

The evolution of a single and a binary helium star of $2.5 M_{\odot}$ up to neon ignition

G.M.H.J. Habets*

Astronomical Institute “Anton Pannekoek”, University of Amsterdam, Roeterstraat 15, 1018 WB Amsterdam, The Netherlands

Received October 8, 1985; accepted February 24, 1986

Summary. The evolution of a single helium star of $2.5 M_{\odot}$ ($Z = 0.03$) is calculated from the helium Zero Age main sequence up to off-centre neon ignition.

In order to fulfil the Schwarzschild criterion for the onset of convection we allow the convective helium-burning core to grow in mass. This implies that we will find a larger initial $C - O$ core mass after core-helium burning than in calculations with fixed convective helium-burning core mass.

A mass-exchange phase to a $17 M_{\odot}$ hydrogen star companion and the subsequent evolution up to neon ignition of the $2.2 M_{\odot}$ mass-exchange remnant are calculated. The $2.5 M_{\odot}$ helium star produces a $C - O$ core mass larger than the Chandrasekhar limiting mass and a steep density gradient near the edge of this core is built up during carbon-shell burning (both as a single star and in the binary). Such a massive $C - O$ core will collapse whereby the steep density gradient may favour the development of a supernova explosion (SN). The single and binary helium stars are both expected to terminate life as a neutron star.

The orbital dimensions of the binary system after a symmetric SN explosion of the helium star are calculated (neglecting effects of the impact of the SN shell on the companion). These dimensions appear to resemble those of low-orbital-eccentricity B-emission (Be)/X-ray binaries. It is shown that these binaries may be formed after a case BB mass-exchange phase. For the known Be/X-ray and some other X-ray binaries the pre-SN orbit and the neutron star progenitor masses are calculated (assuming circular pre-SN orbits, a symmetric SN explosion, and neglecting the effects of impact of the SN shell). The assumption of a circular pre-SN orbit is found to be consistent with the observations. Our calculations thus confirm the model of the formation of Be/X-ray binaries by evolution with mass transfer in intermediate-mass binaries.

Key words: structure of stars – evolution of stars – close binary evolution – case BB of mass exchange – B-emission/X-ray binaries

1. Introduction

Paczynski (1971a), Arnett (1972a, b; 1978a, b), Savonije and Takens (1976), Woosley et al. (1980, 1984), Nomoto (1980a, b; 1981; 1982a, b; 1984a, b, c, e; 1985), and Delgado and Thomas

(1981) have calculated the advanced evolution of helium stars with masses of $2.0 M_{\odot}$ and larger or of hydrogen stars with helium cores in the same mass range. In the lower mass range (up to about $3 M_{\odot}$) the outer radii of the helium stars undergo considerable expansion, which in a binary can cause the onset of a second phase of mass transfer [De Greve and De Loore (1977); Delgado and Thomas (1981); see also Iben and Tutukov (1985) and Iben (1985) for cases of common envelope evolution].

An important question is: for which masses do these stars – even after a second phase of mass transfer in a binary – still collapse to a neutron star. As a first step to the study of this problem we calculate in full detail the behaviour with time of the radius, the core mass, and other parameters of a $2.5 M_{\odot}$ single helium star during its advanced evolution and compare this evolution with that of a $2.5 M_{\odot}$ helium star in a binary system, which during its advanced evolution appears to transfer mass to its companion.

We investigate whether or not the cores of the $2.5 M_{\odot}$ helium stars will collapse and form neutron stars [as is suggested by the calculations of Woosley et al. (1980), Hillebrandt (1982), Hillebrandt et al. (1984), Iben and Tutukov (1985), Iben (1985), Wilson et al. (1985), cf. Habets (1984), but contradicted by the results of Delgado and Thomas and by De Greve and De Loore]. Our results (Sects. 3 and 4 below) confirm the evolution towards core collapse.

The evolution from the helium Zero Age main sequence to the onset of neon burning of the $2.5 M_{\odot}$ helium stars is described in Sects. 3 and 4. The mass exchange in a binary consisting of a $2.5 M_{\odot}$ helium star and a $17 M_{\odot}$ hydrogen star is calculated for the case in which the $2.5 M_{\odot}$ star fills its Roche lobe during the carbon-shell burning phase (Sect. 4). The orbital parameters of this binary were chosen such that the progenitor system could have been a $13 + 6.5 M_{\odot}$ system which underwent case B mass transfer (see the evolutionary scenario in Sect. 4). After the second phase of mass transfer (case BB) the evolution of the remnant of the $2.5 M_{\odot}$ helium star is calculated until neon ignites off-centre.

In Sect. 5 we compute the orbital dimensions after the SN explosion of the $2.5 M_{\odot}$ helium star and in Sect. 6 we compare these with observed orbital dimensions of B-emission (Be) and other X-ray binaries, and Be-star binaries with known eccentricities in order to test the suggestion of Rappaport and Van den Heuvel (1982) that Be/X-ray binaries are formed by a mass-transfer dominated evolution in an intermediate-mass binary.

Our results are discussed in Sect. 7 which contains also the main conclusions.

Before describing the results of the above-mentioned calculations we outline, in Sect. 2, the treatment of convection, the determination of the boundary of the convective helium-burning

Send offprint requests to: G.M.H.J. Habets

* Present address: Sterrewacht “Sonnenborgh”, University of Utrecht, Zonnenburg 2, 3512 NL Utrecht, The Netherlands

core, and briefly summarize part of the computational method in the stellar evolution code. This code (Habets, 1985a, 1986b) was developed to calculate the advanced evolution of $2.0 - 4.0 M_{\odot}$ helium stars [to be described in a subsequent paper (Habets, 1986a)].

2. The treatment of convective mixing and the computational method

The computations of the evolutionary sequences of helium stars were started with a program originally based on a scheme described by Paczynski (1970), which was considerably modified by Savonije and Takens (1976), and subsequently by Van der Linden (1982, 1985) and Habets (1985a).

In our version the four structure equations (see e.g. Savonije and Takens) were solved simultaneously with a diffusion equation for the changes with time of the composition variables. We use a fully implicit reaction network and initial composition of 97% ^4He , 0.75% ^{12}C , 0.75% ^{16}O , and 1.5% ^{14}N (thus the initial heavy metal content $Z = 0.03$).

We now turn to a description of the method for the determination of convective boundaries. We do not include the effects of convective overshooting, because the theory of convection is as yet too limited to be able to predict the extent of real convective overshoot, see Renzini (1977). It seems, however, useful to describe here in some detail the difference between our approach and approaches in which some type of overshoot is taken into account. The term convective overshoot will be used here for the phenomenon that a convective element may enter a radiative region due to its possible non-zero velocity (but zero acceleration) at the convective boundary. [This border is given by the point where the radiative temperature gradient ∇_{rad} , and adiabatic gradient, ∇_{ad} , are exactly equal; see also Sreenivasan and Wilson (1985)]. The extent of overshoot is, in general, estimated from a non-local or local mixing-length theory of convection. (Nowadays, the term overshooting is usually connected with non-local mixing-length theory, see e.g. Maeder (1975), Bressan et al. (1981), Doom (1982)).

Castellani et al. (1971a, b; 1985a, b) have argued that as a consequence of overshoot the convective helium-burning core mass will grow in the core-helium burning phase: nuclear burning and convective mixing in the convective core cause the mixed core material to become gradually more opaque and enhance convective overshoot. The overshoot creates a convectively unstable zone outside the core and less opaque material is mixed into the convective core. Thereby the position of the convective core boundary changes with time. This position and the efficiency of overshoot are expected to be determined by the difference $\nabla_{\text{rad}} - \nabla_{\text{ad}}$. On the other hand, the position of the boundary and this difference depend also on the amount of helium-rich material mixed into the convective core. So, the problem is quite complex. Furthermore, at the convective core boundary there is a discontinuity in density and composition which gives rise to the problem of a consistent determination of the boundary of the convective core, see Schwarzschild (1958, p. 167), Savonije and Takens (1976), and Sweigart and Renzini (1979).

In the following, as mentioned, we do not include overshoot as has been done by Maeder (1975), Bressan et al. (1981), and Doom (1982) who propose a rather large extent of mixing outside the convective core. We do assume, however, that fresh material brought into the core will be mixed completely as long as $\nabla_{\text{rad}} >$

∇_{ad} at the convective side of the core boundary, i.e. in order to fulfil the Schwarzschild criterion for the onset of convection. When calculating the position of the convective Schwarzschild core boundary one has to allow the convective core to grow in mass (Habets, 1985a). It should be stressed here that this growth of the convective core is only introduced here to keep fulfilling the Schwarzschild criterion for the onset of convection and that we do not calculate the extent of convective overshoot.

It is still uncertain which criterion for the onset of convection (Ledoux or Schwarzschild) has to be used in regions with a composition gradient [also due to overstable oscillations occurring in such regions, see Stevenson (1979) and references therein]. Here, we adopted the Schwarzschild criterion. When a zone is convective according to this criterion at the start of the first iteration, we mix this zone throughout the iterations even if this zone becomes radiative after mixing. Also, whenever a zone becomes convective after the completion of the first iteration, this zone is not mixed in all further iterations. Before the start of the iterations of the new model we account for the growth of the convective helium-burning core by careful and frequent rezoning (deleting and inserting interpolated mesh points) near the boundary of the convective core in the previous model. The interpolated points may become convective in the first iteration. In this way the convective boundary moves outward in a sequence of models. In Sect. 3.1. it is shown that this treatment gives the same growth of the convective helium-burning core as the implicit mesh point technique which is described by Savonije and Takens, Van der Linden (1982, 1985), and Habets (1985a, Chap. 2). In this latter method the boundary of the convective regions can be determined consistently (by solving implicitly for the mass coordinate of the convective boundary as defined by the condition $\nabla_{\text{rad}} = \nabla_{\text{ad}}$). Semi-convective zones (in which ∇_{rad} almost equals ∇_{ad}) occurring just outside the convective core can be treated implicitly in our procedure, but not by the methods of Savonije and Takens and Castellani et al. (1985a, b). Large non-transient semi-convective zones arise around the convective cores of helium stars less massive than $2 M_{\odot}$, see Castellani et al.

We assume that convective zones are mixed instantaneously. However, this is probably not realistic for semi-convective zones. Convective helium, carbon, or neon zones are mixed completely if the time scale of convective mixing is much smaller than the helium, carbon, or, neon depletion time scale, respectively. The time step chosen in the models was also much smaller than this depletion time scale, but still larger than the convective "turn-over" time scale [typically about 10 years or less for core-helium burning (Renzini, 1977), but as low as 100 seconds at convective neon-core or neon-shell burning (Habets, 1985a)]. Hence, mixing was instantaneous and complete mixing was achieved in all cases, see also Arnett (1974) and Habets (1985a).

3. The evolution of a $2.5 M_{\odot}$ helium star – single and in a binary – up to neon ignition

In Fig. 1 the evolutionary history of the central density, ρ_c , and temperature, T_c , (both in decimal logarithms) is shown. The evolutionary track of the $2.5 M_{\odot}$ helium star in the theoretical Hertzsprung-Russell (HR) diagram is depicted in Fig. 2. The fully drawn curves in Figs. 1 and 2 are the tracks without mass exchange, the dotted curves are with mass transfer to the $17 M_{\odot}$ companion. In Fig. 3 the evolution of the radius versus core mass (both scales are decimal logarithms) is given. The dotted track is

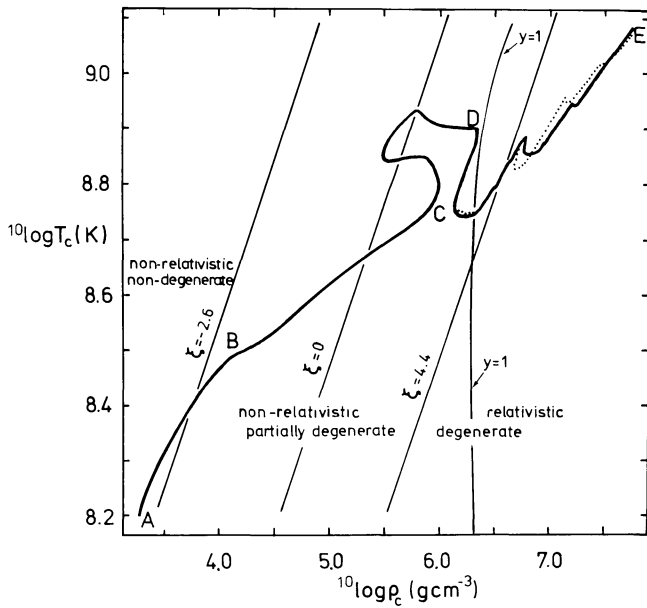


Fig. 1. The central ρ , T history of the $2.5 M_{\odot}$ helium star (with $Z = 0.03$). The solid curve is without mass exchange. The dotted curve is with mass exchange to the $17 M_{\odot}$ companion (see text). The oblique lines of constant electron degeneracy parameter ξ demarcate (1) the non-degenerate/partially degenerate regime ($\xi = -2.6$), (2) the partially degenerate/degenerate regime ($\xi = 4.4$), and (3) the non-degenerate/-degenerate regime ($\xi = 0$). The criteria defining these lines are adopted from Cox and Giuli (1968). The curve $y = 1$ divides the non-relativistic and relativistic regimes. The letters along the solid and dotted curves are explained in the text

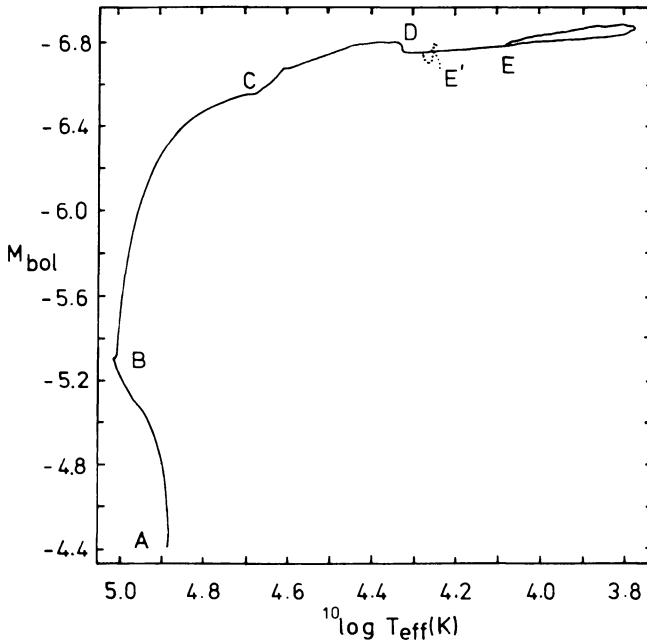


Fig. 2. The evolutionary track of the $2.5 M_{\odot}$ helium star (with $Z = 0.03$) in the HR diagram (solid curve). The dotted curve represents the part of the track with mass exchange to the $17 M_{\odot}$ companion. For the meaning of the letters see the text

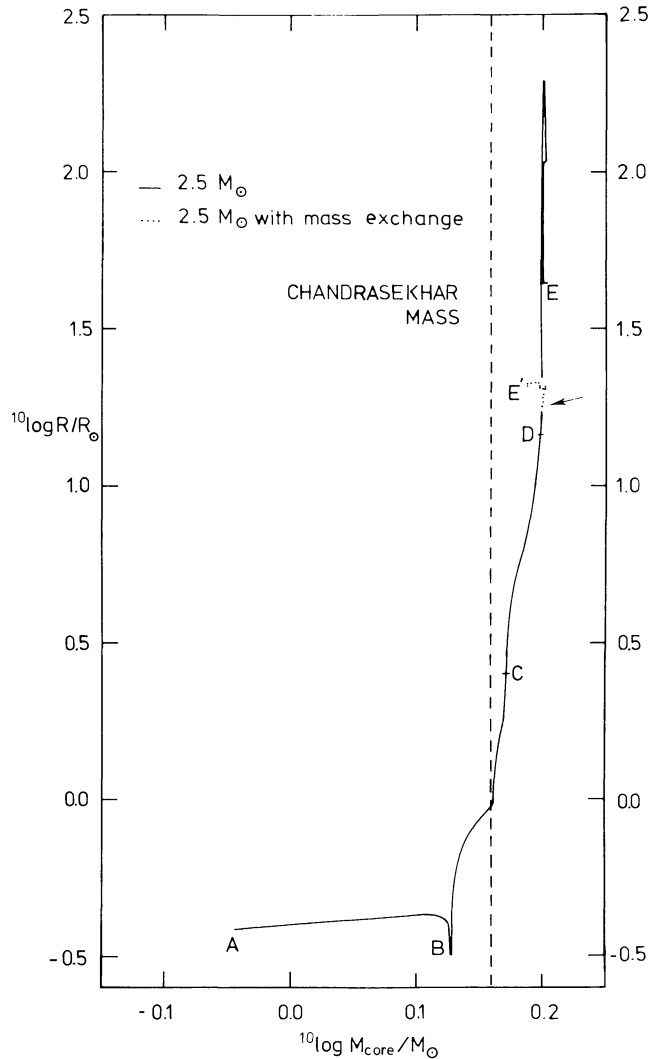


Fig. 3. The change of the outer radius with core mass in the $2.5 M_{\odot}$ helium star. During the convective helium-burning phase (from letters A to about B) the core mass is defined as the mass of the convective helium-burning core. After this phase (from B to E) the core mass is defined as the mass of the core which is limited by the helium-burning shell with maximum energy generation. This core mainly consists of ^{12}C , ^{16}O , and possibly ^{20}Ne and ^{24}Mg , and some ^4He at its outer boundary. The dotted curve represents the part of the track with mass exchange to the $17 M_{\odot}$ companion. The dashed line indicates the position of the Chandrasekhar limiting mass. The arrow marks the onset of mass exchange in the $2.5 + 17 M_{\odot}$ binary system. The letters have the same meaning as in Figs. 1 and 2 (see the text)

with mass exchange, the deviation from the solid curve is marked with an arrow. The oblique lines in Fig. 1 are the approximate lines of constant electron degeneracy parameter ξ as defined by Cox and Giuli (1968). The curve $y = 1$ is the approximate demarcation line of the non-relativistic and relativistic regimes. The meaning of the letters in the Figs. 1-3 is as follows: A – B, the helium main-sequence phase and the exhaustion of helium in the core; B – C, the radiative helium-shell burning phase and the gravitational contraction of the $^{12}\text{C} - ^{16}\text{O}$ core; C – D, the ignition of carbon, radiative and convective core-carbon burning and radiative carbon-shell burning; D – E, the convective carbon-shell burning phase and the gravitational contraction of

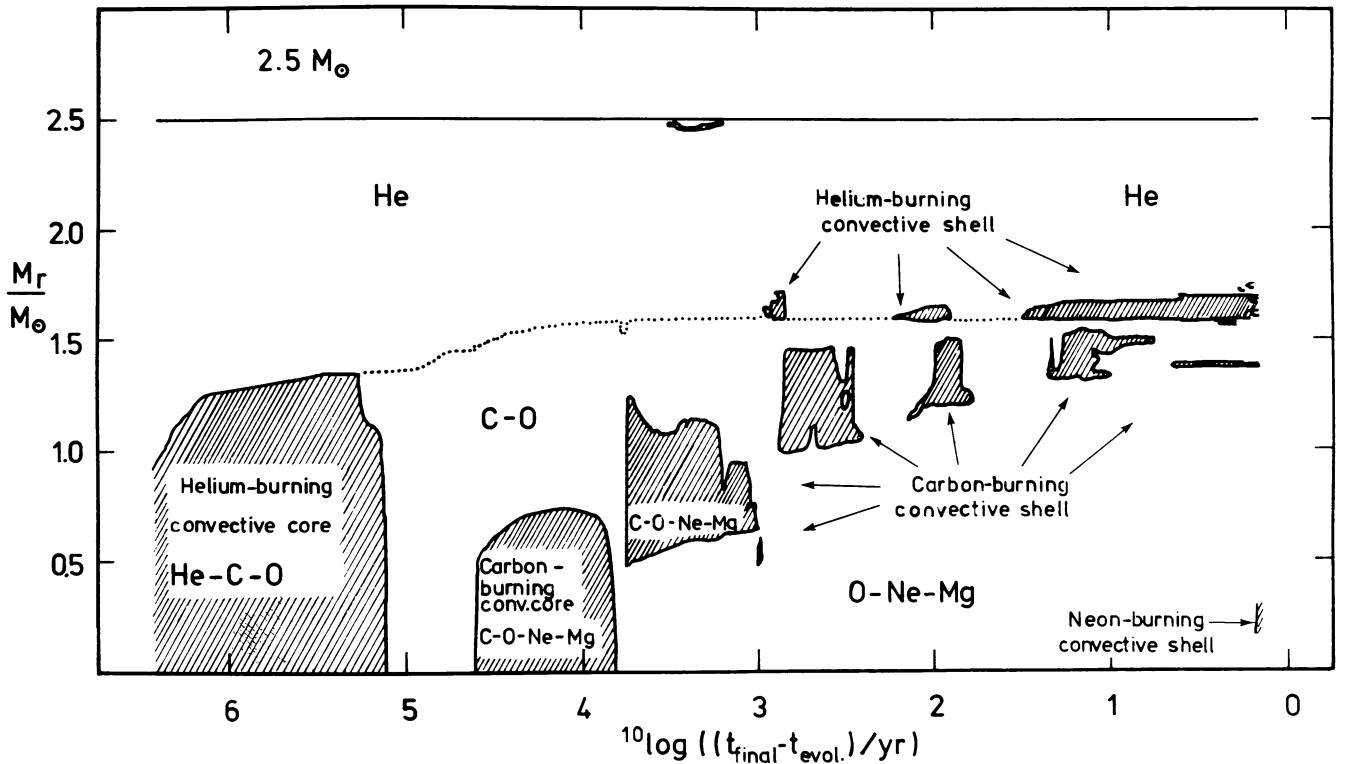


Fig. 4. The evolution of the interior regions of the $2.5 M_{\odot}$ helium star without mass exchange. Hatched regions are convective, the dotted line demarcates the core and the helium-burning layer with maximum net energy generation

the O-Ne-Mg core; E or E' (in case of mass exchange), the radiative off-centre ignition of ^{20}Ne , and convective off-centre neon burning.

The Figs. 4 and 5 show the evolution of the inner regions with time. Figure 5 is for the case with mass exchange. Time is here defined as $^{10}\log((t_{\text{final}} - t_{\text{evol}})/\text{yr})$, where t_{evol} is the evolutionary time in years elapsed since the arrival of the $2.5 M_{\odot}$ helium star on the helium main sequence and t_{final} is chosen to be (about) one year later than the t_{evol} in the final model. Thus, t_{evol} is zero in the first model. The dotted lines in Figs. 4 and 5 indicate the boundary of the core which is defined by the region interior to the helium-burning shell with maximum net energy generation. Convective regions are shown hatched.

In Tables 1 and 2 the main data for the $2.5 M_{\odot}$ helium star without and with mass exchange, respectively, are presented at special points in the evolution: number of the model in the sequence, t_{evol} (in years), $^{10}\log T_{\text{eff}}$ (T_{eff} in K), bolometric magnitude M_{bol} , outer radius R (in $R_{\odot}$), $^{10}\log T_c$ (T_c in K), $^{10}\log \rho_c$ (ρ_c in g cm^{-3}), the central helium (^4He), carbon (^{12}C), oxygen (^{16}O), and neon (^{20}Ne) abundances by mass, and in case of mass exchange (Table 2): the mass-transfer rate $\dot{M}$ in $M_{\odot} \text{ yr}^{-1}$, mass of the mass-losing component in $M_{\odot}$, orbital separation (in $R_{\odot}$), and the orbital period in days. Detailed versions of these tables are published in Habets (1985a).

In the following paragraphs a more detailed description of the phases A – B, ..., to E is given.

3.1. Core-helium burning (phase A – B in Figs. 1–3)

Helium burns non-degenerately in the core (Fig. 1). The convective core grows about 50% in mass and contains at maximum

$1.35 M_{\odot}$ (Fig. 4). This result was obtained by employing a diffusion equation of convective mixing. As an independent check on our results Th.J. Van der Linden computed part of the core-helium burning phase of the $2.5 M_{\odot}$ helium star with his code. He used the same diffusion equation together with the implicit mesh point technique. He found a maximum extent of the convective helium-burning core of $1.356 M_{\odot}$. But, although practically the same input physics was used, the different techniques for determining the convective boundary appear not to give the same abundance variations with time. Figure 6 illustrates this. This Figure depicts the changes in the central abundance (by mass) with time (in years) of the elements ^4He , ^{12}C , and ^{16}O . The solid curves are our results; Van der Linden's results are shown by the dashed curves. The change of the central helium abundance with time obtained by Savonije and Takens (1976) is drawn by a dotted line, the variation with time of the central net energy generation rate ε_{net} (in $\text{erg g}^{-1} \text{ s}^{-1}$) is indicated by the crosses. With different opacities, with another triple-alpha burning rate (0.8 times smaller than our rate), and with some minor differences in other input physics Savonije and Takens obtained a maximum convective helium-burning core mass of $1.23 M_{\odot}$ for a $2.5 M_{\odot}$ helium star just after helium exhaustion in the core. The use of different opacity tables by Savonije and Takens and in particular the different triple-alpha rate are most probably the cause of the much smaller growth of the convective helium-burning core in their calculations than in those of ours and of Van der Linden. For the C – O core we find the following final abundances in the centre after helium exhaustion: $^{12}\text{C} = 0.4905$, $^{16}\text{O} = 0.4945$, i.e. quite close to the values of $^{12}\text{C} = 0.463$ and $^{16}\text{O} = 0.507$ found by Savonije and Takens, see also Habets (1985a, b). If one regards only the behaviour with time, the abundance variations found in

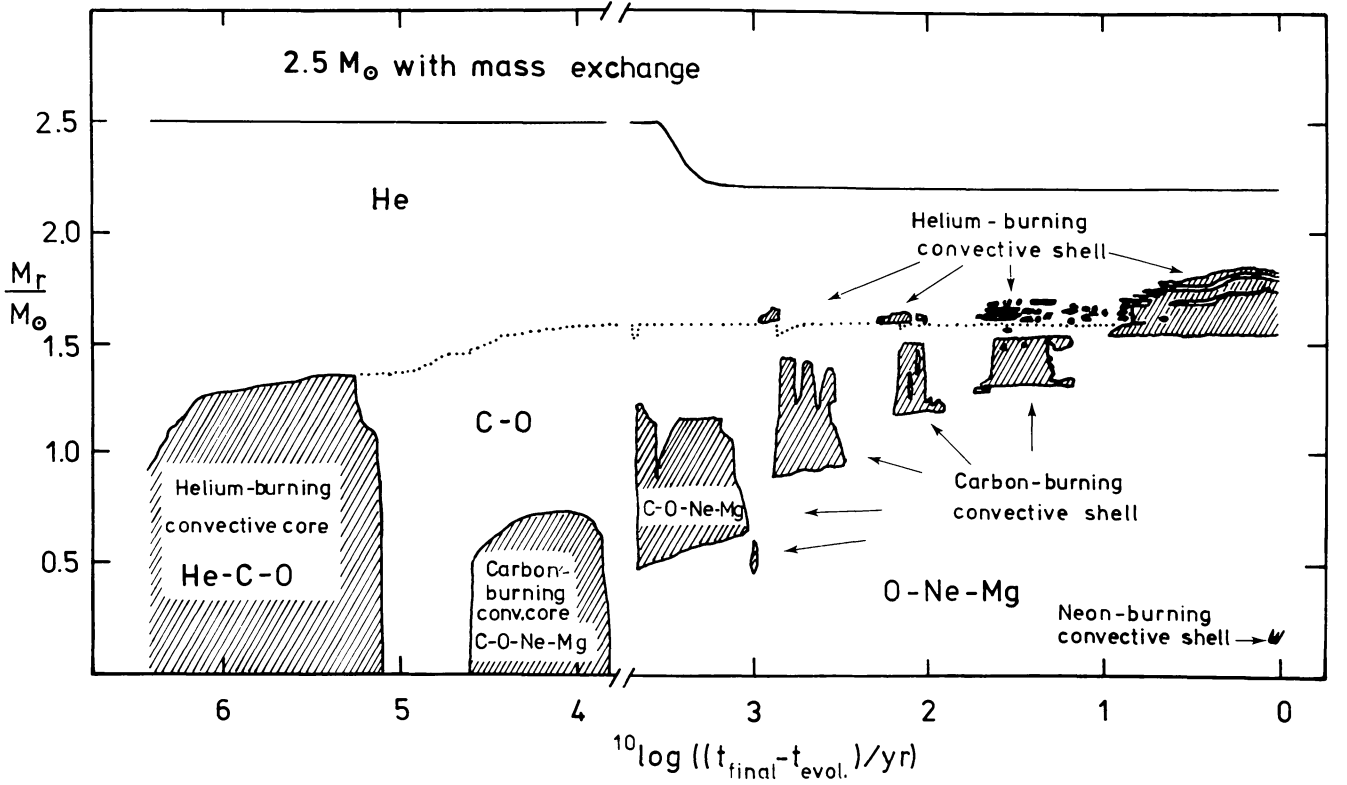


Fig. 5. Same as Fig. 4 with case BB mass transfer from the $2.5 M_{\odot}$ helium star to the $17 M_{\odot}$ companion (see Sect. 4)

Table 1

Model No.	Time (years)	T_{eff} $10 \log$	M_{bol}	$R/R_{\odot}$	T_c $10 \log$	ρ_c $10 \log$	^4He	^{12}C	^{16}O	^{20}Ne	Stage
2	0.620597E + 04	4.8854	-4.4157	0.3825E + 00	8.1921	3.2685	0.9672	0.0103	0.0075	0.0000	A
173	0.263284E + 07	5.0146	-5.3030	0.3174E + 00	8.4865	4.0967	0.0000	0.4905	0.4945	0.0000	B
240	0.271390E + 07	4.7014	-6.5507	0.2387E + 01	8.7836	5.9666	0.0000	0.4802	0.4887	0.0130	C
385	0.274787E + 07	4.3473	-6.7979	0.1366E + 02	8.9252	5.8188	0.0000	0.0132	0.2812	0.5347	1
413	0.274857E + 07	4.3295	-6.7810	0.1471E + 02	8.9010	6.3245	0.0000	0.0022	0.2773	0.5443	D
651	0.275173E + 07	3.7728	-6.8683	0.1988E + 03	8.7425	6.2573	0.0000	0.0019	0.2772	0.5445	2
867	0.275433E + 07	4.0902	-6.7757	0.4416E + 02	9.0866	7.7481	0.0000	0.0000	0.2771	0.5447	E

Notes: 1. End of convective core-carbon burning, 2. Maximum radius

Table 2

Model No.	Time (years)	T_{eff} $10 \log$	M_{bol}	$R/R_{\odot}$	T_c $10 \log$	ρ_c $10 \log$	^{12}C	^{16}O	^{20}Ne	$\dot{M}$ $10 \log$	$M_1/M_{\odot}$	$A_{\text{orb}}/R_{\odot}$	P_{orb} (days)	Stage
537	0.274960E + 07	4.2826	-6.7488	0.1799E + 02	8.7482	6.1355	0.0019	0.2772	0.5445	-4.9443	2.4999	8.420E + 01	2.029E + 01	1
580	0.275047E + 07	4.2632	-6.7059	0.1928E + 02	8.7472	6.2171	0.0019	0.2772	0.5445	-3.6439	2.3640	9.268E + 01	2.343E + 01	2
669	0.275246E + 07	4.2521	-6.7843	0.2104E + 02	8.8704	6.7204	0.0019	0.2771	0.5446	-5.9402	2.2050	1.046E + 02	2.808E + 01	3
767	0.275318E + 07	4.2463	-6.7265	0.2104E + 02	9.0185	7.4493	0.0000	0.2765	0.5462	-5.4921	2.2049	1.046E + 02	2.808E + 01	4
773	0.275318E + 07	4.2460	-6.7236	0.2104E + 02	9.0185	7.4586	0.0000	0.2765	0.5462	-5.7932	2.2049	1.046E + 02	2.808E + 01	5
844	0.275322E + 07	4.2416	-6.6799	0.2105E + 02	9.0842	7.7909	0.0000	0.2772	0.5445		2.2049	1.046E + 02	2.808E + 01	E

Notes: 1., 4. Onset of mass-exchange phases during carbon-shell burning, 3., 5. End of mass-exchange phases during carbon-shell burning, 2. Maximum mass-exchange rate

our work agree better with those of the latter authors than with those of Van der Linden. We guess that an error in the programming of the implicit mesh point technique by Van der Linden caused this difference. The kink in our ^4He curve at $t_{\text{evol}} = 6 \cdot 10^5$ yr

is due to a somewhat coarse distribution of our mesh points. When $Y (= ^4\text{He}) = 0.27$ (Fig. 6) one reaches the maximum net energy generation rate ϵ_{max} in the centre ($2.7 \cdot 10^5 \text{ erg g}^{-1} \text{ s}^{-1}$, the same as the one found by Van der Linden when Y was ~ 0.36).

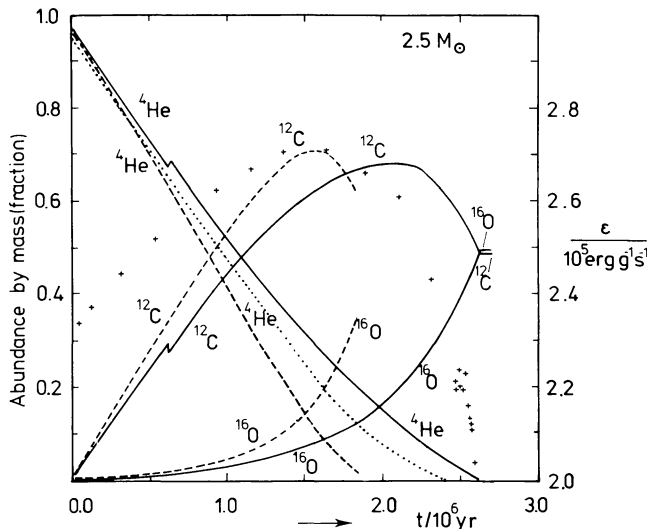


Fig. 6. The variation with time of the central abundances by mass of ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$ and the variations with time of the central net energy generation rate ε_{net} (crosses) during convective core-helium burning in the $2.5 M_{\odot}$ helium star. The solid curves are from the present investigation. Van der Linden's test calculations are shown by the dashed lines. The ${}^4\text{He}$ curve of Savonije and Takens (1976) is given by the dotted line. For all curves $Z = 0.03$, initially

Up to that time the radius increases (Fig. 3). The star begins to shrink shortly afterwards. When helium is becoming exhausted in the core the rate drops quickly to $3 \cdot 10^4 \text{ erg g}^{-1} \text{ s}^{-1}$. The spread of ε_{net} (crosses) at $Y = 0.02$ is due to the mixing of fresh fuel into the core. A finer grid of mesh points on both sides of the convective boundary at that epoch did not produce such a spread. This demonstrates that the evolution of the $2.5 M_{\odot}$ helium star is very sensitive to the mesh point distribution and to the precise determination of the convective boundary.

3.2. Helium-shell burning (phase B – C in Figs. 1–3)

As soon as helium is burning radiatively in a shell with $\varepsilon \approx 10^5 \text{ erg g}^{-1} \text{ s}^{-1}$, the radius begins to increase again. The core (defined here as being limited by the helium-burning shell with maximum energy generation) grows in mass due to the nuclear burning in this helium shell and becomes larger than the Chandrasekhar limiting mass (Figs. 3 and 4). The bolometric luminosity increases by about a factor 3 (see also Figure 2). The maximum net energy generation rate ε_{max} achieved in the radiative helium-burning shell is $\sim 4.2 \cdot 10^5 \text{ erg g}^{-1} \text{ s}^{-1}$, i.e. similar to the maximum in the previous phase. The C – O core contracts: the central density changes from $1.0 \cdot 10^4 \text{ g cm}^{-3}$ to $4.6 \cdot 10^5 \text{ g cm}^{-3}$, but no temperature inversion occurs. Throughout phase B – C the neutrino losses are not large enough to outweigh the gravitational energy production by the contraction of the core. In phase A – B the neutrino losses are negligible.

3.3. Core-carbon burning and radiative carbon-shell burning (phase C – D in Figs. 1–3)

Carbon ignites radiatively in the centre at $T_c = 5.2 \cdot 10^8 \text{ K}$, $\rho_c = 4.6 \cdot 10^5 \text{ g cm}^{-3}$, and $\xi \sim 0.7$. From that moment nuclear burning of carbon by the reactions ${}^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{20}\text{Ne} + {}^4\text{He}$ and ${}^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{23}\text{Na} + {}^1\text{H}$ becomes the dominating energy source in the radiative core. The core-carbon burning takes place in the par-

tially degenerate regime and is therefore non-violent (Fig. 1). The temperature rises quickly after the ignition, the C – O core becomes convective at $T_c = 6.13 \cdot 10^8 \text{ K}$, $\rho_c = 9.28 \cdot 10^5 \text{ g cm}^{-3}$, $\xi \sim 1.05$, and central carbon abundance X_c of 0.48, and contraction ceases. At first the density in the core decreases due to the convective inwards mixing of fresh carbon. Since the nuclear energy production rate of the carbon-burning reactions is proportional to ρ , also the temperature in the core gradually decreases. As soon as the energy loss by gravitational expansion of the core can no longer diminish the total energy production rate, i.e. if no more fuel is mixed inwards, the central temperature of the core begins to increase again. Just as between points A and B, there is a steady nuclear burning stage ($\varepsilon_{\text{max}} = 1.2 \cdot 10^7 \text{ erg g}^{-1} \text{ s}^{-1}$) during which both the central density and the central temperature increase while the bolometric luminosity becomes larger. The maximum net energy generation in phase C – D occurs at the beginning of this phase when the central temperature reaches its (first) maximum (see Fig. 1, at ${}^{10}\log \rho_c \sim 5.8$ and $X_c \sim 0.47$): $\varepsilon_{\text{max}} > 2.6 \cdot 10^7 \text{ erg g}^{-1} \text{ s}^{-1}$. The maximum size of the convective carbon-burning core is $0.74 M_{\odot}$. When the carbon abundance is low in the core, the central density still keeps increasing, but the central temperature drops. The main energy source at that time is helium burning in a shell ($\varepsilon_{\text{max}} = 1.5 \cdot 10^6 \text{ erg g}^{-1} \text{ s}^{-1}$). The central contraction finally results in a temperature increase. At this time, neutrino losses are no longer negligible. We used the neutrino rates uncorrected for neutral current effects as given by Beaudet et al. (1967). In the later evolutionary stages we corrected the rates of the pair-neutrino, the photoneutrino, and the plasma-neutrino processes for the effects from the electron- and muon-neutrinos according to the prescription of Dicus (1972).

3.4. Convective carbon-shell burning up to neon ignition (phase D – E in Figs. 1–3)

At point D the first carbon-burning convective shell has developed. This shell produces at maximum $2.3 \cdot 10^8 \text{ erg g}^{-1} \text{ s}^{-1}$. The carbon shell provides the energy necessary to maintain the luminosity of the star achieved in the foregoing phase. The core expands slightly and the neutrino losses control the cooling of the core during the development of the convective layer. As soon as the energy production in the shell slows down, the core contracts and the central temperature rises again. The stellar centre passes the non-relativistic/relativistic border line (the curve $y = 1$ in Fig. 1) and enters the relativistic degeneracy regime. Once the convective shell has disappeared, the temperature and density throughout the core increase. This process continues until a second carbon-burning radiative layer at a larger mass coordinate becomes unstable to convection. This shell produces initially enough energy to stop the core contraction. The central regions cool again due to neutrino losses. The above described behaviour repeats itself a number of times (Figs. 1, 4 and 5).

3.5. Comparison with results of other calculations

Details of a similar post-carbon-burning stage have been presented by Kutter and Savedoff (1969; for pure carbon stars), Ikeuchi et al. (1971, 1972; for stars consisting of 50% ${}^{12}\text{C}$ and 50% ${}^{16}\text{O}$) and Boozer et al. [1973; for stars $(1 - 1.5 M_{\odot})$ composed of 50% ${}^{12}\text{C}$ and 50% ${}^{16}\text{O}$ without a helium envelope; Joss et al. (1973) reported a similar behaviour for such stars with a helium envelope]. These authors detected a series of carbon shell flashes which each produced a convective zone. Each successive

flash and its associated convective region occurred farther away from the centre. The evolutionary tracks calculated (but not published by Boozer et al.) were erratic. In our $2.5 M_{\odot}$ helium-star sequence the maximum total luminosity in a mass zone of the first convective carbon shell was $\sim 4 \cdot 10^6 L_{\odot}$, i.e. similar to the values of L_n (the rate of energy generation by nuclear reactions) in the shell flashes of the $1.37 M_{\odot}$ model of Boozer et al. The latter authors describe the shell-burning phase as a sequence of thermal relaxation oscillations. [Ikeuchi et al. and Kutter and Savedoff found such a behaviour (with an explicit and an implicit method, respectively), even though the last-mentioned authors used an incorrect equation of energy conservation from Kutter and Savedoff (1967, cf. Strittmatter et al., 1970)].

3.6. Convective helium regions in the outer layers and the evolution of the outer layers

During the first carbon-burning convective shell phase the outer radius of the star grows to its largest value of $200 R_{\odot}$ (Fig. 3). A density inversion and convection occur in the envelope in the region $^{10}\log T = 4.15 - 4.5$ and $^{10}\log \rho = -8.3 \pm 0.3$. The convective layer is at maximum about $2 \cdot 10^{-2} M_{\odot}$ thick. The density inversion extends over $1.5 \cdot 10^{-2} M_{\odot}$ at maximum. In these low-temperature regions partial ionization of helium dominates in the equation of state [cf. Law (1982); our equation of state for a helium mix (Iben V) over the range of ρ and T in a helium red-giant envelope agrees perfectly with the tabulated equation of state of Fontaine et al. (1977)]. When the radius is close to $200 R_{\odot}$ the envelope becomes also convectively unstable over a mass range of at most $1.5 \cdot 10^{-2} M_{\odot}$ at $^{10}\log T \sim 5.2 - 5.45$ and $^{10}\log \rho = -6.3$ to -7.1 , see Fig. 4. This is the temperature-density regime in which helium is nearly completely ionized, but in which no density inversion occurs. The inversion at the lower temperatures and densities is probably due to an opacity peak in the helium ionization zone. Maeder (1981) showed that the effect of the density inversion in a hydrogen envelope will be a Rayleigh-Taylor instability. Maeder's results for $60 M_{\odot}$ and $30 M_{\odot}$ hydrogen stars are that the internal structure is unaffected by his treatment of the Rayleigh-Taylor instability and that T_{eff} is lowered by $\Delta^{10}\log T = 0.09$. By comparison with Maeder's test calculations we may expect that the evolution through the "red-giant" phase is only slightly influenced by the instability resulting from the density inversion in the envelope. In contrast to the result for our $2.5 M_{\odot}$ star, Law (1982) does not mention a density inversion in the red-giant phase of a $1 M_{\odot}$ helium star. Before returning to the blue part of the HR diagram the luminosity of the star increases (Fig. 2). If the results of Law can be extrapolated, the helium Hayashi track of fully convective models of $2.5 M_{\odot}$ will lie close to this turning point of our model in the HR diagram.

The evolution of the interior regions at neon ignition of the $2.5 M_{\odot}$ helium star is described in paragraph 4, see also Habets (1985a) and Appendix B in Habets (1986b).

4. Roche-lobe overflow from a $2.5 M_{\odot}$ helium star in the carbon-shell burning phase

4.1. Evolution before neon ignition (phase D – E' in Figs. 1–3)

In the following the evolution of a $2.5 M_{\odot}$ helium star in a binary up to neon ignition is described. The mass transfer is assumed to be "conservative" (Paczynski, 1971b) and is treated in a way

analogous to a procedure developed by Van der Linden (1982). As a companion we took a $17 M_{\odot}$ hydrogen star at an initial orbital separation of $84.2 R_{\odot}$ ($P_i = 20^d.29$). Such a binary can be a remnant of case B mass transfer ($P_0 = 2^d.58$) in a system with initial component masses of $13 + 6.5 M_{\odot}$ which evolved conservatively. The $2.5 M_{\odot}$ helium star appears to fill its Roche lobe several times in phase D – E. Once the radius of the $2.5 M_{\odot}$ helium star is $\sim 18 R_{\odot}$ a major amount of mass ($0.3 M_{\odot}$) is exchanged. Figure 7 shows the mass-loss rate $\dot{M}$ in $M_{\odot} \text{ yr}^{-1}$ as a function of time (in years) during the first convective carbon-burning shell phase. In view of the new boundary conditions during the mass-transfer phase one obtains a smaller convective carbon-burning shell (cf. Figs. 4 and 5). In 2860 years $0.3 M_{\odot}$ is lost to the companion. The Kelvin-Helmholtz time scale of the star in this phase (i.e. neglecting neutrino losses) is $\sim 240 \text{ yr}$. At the onset of mass transfer the convective part of the stellar envelope is thermally unstable and is expanding on roughly this time scale. On the other hand mass transfer causes also the critical radius to increase, because mass is exchanged from the less massive to the more massive star. Initially, the change of the critical radius cannot follow the growth of the stellar radius and a sharp rise in $\dot{M}$ occurs (Fig. 7). A maximum $\dot{M}$ of $2.3 \cdot 10^{-4} M_{\odot} \text{ yr}^{-1}$ is reached. The helium star detaches from the Roche lobe at $P_f = 28^d.09$; the orbital separation at that time is $104.6 R_{\odot}$. In the fourth convective carbon-burning shell phase the helium star again fills its Roche lobe. In about 4 yr of Roche-lobe overflow a maximum $\dot{M}$ of $2 \cdot 10^{-5} M_{\odot} \text{ yr}^{-1}$ is reached. (This exchange phase lasts much shorter than the foregoing one, because the evolution of the interior regions proceeds much faster just before neon ignites.) A third mass-transfer phase lasting about 10 yr may occur after the disappearance of this convective shell. This phase was not calculated. [The total amount of mass loss in the last-mentioned two phases is negligible ($< 10^{-4} M_{\odot}$)].

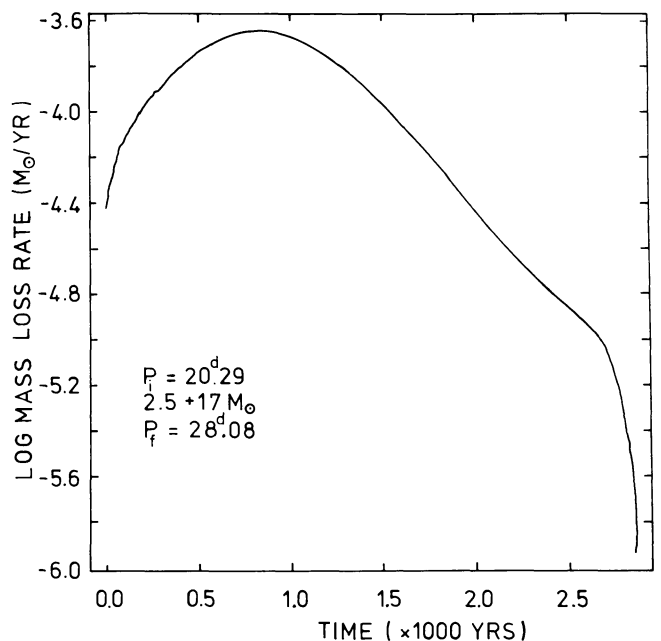


Fig. 7. The variations with time of the mass-loss rate $\dot{M}$ in $M_{\odot}/\text{yr}$ during the first mass-transfer phase in the $2.5 + 17 M_{\odot}$ binary system. The initial orbital period (P_i) and final orbital period after the mass exchange (P_f) are indicated

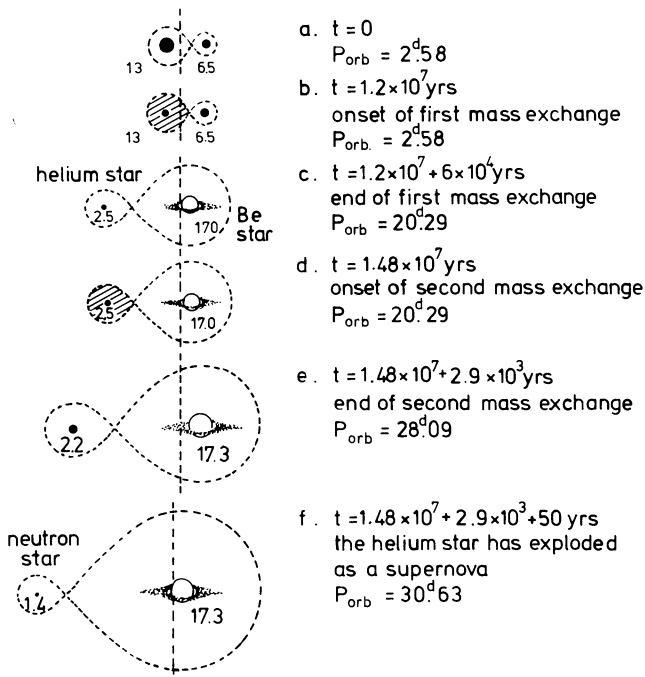


Fig. 8. The conservative case BB evolutionary scenario for the formation of a Be/X-ray binary from a $13.0 + 6.5 M_{\odot}$ binary system. The initial more massive primary first transfers mass to the secondary due to hydrogen-shell burning (at $t = 1.2 \times 10^7 \text{ yr}$) and later on (but now being the less massive, helium-star component) due to helium-shell burning in the carbon-shell burning phase (at $t = 1.48 \times 10^7 \text{ yr}$); the original secondary becomes a rapid rotating Be star as a result of the first mass exchange by which mass and angular momentum is accreted through a disk; finally the original primary star undergoes core collapse and forms a neutron star. As a result of the mass-exchanges the binary orbit becomes wide. A symmetric SN explosion makes the orbit slightly wider and gives it a small eccentricity

The conservative evolutionary scenario for the original binary system and some time scales are given in Fig. 8, see also Sect. 5.1.

The expansion of the outer helium layers was not large enough to cause the formation of a thin convective layer at $^{10}\log T \sim 5.2 - 5.5$ or to cause a density inversion in the stellar envelope. Several helium-burning convective and semi-convective shells were formed in a region at the base of the helium-burning shell with maximum net energy generation (the dotted curve in Fig. 5). At the end of the calculations the base of this shell is convectively burning helium as well as carbon (see Fig. 5): a convective helium-burning layer of the helium-burning shell penetrates the core region and merges together with a helium- and carbon-burning convective layer in the core. This causes a slight reduction of the core mass (Figs. 3 and 5). In case of no mass loss such a penetration is avoided and the convective shell becomes radiative (Fig. 4).

In the binary as well as in the single star computations a temperature inversion develops in the centre. A second small temperature inversion occurs in a carbon-burning layer with a mass coordinate of $1.38 M_{\odot}$.

4.2. Evolution at neon ignition (phase E, E' in Figs. 1–3)

Neon ignites off-centre at a mass coordinate of $\sim 0.16 M_{\odot}$ in the $2.5 M_{\odot}$ helium star and in the $2.2 M_{\odot}$ mass-exchange remnant. The computations were terminated a few models after the

moment at which the neon-burning shell became convectively unstable. Figure 9 depicts the thermodynamic structure of the interior of the $2.5 M_{\odot}$ and the $2.2 M_{\odot}$ stars in the last models (at the time of neon burning in a convective shell). At the edge of the core (at $m_r = 1.54 M_{\odot}$ and $1.58 M_{\odot}$ for the $2.2 M_{\odot}$ and $2.5 M_{\odot}$ helium star, respectively) the density and temperature fall off extremely sharply (the solid and dashed-dotted lines of $T/10^8 \text{ K}$ are of the $2.5 M_{\odot}$ and $2.2 M_{\odot}$ helium stars, respectively; the dashed and dotted lines of $^{10}\log \rho$ are of the $2.5 M_{\odot}$ and $2.2 M_{\odot}$ helium stars, respectively). The pressure gradient is also extreme (the pressure gradient of the $2.2 M_{\odot}$ mass-exchange remnant is depicted), the solid and dashed-dotted lines of $^{10}\log r/R_{\odot}$ are of the $2.5 M_{\odot}$ and $2.2 M_{\odot}$ helium stars, respectively. Figures 10 and 11 show the composition structure of the last models in the $2.5 M_{\odot}$ and $2.2 M_{\odot}$ helium stars. The compositional gradients were produced by nuclear burning as well as convective mixing. The abundance of ^{14}N is kept constant in all computations. Notice the large extent of the helium-burning layer in the $2.2 M_{\odot}$ star just before neon ignites; the inner $1.4 M_{\odot}$ is not much affected by the mass exchange, cf. Figs. 4, 5, 10, and 11.

Similar steep density, temperature, and composition gradients are found, after carbon-shell burning, in hydrogen stars with helium cores of $2.0 - 2.6 M_{\odot}$ by Woosley et al. (1980), Nomoto (1984a), Wilson et al. (1985), and in a $2.2 M_{\odot}$ helium star (Habets, 1985a, 1986a) [but not in helium stars of $2.9 - 4.0 M_{\odot}$; see also the $15 M_{\odot}$ hydrogen star model of Wilson et al. (Fig. 3)].

5. The orbital dimensions after the supernova explosion of the $2.2 M_{\odot}$ remnant

The supernova (SN) explosion takes place some 10 yr after neon ignition in the core (see Woosley et al., 1980). In order to derive the orbital dimensions after the SN explosion we first neglect the impact of the SN shell on the Be-star companion and on the orbit. The effects of impact will be estimated under the assumption of a symmetric explosion.

5.1. Orbital parameters after an explosion with symmetric mass ejection, without impact

In this case, if we assume a circular pre-SN orbit, Eqs. (1)–(8) of Flannery and Van den Heuvel (1975) yield the final period, P_f , the final eccentricity e_f , and the final orbital separation, a_f , as a function of μ_f , the final total mass of the system in units of the initial total mass. These parameters are given by:

$$P_f = (\mu_f / (2\mu_f - 1)^{3/2}) P_0; e_f = (1 - \mu_f) / \mu_f; a_f = a_0 \mu_f / (2\mu_f - 1), \quad (1)$$

where $\mu_f = \frac{M_1^f + M_2^f}{M_1^0 + M_2^0}$, M_1^0 , M_2^0 are the pre-SN masses of the exploding and the non-exploding component, and P_0 and a_0 are the initial orbital period and initial orbital separation, respectively.

With $P_0 = 28^{\text{d}}09$, $M_1^0 = 2.2 M_{\odot}$, $M_2^0 = 17.3 M_{\odot}$, $M_1^f = M_2^f$, $a_0 = 104.6 R_{\odot}$ and assuming that the SN explosion of the $2.2 M_{\odot}$ star leaves a $1.4 M_{\odot}$ compact remnant we readily obtain: $P_f = 30^{\text{d}}63$, $e_f = 0.043$, and $a_f = 109.3 R_{\odot}$, see also Figure 8 and Van den Heuvel (1985).

5.2. The effects of impact on the orbital dimensions

De Cuyper (1982, 1984) derived expressions for P_f , e_f , and a_f if the SN shell impacts on the companion:

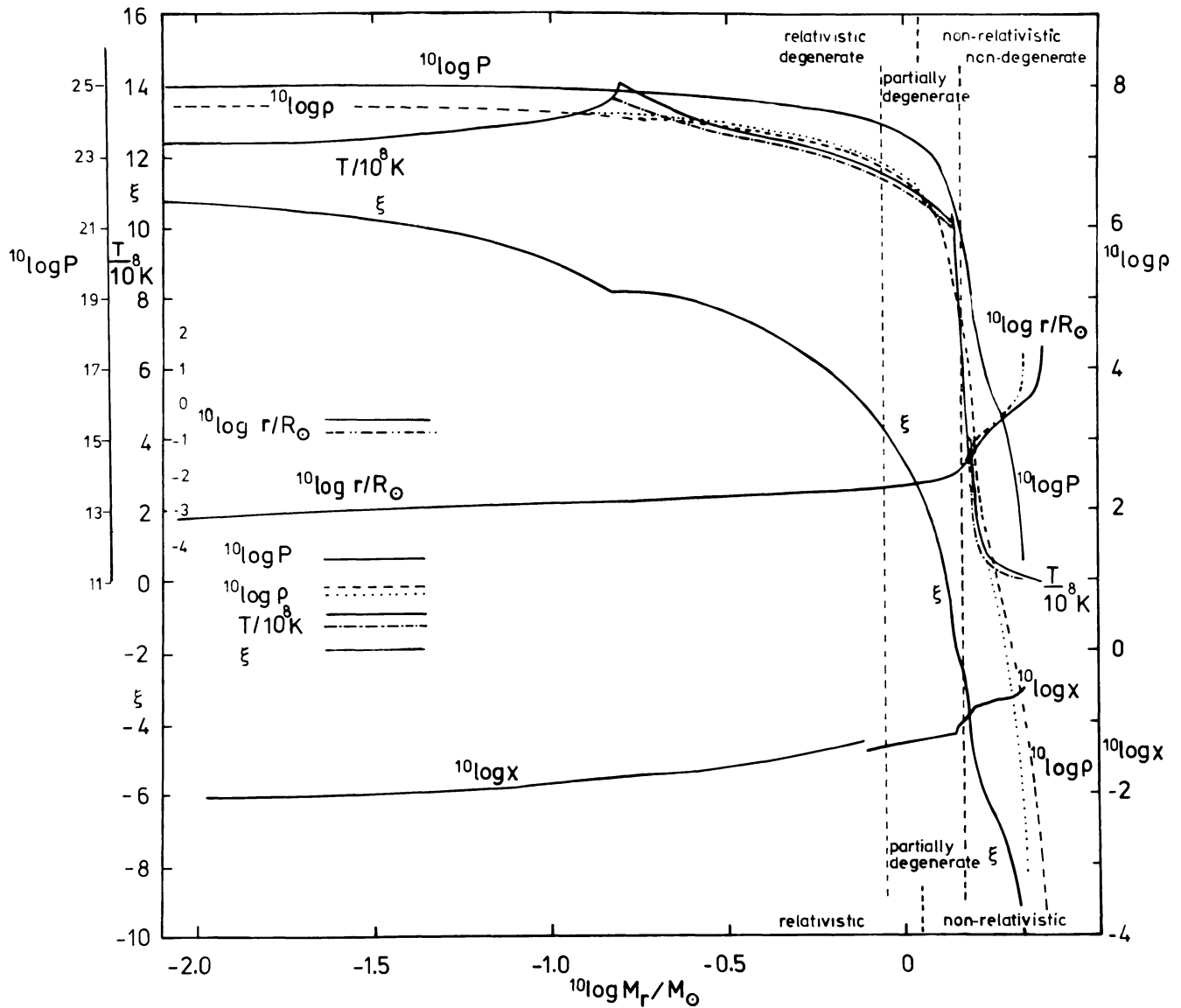


Fig. 9. The thermodynamic structure in the 2.5 and $2.2 M_{\odot}$ stars in the last calculated models with convective neon-shell burning. The run of the temperature ($T/10^8$ K), density ($^{10}\log \rho$; ρ in g cm^{-3}), and radius ($^{10}\log r/R_{\odot}$) are depicted for both stars, see the text. Also the variation of the total pressure ($^{10}\log P$; P in dyn cm^{-2}), of the electron degeneracy parameter ξ , and of the opacity ($^{10}\log \kappa$; κ in $\text{cm}^2 \text{s}^{-1}$) through the $2.2 M_{\odot}$ star are shown

$$P_f = \frac{\mu_f}{S^{3/2}} P_0; e_f = \left\{ 1 - \frac{S}{\mu_f^2} \right\}^{1/2}; a_f = \mu_f \cdot \frac{a_0}{S}, \quad (2)$$

where $S = 2\mu_f - 1 - \left(\frac{v_{\text{im}}}{v^0} \right)^2$, v_{im} and v^0 are the velocity of impact of the SN shell onto the companion and the relative velocity in the initially circular orbit of the system, respectively. These velocities are given by:

$$v_{\text{im}} = I_f \cdot F_{\text{IN}} \cdot v_{\text{SNshell}} \quad \text{and} \quad v^{02} = \frac{G(M_1^0 + M_2^0)}{a_0},$$

where I_f is the ratio of the momentum effectively imparted to the companion to the momentum of the impacting part of the SN shell. I_f is adopted to be 0.3 [see Fryxell (1979) and Fryxell and Arnett (1981), and De Cuyper (1982)]. Note that Eqs. (2) reduce to Eqs. (1) in the case $v_{\text{im}} = 0$. The ratio of the mass of

the SN shell which interacts with the companion to the total mass of the companion, F_{IN} , is:

$$F_{\text{IN}} = \frac{(M_1^0 - M_1^f) \left(\frac{R_2^0}{a_0} \right)^2}{4M_2^0}.$$

Substituting the values of M_1^0 , M_1^f , a_0 , M_2^0 and assuming $R_2^0 = 5.5 R_{\odot}$ for the Be star (the radius of a main-sequence star of $17.3 M_{\odot}$ with $Z = 0.03$ according to Van der Linden, 1982) we obtain $F_{\text{IN}} = 3.2 \times 10^{-5}$. Hence, $5.5 \cdot 10^{-4} M_{\odot}$ of the SN shell of $0.8 M_{\odot}$ interacts with the Be star. De Cuyper gives also an expression for M_2^f [which is a fit to analytic impact calculations for a star with polytropic index 3 by Wheeler et al. (1975)]:

$$M_2^f = M_2^0 \left[1 - \left\{ \frac{\log \phi + 3}{4} \right\}^c \right],$$

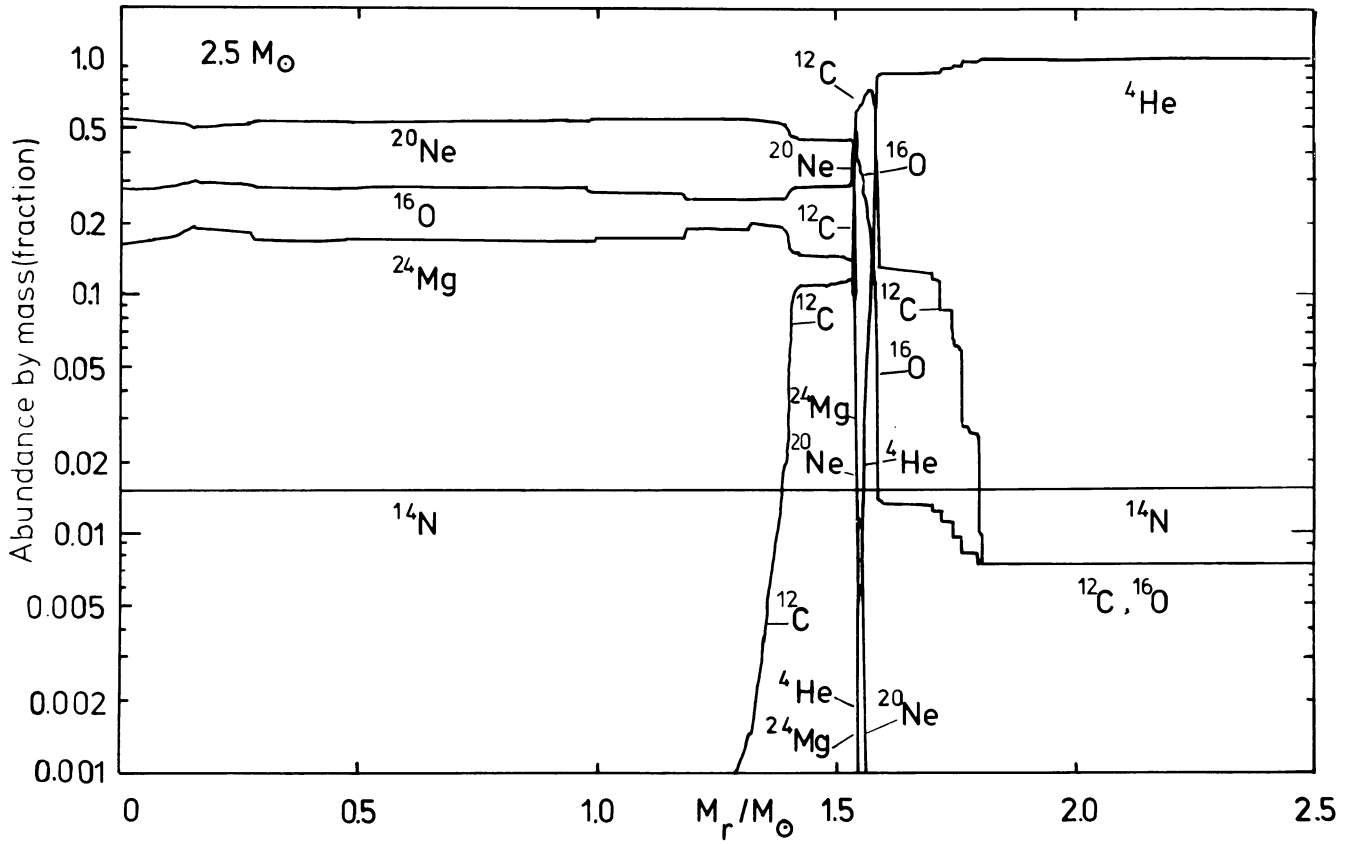


Fig. 10. The composition structure of the interior regions in the last calculated model of the $2.5 M_{\odot}$ helium star at the beginning of convective neon burning in a shell

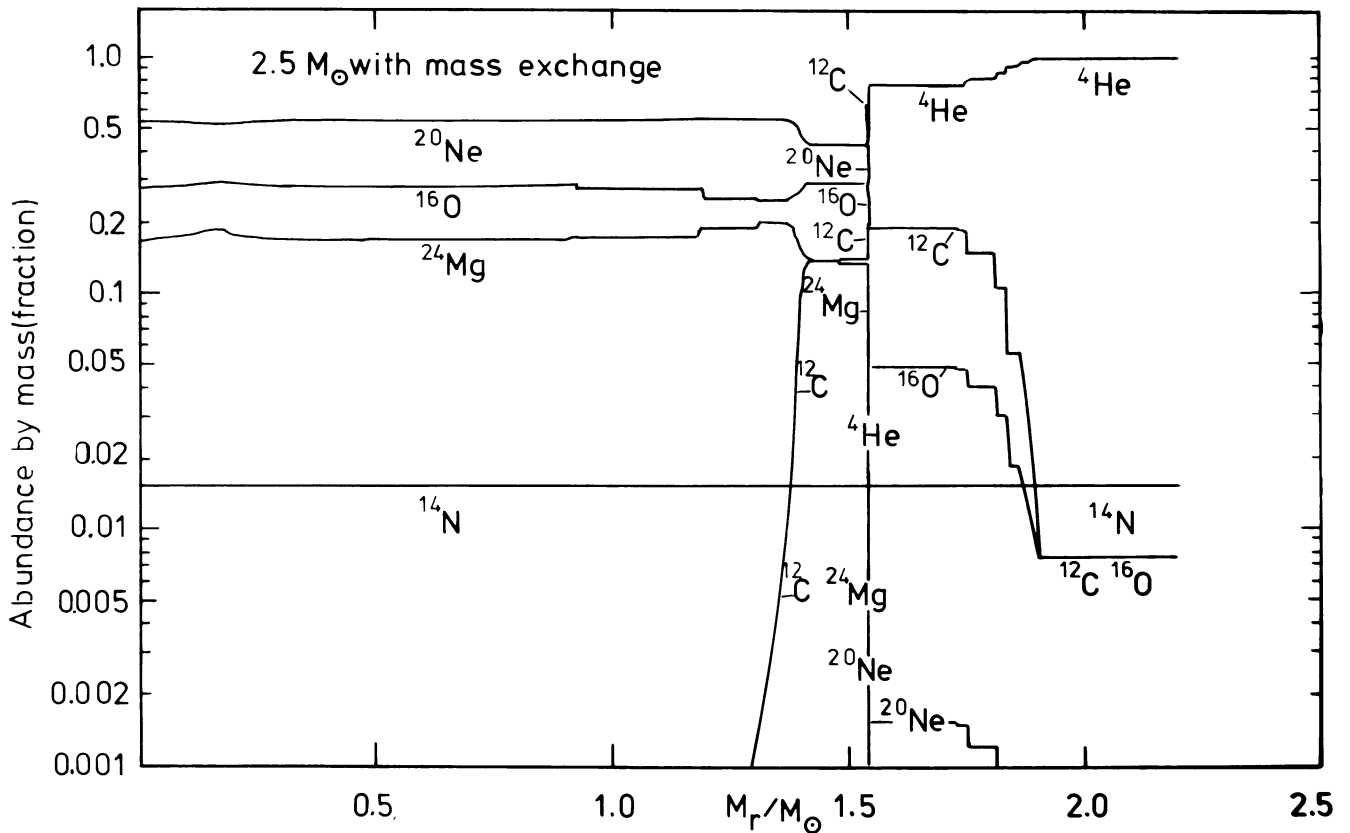


Fig. 11. The same (as in Fig. 10) for the case BB mass-transfer remnant of $2.2 M_{\odot}$

where $\phi = F_{\text{IN}} \left[\frac{v_{\text{SNshell}}}{v_{\text{es}}} - 1 \right]$ with v_{SNshell} being the ejection velocity of the SN shell and v_{es} is the escape velocity from the surface of the companion:

$$v_{\text{es}} = \left(\frac{2GM_2^0}{R_2^0} \right)^{1/2}.$$

This equation only holds for $^{10}\log \phi > -3$; for $^{10}\log \phi \leq -3$ holds in very good approximation: $M_2^f = M_2^0$. The parameter c has the following values:

$c = 4.3$ if $\phi > 0.01$ and $c = 3.3$ if $0.001 < \phi < 0.01$.

Substituting the above-mentioned values of M_2^0 and R_2^0 we derive for v_{es} : 1095.3 km s^{-1} . As $F_{\text{IN}} < 5 \cdot 10^{-5}$ we have to invoke extremely large ejection velocities in order to obtain deviations from a circular orbit due to impact, i.e. v_{SNshell} should be larger than $3.5 \cdot 10^4 \text{ km s}^{-1}$. However, the observed values of v_{SNshell} are in the range 0.5 to $3 \cdot 10^4 \text{ km s}^{-1}$, cf. Schatzman (1965), Zwicky (1965), Shklovsky (1968), and Minkowski (1969). Hence, we can safely neglect the effects of impact on the companion and on the orbit of the system in the case of a symmetric SN explosion. De Cuyper's (1985) results for an asymmetric explosion indicate that low kick velocities ($v_{\text{kick}} < 100 \text{ km s}^{-1}$) are sufficient to yield large eccentricities (> 0.4) for the final orbit (neglecting impact of the SN shell).

6. Comparison with observations of Be/X-ray binaries

Kelley et al. (1983) obtained orbital dimensions for the transient X-ray binary 2S1553 – 542. These authors suggest that the system 2S1553 – 542 contains a Be star with a mass of $5 - 10 M_{\odot}$ according to the mass function. The orbital period of the system is $30^{\text{d}}6 \pm 2^{\text{d}}2$. The orbital eccentricity is at most 0.09. Kelley et al. conclude from this small eccentricity that either the system was tidally circularized or that the SN explosion left the orbit nearly circular. The first possibility is very unlikely, because the tidal circularization time scale is very large in a binary system with $P > 5 - 10$ days, i.e. $> 10^8 \text{ yr}$, according to Savonije and Papaloizou (1983). The second possibility, if true, would require, according to Kelley et al., that the mass of the progenitor of the neutron star was less than $4 M_{\odot}$ (under the assumptions of an upper limit of $20 M_{\odot}$ for the mass of the companion, a mass of the neutron star $< 2 M_{\odot}$, and a SN mass loss in the system $\leq 2 M_{\odot}$). The final orbital period and eccentricity after the SN explosion which we obtained for the $2.2 + 17.3 M_{\odot}$ system happen to be quite similar to those of 2S1553 – 542. Hence, a case BB mass transfer as proposed in this paper could very well have taken place in the progenitor system of 2S1553 – 542.

Table 3 lists the orbital periods and orbital eccentricities of 2S1553 – 542, some other Be/X-ray binaries, some Be-star binaries, and GX301 – 2. [In the case of A0535 + 26 the X-ray flare, the optical outburst, and radial velocity period are probably compatible and yield the orbital period. However, Aab (1984) derived

Table 3. The orbital periods and eccentricities (e) of some Be/X-ray binaries, some Be-star binaries, and the X-ray binaries V0332 + 53, GX301 – 2, and 2S1553 – 542

System	Period (days)	e	Type	References
4U0115 + 63	24.31	0.34	Be/X	Rappaport et al. (1978)
V0332 + 53	34.2 ± 1.5	$0.35 (+0.25, -0.10)$	Be/X?	White et al. (1984)
A0535 + 26	111.0 ± 0.4	$0.3 - 0.4$ (?)	Be/X	Nagase et al. (1982) Priedhorsky and Terrell (1983) Hutchings (1984)
A0538 – 66	111.5 $16.6 \sim 0.7, > 0.4$	0.65 ± 0.17	Be/X	Janot-Pacheco et al. (1985) Charles et al. (1983) Skinner et al. (1982)
GX301 – 2	41.5	0.47	B2Ia/X	Rappaport and Joss (1983) White and Swank (1984)
2S1553 – 542	30.6 ± 2.2	< 0.09	Be/X?	Kelley et al. (1983)
CX Dra	6.697	0.0	Be	Koubsky (1978)
η Ori	7.98	0.0	Be	Batten et al. (1978)
β Lyr	12.93	0.0	Be	Batten et al. (1978)
HD 187399	27.97	0.39 (?)	Be	Batten et al. (1978)
RX Cas	32.3	$0.0 - 0.16$	Be	Kriz et al. (1980)
HD 698	55.93	0.03	Be	Batten et al.
ϕ Per	126.7	0.15	Be	Batten et al.
ζ Tau	132.97	$0.18 (+0.11, -0.05)$	Be	Harmanec (1984)
AX Mon	232.5	0.02	Be	Batten et al.
17 Lep	260	0.13	Be	Batten et al.
ζ Aur	972.2	0.41	Be	Batten et al.
32 Cyg	1147.8	0.30	Be	Batten et al.
31 Cyg	3784.3	0.22	Be	Batten et al.
VV Cep	7430.5	0.35	Be	Batten et al.
KQ Pup	9752	0.46	Be	Batten et al.
ε Aur	9890	0.20	Be?	Hack (1984) and Batten et al.

a three times smaller orbital period of ~ 35 days from a radial velocity study. This period awaits further confirmation.]. The massive ones among the Be-star binaries are likely candidates for an evolution towards Be/X-ray binaries (cf. Rappaport and Van den Heuvel, 1982).

Since circularization occurs on a long time scale, the large observed orbital eccentricities in the wide Be/X-ray binaries listed in Table 3 are due either to an asymmetric SN explosion or to a symmetric SN explosion with a large SN mass loss from the system. This conclusion holds especially for a circular pre-SN orbit, but is weakened for non-circular initial orbits. If we assume a symmetric SN explosion to occur in an initially circular orbit with no effects of impact on the companion and on the orbit, we can obtain the masses of the progenitor systems and the initial pre-SN period (P_0) of 4U0115 + 63, V0332 + 53, A0535 + 26, A0538 - 66, and GX301 - 2. To this end we assume the mass of the neutron star to be $1.4 M_\odot$ and we estimate the final mass of the secondary M_2^f from the spectral type; in the case of GX301 - 2, M_2^f was taken from Rappaport and Joss (1983). In this way we find for the system 4U0115 + 63: $P_0 = 11^d 24$, $M_1^0 = 2.9 - 8.68 M_\odot$ if M_2^0 ranges from 3 to $20 M_\odot$ [the mass of an O or Be star, cf. Hutchings and Crampton (1981)]. Taking $M_2^0 = 9 - 15 M_\odot$ for the mass of the B-type optical component in A0538 - 66 and $e_f = 0.4$ we derive: $P_0 = 6^d 5$ and $M_1^0 = 5.56 - 7.96 M_\odot$. However, for these latter two systems the neglect of impact is questionable. If for the system V0332 + 53 we adopt the lower limit of $e_f (= 0.25)$ and the final period of $34^d 2$, and if we take the mass of the optical component to be $3 - 20 M_\odot$ we derive: $P_0 = 19^d 87$, $M_1^0 = 2.5 - 6.75 M_\odot$. These resulting values for the mass of the progenitor of the neutron star in V0332 + 53 suggest that the neutron star originated from a more massive progenitor than in the system 2S1553 - 42 which has a nearly circular orbit and about the same period [under the assumption of a symmetric SN explosion without impact and a circular pre-SN orbit, after a (case B) mass-exchange phase]. Taking $M_2^0 = 15 - 20 M_\odot$ [the mass of the O9.7 (B0) IIIe companion] and $e_f = 0.3$ we obtain for the system A0535 + 26: $M_1^0 = 6.32 - 7.82 M_\odot$, $P_0 = 57^d 0$; however with $e_f = 0.65$ we find for this system $M_1^0 = \sim 12 - 20 M_\odot$ and $P_0 = 18^d$. With $M_2^0 = 40 M_\odot$ (the minimum mass in absence of eclipses in the X-rays), $e_f = 0.47$ and $P_f = 41^d 5$ we obtain for the system GX301 - 2: $P_0 = 13^d 24$ and $M_1^0 = 20.9 M_\odot$.

From the above derived initial orbital periods and initial masses it is likely that the systems 4U0115 + 63, V0332 + 53, A0535 + 26, and GX301 - 2 all originated from progenitor systems which underwent case B mass transfer. In such a system after the mass exchange a Wolf-Rayet (WR) system is formed in which later on the WR star collapsed to a neutron star. The WR star was the originally more massive star in an unevolved early-type binary. Such unevolved early-type binaries often have a mass ratio close to unity [Garmany et al. (1980) and Lucy and Ricco (1979)]. Hence, in general, due to the mass transfer the period of the pre-SN orbit (i.e. after the mass-transfer phase) was larger than the initial (Zero age) period. Therefore also A0535 + 26 may have had an initial (Zero age) orbital period $< 30^d$ if $e_f = 0.3$ and an even smaller one if $e_f = 0.65$. As a consequence of the mass exchange the pre-SN orbit is expected to have been nearly circular. Massey (1982) pointed out that all observed WR + O systems with $P < 30$ days have nearly circular orbits. Therefore, the above-made assumption of an initially circular orbit is plausible for the pre-SN orbits of 4U0115 + 63, V0332 + 53, A0535 + 26,

and GX301 - 2, such that the conclusions about the pre-SN masses and orbits seem warranted.

7. Discussion and conclusions

7.1. Interior and exterior evolution of single stars and mass-transfer remnants

De Greve and De Loore (1977) have argued that the evolution of single hydrogen stars (or single helium stars) would be very much different from the evolution of hydrogen stars with about the same helium core masses undergoing case B of mass transfer (or even case BB before the onset of core-carbon burning). However, De Greve and De Loore derived much smaller initial $C - O$ core masses for case B mass-transfer remnants than those found by Delgado and Thomas (1981) for helium stars (with comparable progenitor hydrogen star masses) or by us when calculating the advanced evolution of case B remnants of binary evolution [Van der Linden (1982) calculated case B of mass exchange in a $12 + 8 M_\odot$ binary system resulting in a $2.3 M_\odot$ helium star remnant with almost no hydrogen in its envelope. Continuing the evolution of this remnant we found a much larger initial $C - O$ core than De Greve and De Loore for about the same helium core mass.]. In addition, De Greve and De Loore's final $C - O$ core masses at the onset of core-carbon burning agree only with those obtained by Delgado and Thomas [see Law and Ritter (1983)] and not with those resulting from the more detailed calculations of Nomoto (1984a, b) and ourselves. From test calculations for a $2.3 M_\odot$ helium star remnant we find evidence that the advanced interior evolution of case B mass-transfer remnants with envelopes that contain almost no hydrogen, is not at all different from the evolution of single helium stars (with the same mass and Z -value). However, as to the mass transfer in a binary, remnants of an earlier phase of transfer which still have some hydrogen in their envelopes may behave differently from pure helium stars according to the results of De Greve and De Loore (for case B) and Iben and Tutukov (1985; for all kinds of mass-exchange phases including common envelope evolution). This difference depends on the initial orbital period and on the initial mass ratio. In addition, Iben and Tutukov find that the evolution of the $C - O$ cores and helium-burning shells in single stars is very similar to that of such cores and shells in single helium stars (without a hydrogen envelope).

In the carbon-shell burning phase the $2.5 M_\odot$ helium star becomes a red giant. Previously, this was suggested by Savonije and Takens (1976). However, Delgado and Thomas (see Law and Ritter) find that helium stars of $2.0 - 3.33 M_\odot$ reach red giant radii either during helium-shell burning before core-carbon ignition (for a $2.0 M_\odot$ helium star) or during core-carbon burning (for $2.7 M_\odot$ and $3.33 M_\odot$ helium stars). This difference for helium stars in the mass range $2.0 - 3.33 M_\odot$ is due to not allowing the convective helium core to grow in mass and may be also due to the use of different radiative opacities (Habets, 1985a).

7.2. Final fate of the single $2.5 M_\odot$ helium star and the case BB mass-transfer remnant - scenario for the formation of Be/X-ray binaries

Our calculations of the post-mass-transfer evolution of the $2.5 M_\odot$ helium star indicate that the mass-transfer remnant will terminate life as a neutron star (see below), whereas De Greve

and De Loore, Delgado and Thomas, and Iben and Tutukov (in some cases) expect that such a remnant will end as a white dwarf. The reason for this difference is twofold. Firstly, the advanced evolution of the interior regions is not affected by the second mass transfer in our computations contrary to the findings of Delgado and Thomas (in case of a $2.0 M_{\odot}$ helium star), De Greve and De Loore, and Iben and Tutukov. Secondly, these latter groups of authors obtain smaller final $C - O$ core masses before core-carbon ignition [and these cores are even smaller if the $C - O$ core remains constant ("freezes") during the second mass-transfer phase (De Greve and De Loore; however, see Iben and Tutukov)]. Delgado and Thomas, and Nomoto (1984a, b) neglect the effect of convective mixing in unstable layers around the convective helium-burning core (overshooting). However, Nomoto obtains larger final masses for the $C - O$ cores than Delgado and Thomas. Hence, the size of the $C - O$ core depends on the input physics and the computational method. Because we allow the convective helium-burning core to grow in mass (in order to keep fulfilling the Schwarzschild criterion for the onset of convection, but not calculating the effects of overshooting) we derive even larger final $C - O$ core masses than Nomoto. On the other hand, in a binary the final extent of the $C - O$ core depends on the choice of the initial orbital period and initial mass ratio (Iben and Tutukov, 1985). Due to the larger final $C - O$ core mass, the interior evolution of our $2.5 M_{\odot}$ helium star resembles that of a more massive helium star in the calculations of Delgado and Thomas and of Nomoto.

At core-neon ignition the $C - O$ core masses of the $2.5 M_{\odot}$ helium star and the $2.2 M_{\odot}$ mass-transfer remnant are larger than the Chandrasekhar limiting mass. Hence, these stars will undergo core collapse. Furthermore, in the carbon-shell burning phase, these stars develop a steep density gradient at the edge of the $C - O$ core which, in the core collapse phase, may favour the development of a SN explosion and the formation of a neutron star (Hillebrandt, 1984).

The effects of a symmetric SN explosion on the orbital dimensions in the $2.2 + 17.3 M_{\odot}$ binary system ($P = 28^d.09$) were shown to be small. Therefore, in so far the neglect of impact of the SN shell on the companion and on the orbit is not questionable and with the assumptions of a circular pre-SN orbit and a symmetric SN explosion, we may conclude that Be/X-ray binaries may be formed by case B or case BB mass-transfer dominated evolution in an intermediate-mass binary [cf. Rappaport and Van den Heuvel (1982)].

In forthcoming papers [Habets (1986a, b, and in progress)] an extensive discussion of the evolution of helium stars with masses in the range $2.0 - 4.0 M_{\odot}$ will be given. Part of this paper is chapter IIIA of Habets (1985a).

Acknowledgements. I am indebted to Prof. Dr. E.P.J. van den Heuvel for suggesting this investigation, for his enthusiasm and encouragements, to Dr. Th.J. van der Linden for introducing me to his stellar evolution code and for making further improvements in the code, to Dr. G.J. Savonije for critical remarks on Sect. 2 of the manuscript, and to E. Faverey for preparing the figures and tables. The stellar evolution computations were performed on the two CDC Cybers 175 of the Academic Computer Centre Amsterdam (SARA).

This research was supported in part by the ZWO-Foundation for Astronomical Research in the Netherlands, ASTRON, under project no. 19-21-09.

References

- Aab, O.E.: 1984, *Soviet. Astron. L.* **10**, 386 (*Pis ma Astron. Zh.* **10**, 918)
- Arnett, W.D.: 1972a, *Astrophys. J.* **176**, 681
- Arnett, W.D.: 1972b, *Astrophys. J.* **176**, 699
- Arnett, W.D.: 1974, *Astrophys. J.* **194**, 373
- Arnett, W.D.: 1978a, in *Physics and Astrophysics of Neutron Stars and Black Holes*, R. Giacconi and R. Ruffini (eds.), North-Holland Publ. Co., Amsterdam, New York, Oxford, Proceedings of the International School of Physics "Enrico Fermi", Course LXV, p. 356
- Arnett, W.D.: 1978b, *Astrophys. J.* **219**, 1008
- Batten, A.H., Fletcher, J.M., Mann, P.J.: 1978, *Publ. Dominion Astrophys. Obs. Victoria, B.C.* **15**, 121
- Beaudet, G., Petrosian, V., Salpeter, E.E.: 1967, *Astrophys. J.* **150**, 979
- Boozar, A.H., Joss, P.C., Salpeter, E.E.: 1973, *Astrophys. J.* **181**, 393
- Bressan, A.G., Bertelli, G., Chiosi, C.: 1981, *Astron. Astrophys.* **102**, 25
- Castellani, V., Giannone, P., Renzini, A.: 1971a, *Astrophys. Space Sci.* **10**, 340
- Castellani, V., Giannone, P., Renzini, A.: 1971b, *Astrophys. Space Sci.* **10**, 355
- Castellani, V., Chieffi, A., Pulone, L., Tornambe, A.: 1985a, *Astrophys. J.* **296**, 204
- Castellani, V., Chieffi, A., Pulone, L., Tornambe, A.: 1985b, *Astrophys. J.* **294**, L33
- Charles, P.A., Booth, L., Densham, R.H., Bath, G.T., Thorstensen, J.R., Howarth, I.D., Willis, A.J., Skinner, G.K., Olszewski, E.: 1983, *Monthly Notices Roy. Astron. Soc.* **202**, 657
- Cox, J.P., Giuli, R.T.: 1968, in *Principles of Stellar Structure*, Vol. 1 and 2, Gordon and Breach, Science Publishers, New York
- De Cuyper, J.-P.: 1982, in *Binary and Multiple Stars as Tracers of Stellar Evolution*, Z. Kopal and J. Rahe (eds.), (D. Reidel Publ. Co., Dordrecht-Boston-London), p. 417
- De Cuyper, J.-P.: 1984, Ph.D. Thesis Vrije Universiteit Brussel, Brussels
- De Cuyper, J.-P.: 1985, submitted to *Astron. Astrophys.*
- De Greve, J.-P., De Loore, C.: 1977, *Astrophys. Space Sci.* **50**, 75
- Delgado, A.J., Thomas, H.-C.: 1981, *Astron. Astrophys.* **96**, 142
- Dicus, D.A.: 1972, *Phys. Rev. D* **6**, Third Series, No. 4, 941
- Doom, C.: 1982, *Astron. Astrophys.* **116**, 303
- Flannery, B.P., Van den Heuvel, E.P.J.: 1975, *Astron. Astrophys.* **39**, 61
- Fontaine, G., Graboske, H.C., Jr., Van Horn, H.M.: 1977, *Astrophys. J. Suppl.* **35**, 205
- Fryxell, B.A.: 1979, *Astrophys. J.* **234**, 641
- Fryxell, B.A., Arnett, W.D.: 1981, *Astrophys. J.* **243**, 994
- Garmany, C.D., Conti, P.S., Massey, P.: 1980, *Astrophys. J.* **242**, 1063.
- Habets, G.M.H.J.: 1984, in *Observational Tests of the Stellar Evolution Theory*, A. Maeder and A. Renzini (eds.), IAU Symp. **105**, D. Reidel Publ. Co., Dordrecht-Boston-Lancaster, p. 399
- Habets, G.M.H.J.: 1985a, in *Advanced Evolution of Helium Stars and Massive Close Binaries*, Ph.D. Thesis University of Amsterdam
- Habets, G.M.H.J.: 1985b, in *Proceedings of the workshop on Nuclear Astrophysics*, May 20–24, at Ringberg Castle, Tegern-

- see, preprint Max-Planck-Institut für Astrophysik, Institut für Physik und Astrophysik Munich, No. 199, p. 9
- Habets, G.M.H.J.: 1986a, submitted to *Astron. Astrophys.*
- Habets, G.M.H.J.: 1986b, submitted to *Astron. Astrophys. Suppl. Ser.*
- Hack, M.: 1984, *Sci. Am.* **251**, 92
- Harmanec, P.: 1984, *Bull. Astron. Inst. Czech.* **35**, 164
- Hillebrandt, W.: 1982, *Astron. Astrophys.* **110**, L3
- Hillebrandt, W.: 1984, *Ann. N.Y. Acad. Sci.* **422**, p. 197, 11th Texas Symposium on Relativistic Astrophysics, New York Academy of Sciences (Publishers)
- Hillebrandt, W., Nomoto, K., Wolff, R.G.: 1984, *Astron. Astrophys.* **133**, 175
- Hutchings, J.B.: 1984, *Publ. Astron. Soc. Pacific* **96**, 312
- Hutchings, J.B., Crampton, D.: 1981, *Astrophys. J.* **247**, 222
- Iben, I. Jr.: 1985, submitted to *Astrophys. J.*
- Iben, I. Jr., Tutukov, A.V.: 1985, *Astrophys. J. Suppl. Ser.* **58**, 661
- Ikeuchi, S., Nakazawa, K., Murai, T., Hoshi, R., Hayashi, C.: 1971, *Prog. Theor. Phys.* **46**, 1713
- Ikeuchi, S., Nakazawa, K., Murai, T., Hoshi, R., Hayashi, C.: 1972, *Prog. Theor. Phys.* **48**, 1870
- Janot-Pacheco, E., Motch, C., Mouchet, M.: 1985, submitted to *Astron. Astrophys.*, January
- Joss, P.C., Katz, J.I., Malone, R.C., Salpeter, E.E.: 1973, *Astrophys. J.* **181**, 409
- Kelley, R., Rappaport, S., Ayasli, S.: 1983, *Astrophys. J.* **274**, 765
- Koubsky, P.: 1978, *Bull. Astron. Inst. Czech.* **29**, 288
- Kriz, S., Arsenijevic, J., Grygar, J., Harmanec, P., Horn, J., Koubsky, P., Pavlovski, K., Zverko, J., Zdarsky, F.: 1980, *Bull. Astron. Inst. Czech.* **31**, 284
- Kutter, G.S., Savedoff, M.P.: 1967, *Astron. J.* **72**, 810
- Kutter, G.S., Savedoff, M.P.: 1969, *Astrophys. J.* **157**, 1021
- Law, W.-Y.: 1982, *Astron. Astrophys.* **108**, 118
- Law, W.-Y., Ritter, H.: 1983, *Astron. Astrophys.* **123**, 33
- Lucy, L.B., Ricco, E.: 1979, *Astron. J.* **84**, 401
- Maeder, A.: 1975, *Astron. Astrophys.* **40**, 303
- Maeder, A.: 1981, *Astron. Astrophys.* **99**, 97
- Massey, P.: 1982, In *Wolf-Rayet Stars: Observations, Physics, Evolution*, C.W.H. de Loore and Willis, A.J. (eds.), IAU Symp. **99**, D. Reidel Publ. Co., Dordrecht-London-Boston, p. 251
- Minkowski, R.: 1969, in *Supernovae and their Remnants*, P.J. Brancario and A.G.W. Cameron (eds.), Gordon and Breach Science Publishers, New York-London-Paris, p. 23
- Nagase, F., Hayakawa, S., Kunieda, H., Makino, F., Masai, K., Tawara, Y., Inoue, H., Kawai, N., Koyama, K., Makishima, K., Matsuoka, M., Murakami, T., Oda, M., Ogawara, Y., Ohashi, T., Shibazaki, N., Tanaka, Y., Miyamoto, S., Tsunemi, H., Yamashita, K.: 1982, *Astrophys. J.* **263**, 814
- Nomoto, K.: 1980a, *Space Sci. Rev.* **27**, 563
- Nomoto, K.: 1980b, in *Type I Supernovae, Proceedings of the Texas Workshop on Type I Supernovae*, J.C. Wheeler (ed.), Univ. Texas, Austin and McDonald Obs., Austin, Texas, p. 164
- Nomoto, K.: 1981, in *Fundamental Problems in the Theory of Stellar Evolution*, D. Sugimoto, D.Q. Lamb, and D.N. Schramm (eds.), IAU Symp. **93**, D. Reidel Publ. Co., Dordrecht-Boston-London, p. 295
- Nomoto, K.: 1982a, *Astrophys. J.* **253**, 798
- Nomoto, K.: 1982b, *Astrophys. J.* **257**, 780
- Nomoto, K.: 1982c, *Comments Astrophys.* **9**, 301
- Nomoto, K.: 1982d, in *Supernovae: A Survey of Current Research*, M.J. Rees and R. J. Stoneham (eds.), (D. Reidel Publ. Co., Dordrecht-Boston-London), p. 205
- Nomoto, K.: 1984a, *Astrophys. J.* **277**, 791
- Nomoto, K.: 1984b, in *Problems of Collapse and Numerical Relativity*, Toulouse, D. Bancel and M. Signore (eds.), D. Reidel Publ. Co., Dordrecht-Boston-London, NATO-ASI C134, p. 89
- Nomoto, K.: 1984c, in *Stellar Nucleosynthesis*, C. Chiosi and A. Renzini (eds.), D. Reidel Publ. Co., Dordrecht-Boston-London, p. 205
- Nomoto, K.: 1984d, in *Nucleosynthesis: Challenges and New Developments*, W.D. Arnett and J.W. Truran (eds.), Yerkes Observatory, Univ. Chicago Press, preprint
- Nomoto, K.: 1984e, in *Stellar Nucleosynthesis*, C. Chiosi and A. Renzini (eds.), D. Reidel Publ. Co., Dordrecht-Boston-London, p. 239
- Nomoto, K.: 1985, in *Proceedings XIIth Texas Symposium on Relativistic Astrophysics*, New York Academy of Sciences (Publishers), preprint.
- Paczynski, B.: 1970, *Acta Astron.* **20**, 47
- Paczynski, B.: 1971a, *Acta Astron.* **21**, 1
- Paczynski, B.: 1971b, *Ann. Rev. Astron. Astrophys.* **9**, 183
- Priedhorsky, W.C., Terrell, J.: 1983, *Nature* **303**, 681
- Rappaport, S.A., Joss, P.C.: 1983, in *Accretion-driven Stellar X-ray Sources*, W.H.G. Lewin and E.P.J. Van den Heuvel (eds.), Cambridge University Press, p. 1
- Rappaport, S., Van den Heuvel, E.P.J.: 1982, in *Be Stars*, M. Jasehek and H.-G. Groth (eds.), IAU Symp. **98**, (D. Reidel Publ. Co., Dordrecht-London-Boston), p. 327.
- Rappaport, S., Clark, G.W., Cominsky, L., Joss, P.C., Li, F.K.: 1978, *Astrophys. J.* **224**, L1
- Renzini, A.: 1977, in *Advanced Stages of Stellar Evolution*, P. Bouvier and A. Maeder (eds.), 7th "Advanced course of the Swiss Society of Astronomy and Astrophysics", Saas-Fee, Geneva Observatory, p. 149
- Savonije, G.J., Papaloizou, J.C.B.: 1983, *Monthly Notices Roy. Astron. Soc.* **203**, 581
- Savonije, G.J., Takens, R.J.: 1976, *Astron. Astrophys.* **47**, 231
- Schatzman, E.: 1965, in *Stellar Structure*, L.H. Aller and D.B. McLaughlin (eds.), Univ. Chicago Press, Chicago-London, p. 327
- Schwarzschild, M.: 1958, in *Structure and Evolution of the Stars*, Princeton University Press, Princeton, New York
- Shklovsky, I.S.: 1968, in *Supernovae*, Vol. 21 in "Interscience Monographs and Texts in Physics and Astronomy", R.E. Marshak (ed.), Univ. Rochester, New York, John Wiley & Sons Ltd., London-New York-Sydney
- Skinner, G.K., Bedford, D.K., Elsner, R.F., Leahy, D., Weisskopf, M.C., Grindlay, J.: 1982, *Nature* **297**, 568
- Sreenivasan, S.R., Wilson, W.J.F.: 1985, *Astrophys. J.* **292**, 506
- Stevenson, D.J.: 1979, *Monthly Notices Roy. Astron. Soc.* **187**, 129
- Strittmatter, P.A., Faulkner, J., Robertson, J.W., Faulkner, D.J.: 1970, *Astrophys. J.* **161**, 369
- Sweigart, A.V., Renzini, A.: 1979, *Astron. Astrophys.* **71**, 66
- Van den Heuvel, E.P.J.: 1985, in *Birth and Evolution of Massive Stars and Stellar Groups*, W. Boland and H. van Woerden (eds.), D. Reidel Publ. Co., Dordrecht-Boston-Lancaster-Tokyo, p. 107
- Van der Linden, Th.J.: 1982, in *Mass Transfer in Close Binaries*, Ph.D. Thesis, University of Amsterdam
- Van der Linden, Th.J.: 1985, submitted to *Astron. Astrophys.*

- Wheeler, J.C., Lecar, M., McKee, C.F.: 1975, *Astrophys. J.* **200**, 145.
- White, N.E., Swank, J.H.: 1984, *Astrophys. J.* **287**, 856
- White, N.E., Davelaar, J., Parmar, A.N., Stella, L., Van der Klis, M.: 1984, IAU Circ. **3912**
- Wilson, J.R., Mayle, R., Woosley, S.E., Weaver, T.A.: 1985, in *Proceedings XIIth Texas Symposium on Relativistic Astrophysics*, Jerusalem, Israel, New York Academy of Sciences (Publishers), preprint
- Woosley, S.E., Weaver, T.A., Taam, R.E.: 1980, in *Type I Supernovae*, J.C. Wheeler (ed.), Univ. Texas Press, p. 96
- Woosley, S.E., Axelrod, T.S., Weaver, T.A.: 1984, in *Stellar Nucleosynthesis*, C. Chiosi and A. Renzini (eds.), D. Reidel Publ. Co., Dordrecht-Boston-London, p. 263
- Zwicky, F.: 1965, in *Stellar Structure*, L.H. Aller and D.B. McLaughlin (eds.), Vol. 8 in the series "Stars and Stellar Systems", G.P. Kuiper and B.M. Middlehurst (eds.), Univ. Chicago Press, Chicago-London, p. 367