Mon. Not. R. astr. Soc. (1985) **214,** 559–574

Photometry and frequency analysis of line profile variables

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Accepted 1985 January 24. Received 1985 January 15; in original form 1984 July 30

Summary. We have monitored three line profile variable or 53 Per stars and one broad line star for light variations. The observations comprise about 750 measurements of each star and its comparison spread over 28 nights in two seasons. From these results we detect light variability in all programme stars with semi-amplitudes ranging from 0.8 to 8.0 millimag. The coherent periods we have detected do not agree with those found from the line profile variability. We discuss possible reasons for this discrepancy.

1 Introduction

The 53 Per variables are a group of sharp-lined early-type stars showing variations in spectral line profiles. Petrie & Pearce (1962) were the first to call attention to the fact that some early-type stars appear to show line-width variations and suggested they may be β Cep stars. Percy (1970) tested these stars for short-period light variations with negative results, though three of the 14 candidates showed variations over a time-scale of days.

In an investigation primarily directed at studying stellar winds in early-type stars, Smith & Karp (1976) discovered line profile variations in seven of the eight sharp-lined stars they selected. Most of these stars were subsequently studied in detail by Smith and co-workers who proposed a new class, the 53 Per stars, after the best studied object. Smith (1979, 1980a) describes the properties of these stars in detail. We summarize them as follows.

Their spectral types range from B5IV (53 Per) to O9V (10 Lac) and surround the region of β Cep variability. The bright giant ι CMa (B3II) and supergiant ϱ Leo (B1Iab) also show line profile variations and may be related to the 53 Per class.

The variation in line profiles, which is the defining characteristic, takes place with periods in the range 5 to 22 hr. The periods and amplitudes themselves vary with a time-scale of about a month. When the periods change they do so in characteristic factors of 2, but this is not a general rule. The same period is often observed from time to time.

These profile variations are easily modelled on the assumption that they are caused by a travelling wave non-radial oscillation with low spherical harmonic order (l=2 or 3). When fitting the profiles the assumption is made that only one such oscillation is present, though another mode with amplitude less than one third of the predominant one will pass undetected (Smith & McCall 1978).

The idea that the profile variations are caused by non-radial oscillations is generally accepted, but the instability and lengths of the periods are difficult to understand in this framework. The periods are based on no more than 10 profiles and most often on five profiles or less per night. While accepting that the phase in profile modelling is uniquely determined by just one observation (Smith & Stern 1979), this is true only if one mode is present. By analogy with β Cep variables, it is quite possible for more than one profile-distorting mode to be present, in which case the periods and mode identification can be questioned.

This most important problem is best approached by photometric observations of these stars. Only one star, 53 Per, has been studied in some detail by photometry. Buta & Smith (1979) found a pair of frequencies near 0.5 day^{-1} in photometric observations during 1977/78. More recently, Smith *et al.* (1984) showed that this pair of frequencies was still present in 1981. The periods originally found from the early profile modelling could no longer be recovered either in the photometry or the more recent line profiles. Clearly, photometric observations are crucial to resolving the nature of these stars.

For these reasons we embarked on a project to monitor photometrically in detail three line profile variables visible from the southern hemisphere; 22 Ori, v Ori and ι CMa. Here we discuss our results and speculate on the nature of these stars.

2 The observations

All observations were made through the Strömgren b filter using the Peoples photometer attached to the 0.5-m reflector at the Sutherland site of the SAAO. This choice of filter is a compromise between the expected increase in light amplitude towards shorter wavelength (as in the β Cep stars) and the decreasing sensitivity to transparency variations towards longer wavelength. The narrow band also assists in minimizing the latter effect as well as reducing the light intensity from these very bright stars.

Table 1 gives details of the programme stars and their comparisons. Also shown are absolute magnitudes determined from the (β, c_0) calibration of Balona & Shobbrook (1984) and luminosities, effective temperatures, masses and radii from the formulae in Balona (1984). During the first observing season HR2595 was used as a comparison star for ι CMa. This star subsequently turned out to be variable so in the second season we added HR2625 while keeping HR2595 as a programme star. As a result our observations for HR2595 and ι CMa made in the first season are unusable.

Each star was observed in sequence with a total integration time of 60 s. Corrections for

Table 1. Data for programme and comparison stars. The masses, M, and radii, R, are in solar units.

Star	MK type	v sin i	β	c_0	M_V	Log L	Log T_e	M	R
Programme Stars:									
22 Ori	B2IV-V	14	2.625	0.168	-2.62	3.648	4.276	9.1	6.2
v Ori	O9.5V	20	2.597	-0.096	-4.20	4.719	4.471	19.7	8.7
HR2595	B3II-III	248	2.589	0.150	-3.90	4.107	4.254	11.9	11.7
ι CMa	B3II	29	2.582	0.192	-4.36	4.220	4.221	12.9	15.5
Comparison Stars:									
HR1806	B9.5Vn	280	2.817	0.834	0.49	1.922	4.060	2.9	2.3
HR1848	B2V	25	2.676	0.201	-1.54	3.211	4.273	7.1	3.8
HR2625	B4III		2.681	0.316	-1.33	3.017	4.223	6.1	3.8

Table 2. Observing log, showing mean nightly magnitude of one of the comparison stars, HR 1806, relative to the mean of HR 1806 and HR 1848. The number of hours observed and the number of integrations for each star is also shown.

Julian Day 2440000+	Hours	HR1806 (millimag)	N
5300	6.0	38.1	22
5301	6.0	37.6	24
5302	6.5	37.5	29
5304	6.7	37.8	23
5305	3.1	37.8	14
5306	6.6	37.6	29
5307	6.7	37.8	36
5675	2.5	37.6	18
5676	2.5	36.3	21
5677	5.8	38.7	36
5679	7.0	37.5	51
5680	6.7	37.7	47
5681	6.7	37.1	47
5682	6.2	38.8	35
5683	3.4	37.9	29
5684	3.0	38.3	15
5686	4.3	37.2	29
5687	6.7	37.6	43
5688	6.7	37.7	36
5691	7.0	38.0	25
5692	5.8	36.9	22
5693	2.0	38.0	12
5695	6.7	37.1	27
5698	2.9	38.4	10
5706	3.8	38.9	17
5707	1.0	38.8	5
5708	6.7	37.4	27
5709	6.7	38.4	25

transparency variation were made using the nearby comparison stars. Table 2 gives the observing log and the mean magnitude of HR 1806 (one of the comparison stars) relative to the mean of HR 1806 and HR 1848. The number of observations of this star, which is typical of all the others, is also shown. It can be seen that the nightly variations of HR 1806 do not exceed 2 millimag, attesting to the constancy of the comparison stars. Further tests of constancy are described below.

The quality of our observations can be judged from the rms scatter of the comparison stars relative to each other. This amounts to 1.5–3.0 millimag per observation, depending on the quality of the night.

3 Frequency determination

A standard discrete Fourier transform periodogram analysis for unequally spaced data was used for extracting frequencies. After one frequency was identified, the data were prewhitened using the simultaneous multiperiodic least-squares Fourier coefficients. Scargle (1982) has shown that the signal-to-noise ratio, z, in the power at which a peak is detected with 99 per cent probability is given approximately by $z = 4.6 + \ln(N)$, where N is the number of frequencies searched. Since N never exceeded 2, a rule we have used is to consider peaks which are at least 2.5 times larger than the noise level in amplitude. While this is a useful criterion, in general we divided our data set into independent parts and considered an oscillation as definitely real only if it was present in all the

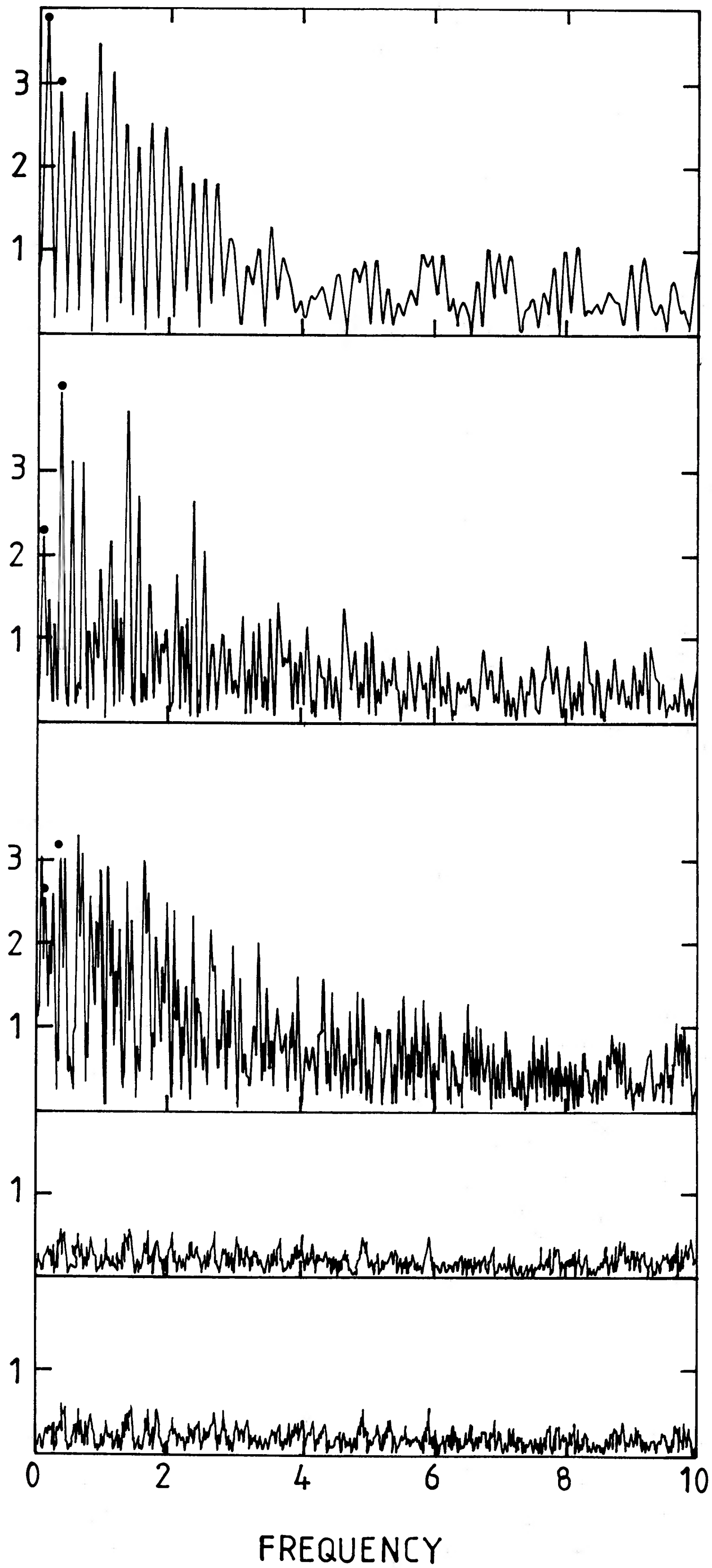

Figure 1. Periodograms of 22 Ori for three different data subsets. Semi-amplitudes in millimagnitudes are shown. The upper, middle and lower panels correspond to data sets (i), (ii) and (iii) in the text. The dots show the adopted pair of frequencies. The lower two panels show the periodograms for the comparison stars (upper HR 1806, lower HR 1848) on the same scale.

data sets. These sets are (i) JD2445300–JD2445307; (ii) JD2445675–JD2445688; (iii) JD2445691–JD2445709.

Since the stars could only be observed for a maximum of 7 hr each night, the spectral window of all data sets has a strong 1 cycle day^{-1} alias which is 85 per cent the strength of the main peak. Other aliases at 2 and 3 cycle day^{-1} had strengths of 56 and 23 per cent respectively.

All the frequencies are estimated to be accurate to about 0.002 day^{-1}. We could not refine them using the year gap between seasons because of cycle count ambiguities.

4 The comparison stars

The problem of detecting millimag variations is not very great when the period to be found is very short since many cycles can then be sampled. To detect such variations with periods typical of 53 Per stars demands that the stars used as comparisons be examined in detail for any signs of variability. We have shown in Table 2 how the nightly variation of one of the standards compares relative to another standard. Within the errors this test does show that HR 1806 and HR 1848 are sensibly constant to within 1 or 2 millimag per night. This test cannot be applied to HR 2625 since it lies in a different region of the sky. In this case the constancy was tested by comparing the frequency spectra of the programme stars. We could find no frequency common to both programme stars for which HR 2625 was used as a standard, as might have been expected if HR 2625 was a variable.

We have also applied the periodogram analysis to HR 1806 and HR 1848. These show a power spectrum having a mean noise level of 0.2 millimag and no peak greater than 0.5 millimag. Both periodograms appear to be typical noise spectra (see Fig. 1).

5 HR 1765 (22 Ori)

Smith (1980b) obtained 53 high-dispersion line profiles during the 1976–78 observing seasons and also analysed some 21 ultraviolet *Copernicus* profiles taken over four days. His analysis showed unstable mode amplitudes and periods characteristic of these stars. Four periods were observed at different times, each of them at least twice. In the best case a period of 8.95±0.06 hr was found from 10 profiles spread over four consecutive nights. In the worst case, a total of three profiles taken on two nights suggested periods of 4.5, 9 or 14 hr. On two occasions a period of 22.9 hr was found from three and seven profiles.

For most profiles, Smith (1980b) obtains good fits assuming a single retrograde quadrupole mode (l=2, m=−2), but sometime between 1978 and 1979 a change took place to a prograde (m=+2) travelling wave. During one run, the best fit demanded spherical harmonic orders with l=3, m=−3. The analysis of the *Copernicus* data was not very conclusive but tended to support periods of 9.0 and 4.5 hr.

Our photometric data do not confirm any of these periods, though the star is clearly variable. Fig. 1 shows the periodograms of the three independent data sets and of the comparison stars on the same scale. While variability is certainly present, the aliasing problem obscures the true frequencies. This is made clear in the periodogram of the combined data (Fig. 2). The top panel of this figure indicates that at least two periods are present. The largest peak at ν_0=0.342 day^{-1} is five times higher than the mean noise level. The true peak can be distinguished from its aliases at $1-\nu_0$ and $1+\nu_0$ unambiguously. The bottom panel of Fig. 2 shows the periodogram after the combined data were prewhitened for ν_0. The identification of ν_1=0.091 day^{-1} is again unambiguous; the aliases at $1-\nu_1$, $1+\nu_1$, $2-\nu_1$, $2+\nu_1$ are clearly visible. This peak is four times higher than the mean noise level. Further prewhitening leads to a spectrum in which no particular frequency appears to dominate and where no peak is higher than 2.5 times the noise level.

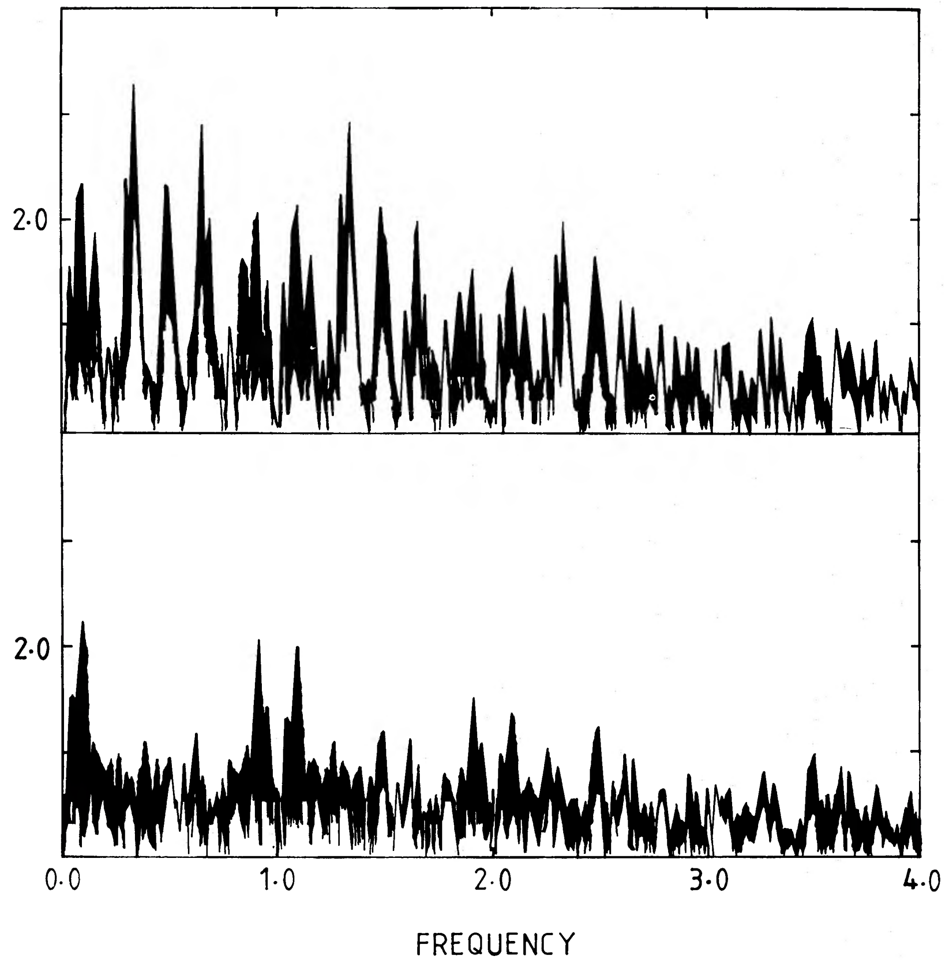

Figure 2. Periodograms of the combined data for 22 Ori (top) and after prewhitening by a frequency of 0.342 day^{-1} (bottom). Semi-amplitudes in millimagnitudes are shown.

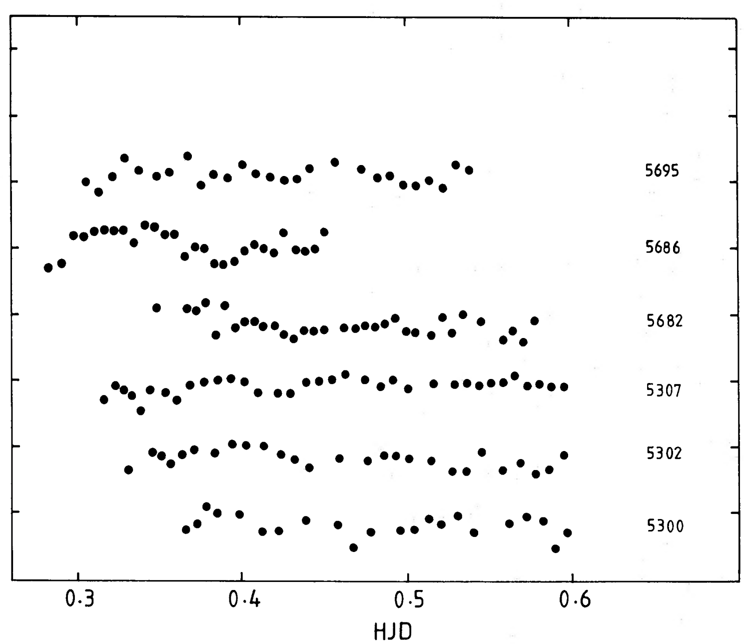

Figure 3. Short-period transient oscillations visible in the light curve of 22 Ori. The Julian day is marked; tick marks are spaced at intervals of 0.02 mag.

Both frequencies exist in all the independent data sets (Fig. 1), though they vary in power relative to each other. This test gives added confidence to the reality of these oscillations. In none of the sets do we find frequencies which match those found from the line profiles with a significant signal-to-noise ratio.

Of particular interest are apparent short-period oscillations sometimes visible in the light curve (Fig. 3). These consist of a few cycles having a characteristic period of about 2.5 hr and amplitude of several millimagnitudes. These variations cannot be seen in the comparison stars or indeed in any star observed on the same night. Since the amplitudes of the oscillations are considerably larger than can be accounted by the observational errors, we are confident of their reality. They are not visible in the power spectra of the combined data or of the individual data sets, so the oscillations must be non-coherent.

22 Ori has been noted to vary in velocity by several authors. Abt & Levy (1978) determined marginal orbital elements, obtaining an orbital period of 293 day. A power spectrum of their data

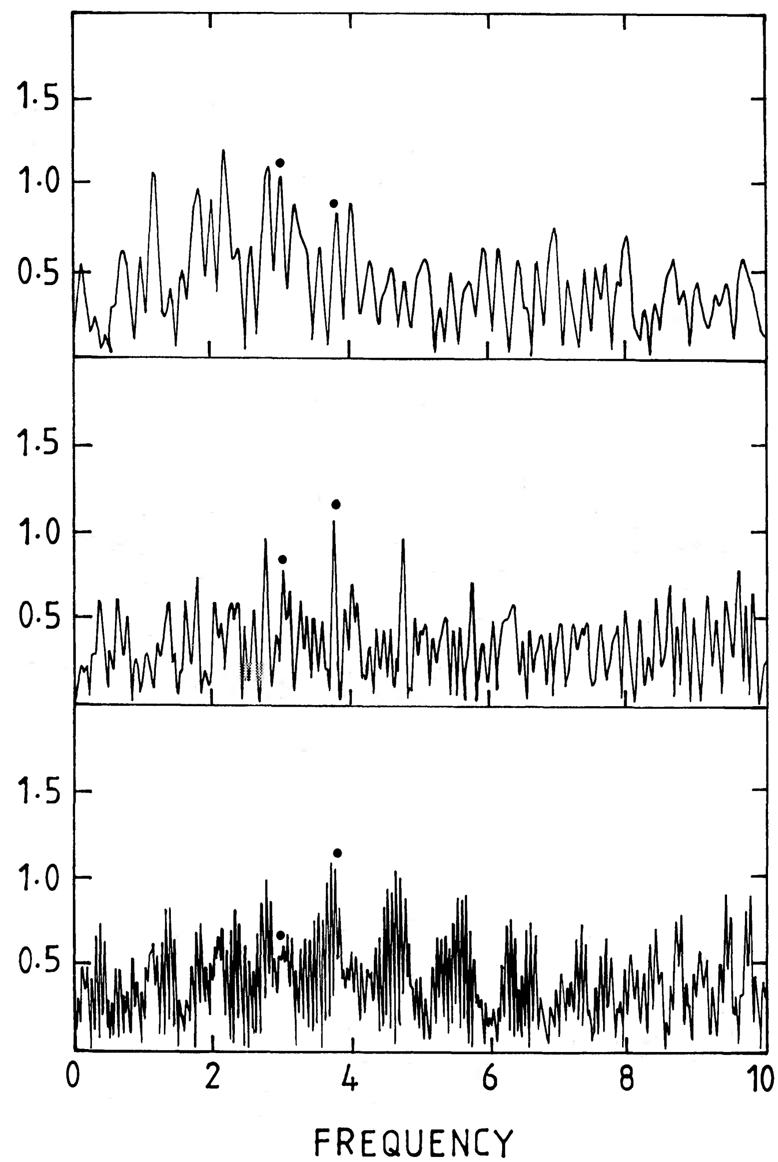

Figure 4. Periodograms of υ Ori for three different data subsets. Semi-amplitudes in millimagnitudes are shown. The upper, middle and lower panels correspond to data sets (i), (ii) and (iii) in the text. The dots show the adopted pair of frequencies.

actually shows the largest peak at 8.593 day, though the signal-to-noise ratio is very low. This corresponds to a frequency of 0.116 day^{-1}, not too different from ν_1. Taken as a spectroscopic binary, we obtain a radial velocity semi-amplitude of 3 km s^{-1} and a mass function of 0.000025. This leads to rather unrealistic orbital characteristics.

6 HR1855 (υ Ori)

Smith (1981) obtained 37 high-resolution line profiles on 19 nights during the 1976, 1978 and 1979 seasons. His analysis shows that a period of 12.3±0.1 hr is present most of the time except for two runs when a period of 23.52±0.05 was determined. All profiles were well described by a single non-radial mode with $l=2$, $m=-2$.

The periodograms of the three independent data sets are shown in Fig. 4, while that of the combined photometry is shown in the top panel of Fig. 5. There appear to be at least two frequencies present. The main oscillation has a frequency of either 2.76 or 3.76 day^{-1} with a signal-to-noise ratio of about 3.5. If the power spectra of the individual data sets are examined, we find a very clear pattern in sets (ii) and (iii) indicating that the correct choice of the principal frequency is ν_0=3.762 day^{-1}. In data set (i) ν_0 is present but has a much lower signal-to-noise ratio. The spectrum in this case is dominated by a system of peaks generated by frequencies of 2.17 and 2.33 day^{-1}. These also have rather small signal-to-noise ratios, so it is not clear whether the modal content of this star has really changed between the two seasons.

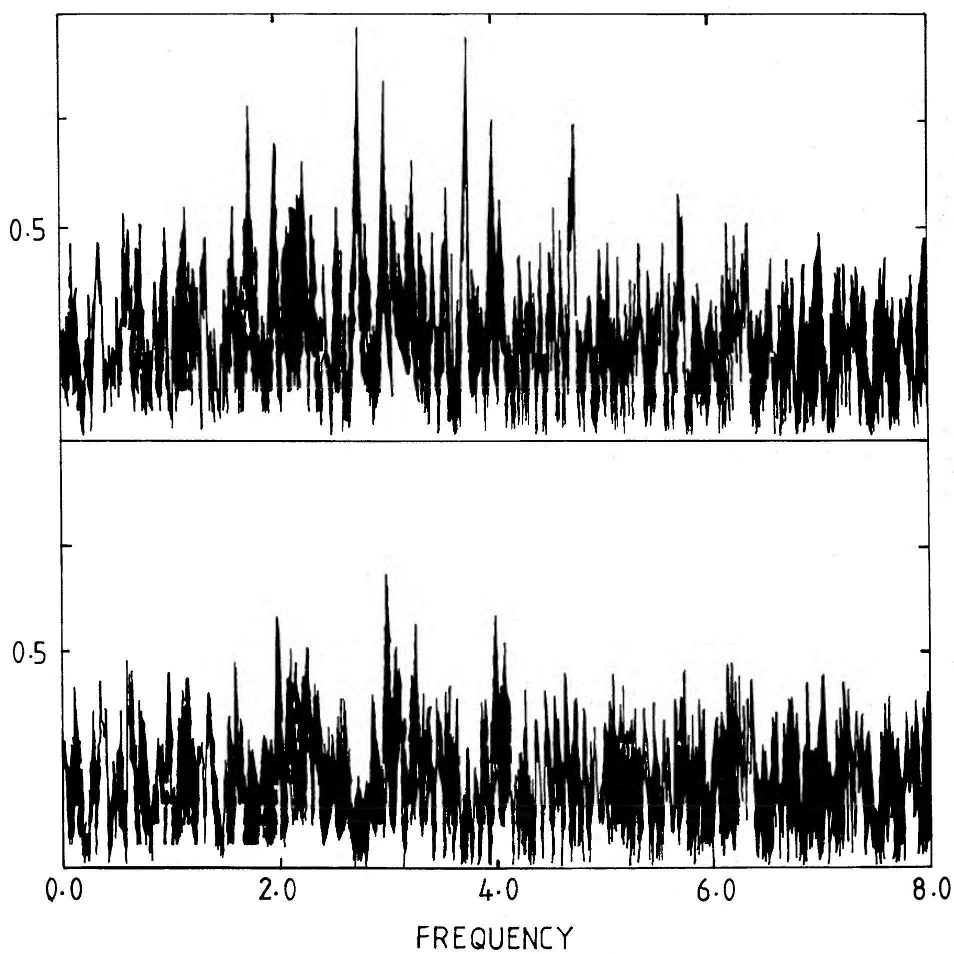

Figure 5. Periodograms of the combined data for υ Ori (top) and after prewhitening by a frequency of 3.762 day^{-1} (bottom). Semi-amplitudes in millimagnitudes are shown.

The bottom panel of Fig. 5 shows the periodogram of the combined data after prewhitening by ν_0. The frequency ν_1=2.995 day^{-1} is unambiguous. This peak is only 2.4 times the height of the noise, but the same frequency is also present in all three data sets, so we are convinced it is a real oscillation. In fact, ν_1 is almost as strong as the other peaks in data set (i). It is possible that the near commensurability of this frequency with 1 cycle day^{-1} may be responsible for the different pattern seen in data set (i). In no case could we detect any frequencies similar to those found from the profile variations.

7 HR 2595

This star has no known history of variability; Balona (1977) used it as a standard. Campbell & Moore (1928) commented on the broad spectral lines in this star, while Huang (1953) measured their plates and obtained $v \sin i$=248 km s^{-1}.

We discovered the variability of this star while using it as a comparison for ι CMa during the 1982 season. Because it appeared to be an interesting star, we included it in our observation programme for the following season. The periodogram of this star simply shows a rather sharp increase in noise level towards low frequencies with no predominant peaks. Examination of the night-to-night mean magnitude level shows rather erratic long-term behaviour (Fig. 6). No clear evidence of periodicity could be found at any frequency.

8 HR 2596 (ι CMa)

This star has been observed to show line profile variations (Smith & Karp 1976), but no detailed analysis has been undertaken. Balona (1977) could find no variability over a time-scale of a few hours, but Burki (1983) found quite a high scatter in the mean night-to-night magnitude.

It was apparent after the first observing season that the variability of this star was the largest in our sample. Unfortunately no use could be made of the data from this season for reasons mentioned above. The periodogram of the data from the second season (Fig. 7) shows a gradual increase in noise towards low frequencies with a few high peaks. The highest peak at ν_0=0.717 day^{-1} is clearly present in data sets (ii) and (iii) and is the highest peak in an intermediate data set formed by grouping data in the interval JD2445681–JD2445698. For these reasons we are reasonably confident of its reality. After prewhitening, no further peaks can be isolated with confidence. It is clear, however, that the star is active on a time-scale longer than

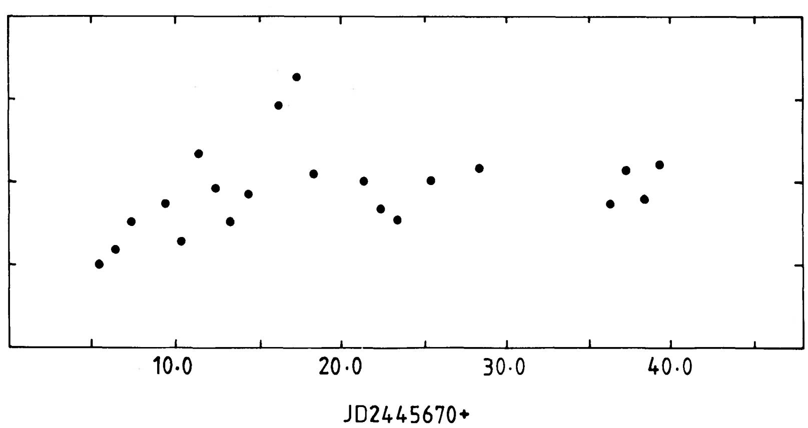

Figure 6. Night-to-night mean magnitude of HR 2595 showing irregular variations. Tick marks are spaced at intervals of 0.01 mag.

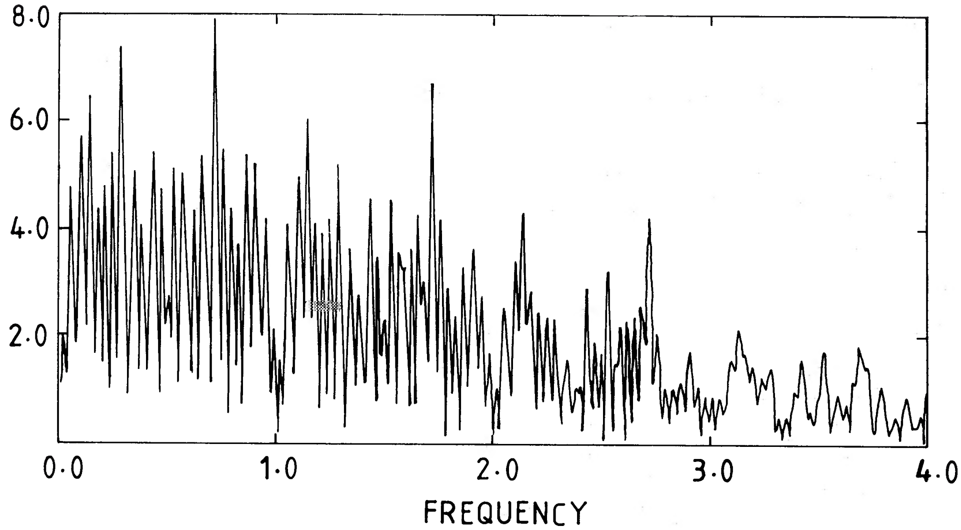

Figure 7. Periodogram of data for ι CMa obtained during the second season. Semi-amplitudes in millimagnitudes are shown.

half a day. These irregular variations contribute significantly to the power at low frequencies. In this way it resembles supergiants of all spectral types which appear to vary on many time-scales.

Table 3 summarizes the frequencies and amplitudes, obtained from a multiperiodic least-squares analysis, for all the programme stars.

9 Relationship between light and profile variations

For the two 53 Per stars which we observed and for which periods have been determined from profile fitting, we find that the photometric periods do not agree with the profile periods. We note that this disagreement also applies to 53 Per itself (Buta & Smith 1979; Smith *et al.* 1984), though the authors argue for a period change. There are three possible reasons for this discrepancy: (i) the periods have changed; (ii) the light and profile variations are caused by independent oscillations which are mutually unobservable; or (iii) the periods estimated from the photometry or from the profiles are incorrect.

If the periods obtained from the line profiles and from the photometry are both correct and if the variations in line profile do lead to observable photometric variations and vice versa, then we

Table 3. Summary of frequencies ν (in cycles day^{-1}) and semi-amplitudes (in millimagnitudes) of oscillations in the programme stars obtained by a least-squares fit to the data. The dimensionless squared frequency, σ^2, is calculated using the masses and radii of Table 1.

Star	ν	Semi-amplitude	σ^2
22 Ori	0.342	3.6 ± 0.2	0.042
	0.091	1.8 ± 0.2	0.003
υ Ori	3.762	0.9 ± 0.2	6.339
	2.995	0.8 ± 0.2	4.019
HR2595	irregular	20.0	
ι CMa	0.717	8.0 ± 0.2	2.003

must conclude that in three stars a modal change leading to quite different frequencies must have occurred between 1977 and 1983. While this cannot be ruled out, it seems most improbable. The available evidence would indicate a greater measure of stability (e.g. 53 Per) and a recurrence of particular frequencies from time to time.

Proceeding on the assumption that a simultaneous change of mode in three stars is not the explanation, we now examine the question of how great a light variation can be expected for a given pulsational velocity obtained from profile modelling.

Let us consider a sinusoidal oscillation such that the radius R is determined by

$$R=R_0\{1+\varepsilon P_{lm}(\cos\theta)\cos\omega t\}$$

where R_0 is the equilibrium stellar radius, ε is the radial semi-amplitude and $P_{lm}(\cos\theta)$ is the spherical harmonic of orders l and m which depends on the colatitude θ. The local pulsational radial velocity is given by

$$\Delta V_{\mathrm{pul}}=\frac{dR}{dt}=-2\pi\nu\varepsilon R_0 P_{lm}(\cos\theta)\sin\omega t.$$

Dziembowski (1977), Balona & Stobie (1979) and Balona (1981) show that the light variation arising from such a non-radial oscillation is given by

$$\Delta m=1.084\varepsilon P_{lm}(\cos\theta)\ F\cos(\omega t+\alpha)$$

where α is a phase angle and

$$F^2=f^2b_l^2+2fb_lg_l\cos\psi+g_l^2.$$

In this equation f is the ratio of surface brightness (in the appropriate wavelength band) to radius variation and ψ is the phase between the same two quantities. The constants b_l and g_l are given in Balona (1981) and are functions of the harmonic order, l. It is expected that maximum temperature will occur at minimum radius (as in the β Cep stars), so we shall assume $\psi=180°$.

Table 4. (a) Light semi-amplitudes (in millimagnitudes) expected from a pulsational radial velocity semi-amplitude of 3 km s^{-1} for line profile variations of different spherical harmonic orders, l. (b) Maximum pulsational velocity semiamplitudes to be expected from the observed light variations for different spherical harmonic orders.

(a)

Star	ν (day^{-1})	ℓ = 2	ℓ = 3	ℓ = 4
22 Ori	5.33	2.5	1.2	0.8
	1.05	12.8	5.9	3.9
υ Ori	1.95	4.9	2.3	1.5
	1.02	9.4	4.3	2.9

(b)

Star	ν (day^{-1})	ℓ = 2	ℓ = 3	ℓ = 4
22 Ori	0.342	0.3	0.6	0.9
	0.091	0.05	0.1	0.1
υ Ori	3.762	1.1	2.3	3.5
	2.995	0.7	1.6	2.5
ι CMa	0.717	3.2	7.0	10.5

Thus the ratio of light amplitude to pulsational velocity amplitude is

$$\frac{\Delta m}{\Delta V_{\text{pul}}}=\frac{1.084\,(fb_l-g_l)}{2\pi v R_0}.$$

The greatest uncertainty in this relationship is what value of f should be adopted in any given case. If geometric effects predominate then f is very small. It is easy to show from the above formulae that the minimum light amplitude is given by putting $f=0$ except for the radial mode. For the dipole mode ($l=1$), an oscillation with $f=0$ will give rise to no light variation but with observable radial velocity and profile variations. The variations seen in 53 Per stars cannot be due to radial pulsation because the periods are far too long. Moreover, none of the profiles which Smith has modelled fits $l=1$. We can therefore compute the minimum light amplitude for quadrupole and higher orders by setting $f=0$. In Table 4 we show these results using the radii of Table 1 and assuming a typical pulsational amplitude of 6 km s^{-1} and two extreme periods found by Smith. The conclusion is that even for high spherical harmonic order ($l>4$), these oscillations should have been easily detected in our photometry.

We now address the converse problem: what pulsational amplitude can be expected from the photometric oscillations that we have detected. It is clear that the assumption $f=0$ leads to maximum pulsational velocities for quadrupole and higher orders. Table 4 again shows the results. The indication is that the photometric oscillation in 22 Ori should produce no detectable profile variation unless the order is very high (in which case the light variation will be unobservable). For v Ori and ι CMa, profile variations should be observed. Note that if $l=1$. A sufficiently large pulsational velocity can be produced by suitable choice of f.

Note that the above arguments are quite general. We are considering the light variations caused by geometric effects alone. Under our assumptions this will lead to minimum light amplitude whether the oscillation is adiabatic or not. The conclusion is that the profile variations found by Smith should manifest in easily detectable light variations if his periods and mode identifications are correct. Conversely, detectable profile variations should occur for v Ori and ι CMa for $l=2$ or 3. For 22 Ori detectable profile variations will occur if $l=1$. Some evidence for a low order is the variable radial velocity of this star which is not easily interpreted as binary motion (see Section 5).

Assuming then that these stars have not changed modes simultaneously and that the light and profile variations are not mutually unobservable, we ask the question: is there reason to believe that the periods obtained from the line profile analysis are in error? In our opinion, the possibility that this is indeed the case cannot be ignored. The most compelling evidence comes from the photometric observations, where at least two oscillations of comparable amplitudes are detected in 53 Per, 22 Ori and v Ori. According to Smith & McCall (1978) both oscillations will contribute to the profile variations, whereas in the profile modelling only one period is assumed. It does not seem at all likely that only one dominant period would have existed in all three stars prior to 1978. In such a case the period obtained from the profiles would be some alias of the true period. This argument alone is sufficient reason to question the profile periods, but we may also point out the unavoidable paucity of profile data used in the analyses.

10 The case of 53 Per

As already mentioned, Buta & Smith (1979) and Smith *et al.* (1984) showed that the photometric behaviour of 53 Per in 1977/78 and 1981/82 is consistent with a pair of frequencies near 0.5 day^{-1}. This is quite different from the 3.6 to 14.6 hr periods found from line profiles prior to 1977.

For the 1981/82 season, Smith *et al.* (1984) obtain $\nu_0=0.4673$, $\nu_1=0.5921$ day^{-1}, both with semi-amplitudes of about 7.5 millimag. We re-analysed their published data and found a significantly different frequency for $\nu_0=0.4302\,\text{day}^{-1}$. This discrepancy is not understood since we obtain the same value for $\nu_1=0.5940\,\text{day}^{-1}$. But more important, we find after further analysis that two more frequencies are present: $\nu_2=1.161$, $\nu_3=1.471\,\text{day}^{-1}$. There is an aliasing problem and these two frequencies might be in error by $\pm1\,\text{day}^{-1}$. The four frequency model fits the photometry with an rms error of 5.6 millimag. The following semi-amplitudes (millimag) are obtained: $A_0=9.0$, $A_1=8.9$, $A_2=5.3$, $A_3=5.0$. The presence of these additional frequencies throws some uncertainty in the parameters deduced from the profile modelling.

An interesting and very puzzling discovery made by Smith *et al.* (1984) is the necessity of postulating predomiantly radial motions for the two-day modes. Theoretically, the ratio of horizontal to vertical pulsational velocity, K, should exceed 100. Smith *et al.* (1984) find that the profiles cannot be described unless K is small. They use $K=0.15$ and suggest that non-linearities may be responsible for the discrepancy. While this might be the case, the meaning of linear profile modelling then becomes unclear. As evidence in support of non-linearities they cite the fact that 53 Per is cooler when fainter, whereas for g-modes with the given periods, the opposite is expected to occur. However, as Saio & Cox (1980) point out, even for high-order g-modes the phase lag between temperature and luminosity is small though the phase lag of maximum luminosity behind minimum radius is about half a cycle. Pesnell (1985) also makes this point clear.

We suggest that the exclusion of ν_2 and ν_3 from the profile analysis may go some way to explain the apparently small value of K. There is also another effect which merits serious consideration. Carroll & Hansen (1982) point out that the observed velocity of a mass element in rotating, oscillating star is not simply the vector sum of the zero-rotation oscillation and the velocity of rotation as assumed in all profile modelling. For long-period oscillations such as those in 53 Per, this omission could possibly lead to a spuriously small K value.

In considering what light amplitude is expected from the observed profile distortions in 53 Per, we first take at face value the pulsational amplitude of $12\,\text{km s}^{-1}$ found by Smith *et al.* (1984). If we adopt a radius of four solar radii for this star, a sinusoidal radial pulsation with this velocity amplitude leads to a 25 per cent radius change. This will of course be reduced if $l=3$ as suggested, but even so, it is difficult to understand the small light amplitude. A more quantitative approach on the lines discussed above gives: $\Delta m/\Delta V_{\text{pul}}=0.014$, 0.007, 0.004 for $l=2$, 3, 4 respectively assuming $f=0$ (geometric effects only) and if the velocity is measured in km s^{-1}. For $l=3$, $\Delta m=0.04$ mag is predicted – much larger than actually observed. The predicted maximum pulsational velocity giving rise to the observed light amplitude is only 2 or $4\,\text{km s}^{-1}$ total amplitude, though it can be much larger for $l=1$.

In conclusion, there are reasons for doubting the profile analysis of Smith *et al.* (1984). The question as to what is really happening in this star must remain open until more observations are available. It would be of much interest to model the existing profiles with all the frequencies indicated by the photometry and taking into account the proper vector addition for the observed velocity of a mass element.

11 Discussion

There has recently been much interest in early-type stars which show strictly periodic behaviour with periods too long to be explained in terms of p-mode oscillation but too short to be caused by binary motion. The best observed of these stars are shown in Table 5. These stars have often been discussed in the context of the 53 Per variables, especially as some of them do show profile variations. Tables 3 and 5 list the dimensionless frequency given by $\sigma^2=4\pi^2\nu^2R^3/GM$.

In Table 3 masses and radii from Table 1 were used; in Table 5 these quantities were estimated from the spectral type and luminosity class. The value $\sigma=1$ describes the critical rotational

Table 5. Stars with strictly periodic light or radial velocity variations related to the 53 Per variables.

Star	MK Type	ν (day^{-1})	σ^2	Reference
EM Cep	B1IV	1.24	1.0	Breinhorst & Karimie (1982)
λ Eri	B2IV	1.43	0.8	Bolton (1981)
28 CMa	B2.5IV	0.73	0.2	Baade (1982)
HR3562	B3IV	0.58	0.1	Burki (1983)

frequency of the star or the orbital frequency of a massless satellite at the photosphere. The error in σ^2 is mostly due to uncertainties in the radii and is estimated to be about 0.05.

The closeness of σ^2 to unity for most stars in the table is the principal reason for eliminating the binary hypothesis. In most cases also, there are no large radial velocity variations which one might expect of a close binary. Values of the dimensionless frequency for polytropes (Saio 1981) or for more realistic stellar models (Carroll & Hansen 1982) have been calculated for several harmonic orders and overtones. A comparison shows that only high-overtone g-modes have frequencies comparable to those present in these stars. In fact as the star evolves from the main sequence the frequencies of the g-modes increase fairly rapidly, so that near the end of core hydrogen burning only fifth or higher overtone g-modes with $l=2$ or 3 have $\sigma^2<1$. This poses a severe problem for the interpretation of these oscillations as g-modes, since it is difficult to understand why such high overtones should be excited. Moreover, these modes will have very low amplitudes at the surface. An explanation of these oscillations as p-modes split by rotation (Baade 1982) cannot be valid as the second-order rotational perturbations calculated by Saio (1981) produce a splitting which is much too small to explain to low frequencies.

Apart from p- and g-modes, the toroidal or r-modes can also be excited if the star is rotating. To first order the frequencies are determined purely by the frequency of rotation, Ω:

$$\nu=m\Omega\left\{\frac{2}{l(l+1)}-1\right\}.$$

Berthomieu & Provost (1981) have shown that these modes are not very effective in producing radial velocity variations. They assume that the radial velocities are measured from the centroid of the line profile. If the point of maximum photographic density is measured, then radial velocity variations similar to those found in 28 CMa (Baade 1982) could possibly be detected. Like g-modes, the r-modes may give rise to detectable light variations as these surface waves generate patches of different brightness on the star.

The long periods found for 22 Ori make it most unlikely that the oscillations are due to g-modes. Rotational effects or the presence of a companion of small mass seems more probable. However, it is interesting to note that the period ratio in this star is almost exactly what one might expect for r-mode oscillations with $l=2$, $m=-1$ and $l=3$, $m=-3$. The resulting rotational period is 7.3 day giving an equatorial velocity of about 40 km s^{-1}. The observed $v \sin i=14$ km s^{-1} then implies an inclination of about 20°.

Of particular interest are the β Cep-like pulsations which appear from time to time in 22 Ori. This star lies on the cool edge of the β Cep instability strip and it is conceivable that it may be on the point of becoming unstable to such pulsations. We note that in at least two β Cep stars in NGC 6231 Balona & Shobbrook (1983) found both long-period 53 Per type behaviour as well as the conventional β Cep pulsations.

The oscillations discovered in v Ori have periods very near to those occurring in β Cep stars. Its main period of about 6.4 hr is only slightly longer than the longest periods found in β Cep variables, and indeed is expected for such a hot star. On this basis we can classify it as a β Cep variable. We note with interest that the period ratio $P_1/P_0=0.80$, is close to the value expected for first overtone and fundamental radial pulsation. Radial velocity variations originating from these pulsations will be only about 0.5 km s^{-1}, assuming the normal value of 500 for the radial velocity to light amplitude ratio found in these stars, and would not have been detected. This suggests that the profile variations found by Smith (1981) could possibly be due to two radial pulsations. If this is confirmed, the period ratio may be used to estimate the mass in the same way as has been done for double-mode Cepheids.

12 Conclusions

We have discovered coherent light variations in two 53 Per stars, 22 Ori and v Ori, and one bright giant, ι CMa, known to show line profile variations. Non-coherent short-period light variations are suspected for 22 Ori. For a rapidly rotating mid-B giant, HR 2595, we find irregular variability with a time-scale of several days.

None of the periods obtained from line profile matching can be confirmed by the light variations. Unless three stars have all changed their modes in 1977–83, which we consider unlikely, the only explanation we can suggest is that the profile periods may be aliases of the true periods due to the fact that only one mode was considered in the profile modelling. It does not seem possible to explain the discrepancy in terms of mutually unobservable light and profile variations. For 53 Per itself recent observations show that this is not the case.

One of the line profile variables, v Ori, can be classified as a β Cep star from the short closely spaced periods. The ratio of the periods suggests fundamental and first-overtone radial pulsation.

We have outlined the difficulties of non-radial oscillation as an explanation for the light and profile variability. To make progress in this direction, simultaneous line profile and multicolour photometric observations will be required.

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