

of Weidenschilling⁶, is $P_E \approx 1.9 \times 10^{-10}$ per year. However, the increased velocity decreases the gravitational focussing factor, and the collision probability is reduced to $P_c \approx 0.23 \times 10^{-10}$ per year. The total probability of elimination (collision or ejection) is therefore about twice as likely if Zagreus were retrograde than if it were prograde. The angular deflection per encounter (root-mean-square) is only ~ 0.1 at this higher relative velocity, so to accumulate a deflection of 90° takes $\approx 3 \times 10^9$ years. This compares with an elimination time-scale of $1/(0.23 \times 10^{-10} + 1.9 \times 10^{-10}) \approx 5 \times 10^9$ years. Therefore many of the retrograde objects would be eliminated by Neptune, but almost all of those which are prograde would be passed to the control of another planet. It therefore appears that Pluto exists in its Neptune-crossing orbit, and has not been lost in a collision or as a result of a series of close encounters, solely on account of its stable resonance and position in its orbit.

References

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NOTES FROM OBSERVATORIES

THE COSMOLOGICAL CONSTANT IN THE MCCREA-MILNE
COSMOLOGICAL SCHEME

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A natural way of introducing the cosmological constant into the equations of motion in the McCrea-Milne approach is demonstrated.

It is known that a significant understanding of the structure of the Universe can be reached from the General Theory of Relativity. Simultaneously, of a remarkable heuristic interest is the possibility shown by McCrea & Milne¹ of description of the main properties of the observed Universe (in not too large scales) without proceeding from the relativistic equations.

The equation of motion of a test particle on the surface of an expanding sphere

$$\ddot{a} = - \frac{(4\pi/3)\rho a^3}{a^2} \tag{1}$$

rewritten in the form

$$\dot{a}^2 = \frac{1}{a} (\text{const} - \kappa a) \tag{2}$$

(const and κ determine the sphere's mass and initial energy) coincides with the Friedmann equation

$$\dot{a}^2 = \frac{1}{a} \left(\text{const} - \kappa a + \frac{\Lambda}{3} \right) \tag{3}$$

written for the Robertson–Walker metric when $\Lambda = 0$ (for simplicity, we speak about the Universe filled with dust: $P = 0$). In other words, the McCrea–Milne approach based on the homogeneity and isotropy of space–time corresponds to a Universe represented by the relativistic equations with a zero value of the cosmological constant.

However, one can consider eq. (1) not as a simple consequence of the law $1/r^2$, but as a reflection of the fundamental fact of Newtonian theory of gravitation: the sphere of a mass M interacts with test particles as a point mass M located at the sphere's centre (if, say, $\rho = \text{const}$). There is a suggestion that the difficulties connected with this theorem forced Newton to postpone for almost 20 years the publication of the law of universal gravitation.

Considering as an initial point not the law $1/r^2$, but this principle, one can inquire into the problem of searching for the most general function satisfying this principle. It is not difficult to bring the problem to the equation for the force

$$\int_0^{2\pi} \int_0^\pi \frac{F(r)}{r} (a - b \cos \theta) \sin \theta \, d\theta d\phi = 4\pi F(a),$$
$$r^2 = b^2 + a^2 - 2ab \cos \theta$$

or

$$(a^2/2) F''(a) + aF'(a) - F(a) = 0 \tag{4}$$

the general solution of which has a form

$$F(a) = Aa^{-2} + (\Lambda/\sigma)a. \tag{5}$$

Note that now the field inside the sphere is a non-force one only at the centre, but the oscillator ('confinement') term does not change the initial $o(4)$ symmetry of the Newtonian field. Moreover, that term corresponds to the only field possessing the property of the Newtonian one: the closing of orbits at any negative value of energy, *i.e.* the coincidence of the period of variation of the value of radius vector with that of its revolution by 2π (resonance principle).

Combining (1) and (5) we come to an equation coinciding with Friedmannian one (3). It is remarkable that besides the cosmological-type member no other ones appear from (4).

Thus the transition from $1/r^2$ law to (5) within the McCrea–Milne approach corresponds to introducing a cosmological member into the equation of motion.

The smallness of the cosmological constant evidently excludes the checking of (5) by means of any experimental methods; however, the contribution of the second member in (5) can be evaluated from the analysis of the structure of galaxy clusters, their haloes, *etc.* The possibility of the existence of a long-range force of the above type may affect in a certain way the ideas concerning the future of the open Universe².

References

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