# CELESTIAL REFERENCE COORDINATE SYSTEMS: SUBMILLIARCSECOND PRECISION DEMONSTRATED WITH VLBI OBSERVATIONS

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## ABSTRACT

VLBI observations from the POLARIS and IRIS projects have been used to determine positions for 26 extragalactic radio sources. Nearly all the formal errors of the estimated coordinates are less than one millisecond of arc. Repeatability tests using subsets of the total data set indicate that systematic errors in the source coordinates are of the order of one half millisecond of arc. In order to attain submillisecond repeatability it was necessary to correct millisecond-level errors in the adopted IAU 1980 nutation model. Further improvements are expected as the number and distribution of VLBI baselines improve and as techniques are improved for eliminating atmospheric refraction effects.

## INTRODUCTION

The MERIT (Monitor Earth Rotation and Intercompare Techniques of Observation and Analysis) project (Wilkins 1980), set up by the International Astronomical Union (IAU) and the International Union of Geodesy and Geophysics to evaluate modern techniques for determining Earth orientation, recommended that a Conventional Celestial Reference System (CCRS) be defined in terms of the locations of specified extragalactic radio sources. In this paper we demonstrate the feasibility of defining such a coordinate system with errors of less than a millisecond of arc using very-long-baseline interferometry (VLBI) observations. Other recent work on radio-source coordinate determinations has demonstrated accuracies at the level of one to five milliseconds of arc (Fanselow *et al.* 1984). The nearly order-of-magnitude improvement reported here results from a combination of improved data and improved processing algorithms. The only other astrometric work in the literature that has exceeded millisecond-of-arc accuracy involved observations of close pairs of radio sources (Shapiro *et al.* 1979) and has not attempted the type of global sky coverage discussed here.

The VLBI data used in this study were obtained under projects POLARIS (Polar Motion Analysis by Radio Interferometric Surveying) (Carter *et al.* 1984) and IRIS (International Radio Interferometric Surveying) (Carter and Robertson 1984), whose objectives include long-term monitoring of variations in Earth rotation and nutation and establishment of reference-coordinate systems in celestial and terrestrial frames. The observations began in 1980 with a single baseline from MIT's Haystack Observatory in Massachusetts to Harvard's George R. Agassiz station in Texas, with 24 hr observing sessions every two weeks. The Onsala Space Observatory in Sweden participated about once per month. In 1981, with the completion of the Westford Observatory in Massachusetts, observing sessions were increased to once per week, and in September 1983 increased to once every 5 days. Since January 1984, with the completion of the Richmond POLARIS observatory near Miami, Florida, and the Wettzell Observatory in Bavaria, Federal Republic of Germany, the observing sessions have included four stations.

The observing stations are located exclusively in the northern hemisphere, and none of the sources observed are far south of the celestial equator. Most of the baselines are oriented predominantly in an east-west direction. This orientation is optimal for determining right ascensions, and is fairly good for determining declinations far from the celestial equator. However, sensitivity to declinations close to the equator requires baselines with large north-south extents, which are lacking in this data set. The effect of the lack of north-south baseline extent is seen in both the formal errors and the repeatability of declination values.

All the VLBI observations discussed here were made using the Mark III VLBI system (Rogers *et al.* 1983). The data were recorded simultaneously at two frequency bands ($X$ and $S$ band) for the purpose of removing dispersive propagation medium (e.g., ionosphere) effects.

The data were processed using a program developed at the National Geodetic Survey (NGS) called SOLVE-3, which runs on an HP-A900 minicomputer (Dillinger and Robertson 1986). Using efficient matrix-handling procedures and arithmetic and vector operations in hardware, SOLVE-3 is capable of processing the entire POLARIS/IRIS data set in about 2 hr, fitting some 5000 parameters to more than 80 000 observations. The efficiency of this program has enabled us to try a variety of ways of processing the data in a reasonable amount of time and therefore explore in some detail any systematic errors resulting from choices of different subsets of the data.

In addition to the source coordinates, the parameters estimated in processing the VLBI data included station coordinates, polar motion and UT1 corrections, nutation corrections, clock-error polynomial coefficients, and a correction to the zenith atmospheric refraction at each station for each observing session.

The data were processed with algorithms generally consistent with the MERIT standards (Melbourne *et al.* 1983), which specify that celestial coordinates are to be calculated at epoch J2000 using the IAU 1976 precession model and the IAU 1980 nutation model (with no aberration $E$ terms). The MERIT standards also specify the use of general relativistic corrections for propagation and time, the Wahr model for solid Earth tide displacements (Wahr 1981a), and a model for ocean loading site displacements. The only important deviations from the MERIT standards in our data-processing procedures are the absence of ocean loading corrections to the site positions, and the corrections that were made to the values given by the standard nutation series.

The need for corrections to the MERIT standard nutation series (Wahr 1981b) was first detected using the POLARIS/IRIS data (Carter *et al.* 1985; Herring *et al.*

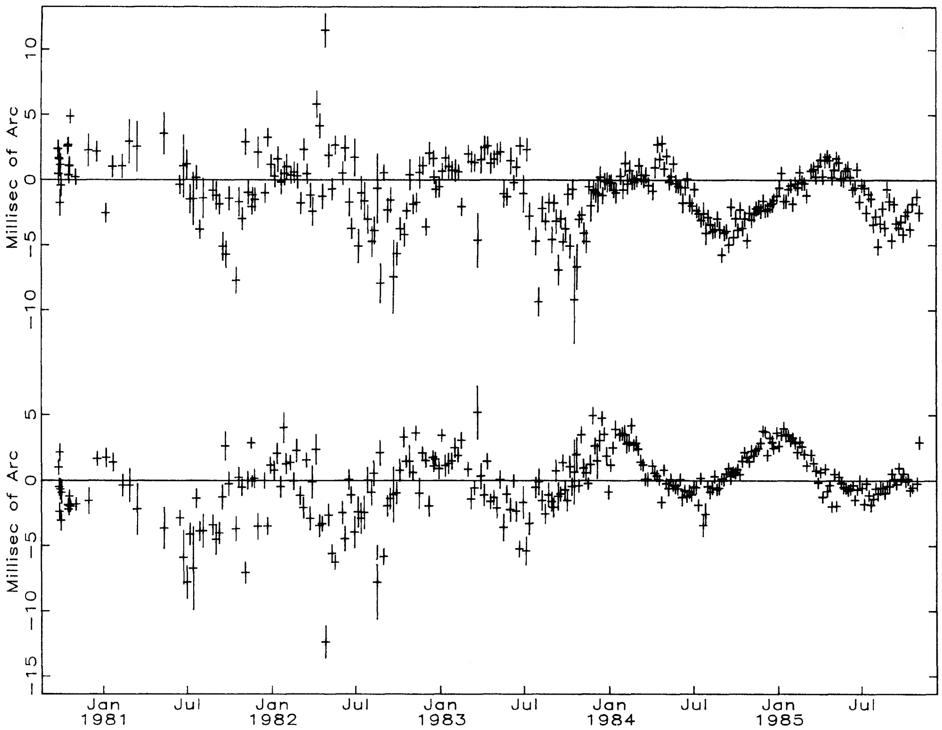


FIG. 1. Estimated corrections to the IAU 1980 nutation series. The corrections in longitude, scaled by the cosine of the mean obliquity, are shown in the upper plot, and corrections in obliquity are shown in the lower. The scatter of the corrections reduced substantially when intercontinental baselines became routine in January 1984.

1986). The errors in the standard series have been attributed in part to an error in the assumed value for the ellipticity of the core-mantle interface (Gwinn *et al.* 1986). The nutation corrections used in processing the VLBI data were determined by SOLVE-3 simultaneously with the source coordinates. These corrections, one value for nutation in longitude and one for nutation in obliquity for each 24 hr observing period, are shown in Fig. 1. The primary correction has an annual signature with an amplitude of about 2 ms of arc. There is also an apparent small secular drift probably resulting from an error in the long-period (18 yr) nutation component, an error in the precession constant, or both. Obviously, nutation errors of this magnitude must be removed in order to attain the submillisecond accuracy that the VLBI data are capable of achieving. Most of the millisecond-level differences between the source coordinates we have previously published (Carter *et al.* 1985) and the corresponding values in this paper result from the neglect of nutation corrections in the source coordinate results in the earlier paper.

One of the models included in the MERIT standards that is sometimes neglected in astrometric work is the deflection of electromagnetic radiation caused by the solar gravitational field. This effect is not small compared to the desired submillisecond level accuracy. It amounts to about 4 ms at 90 deg from the Sun, and does not reach 1 ms until the source is more than 152 deg from the Sun (see, for example, Robertson and Carter 1984).

Atmospheric refraction effects have been calculated from a model developed by Marini (Marini 1974) that employs surface meteorological measurements at each station. In addition, an additive correction term for the zenith refraction at each station was estimated from the VLBI observations simultaneously with the estimation of the other parameters.

No correction has been made for possible effects of non-point-like structure in the brightness distribution of the radio sources. Such structure could introduce time-varying systematic errors at the level of a fraction of a millisecond of arc, and could place fundamental limits on the ultimate accuracy of celestial radio-source coordinate frames (Cotton 1980; Robertson 1981; Kellerman 1982). These limits could be minimized by modeling the structures or by selecting can be reduced is beyond the scope of this paper.

## RESULTS

The source coordinate values and formal standard errors resulting from a combined solution including all of the POLARIS/IRIS data are shown in Table I. The origin of right ascension was defined by fixing the right ascension of 3C 273B. The distribution of the observed radio sources in the sky is shown in Fig. 2. The sources are well distributed in the northern hemisphere; observing sessions are currently being planned to use observatories in the southern hemisphere in order to observe radio sources located below the celestial equator.

The formal standard errors of the estimated source coordinates are in the range of a few tenths of a millisecond of arc,* for those sources that have more than about a hundred observations. Of course, formal standard errors are not a reliable guide to the total errors in the determination of a parameter. Rather, they represent a lower bound on the error, the bound that would be achieved in the absence of unmodeled systematic errors. To obtain a better understanding of the total errors in the source coordinates, we have divided the observations into subsets and examined the repeatability of the coordinates determined from these different subsets. Repeatability of the results from different subsets of data is also not a perfect measure of the true errors in the coordi-

*The errors or standard deviations in both right ascension and declination will be specified in milliseconds of arc in the remainder of this paper, and right ascension errors and standard deviations will be scaled by the cosine of source declination.

TABLE I. Source coordinate results from POLARIS/IRIS VLBI observations. The column labeled "sigma" shows the formal standard deviations of the estimates. The column labeled "scaled sigma" shows the right ascension standard deviations scaled by the cosine of the source declinations and converted to seconds of arc. The sources currently being observed are indicated by asterisks in the last column.

| Source name | | Right Ascension | | | | | Declination | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| IAU | Alternate | h | m | s | Sigma time(s) | Scaled Sigma (Arc sec) | d | m | s | Sigma (Arc sec) | Number of observations |
| 0106 + 013 | | 1 | 08 | 38.77107 | 0.00001 | 0.0001 | 1 | 35 | 00.3210 | 0.0006 | 4226* |
| 0212 + 735 | | 2 | 17 | 30.81339 | 0.00008 | 0.0003 | 73 | 49 | 32.6231 | 0.0004 | 5833* |
| 0224 + 671 | 4C 67.05 | 2 | 28 | 50.05152 | 0.00006 | 0.0003 | 67 | 21 | 03.0304 | 0.0005 | 525 |
| 0229 + 131 | | 2 | 31 | 45.89406 | 0.00001 | 0.0001 | 13 | 22 | 54.7188 | 0.0005 | 2029* |
| 0234 + 285 | | 2 | 37 | 52.40570 | 0.00001 | 0.0002 | 28 | 48 | 08.9917 | 0.0005 | 2486 |
| 0235 + 164 | | 2 | 38 | 38.93010 | 0.00001 | 0.0002 | 16 | 36 | 59.2779 | 0.0006 | 763 |
| 0300 + 470 | | 3 | 03 | 35.24221 | 0.00004 | 0.0004 | 47 | 16 | 16.2768 | 0.0006 | 101 |
| 0335 + 508 | NRAO 150 | 3 | 59 | 29.74728 | 0.00003 | 0.0003 | 50 | 57 | 50.1629 | 0.0004 | 3343 |
| 0528 + 134 | | 5 | 30 | 56.41674 | 0.00001 | 0.0001 | 13 | 31 | 55.1509 | 0.0004 | 2604* |
| 0552 + 398 | | 5 | 55 | 30.80565 | 0.00003 | 0.0003 | 39 | 48 | 49.1658 | 0.0003 | 7283* |
| 0742 + 103 | | 7 | 45 | 33.05949 | 0.00009 | 0.0014 | 10 | 11 | 12.6893 | 0.0025 | 54 |
| 0851 + 202 | OJ 287 | 8 | 54 | 48.87492 | 0.00001 | 0.0002 | 20 | 06 | 30.6405 | 0.0004 | 6394* |
| 0923 + 392 | 4C 39.25 | 9 | 27 | 03.01389 | 0.00002 | 0.0003 | 39 | 02 | 20.8513 | 0.0004 | 5903* |
| 1226 + 023 | 3C 273B | 12 | 29 | 06.6997 | — | — | 2 | 03 | 08.5988 | 0.0006 | 4873* |
| 1253 − 055 | 3C 279 | 12 | 56 | 11.16650 | 0.00007 | 0.0011 | − 5 | 47 | 21.5226 | 0.0025 | 12 |
| 1404 + 286 | OQ 208 | 14 | 07 | 00.39434 | 0.00001 | 0.0002 | 28 | 27 | 14.6889 | 0.0005 | 6814* |
| 1637 + 574 | | 16 | 38 | 13.45617 | 0.00005 | 0.0004 | 57 | 20 | 23.9792 | 0.0004 | 2025 |
| 1642 + 690 | | 16 | 42 | 07.84832 | 0.00007 | 0.0004 | 68 | 56 | 39.7564 | 0.0004 | 1036 |
| 1641 + 399 | 3C 345 | 16 | 42 | 58.80985 | 0.00002 | 0.0003 | 39 | 48 | 36.9940 | 0.0004 | 7901* |
| 1741 − 038 | | 17 | 43 | 58.85610 | 0.00003 | 0.0005 | − 3 | 50 | 04.6127 | 0.0015 | 103 |
| 1803 + 784 | | 18 | 00 | 45.68339 | 0.00015 | 0.0004 | 78 | 28 | 04.0183 | 0.0003 | 3467* |
| 1928 + 738 | | 19 | 27 | 48.49438 | 0.00019 | 0.0008 | 73 | 58 | 01.5712 | 0.0010 | 57 |
| 2134 + 004 | 2134 + 00 | 21 | 36 | 38.58630 | 0.00001 | 0.0001 | 0 | 41 | 54.2171 | 0.0005 | 4023* |
| 2200 + 420 | VR 422 201 | 22 | 02 | 43.29128 | 0.00002 | 0.0003 | 42 | 16 | 39.9818 | 0.0004 | 5741* |
| 2216 − 038 | | 22 | 18 | 52.03765 | 0.00002 | 0.0004 | − 3 | 35 | 36.8763 | 0.0010 | 94 |
| 2251 + 158 | 3C 454.3 | 22 | 53 | 57.74790 | 0.00001 | 0.0001 | 16 | 08 | 53.5636 | 0.0005 | 5084* |

nates, since there could be errors common to all of the subsets. However, processing separate subsets of data is a valuable technique for detecting seasonal or other time-varying systematic error components.

The differences between the source coordinates that resulted from selecting various subsets of the data are summarized in Table II. Three different experimental strategies were employed to select subsets of data. In the first experiment the data were divided into three sets which were interleaved in time. Each subset contained every third observing session. The second experiment was designed to look for time variations in the source coordinate values. The data were divided into uniform 2 month intervals starting at the beginning of 1984 when intercontinental baselines were first available in nearly all data sets. The final experiment was designed to detect seasonal effects. The data were divided into four sets

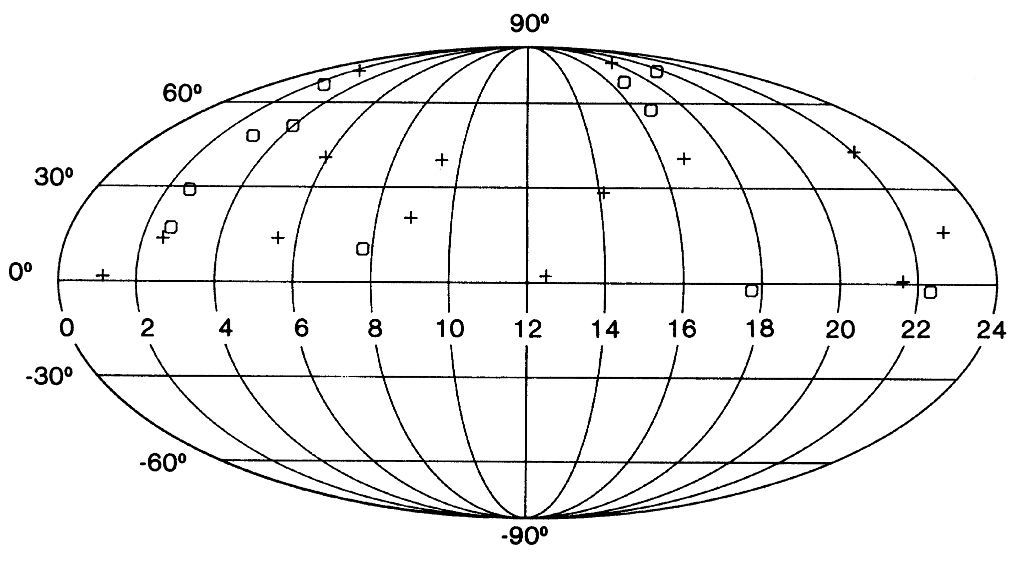


FIG. 2. Chart of the locations of the radio sources shown in Table I. The crosses represent sources that are presently being observed. The circles represent sources that have been observed but for various reasons are not presently included in the observing schedule.

TABLE II. Rms scatter in source coordinates from different subsets of the IRIS VLBI data set.

| | rms scatter in R. A. (milliseconds of arc) | rms scatter in Dec. (milliseconds of arc) | Approx. no. of observations per set |
|---|---|---|---|
| Interleaved experiment | 0.2 | 0.5 | 26 000 |
| Time-sequence experiment | 0.3 | 0.6 | 8 000 |
| Seasonal experiment | 0.4 | 0.5 | 20 000 |

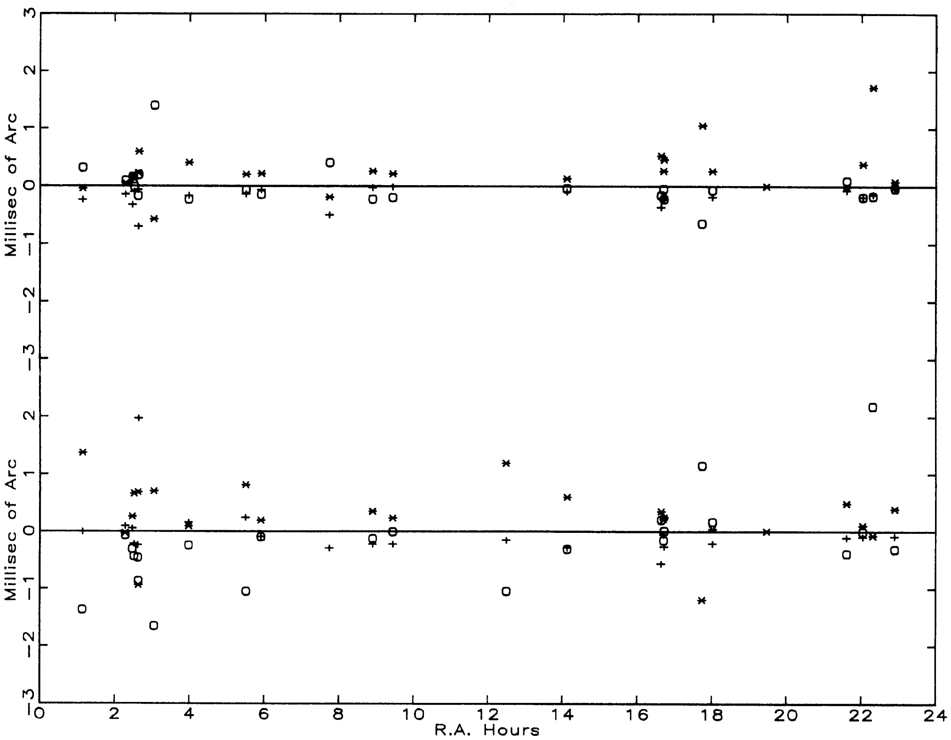


FIG. 3. Scatter of the source coordinate values from the three interleaved subsets, plotted as differences from the mean against right ascension. Righ ascension is shown in the upper plot, declination in the lower. The different symbols distinguish values from the three different solutions.

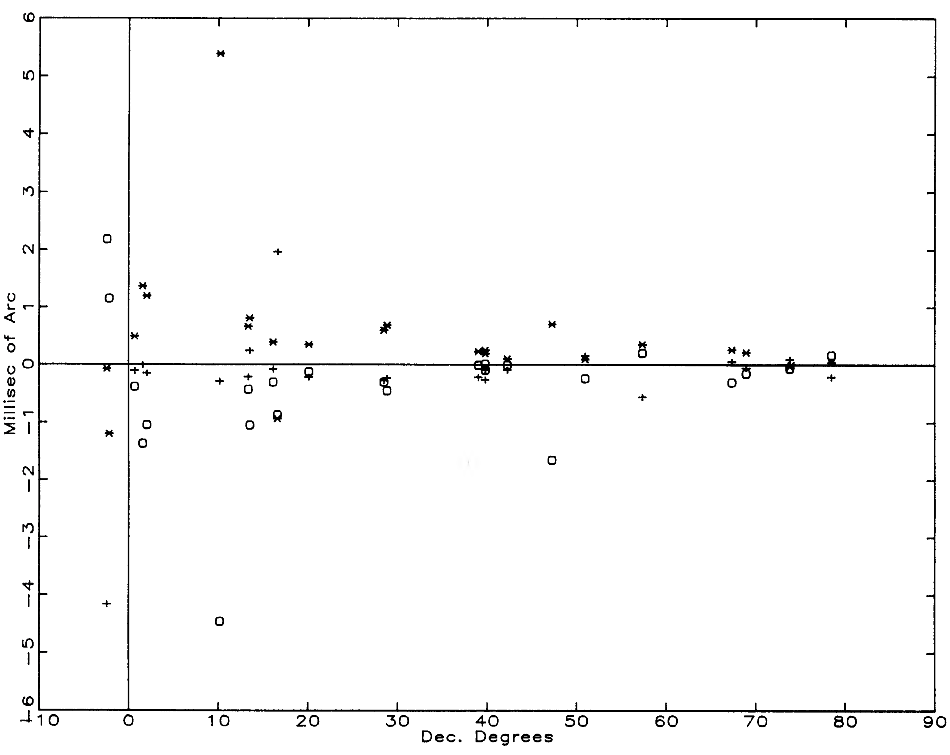


FIG. 4. Scatter of source declinations from the interleaved sets plotted against declination, showing the increased uncertainty in declination near the celestial equator.

according to the months in which the observing sessions were made; one set included all data taken from March through May, the second June through August, the third September through November, and the last December through February. In order to maintain a consistent origin for the nutation corrections in the separate solutions, each solution also contained a single reference day in common. The values of weighted root-mean-square (rms) scatter listed in Table II suggest that the errors in the source coordinates determined from the full set of observations are less than half a millisecond of arc.

The scatter of the three sets of coordinates resulting from the interleaved subset solutions is shown graphically in Fig. 3, plotted as a function of source right ascension. The declination dependence of the values is shown in Fig. 4, and the expected poorer sensitivity at the celestial equator is clearly seen. Similar determinations for the two other subsets are shown in Figs. 5 and 6. The plots exhibit some seasonal or time-varying component to the systematic errors in the source coordinates, but only at the level of a fraction of a millisecond of arc, not much above the noise level of the values. A major part of the systematic errors probably results from inadequacies in the atmospheric refraction models used in processing the data. No water vapor radiometer data were employed (Fallon 1986); the refraction was calculated from a model based only on surface meteorological measurements.

In spite of the relatively crude atmospheric refraction modeling employed in reducing the VLBI data the systematic errors in the resulting source coordinates as determined by these comparisons were substantially less than one millisecond of arc, corresponding to an average error in the measured VLBI time delays of only a few centimeters.

A better measure of the true errors in the source coordinates would be obtained if it were possible to compare values from completely different measurement techniques. Regrettably, conventional ground-based astrometry does not come within orders of magnitude of the accuracy necessary for such comparisons. Even space-based observing techniques, such as the *Hubble Space Telescope* and the *Hipparcos* astro-

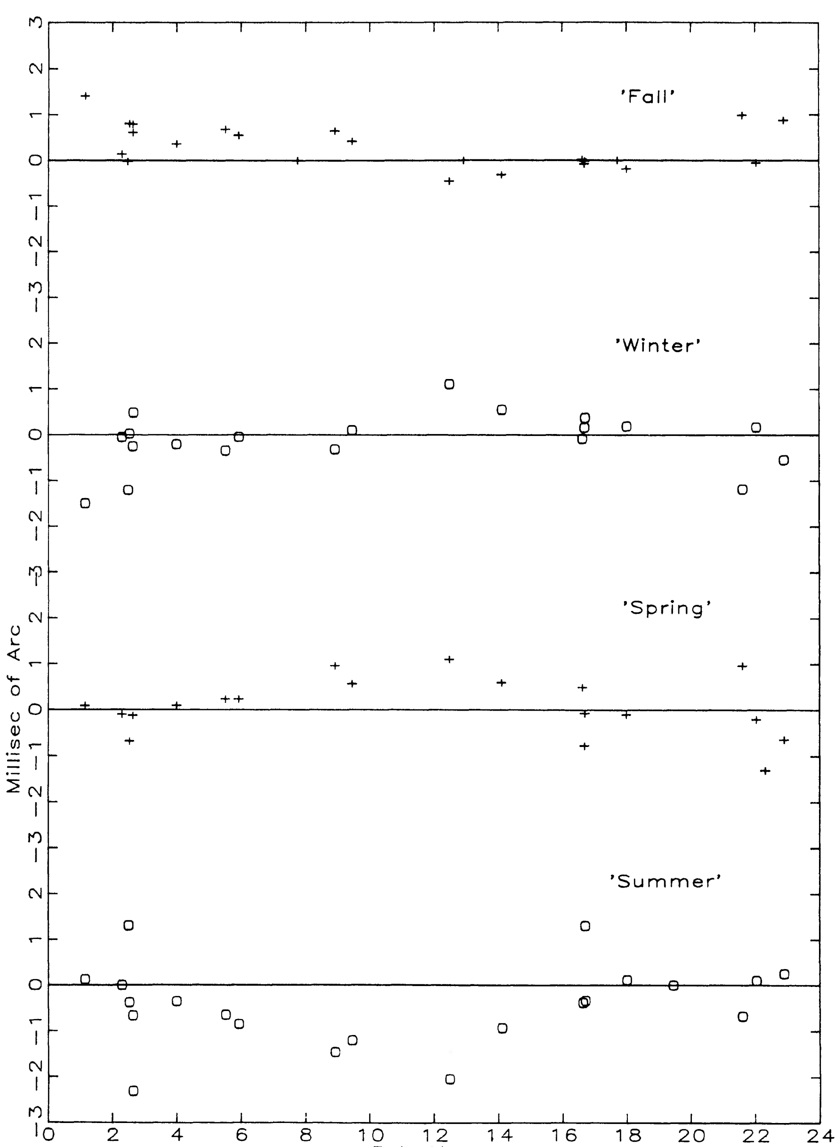


FIG. 5. Scatter of the source declinations for each of the four "seasonal" subsets. Crosses and circles are used alternately to separate the groups more clearly.

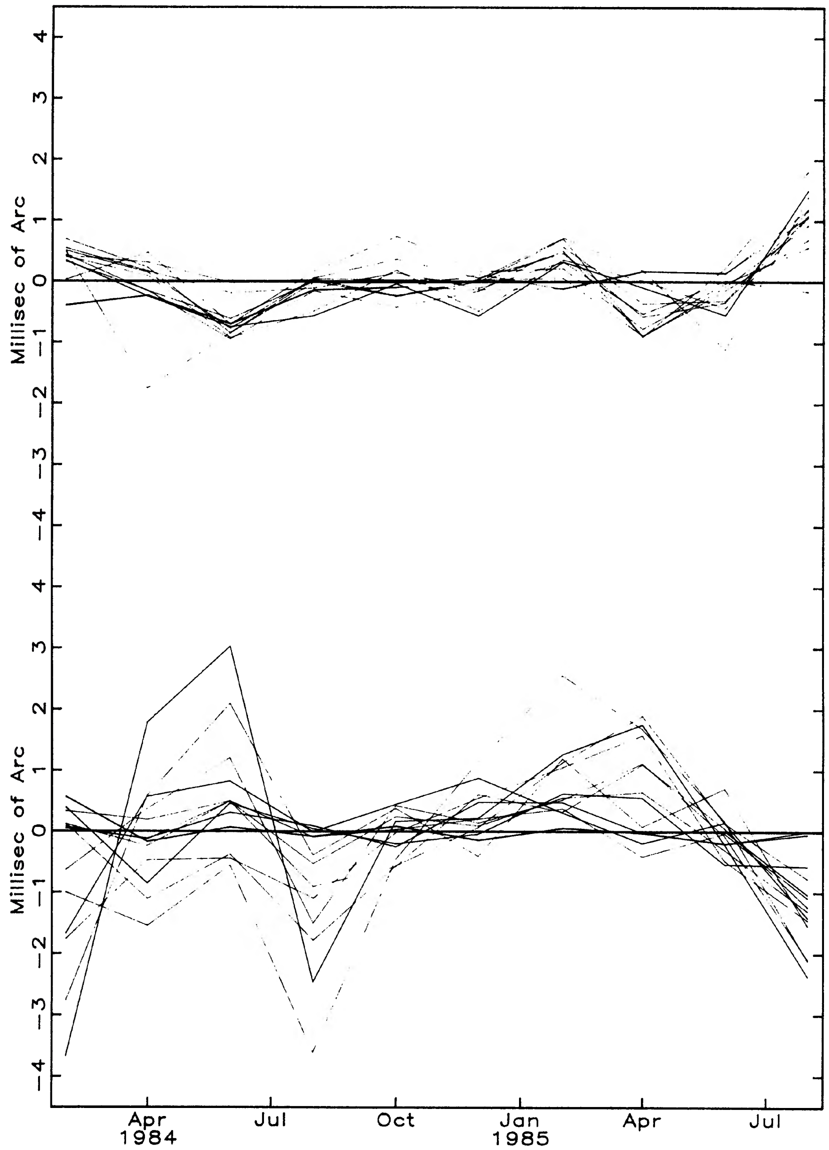


FIG. 6. Scatter of the source coordinates for 14 sources from the time-sequenced subsets. Right ascension is shown in the upper plot, declination in the lower. The values for each individual source are shown by a connected line.

metric satellite, are only expected to attain accuracies of the order of several milliseconds (Hemenway *et al.* 1985). Therefore VLBI observations represent the only measurements capable of reaching submillisecond accuracy in the foreseeable future. Comparisons of subsets of VLBI observations are thus the best means now available for demonstrating submillisecond accuracy levels. Only with the advent of large space telescopes or interferometers (see, for example, Reasenberg 1985) will optical observations exceed the accuracy demonstrated with ground-based VLBI observations. Of course, the problem of whether the optical and radio emissions are spatially coincident would then need to be addressed.

## CONCLUSIONS

Our solutions demonstrate the ability of VLBI observations to determine a set of radio-source coordinates with repeatability at the level of a fraction of a millisecond of arc. This repeatability is substantially better than any previously reported, and is orders of magnitude better than can be obtained with conventional optical techniques.

The coordinates given here must be considered to be preliminary, because of the *ad hoc* treatment of the nutation errors. A final definition of a celestial coordinate system with accuracies at the submillisecond level will have to wait until international agreement is achieved on how best to make these corrections, especially corrections to the long-period (18 yr) terms.

Further refinements in radio-source coordinates, especially the declinations of low-declination sources, can be expected in the near future as baselines with greater north-south extent become available. In pursuit of this objective, observing sessions including Hartebeesthoek Observatory, South Africa, and European and North American stations are planned for January and February 1986. Major improvements in accuracy can be expected only if efforts to sense or model atmospheric refraction effects at the centimeter level are successful, in which case an additional order-of-magnitude improvement in accuracy might be atttained.

We wish to acknowledge the pioneering contributions of the Massachusetts Institute of Technology, Haystack Observatory, and NASA Goddard Space Flight Center to the development of VLBI. The observational phases of these projects require dedicated and creative efforts by too many individuals to list here, but we want to recognize the critical contributions of Jesse James and the staff at the George R. Agassiz Station, John Webber and the staff at the Westford Observtory, Jim Martin and the joint NGS and U. S. Naval Observatory staff at the Richmond Observatory, Richard Kilger and the staff at the Wettzell Observatory, and B. Ronnang and the staff at the Onsala Space Observatory. We thank Robert Phillips and James Campbell and their staffs at the Haystack and Max Planck Institute correlator facilities for their efficient processing of this massive set of observational data. We also wish to thank Mike Abell and the staff of the NGS VLBI Data Reduction Center for their efforts in processing the massive quantity of data.

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