CHRISTIAAN HUYGENS' MEASUREMENT OF THE DISTANCE TO THE SUN

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The methods for finding the distance between the Earth and the Sun that we inherited from the ancient Greeks never succeeded in giving an accurate result, but rather lower limits to the distance. Astronomical history books tell us that the first measurement came from observations of the parallax of Mars in 1672 by Cassini and Richer. There was, however, a determination in 1659 by Christiaan Huygens that is both ingenious and accurate and which deserves to be better known. It is described in Latin in *Systema Saturnium*[1], and in French in his *Oeuvres Complètes*[2], and also in a new book[3] by Van Helden. In 1656 Huygens, living in Paris, had determined the shape of Saturn and its ring with the aid of a superior telescope and his own analytical skills[4]. (Cassini's discovery that the ring was divided into two parts was more than a decade in the future.) Huygens wanted to give the sizes of these objects relative to the size of the Earth. He knew the relative distances of the planets, but in order to convert his angular measurements into Earth diameters, he needed to find the distance from the Earth to the Sun in Earth diameters. Huygens solved this problem with an assumption. If he supposed that the planet Venus had the same size as Earth, he could calculate the distance in Earth diameters from his measurement of its angular size. Then with the Copernican theory he could calculate for the time of the observation a theoretical distance to the planet in terms of the radius of the Earth's orbital circle (the astronomical unit in our terms). Knowing the same distance in astronomical units and in Earth diameters, he might then calculate the length of an astronomical unit in Earth diameters. Then he might repeat the assumption and the calculation for Mars.

When one averages the results obtained from Venus with those obtained from Mars, one replaces the assumption that both planets are of the same size as Earth with the assumption that Earth has a size that is the average of the sizes of Venus and Mars. This is the only assumption that Huygens described explicitly. He noted that it is harmonious for Earth, having a place between the places of Venus and Mars, to have a size that is between the sizes of Venus and Mars. It sounds plausible to our modern ears—just the sort of assumption that a present-day astrophysicist would make. It must have been a bold step for a seventeenth century scientist, however.

Huygens' descriptions of his calculations are a little disorganized. He gives us the details of his measurement of Venus including a sketch of its crescent form. He tells us that he also measured Mars but does not give us particulars. He states the result 1 AU = 12543 diameters, a value that differs by about 7 per cent from the currently-used ratio of the AU to the equatorial diameter, 11728.

About 14 years later Cassini in France (assisted by Picard and Roemer) and Richer in Cayenne measured the parallax of Mars, and could then find the length of the astronomical unit without any assumption about the sizes of planets. Cassini derived the size of the Solar System and described his results in two papers[5,6] without any reference to Huygens. Cassini's value for the astronomical unit was 11,000 diameters in the earlier paper. Since Huygens and Cassini were working in the same scientific institute they could scarcely have been ignorant of one another's work.

Newton in the *Principia* did not refer to either Huygens' or Cassini's measurement, although he was well aware of the French observations since he used Richer's report from Cayenne to calculate the oblateness of the Earth.

Huygens[7] referred to his own measurement in the *Treatise on Light* written in French in 1678 and in Latin in 1690, rounding down the distance to 12,000 diameters. He did not express his result in terms of the solar parallax, the usual astronomical way, nor did he refer to any other measurements of the astronomical unit. Dr. John Stanley, Mr. Robert Emmering and I have checked Huygens' measurement of the angular size of Venus. Huygens found 51 arcseconds for the diameter and from his sketch I determine that the width of the crescent is 31 per cent of the diameter. Our calculation from the DE102 ephemeris of the Jet Propulsion Laboratory gives 36 arcseconds for the diameter and 33 per cent for the width of the crescent at the time of the observation. Since there is a natural tendency to over-estimate the size of a bright object, I take these values to be in agreement.

One application for Huygens' method is to serve as a laboratory exercise for college students. It offers a chance to measure the heavens. In 1984 February, 22 University of Virginia students watched Mars and Venus in the early morning to measure their angular sizes. They used a 10-inch equatorially mounted reflector (*f*/12) with a medium-power eyepiece having a reticle giving 1-arcminute divisions. The students also checked the size of the reticle divisions by timing the apparent motion of a star with the telescope drive turned off.

With the assumption that a planet has the same size as Earth, its distance in Earth diameters is easily calculated. To find the distance to each planet in astronomical units, I assumed that Venus, Earth, and Mars had circular orbits in the same plane, and I gave the students the mean synodic motion for Venus and Mars and the mean orbital radii for all three planets. From the *Astronomical Almanac*, I supplied them with the dates and time to the nearest hour of the nearest inferior conjunction for Venus and the nearest opposition of Mars.

With this material and the plane law of cosines the students were required to calculate the distance to each planet in astronomical units. They had previous experience with the law of cosines and had little difficulty in making the calculation. It is a much harder task to learn to use the tables in *De Revolutionibus*, the description Copernicus made of his planetary theory. The next step was to set up for each planet an equation relating the measured distance in Earth diameters to the calculated distance in astronomical units, then to solve for the number of Earth diameters in an astronomical unit. Several students needed assistance with this calculation. The last step was to average the results for the two planets.

Huygens' value (12,543 diameters) was closer to the present value than those of most of the Virginia students. When Huygens rounded off the distance to 12,000 diameters in the *Treatise on Light*, he had greater accuracy than any of them.

Acknowledgements. I thank my co-workers, John Stanley and Robert Emmering, and P. D. Hingley of the RAS library, for their valuable assistance. I also thank Albert Van Helden for sending me page proofs of his book in 1984 October, and Dr. Martha Carpenter for reading the manuscript.

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