\right|_{t_0}, \quad n \text{ order of the method.}$$

If we write the equations of motion in the form:

$$\ddot{\boldsymbol{r}}_i = -k^2 \sum_{j=1}^{N}{}^{(i)} m_j \boldsymbol{r}_{ij} s_{ij}, \quad i = 1, 2, \ldots, N, \tag{3}$$

where:

$$s_{ij} = r_{ij}^{-3}, \qquad r_{ij} = |\boldsymbol{r}_{ij}|, \qquad \boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j, \tag{4}$$

and then use Eqs. (3) and the auxiliary equations:

$$\begin{aligned} r_{ij}\dot{r}_{ij} &= \boldsymbol{r}_{ij}\cdot\dot{\boldsymbol{r}}_{ij}, \\ r_{ij}\dot{s}_{ij} &= -3\dot{r}_{ij}s_{ij}, \end{aligned} \tag{5}$$

as well as the Leibnitz formula for n-th derivative of the product of two functions:

$$(uv)^{(n)} = \sum_{i=0}^{n} \binom{n}{i} u^{(n-i)} v^{(i)}, \tag{6}$$

we obtain the following recurrence relations for any order derivative $\boldsymbol{r}_i^{(j)}$ occurring in the solutions (2):

$$\begin{aligned} \boldsymbol{r}_i^{(n+2)} &= -k^2 \sum_{j=1}^{N}{}^{(i)} m_j \sum_{p=0}^{n} \binom{n}{p} \boldsymbol{r}_{ij}^{(p)} s_{ij}^{(n-p)}, \\ r_{ij}^{(0)} s_{ij}^{(n+1)} &= -3r_{ij}^{(n+1)} s_{ij}^{(0)} - \sum_{p=1}^{n} 3\left[\binom{n}{p-1} + \binom{n}{p}\right] r_{ij}^{(p)} s_{ij}^{(n-p+1)}, \\ r_{ij}^{(0)} r_{ij}^{(n+1)} &= \boldsymbol{r}_{ij}^{(0)} \boldsymbol{r}_{ij}^{(n+1)} + \sum_{p=1}^{n} \binom{n}{p} \left[\boldsymbol{r}_{ij}^{(p)} \boldsymbol{r}_{ij}^{(n-p+1)} - r_{ij}^{(p)} r_{ij}^{(n-p+1)}\right]. \end{aligned} \tag{7}$$

The above derivatives may be calculated when $\boldsymbol{r}_i^{(0)}(t_0)$ and $\boldsymbol{r}_i^{(1)}(t_0)$ are known. The subroutine in Fortran for the calculation of these formulae was written taking into account many earlier experiences and practical conclusions concerning the Taylor series method, described in (Black, 1973), (Roberts, 1975), (Sitarski, 1979), (Emslie, Walker, 1979), (Schwarz, Walker, 1982), (Fox, 1984) and others.

The automatic step length adjustment was carried out by the method proposed by Sitarski (1979). Basing on testing calculations concerning the optimization of machine time and accuracy for a given precision factor $\varepsilon = 10^{-14}$ at each step, we obtained the optimum order of the method equal to 20.

The parameters of the method, accepted, and kept constant in the whole integration) permitted the execution of one integration step in 5.5 seconds (JS-1032 computer, FORTHCLG compiler, double precision). Correctness of the integration was globally controlled by calculating and recording the values of the barycenter integral after every 100 steps. The initial and final values of this integral were 1×10^{-18} and 1×10^{-13} AU, respectively.

After each step we recorded on the magnetic disc the epoch and first ten derivatives $(\boldsymbol{r}_i^{(0)}, \boldsymbol{r}_i^{(1)}, \ldots, \boldsymbol{r}_i^{(9)})$ for each body. That took about 1.1

kbytes of the stored information. Such arrangement of the output data makes it possible to obtain the position and velocity vectors of each integrated body for arbitrary moment of time in a very simple way, without using any interpolation formulae.

3. Results

During the integration 65000 steps have been executed with mean length of about 22.5 days, within the interval from JED = 2485315.5 to JED = 1020632.5 (about 4000 years). There were two integrations carried out from the common initial epoch JED = 2440000.5 — forward and backward. In Table 2 the components of position and velocity vectors are presented for three selected epochs.

The results of integration are also presented in graphical form. Fig. 1 to Fig. 10 show the changes in osculating heliocentric ecliptical elements of orbits of the integrated bodies within the whole interval of four thousand years (the zero point on the time scale corresponds to the standard epoch J2000).

The graphs presented below were obtained by plotting every hundreth point of the ALPL-1 ephemeris expressed in orbital elements and connecting each of them with the following one by sections. The jump change in angular elements of the Earth — Moon barycenter follows from the passing of its orbital plane through the referrence one.

4. ALPL-1 accuracy estimation

A comparison between ALPL-1 and DE 102 ephemerides was made for selected epochs, for which we had DE 102 positions (Standish, 1984). The results of this comparison expressed in heliocentric equatorial spherical coordinates are presented in Table 3.

There are a few reasons for so significant differences between the positions of inner planets i.e.: the essentially restricted ALPL-1 force model when compared to the DE 102 one, the omission of Mercury (in the sense described above in the Section 2) and the integration of the Earth and Moon as a single body.

5. Conclusions

The ALPL-1 ephemeris is the first planetary ephemeris of that kind obtained at the Astronomical Observatory of A. Mickiewicz University. In spite of necessary restrictions and simplifications mainly connected with our numerical possibilities the ALPL-1 ephemeris is useful for evolu-

Table 2

Barycentric equatorial states of every integrated point-masses for selected epochs. Units are AU and AU/day

EPOCH JED = 2485315.5 (2092 JUN 17.0)

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
VEN	.20191181E+0	-.63162793E+0	-.29768122E+0	.19271213E-1	.55563762E-2	.12911544E-2
EMB	-.93515456E-1	-.92823445E+0	-.40210856E+0	.16851647E-1	-.14754062E-2	-.63977288E-3
MAR	.69729513E+0	.12147131E+1	.53914738E+0	-.11845799E-1	.68884457E-2	.34767656E-2
JUP	.35731151E+1	-.32632320E+1	-.14866379E+1	.52577802E-2	.52694493E-2	.21318648E-2
SAT	-.16528045E+1	.81686382E+1	.34515173E+1	-.57855648E-2	-.10446749E-2	-.18110312E-3
URA	.19718557E+2	-.34425174E+1	-.17860662E+1	.72604774E-3	.33718200E-2	.14671353E-2
NEP	-.25300295E+2	.14908406E+2	.67440828E+1	-.17219118E-2	-.24366927E-2	-.95479371E-3
PLU	.42229215E+2	.23011088E+2	-.55293139E+1	-.66628371E-3	.19094943E-2	.80520627E-3
SMB	-.25058654E-2	.15539922E-3	.16524874E-3	-.34030501E-5	-.47658003E-5	-.20010256E-5

EPOCH JED = 1762075.5 (112 APR 20.0)

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
VEN	.54687490E+0	.43837023E+0	.15948109E+0	-.13128368E-1	.13724622E-1	.69438319E-2
EMB	-.60378935E+0	-.74883415E+0	-.32833849E+0	.13578597E-1	-.93488003E-2	-.41003996E-2
MAR	-.11315907E+1	.10921821E+1	.53544890E+0	-.95651965E-2	-.77898498E-2	-.32698723E-2
JUP	.47529844E+1	-.13418539E+1	-.69868638E+0	.22121872E-2	.69370031E-2	.29281643E-2
SAT	.90258770E+1	.21666821E+1	.52424256E+0	-.16477338E-2	.50384850E-2	.21339898E-2
URA	-.14794767E+2	.99623473E+1	.45870460E+1	-.23666508E-2	-.30686611E-2	-.13116424E-2
NEP	-.23044116E+2	.17653626E+2	.78097427E+1	-.20356347E-2	-.22290075E-2	-.86202637E-3
PLU	.41920633E+2	.23406354E+2	-.53160163E+1	-.69492356E-3	.19003152E-2	.80911563E-3
SMB	-.52914247E-2	-.69483264E-3	-.85501822E-4	-.14380612E-5	-.78174407E-5	-.33075881E-5

EPOCH JED = 1020632.5 (-1918 MAY 5.0)

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
VEN	-.64220693E+0	.28685164E+0	.16685112E+0	-.93820680E-2	-.16798998E-1	-.67440676E-2
EMB	-.18433513E+0	-.91469328E+0	-.40614826E+0	.16619271E-1	-.27656751E-2	-.12405345E-2
MAR	.11435690E+1	.88299434E+0	.36240369E+0	-.80844464E-2	.10532930E-1	.50909373E-2
JUP	.20143811E+1	-.43124710E+1	-.19140067E+1	.68990080E-2	.30774931E-2	.11408650E-2
SAT	.59828482E+1	.62271318E+1	.23195248E+1	-.45033774E-2	.35131087E-2	.15982684E-2
URA	.28301968E+1	.17286844E+2	.75506856E+1	-.39319680E-2	.33999439E-3	.20912430E-3
NEP	.27149918E+2	.11646665E+2	.40929365E+1	-.13156407E-2	.26521235E-2	.11192483E-2
PLU	.28090709E+2	-.20555193E+2	-.14949078E+2	.22025112E-2	.18013845E-2	-.99832068E-4
SMB	-.51707336E-2	.98701108E-3	.62797235E-3	-.50852975E-5	-.40493188E-5	-.15946149E-5

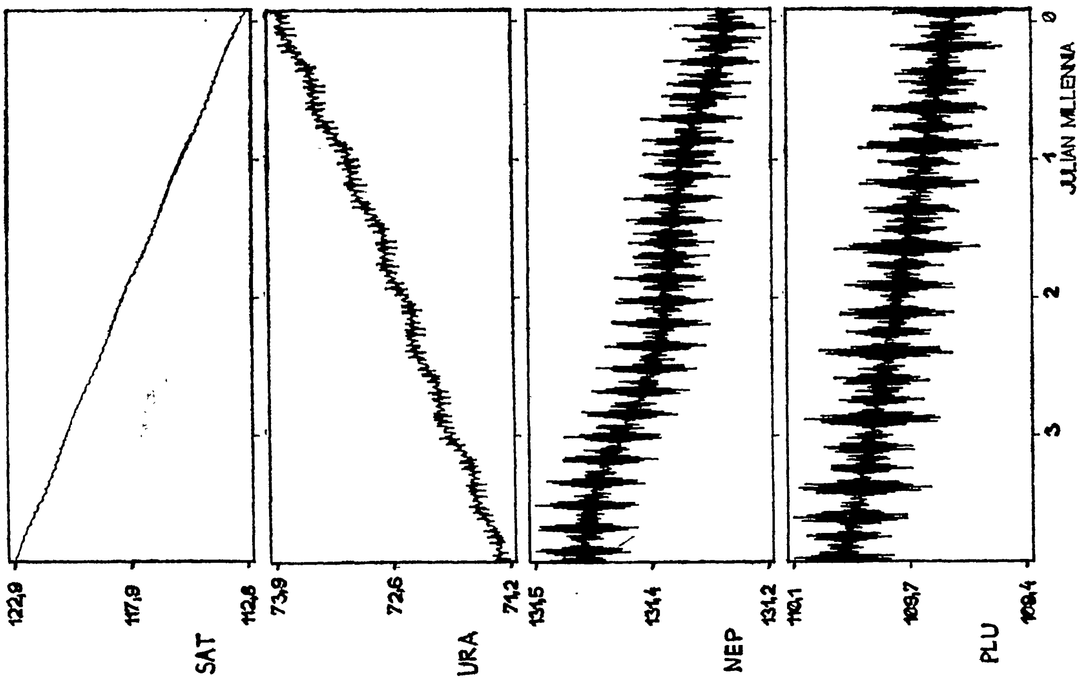

Fig. 2. Changes in the longitude of the ascending node.

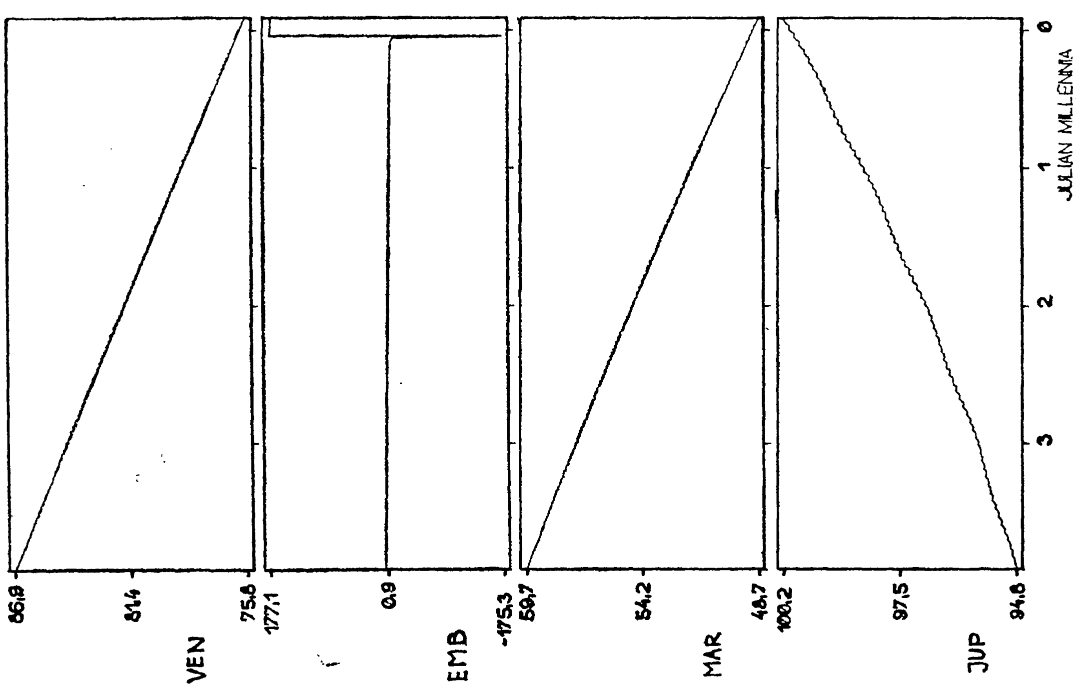

Fig. 1. Changes in the longitude of the ascending node.

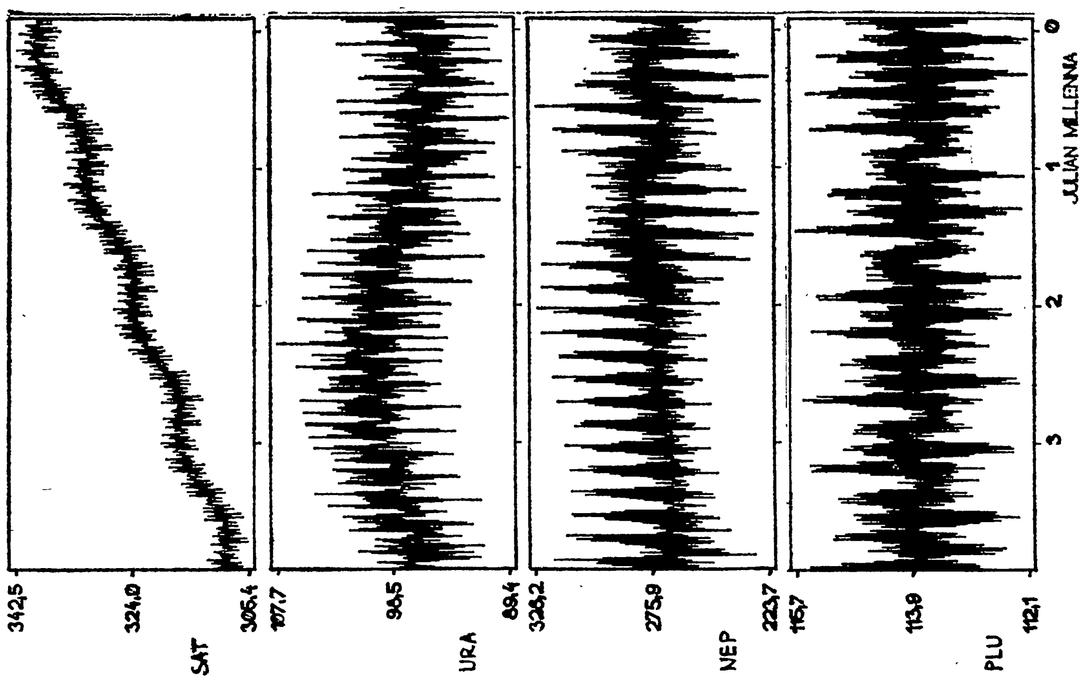

Fig. 4. Changes in the argument of the perihelion.

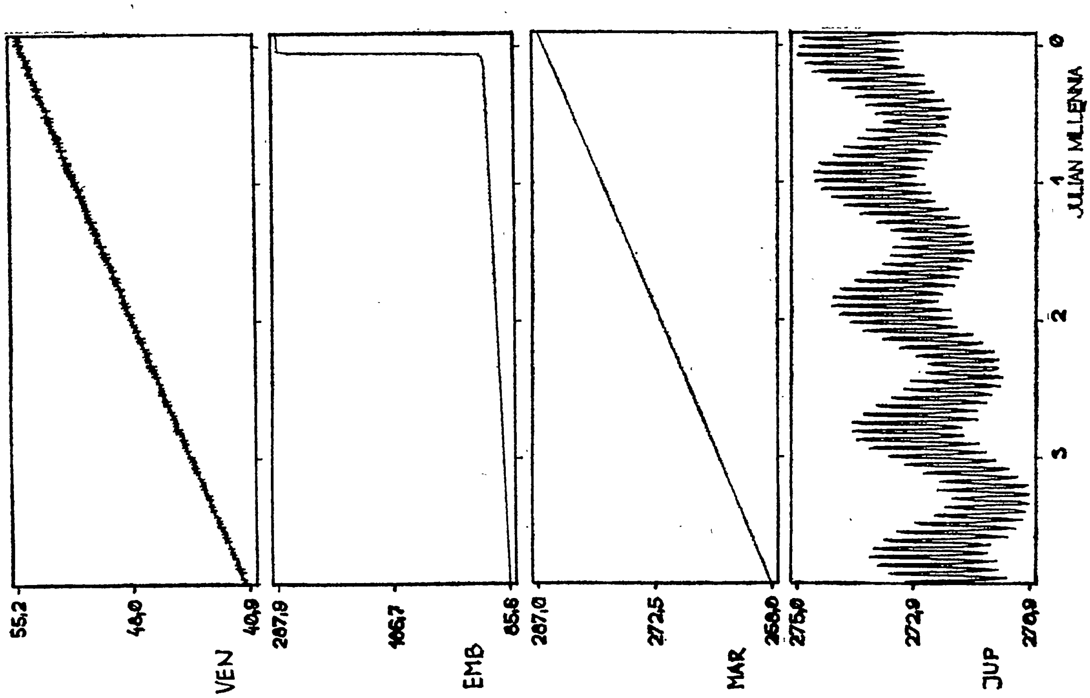

Fig. 3. Changes in the argument of the perihelion.

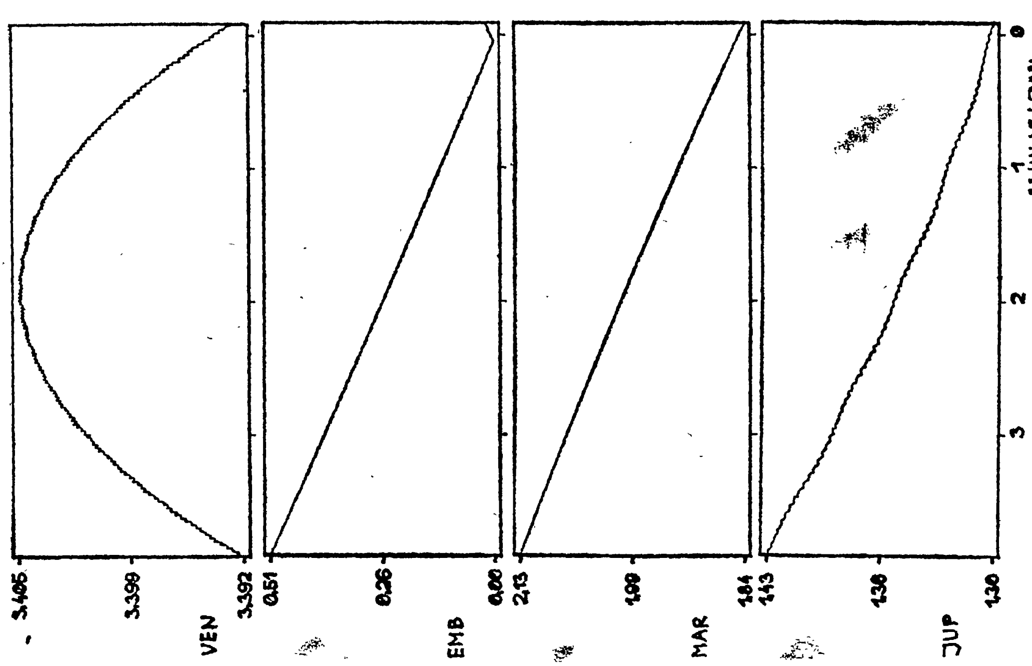

Fig. 5. Changes in the inclination.

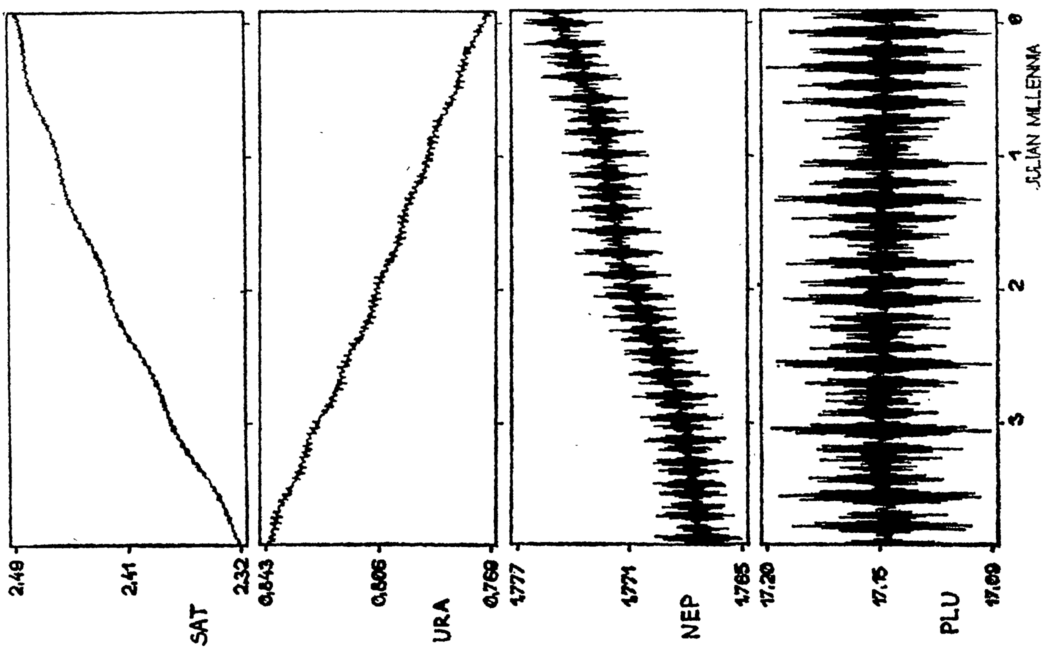

Fig. 6. Changes in the inclination.

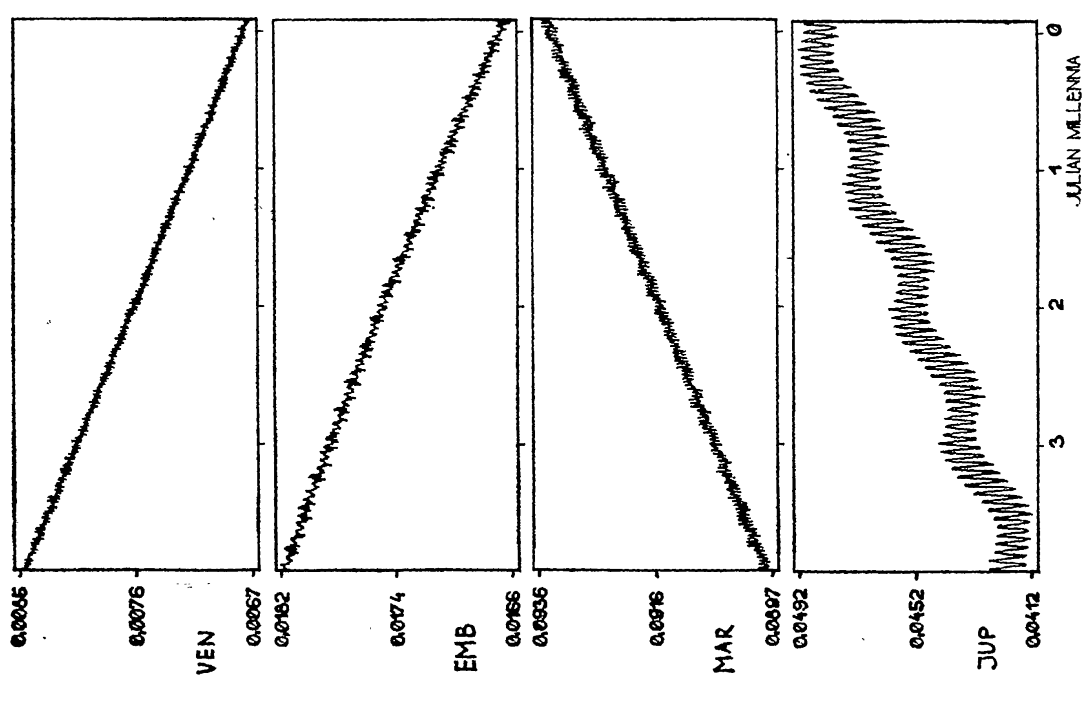

Fig. 7. Changes in the eccentricity.

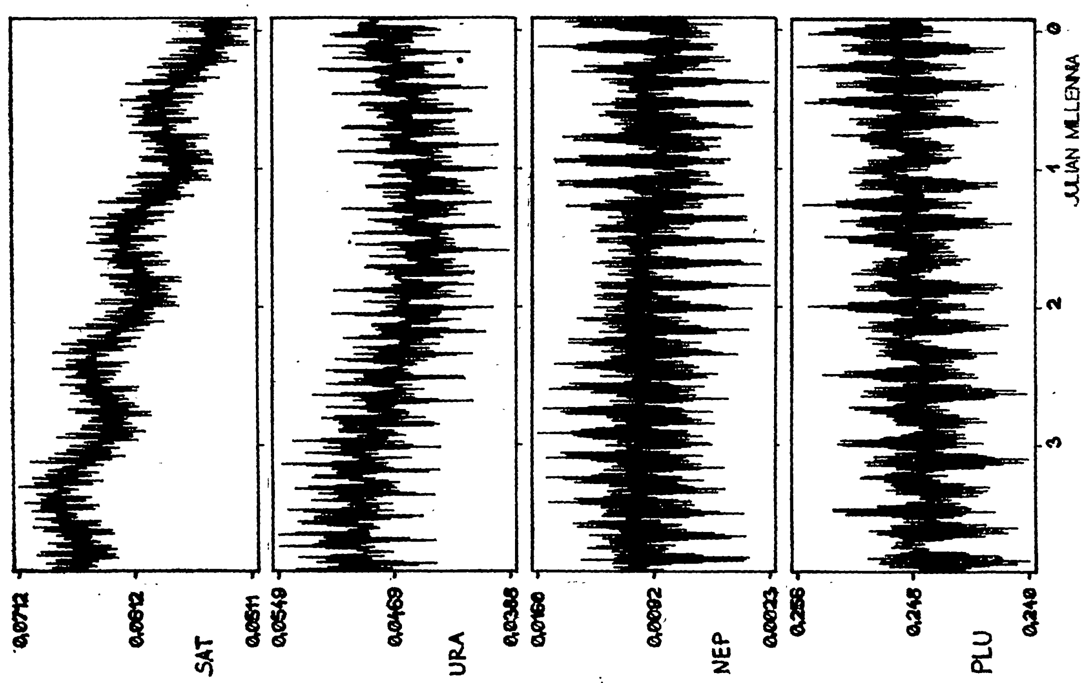

Fig. 8. Changes in the eccentricity.

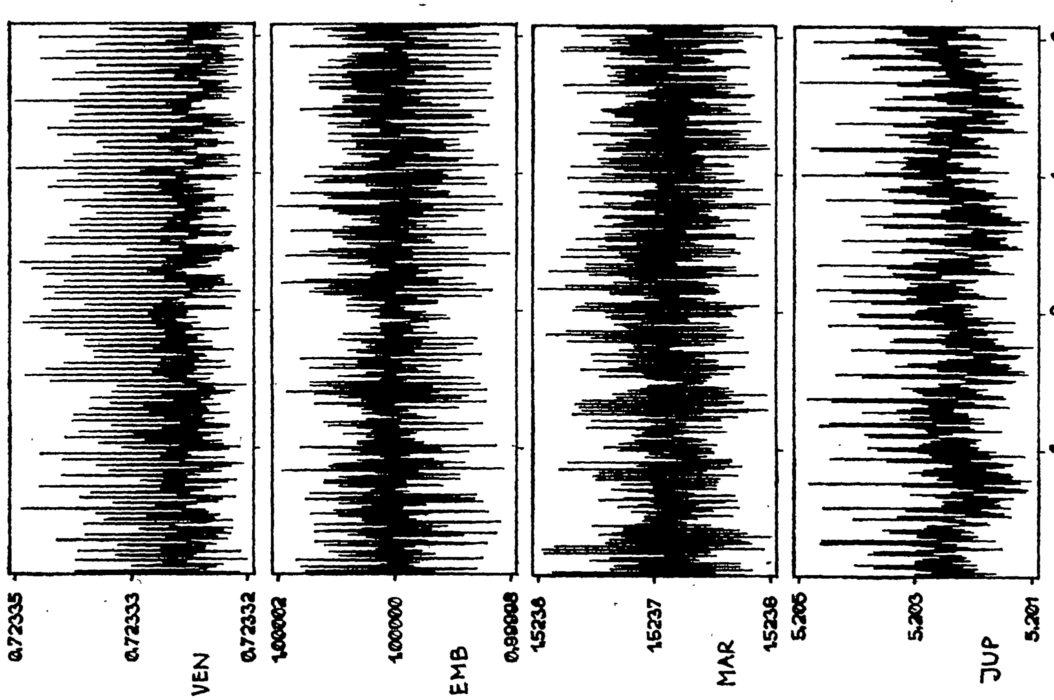

Fig. 9. Changes in the semimajor axis.

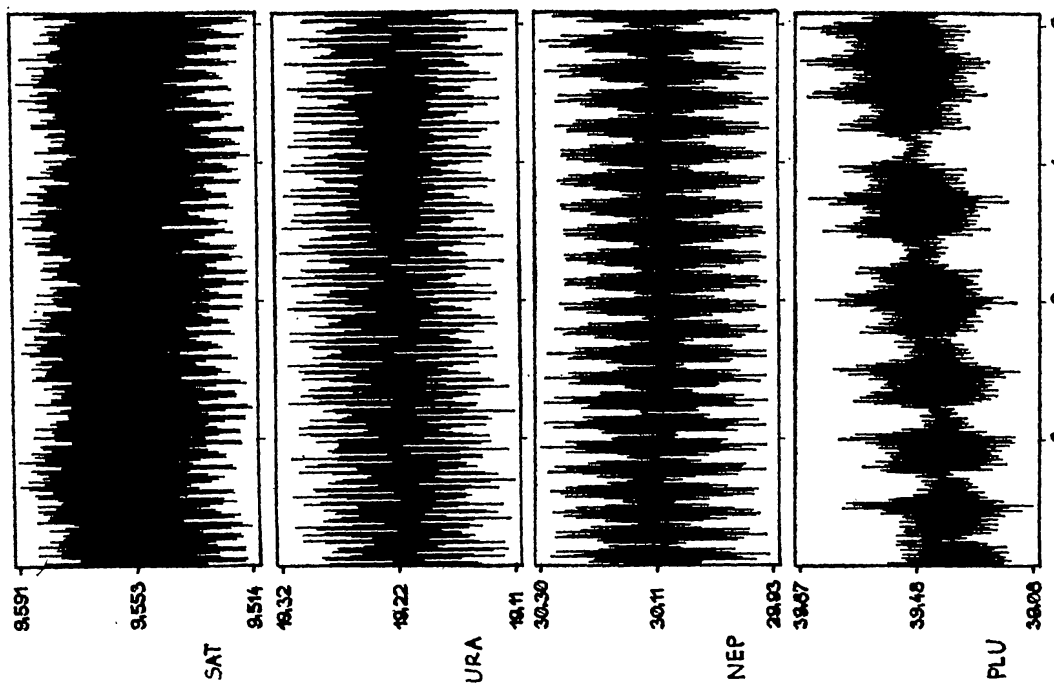

Fig. 10. Changes in the semimajor axis.

Table 3

Differences in right ascension, declination and radius vector of planets obtained from DE 102 and ALPL-1 ephemerides

	JED = 2485315.5 (2092 JUN 17.0)			JED = 1762075.5 (112 APR 20.0)			JED = 1222317.5 (-1366 JUL 11.0)		
	$\|\Delta\alpha\cos\delta\|$	$\|\Delta\delta\|$	$\|\Delta r\|$	$\|\Delta\alpha\cos\delta\|$	$\|\Delta\delta\|$	$\|\Delta r\|$	$\|\Delta\alpha\cos\delta\|$	$\|\Delta\delta\|$	$\|\Delta r\|$
VEN	49″	3″6	5.0E-6	570″	240″	3.8E-5	1040″	504″	1.7E-4
EMB	16.	0.3	3.4E-7	227.	52.	1.1E-5	406.	140.	4.7E-5
MAR	1.3	0.4	2.3E-6	10.	3.	5.5E-6	37.	14.	7.5E-5
JUP	0.01	0.01	6.4E-7	1.4	0.6	1.4E-6	3.5	1.1	7.4E-6
SAT	0.6	0.03	4.7E-7	8.4	3.4	2.5E-5	12.4	3.0	3.1E-5
URA	0.2	0.08	6.1E-6	4.0	1.3	1.0E-5	6.2	1.2	3.0E-5
NEP	0.2	0.07	5.1E-6	1.8	0.5	7.6E-7	3.1	1.1	4.6E-6
PLU	0.1	0.03	2.3E-6	1.2	0.5	3.1E-5	2.0	0.9	4.4E-5

tional investigation and ephemeridal calculation of motion of comets, minor planets and meteors. It has been confirmed by small differences between ALPL-1 and DE 102 positions of outer planets and by first calculations of the cometary motion. In these calculations only the ALPL-1 positions of perturbing bodies were used and the obtained results were in a good agreement with these obtained by other authors (Marsden, 1982).

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