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A Commemoration of Maskelyne at Schiehallion

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At 11 o'clock on the morning of 1983 April 15 a group of 40 people could be seen assembled at Braes of Foss on the road that runs south-east from Kinloch Rannoch to General Wade's Old Military Road and Aberfeldy. A cairn had been erected some months previously on the corner of this road and a forest track running south. It is from this track that the footpath begins to the summit of the mountain Schiehallion which on that morning was wrapped in rolling mist and cloud through which the high snows could be seen from time to time. The purpose of the assembly was to dedicate the cairn made from local rocks on which was a plaque commemorating the classical measurement of the gravitational pull of Schiehallion by Nevil Maskelyne, the fifth Astronomer Royal. This cairn and plaque had been set up by the Royal Society and the Royal Astronomical Society in 1982 on the 250th anniversary of Maskelyne's birth in 1732.

The short ceremony was opened by Professor J.F. Allen of St Andrews University who had provided much of the effort and drive to set up a cairn on this popular scenic and mountain walking route. Sir Andrew Huxley, President of the Royal Society, gave the speech of dedication in which he underscored the fundamental nature of the measurement of the 'attraction of mountains'. The ceremony was concluded with the playing of a lively Scottish reel on a fiddle owned by a descendant of the Duncan Robertson of Struan who assisted Maskelyne in his measurements on the mountain and to whom Maskelyne gave the fiddle after a near-disastrous fire on the mountain in which Duncan's own fiddle was destroyed. The author attended the ceremony on behalf of the Royal Astronomical Society.

Below I set down the background to this classical set of observations by Nevil Maskelyne and I give a short account of the experiment and the significance of the results, the main account of which was published in the *Philosophical Transactions of the Royal Society* in 1775 (Maskelyne 1775) under the title 'A proposal for measuring the attraction of some hill in this Kingdom by astronomical observations'.

1 THE BACKGROUND

The idea that mountains would pull a plumb-line from the vertical was already set out by Isaac Newton (1687) in the third book of his *Principia* where he showed that all three of Kepler's laws were the consequence of the inverse square law of gravitational attraction. He gave as an example a hemispherical mountain 3 miles high and 6 miles broad which he said would deflect a pendulum somewhat less than 2 arcmin (actually 1′ 18″ if the mountain had the same density as the mean density of the Earth). Newton concluded that such deflections by mountains would be too small to measure. However over the next 100 years advances in astronomical and surveying techniques made possible such measurements in the hands of a careful observer of which Nevil Maskelyne was the quintessential example.

Maskelyne's involvement in the measurement of the attraction of mountains may be thought of as having two origins. The first was his interest in acquiring the best instruments and using them to the limit of their accuracy. There is the apparently true story of the dismissal of an observing assistant because his equation of time was significantly (0·5 s) greater than Maskelyne's own. He is regarded as being the major force in reviving British observational astronomy in the late eighteenth century. Among the instruments he improved was the zenith sector used to make accurate measurements of the zenith distances of stars relative to a plumb-line. It is estimated that the boundary (38th latitude) between Maryland and Pennsylvania was established by Charles Mason and Jeremiah Dixon at the instigation of Maskelyne to a consistency of 15–20 yards (0·5–0·7 arcsec). This instrument was at the heart of the measurements on Schiehallion.

The second thread of Maskelyne's interest came from his involvement in measurements of the length of the degree of meridian arc on the surface of the Earth. This was fundamental for determining the figure of the Earth and for obtaining accurate relative latitudes of observatories which was essential for coordinating positional astronomy. Inconsistencies between various observers of the true length of the degree measured at widely spaced locations on the Earth were well-established by 1770. It was known that the rotation of the Earth was one non-instrumental component: this oblateness had been evaluated by Maupertius and others in the 1740s. In assessing these inconsistencies and in reading Bouguer's *La Figure de la Terre* (Paris, 1749) Maskelyne became aware that the attraction of mountains first discussed by Newton might be having a significant effect on measurements of the length of the meridian degree of arc (Forbes 1975). He believed that the extant measurements were not of sufficient accuracy to establish unambiguously the magnitude of this effect and he determined to make a suitable measurement with instruments which he was confident could provide the required accuracy, namely one second of arc or better.

2 MASKELYNE'S PROPOSAL TO MEASURE THE ATTRACTION OF MOUNTAINS

Maskelyne formulated his proposal to measure the attraction of mountains in a paper read to the Royal Society in 1772 (and subsequently published in the *Philosophical Transactions* of 1775 along with his report of the actual measurements).

> If the attraction of gravity be exerted, as Sir Isaac Newton supposes, not only between the large bodies of the universe, but between the minute particles of which these bodies are composed, or into which the mind can imagine them to be divided, acting universally according to that law, by which the force which carries on the celestial motions is regulated; namely that the accelerative force of each particle of matter towards every other particle decreases as the squares of the distances increase, it will necessarily follow, that every hill must, by its attraction, alter the

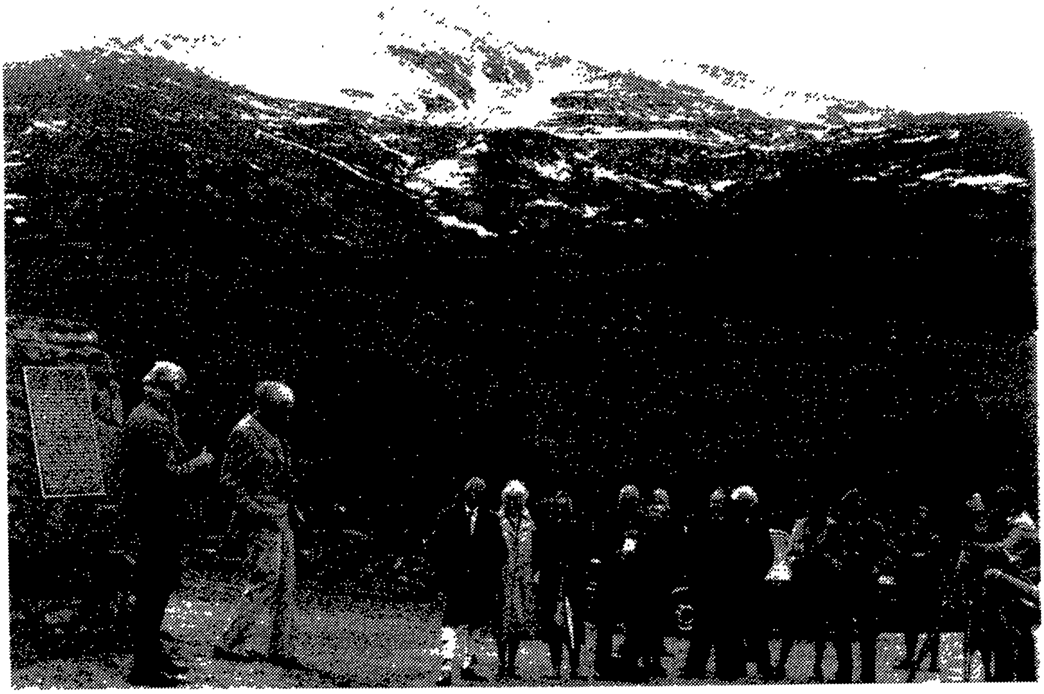

PLATE I. The dedication of the plaque and cairn at Schiehallion commemorating Nevil Maskelyne's measurement of the deflection of the plumb-line by a mountain. The ceremony was performed by Sir Andrew Huxley (left), President of the Royal Society, on 1983 April 15. The mountain Schiehallion (3553 ft) is in the background.

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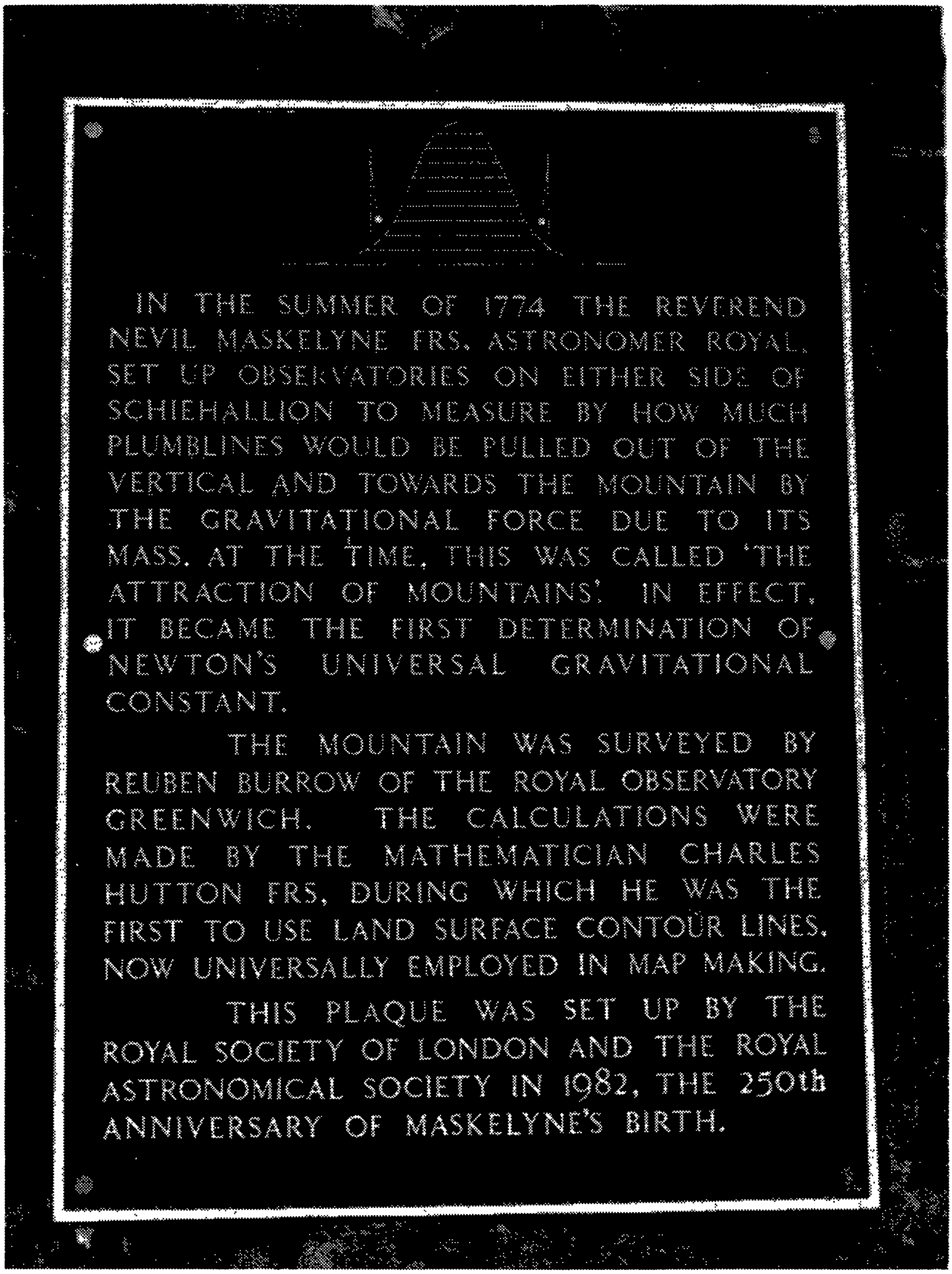

Plate II. The plaque bearing the inscription recalling the measurement at Schiehallion on the gravitational deflection of plumb-lines.

> direction of gravitation in heavy bodies in its neighbourhood from what it would have been from the attraction of the earth alone, considered as bounded by a smooth and even surface. For, as the tendency of heavy bodies downwards perpendicular to the earth's surface is owing to the combined attraction of all parts of the earth upon it, so a neighbouring mountain ought, though in a far less degree, to attract the heavy body towards its centre of attraction, which cannot be placed far from the middle of the mountain.

Such an observation he argued would not only demonstrate the general applicability of the theory of gravity but it would equally importantly clarify fundamental properties of the Earth, namely the total mass of the Earth and the ratio of the density near the surface to the mean density of the whole Earth. He then proceeded to consider hills in England which might be suitable: these included Pendle Hill, Pennygant near Ingleborough and Whernside, all of which would give deviations of the plumb-line in the range 10–40 arcsec, depending on their density. The inverse measurement was also proposed, namely the contrary attraction experienced in the valley between two large hills. The valley of St John's in the Vale lying between Skiddaw and Helvellyn in Cumberland would have afforded a measurement of a sum of deviations of about 10 arcsec.

Accordingly at Maskelyne's instigation a committee of the Royal Society was set up to consider an appropriate hill for this measurement and to ensure that proper arrangements were made to undertake the necessary measurements successfully. Charles Mason, who had been employed on several occasions on astronomical expeditions, was appointed to make a tour through the Highlands of Scotland in the summer of 1773. Maskelyne reported on the success of this tour in the following terms

> Fortunately, however, Perthshire afforded us a remarkable hill, nearly in the centre of Scotland, of sufficient height, tolerably detached from other hills, and considerably larger from East to West than from North to South, called by the people of the low country Maiden-pap, but by the neighbouring inhabitants, Schehallion; which I have since been informed, signifies in the Erse language, Constant Storm; a name well adopted to the appearance which it so frequently exhibits to those who live near it, by the clouds and mists which usually crown its summit.

With the choice of mountain made and with Maskelyne appointed to be in charge of the expedition, arrangements were put in hand for a campaign of observations in the summer of 1774. Funds were made available from the residual grant by George III for the 1769 transit of Venus expedition made in the South Seas by the *Endeavour* with James Cook as captain.

3 THE EXPEDITION TO THE MOUNTAIN SCHIEHALLION

The programme of measurements was in two parts. One was a survey of the mountain to establish its volume and consequently its mass. This task was undertaken by Reuben Burrows, a former assistant of Maskelyne's and William Menzies, a local land surveyor. Charles Hutton, the mathematician, subsequently reduced the survey data and derived the form of the attraction of Schiehallion at the observatories on its two flanks. The other programme under the immediate supervision of Maskelyne himself was the measurement of the deflection of the plumb-line at the northern and southern observatories located about halfway up the side of the 3553 ft (1083 m) mountain.

The principal instrument used by Maskelyne was the 10 ft zenith sector which he had previously used on the transit of Venus observations on St Helena in 1761. With proper use, this instrument could measure the angular distance of a star from the zenith at transit as defined from the plumb-line to a precision better than one second of arc. This accuracy was the result of Maskelyne's improvements in its design, particularly by reducing the diameter of the support pivot pin of the plumb-line suspension. Other important instruments included John Bird's astronomical quadrant, an astronomical clock and the theodolite used in the survey of the mountain.

The basic astronomical experiment was the determination of the apparent latitude at each of the northern and southern sites. This required as a first step the measurement of the meridian for which, characteristically, Maskelyne devised an improved method, stimulated by the lack of clear skies, in which stars of known right ascension were timed with the accurate clock and the offset from the meridian calculated. This technique is now in common use. He further realized that there may be systematic errors in setting and reading the zenith sector and accordingly he made two series of measurements at both sites first with the plane of the zenith sector facing east and then facing west. This again proved to be a decisive factor in obtaining his accurate results. Maskelyne's observations extended from 1774 July to October.

The survey of Schiehallion proved to be critical in converting the measurement of the deflection of a plumb-line into an estimate of the density of the Earth. In itself, the survey was probably not the limiting factor, but rather, the density distribution within the mass of the mountain. Burrows and Menzies used a fundamental baseline 5895·4 ft in length on the Plain of Rannoch north of Schiehallion. This baseline was used to triangulate a polygon surrounding the base of the mountain which in turn was used to observe sections of the mountain from which its detailed shape was reconstructed. This task of painstaking surveying and triangulation extended from spring to autumn 1774 with a few completion measurements in 1775. The survey also yielded the physical separation between the two observatories half-way up the northern and southern faces of Schiehallion.

4 THE RESULTS OF MASKELYNE'S MEASUREMENT OF THE ATTRACTION OF MOUNTAINS

In all, Maskelyne made 337 observations of 43 stars passing near the zenith at Schiehallion (longitude 4°05′W; latitude 56°40′N). From 76 observations of 34 stars at the southern observatory and 100 observations of 37 stars at the northern observatory in both cases with the zenith sector facing east he derived an *apparent* latitude difference of 55·0 arcsec. Likewise from 93 observations on 39 stars at the southern observatory and 68 observations on 32 stars at the northern observatory with the zenith sector facing west he derived an *apparent* latitude difference of 54·2 arcsec. The close agreement between these values confirmed that his method of observation had correctly taken account of the collimation errors in the zenith sector. The mean value of 54·6 arcsec was compared with the *trigonometric* latitude difference of 42·94 arcsec derived from the surveying and led Maskelyne to state 'The difference, 11·6 arcsec, is to be attributed to the sum of the two contrary attractions of the hill'. Basically the observations had shown that a plumb-bob placed half-way up the narrow dimension of Schiehallion is deflected by 5·8 arcsec. The precision of this value as indicated by the tabulated results is better than one second of arc.

In his second *Philosophical Transactions* paper of 1775 Maskelyne sets down his four conclusions in the following form

1. The attraction of Schiehallion was sensible in magnitude; hence every particle of matter is endowed with gravitational force proportional to its mass.
2. This force varies as Newton postulated, as the inverse square of the distance between any two attracting bodies whether they be terrestrial or celestial.
3. The Earth's mean density is at least double that at its surface, implying a greater density nearer the core which result is 'totally contrary to the hypothesis of some naturalists, who suppose the earth to be only a great hollow shell of matter'.
4. In consequence of the deflection in the direction of the plumb-line produced by the less dense superficial parts of the Earth, errors were liable to be incurred in astronomical measurements of meridian arcs.

Maskelyne had shown for the first time that two non-astronomical masses, namely a mountain and a plumb-bob, show a measurable attraction to one another. This was a major new test of Newton's gravitational theory. Newton's gravitational constant G could not be determined directly by these observations. A measurement of the deflection of mountains provides a *ratio* between two gravitational forces and as a consequence is a measure of the ratio of two values of M/R^2 where M is the mass and R is its distance from the point at which the deflection is measured. Since for Schiehallion M_s and R_s are in principle known from the survey and the radius (R_E) of the Earth is known, Maskelyne was basically measuring the mass M_E of the Earth. He chose to express this as a density. Accepting the density of the surface material as between 2·0 and 2·5, his result could be interpreted as indicating that the mean density of the Earth was between 4 and 5.

As for his claim that he had demonstrated that the inverse square of gravitational attraction had been demonstrated by the Schiehallion observations, he could, in reality, only claim that the index was approximately 2·0. Given that he had determined the density and hence the mass of the Earth to a factor of 1·5 and the ratio of the distances of the two gravitating bodies which deflected the plumb-bob was $\sim 10^4:1$, Maskelyne's observations imply an index of 2 with an uncertainty of only 0·044. We see that he was indeed justified in his belief that he had demonstrated the inverse square law of attraction between terrestrial bodies.

The mammoth task of evaluating the density of the Earth from Maskelyne's measurements of the plumb-bob deflection and the survey of Schiehallion was undertaken by Charles Hutton. Hutton's value of 4·5 for the specific gravity of the Earth, based on 2·5 for the mountain, was published in 1778. After a subsequent resurvey of the lithography of the rocks of Schiehallion, the value was assessed to be in the range 4·56–4·87. Some 23 years after Cavendish (1798) had made his successful torsion balance measurement which gave a value of 5·448, Hutton (1821) again re-argued the case for the deflection of the plumb-bob method and revised his value upwards to 4·95, still significantly less than the torsion balance value.

Without detracting from the importance of Maskelyne's results, the limitations of the plumb-bob deflection method are such that an accuracy of 10 per cent is optimistic. The greatest uncertainty lies in the density profile of the mountain. A lesser fractional uncertainty arises from the smallness of the angular deflection.

It is of interest to compare modern values of the mean density of the Earth and of G. Observations of planetary orbits give accurate values of the product GM. G can still only be measured through accurate torsion balance experiments. A recent value for G is $6{\cdot}670 \pm 0{\cdot}015 \times 10^{-11}$ N m^2 kg^{-2}. This leads to a density of the Earth of 5·52 g cm^{-3}.

In summary, the significance of Maskelyne's measurements of the 'attraction of mountains' was two-fold. Firstly, they confirmed and extended the experimental evidence for the Newtonian theory of gravitation. Secondly they underlined the importance of contributions of the Earth's surface features to accurate measurements of the length of the meridian arc. These latter were significant in refining theories of the lunar motion which were dependent upon an accurate knowledge of the Earth's oblateness.

I have described above one of Nevil Maskelyne's contributions to science. He is also remembered for his wider contributions to the revival of British observational astronomy (see, e.g. Forbes (1975) for the context of his contributions). He saw to it that instruments and buildings were kept under good repair and he made all results of the Royal Observatory available to the astronomical community. His emphasis on precise measurement led to the determination of the most accurate positions then available for 36 stars. His contributions to navigation were no less important. He assessed chronometers for longitude determination, he instituted the nautical almanac and introduced the method of lunar distances used in finding longitude at sea for the next century.

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