THE ASTRONOMICAL JOURNAL VOLUME 92, NUMBER 1 JULY 1986

CAPTURE OF $V_\infty = 20\ \mathrm{km\ s^{-1}}$ INTERSTELLAR COMETS BY THREE-BODY INTERACTIONS IN THE PLANETARY SYSTEM

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Received 7 February 1986; revised 4 April 1986

ABSTRACT

Interstellar comets with velocities at infinity of 20 km s^{-1} can be captured by three-body interactions within the planetary system. A semianalytic method is presented indicating that Jupiter is the only planet capable of scattering these comets into bound orbits. The time-averaged cross section for this process seems to be roughly five times the area of Jupiter. With an observational upper limit on interstellar comets of $\sim 10^{13}$ pc^{-3}, the resulting capture rate amounts to only one comet every 60 Myr.

Although no clear example of an interstellar comet has ever been seen, their existence seems undeniable. If comets originated in the Uranus-Neptune region of the solar system (Kuiper 1951; Safronov 1969), then their ejection to the Oort cloud would necessitate a much greater number having been lost to interstellar space altogether. If comets originated in the outskirts of the solar nebula (Cameron 1962), then passing stars and molecular clouds will have removed a considerable number to interstellar space. The fact that no clearly hyperbolic comet has ever been seen allows a liberal upper limit for interstellar comet density to be put at $\sim 10^{13}$ pc^{-3} (Whipple 1975) corresponding to, at the local stellar density, every star having ejected $\sim 10^{14}$ comets.

It has been suggested that these interstellar comets might be captured by the solar system through a couple of processes. Tidal changes in the gravitational sphere of influence during encounters with molecular clouds may capture comets with low relative velocities (Clube and Napier 1984). To be effective, however, this process requires comet velocities at infinity to be less than ~ 1 km s^{-1}. Since the solar motion through the interstellar field is ~ 20 km s^{-1}, it would seem that this process would be rather rare. Scattering to bound orbits during three-body encounters with the planets has been investigated by Monte Carlo numerical techniques (Valtonen and Innanen 1982). There it was found that the capture cross section was very sensitive to the approach velocity V_∞ with appreciable capture taking place only for the lowest velocities ($\leqslant 1$ km s^{-1}). Getting good statistics to assess the capture cross section for velocities $V_\infty = 20$ km s^{-1} by this technique requires a fairly extensive computational program, even with orbits constrained to make close approaches to Jupiter. In this paper a convenient semianalytical method is presented for computing the capture cross section at high velocities.

Neglecting planetary influences for the moment, at a given distance from the Sun r within the solar system the kinetic energy per unit mass is given in solar-fixed coordinates by

$$v^2/2 = v_\infty^2/2 + Gm/r, \tag{1}$$

where v_∞ is the comet's velocity with respect to the solar system at infinity. The velocity at r is then

$$v(r) = (v_\infty^2 + 2GM/r)^{1/2}. \tag{2}$$

In the rare instance when a comet closely approaches a planet, it can exchange energy and lose enough to become bound to the Sun. Scattering, with large energy changes, can take place by virtue of the fact that the velocity in Eq. (2) can have various orientations with respect to the planetary velocity. Thus, for suitable arrangements of incoming velocity direction, planetary motion, and direction of deflection, the planets can scatter comets on unbound orbits into bound orbits. For instance, if incoming comets move parallel to planetary velocity, $\mathbf{v}_\mathrm{p}$ the relative velocity in the center of mass frame ($\sim$planet's frame) can be much less than, say, for a stationary planet. Provided that close encounter results in backscattering, then its velocity following scattering, as measured from the solar frame, can be less than escape velocity from the point in the solar system. The excess energy extracted from the comet is deposited in the orbital energy of the planet. (The exact opposite process was performed by the *Voyager* spacecraft in utilizing the gravitational boosts from Jupiter and Saturn to escape the solar system.) We next proceed to obtain analytical estimates of the capture cross section by transforming to the planet's rest frame, calculating a hyperbolic collision, and transforming back to solar-fixed coordinates.

For what follows we adopt the coordinate system shown in Fig. 1. Assume that the planet moves in a circular orbit around the Sun at velocity $\mathbf{v}_\mathrm{p}$ in the plane of the ecliptic. Denote the ecliptic longitude, measured from the Sun, of the incoming direction of the comet by $\lambda = 0$. Let the ecliptic latitude of the comet's motion be β_1. Since the solar motion is in the general direction of the star Vega($\alpha = 18^\mathrm{h}\,10^\mathrm{m}$, $\delta = +37°$), assume $\beta_1 = +60°$ for the inclination to the ecliptic. In these calculations we are assuming that gravitational deflection by the Sun is negligible. Averaged over an orbital period of a planet, the net deviation from the direction assumed here will be small. The longitude of the planet at encounter is denoted by λ_p. Let λ_2 and β_2 be the longitude and latitude of the comet's velocity following scattering. In solar-fixed coordinates, oriented as shown in Fig. (1) with the negative y axis in the direction of $\lambda = 0, \beta = 0$, we have for the components of the incoming velocity

$$\begin{aligned} v_x &= 0, \\ v_y &= v\cos(\beta_1), \\ v_z &= -v\sin(\beta_1), \end{aligned} \tag{3}$$

while those of the planet are

$$\begin{aligned} v_{\mathrm{p}_x} &= v_\mathrm{p}\cos(\lambda_\mathrm{p}), \\ v_{\mathrm{p}_y} &= v_p \sin(\lambda_\mathrm{p}), \\ v_{\mathrm{p}_z} &= 0. \end{aligned} \tag{4}$$

The relative velocity of the comet with respect to the planet is then $\mathbf{v}^* = \mathbf{v} - \mathbf{v}_\mathrm{p}$. The components of $\mathbf{v}^*$ are then

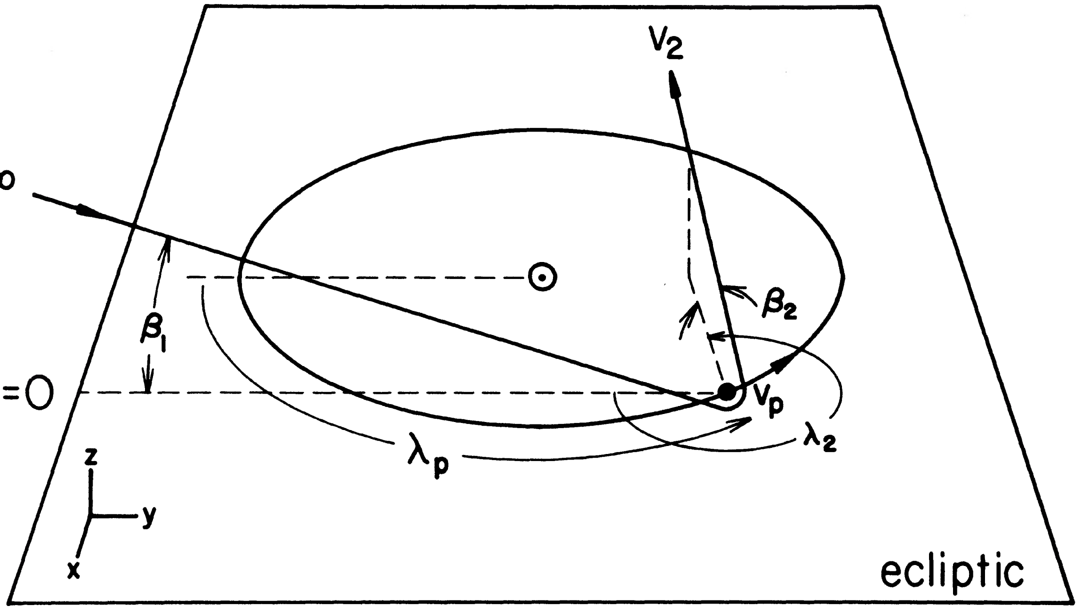

FIG. 1. Adopted coordinate system for comet-planet encounter. Gravitational focusing has been ignored (see the text). Incoming comet direction chosen as longitude $\lambda = 0$ and elevation above ecliptic $\beta_1 = +60°$. Encounter takes place when planet is at longitude λ_p. Scattered direction of comet denoted by λ_2, β_2. Incoming direction chosen to be the direction of the apex of solar motion.

$$v_x^* = -v_p\cos(\lambda_p),\quad v_y^* = v\cos(\beta_1) - v_p\sin(\lambda_p),\quad v_z^* = -v\sin(\beta_1). \tag{5}$$

The incoming comet velocity as measured in the planet frame is then

$$v^* = (v^2 + v_p^2 - 2vv_p\cos\beta_1\sin\lambda_p)^{1/2}. \tag{6}$$

Now, in the scattering by the planet as measured in the planet's frame, the magnitude of the post-scattering velocity v_2^* is equal to the prescattering velocity v^*. The direction of motion, however, may be significantly different. When vectorially added to the velocity of the planet's frame, the resultant velocity in the solar-fixed frame is

$$\mathbf{v}_2 = \mathbf{v}_2^* + \mathbf{v}_p. \tag{7}$$

If $\mathbf{v}^*$ and v_p are of comparable value, then certain scattering orientations can make v_2 even less than either v^* or v_p. The components of $\mathbf{v}_2$ are specified from the geometry of scattering in the solar frame by

$$v_{2_x} = v_2\cos\beta_2\sin\lambda_2,\quad v_{2_y} = -v_2\cos\beta_2\cos\lambda_2,\quad v_{2_z} = v_2\sin\beta_2. \tag{8}$$

The components of v_2^* are thus given by

$$v_{2_x}^* = v_{2_x} - v_{p_x},\quad v_{2_y}^* = v_{2_y} - v_{p_y},\quad v_{2_z}^* = v_{2_z} - v_{p_z}, \tag{9}$$

or

$$v_{2_x}^* = v_2\cos\beta_2\sin\lambda_2 - v_p\cos\lambda_p,\quad v_{2_y}^* = -v_2\cos\beta_2\cos\lambda_2 - v_p\sin\lambda_p,\quad v_{2_z}^* = v_2\sin\beta_2. \tag{10}$$

These components are subject to the constraint that post-scattering velocity equals the prescattering velocity or

$$v_2^* = [(v_{2_x}^*)^2 + (v_{2_y}^*)^2 + (v_{2_z}^*)]^{1/2} = v^*, \tag{11}$$

where v^* is given by Eq. (3). Substituting for the components of $\mathbf{v}_2^*$ and v^* and simplifying gives

$$[v_2^2 + v_p^2 + 2v_2v_p\cos\beta_2\sin(\lambda_p - \lambda_2)]^{1/2} = (v^2 + v_p^2 - 2vv_p\cos\beta_1\sin\lambda_p)^{1/2}. \tag{12}$$

Squaring both sides yields a quadratic equation in v_2:

$$v_2^2 + v_2[2v_p\cos\beta_2\sin(\lambda_p - \lambda_2)] + [2vv_p\cos\beta_1\sin\lambda_p - v^2] = 0, \tag{13}$$

whose solutions are given by

$$v_2 = -v_p\cos\beta_2\sin(\lambda_p - \lambda_2) \pm [v_p^2\cos^2\beta_2\sin^2(\lambda_p - \lambda_2) - 2vv_p\cos\beta_1\sin\lambda_p + v^2]^{1/2}. \tag{14}$$

The positive root corresponds to the physical solution.

For a given planet, scattering to orbits bound to the Sun can occur if

$$v_2 < v_{esc} = (2GM/r)^{1/2}. \tag{15}$$

This is subject to the constraint that the distance of closest approach to the planet in the orbit required to scatter from $\lambda = 0, \beta_1$ to λ_2, β_2 be greater than the planetary radius. Otherwise, comets are absorbed by the planets and do not scatter. In order to find the pericenter distance d in the hyperbolic orbit around a planet of mass M_p whose asymptotes make an angle Ψ, we follow the analysis of Symon (1960). In planet-fixed coordinates, the pericenter distance for a hyperbolic orbit is given by

$$d = a(e-1), \tag{16}$$

where the semimajor axis $a = GM_p m/2E$, and e is the eccentricity and m the comet mass. With encounter velocity v^* we have

$$E = 1/2\, m(v^*)^2. \tag{17}$$

Substituting gives

$$a = GM_p/(v^*)^2. \tag{18}$$

The asymptotes of the hyperbola make an angle α with the line between the planet and the pericenter as shown in Fig. 2. The deflection suffered is then given as

$$\Psi = \pi - 2\alpha. \tag{19}$$

Solving for α gives

$$\alpha = (\pi - \Psi)/2, \tag{20}$$

where α is given by

$$\cos\alpha = 1/e. \tag{21}$$

Thus, $e = 2/\cos(\pi - \Psi)$ and hence

$$d = GM_p/(v^*)^2[2/\cos(\pi - \Psi) - 1]. \tag{22}$$

Now Ψ is the angle between v^* and v_2^* in the planet's reference frame and given by

$$\cos\Psi = \frac{v_x^* v_{2_x}^* + v_y^* v_{2_y} + v_z^* v_{2_z}}{v^* v_2^*}. \tag{23}$$

Since $v^* = v_2^*$, substituting for components gives

$$\cos\Psi = \frac{v_p^2 - vv_2(\cos\beta_1\cos\beta_2\cos\lambda_2 + \sin\beta_1\sin\beta_2) + v_p v_2\cos\beta_2\sin(\lambda_p - \lambda_2) - vv_p\cos\beta_1\sin\lambda_p}{v^2 + v_p^2 - 2vv_p\cos\beta_1\sin\lambda_p}. \tag{24}$$

Thus, for a given initial direction ($\lambda = 0, \beta_1$), a given position of the planet in the orbit (λ_p), and an assumed direction of scattering (λ_2, β_2), one can readily calculate the velocity v_2 with which the comet emerges from its close encounter. Emergent directions for which $v_2 < v_{esc}$ and for which pericenter distance d is greater than the planetary radius then correspond to scattering into bound orbits.

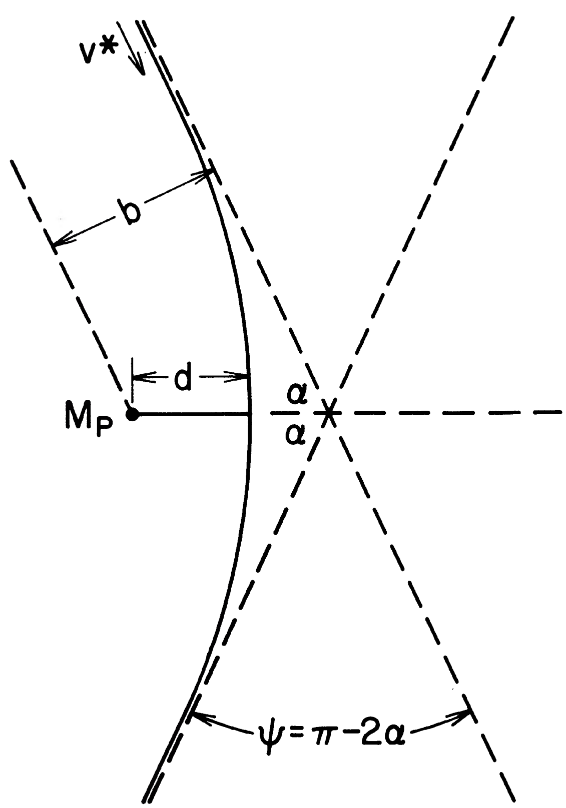

FIG. 2. Geometry of hyperbolic encounter between comet and planet in planet-fixed coordinates. M_p denotes planet position, b impact parameter, d closest approach, and v^* the comet-planet relative velocity. Dotted lines denote asymptotes which make an angle α with respect to the line joining planet and point of comet's closest approach. The angle Ψ is deflection suffered in planet's frame.

Systematically evaluating Eqs. (14), (15), (22), and (24) yields the result that Jupiter is the only planet that can scatter $v_\infty = 20$ km s^{-1} comets into bound orbits for the direction of approach of comets adopted here. Shown in Fig. 3 are the regions in λ_2, β_2 space where capture is possible. Interior to the curves parametrized by λ_p are scattering directions where $v_2 < v_{esc}$. Outside the range of λ_p shown, capture probability is negligible; that is, comet and planet must be moving in the same direction about the Sun for capture to take place.

To estimate the average capture cross section, one needs to compute the impact parameter b for each scattering situation and then add up the time averaged (i.e., averaged over orbital position) area of the regions with impact parameters resulting in bound orbits. Following Symon, the impact parameter is given as

$$b = ae\sin\alpha = a\tan\alpha, \tag{25}$$

or

$$b = GM_p/(v^*)^2\tan(\pi - \Psi)/2). \tag{26}$$

Solving this for the orbital positions (λ_p) of Fig. 3 yields the cross-sectional area depicted in Fig. 4. The time-averaged cross-sectional areas for scattering to bound orbits σ is thus estimated at ~ 5 times the area of Jupiter.

Having obtained a cross section, it is possible to estimate the rate at which interstellar comets are captured. The rate R is given by the product of interstellar comet density n, cross section, and relative velocity, or

$$R = n\sigma v_\infty. \tag{27}$$

With $n \sim 10^{13}$ pc^{-3}, $v_\infty = 20$ km s^{-1}, and $\sigma = 5A_J$ this yields

$$R = 1.62 \times 10^{-8}\ \text{yr}^{-1}, \tag{28}$$

or one every ~ 62 Myr.

The comets captured in this manner will resemble short-period comets. Following scattering, the semimajor axis of a bound comet leaving the orbit of Jupiter at a distance a_J from the Sun at a velocity v_2 is given by

$$a = \left(\frac{2}{a_J} - \frac{v_2^2}{GM_\odot}\right)^{-1}. \tag{29}$$

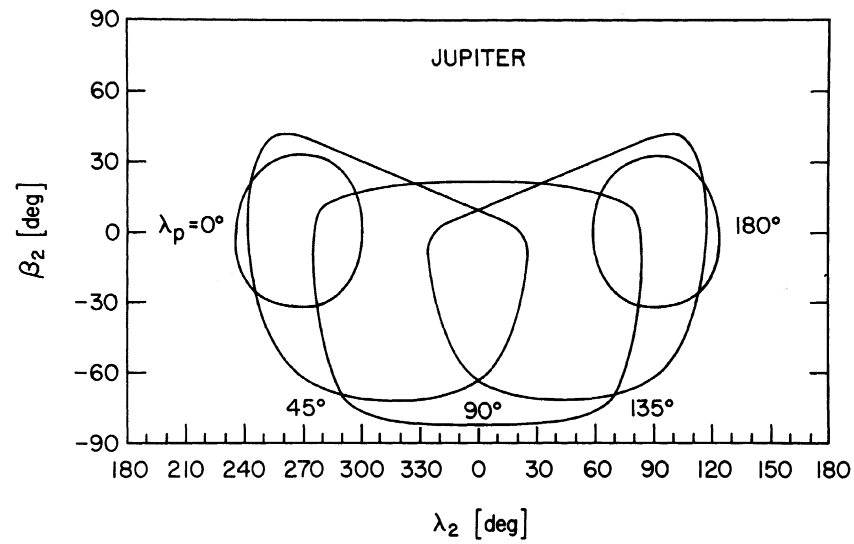

FIG. 3. Locus of scattered directions, λ_2 and β_2 resulting in bound comet orbits for various planetary longitudes λ_p at encounter. Comets scattered with directions inside the curves labeled by λ_p lose enough energy to become bound to the Sun.

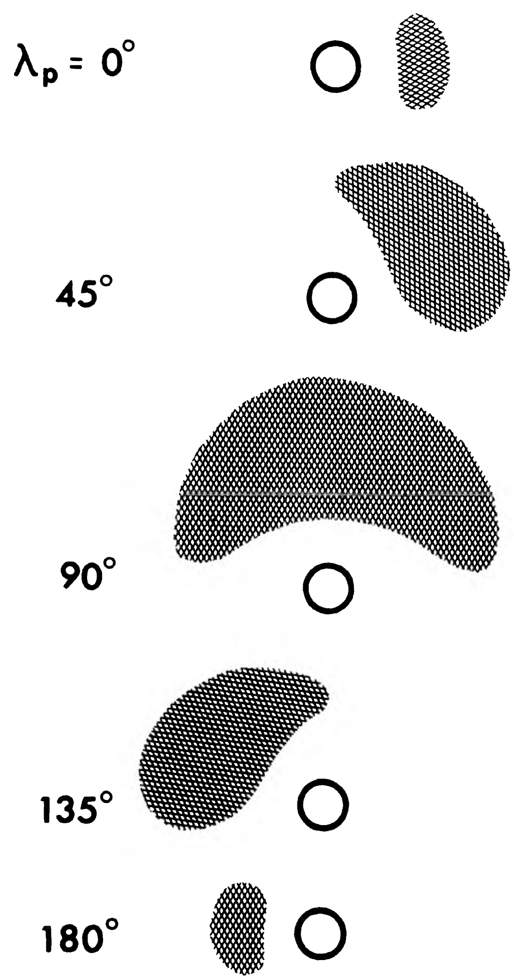

FIG. 4. Cross section for scattering to bound orbits for various encounter longitudes λ_p. The circles represent the disk of Jupiter and the cross-hatched regions represent the area within which scattering to bound orbits can take place. Figures are viewed in the direction of the incoming velocity of the comet and thus cross-hatched areas measured in the plane perpendicular to comet velocity.

Assuming a uniform distribution of $\sim 10\,000$ comets passing through the cross-hatched areas of Fig. 4 results in the distribution of semimajor axes shown in Fig. 5. Thus, the mean semimajor axes of captured comets will be ~ 10 AU.

Employing a crude analytical method to the problem of capture of $v_\infty = 20\ \mathrm{km\ s^{-1}}$ interstellar comets arriving from the direction of the solar motion by the three-body scattering within the planetary system, it was possible to demonstrate that Jupiter is the only planet massive enough to scatter comets into bound orbits. The cross section for this process is roughly five times the area of Jupiter when averaged over an orbital period. Using the maximum density of interstellar comets permitted by the lack of clearly hyperbolic objects, the capture rate can be fixed at a maximum value of one

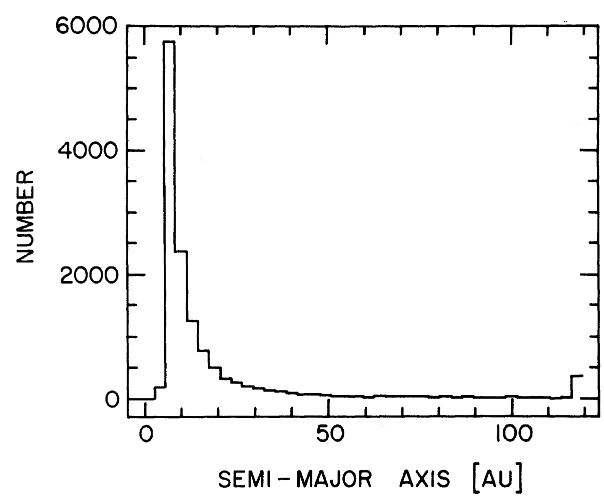

FIG. 5. Distribution of semimajor axes of $\sim 10\,000$ comets from a uniformly distributed population directed at the cross-hatched regions of Fig. 4. The mean semimajor axis of captured comets is roughly 10 AU.

comet per 60 million years. Although this rate may be enhanced within spiral arms or molecular clouds, this capture process would not seem to be important to the general dynamics of comets. Occasional capture of interstellar comets would, however, be capable of explaining the potential discovery of comets with isotopic distributions clearly indicating extrasolar origin.

The author is indebted to K. Innanen for helpful suggestions.

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