Mon. Not. R. astr. Soc. (1985) 213, 103–109

The size, mass, mass loss and age of Halley's comet

David W. Hughes *Department of Physics, University of Sheffield, Sheffield S3 7RH*

Accepted 1984 October 10. Received 1984 October 10; in original form 1984 July 30

Summary. Halley's comet (1910 II, 1982i) has a D^2p_v value of $5.90\pm0.33\,\mathrm{km}^2$, where D km is the diameter of the nucleus and p_v is the geometric albedo. The mass of the nucleus is $7.5\times10^{15}\ \varrho p_v^{-1.5}$ g, where ϱ is the density. Reasonable assumptions as to p_v and ϱ yield diameter and mass values of 9.4 km and 2.2×10^{17} g.

In the 1910 apparition the comet lost a mass of 2.8×10^{14} g which is equivalent to an absolute magnitude change of 9×10^{-4} per apparition. The mass of the meteor stream, produced by the decay of P/Halley, is consistent with the statement that the comet has had 2300 previous close passages of the Sun. The comet will probably disappear altogether after another 2300 close perihelion passages.

1 Introduction

The four quantities mentioned in the title of this paper are all of considerable interest to students of comets and cosmic dust, but at present (1984 October) none can be measured directly. All the quoted results thus depend on a series of assumptions and in the following short review these assumptions will be carefully listed.

2 Size and mass

The size of an inactive comet can be related to its brightness. Halley's comet was recovered on the night of 1982 October 16 (see Jewitt *et al.* 1982). It had a visual magnitude, m_2, of 24.3 ± 0.2 and was at a heliocentric distance, r, of 11.05 AU and a geocentric distance Δ of 10.94 AU. No coma was detected but this was hardly surprising as the surface temperature of the nucleus at that distance was expected to be around 120 K. The brightness of the bare cometary nucleus can be considered to be similar to that of an asteroid so

$$m_2 = V(1,0) + 5\log r\Delta + \phi(\beta) \tag{1}$$

where $V(1,0)$ is the absolute magnitude, at unit distance from both the Sun and Earth and at zero phase, and $\phi(\beta)$ is the phase function. Away from zero phase the magnitudes of asteroids drop off very nearly linearly with phase angle. Halley's comet was recovered when it had a phase angle of 5.2°. Bowell & Lumme (1979) found that very low albedo surfaces (i.e. those of C-type asteroids)

Table 1. Early magnitudes of Halley's comet. *n* is the number of observations taken to give the visual nuclear magnitude, m_2. The comet is at a heliocentric distance of r AU and has a phase of $\beta°$. The absolute visual magnitude is $V(1,0)$.

Date	IAU Circ	n	m_2	r	β	$V(1,0)$**
1982 Oct 16.47	3737	1	24.2±0.2	11.05	5.2	13.33
1982 Oct 19.4	3742	1	24.2 ?	11.03	5.1	13.36
1982 Nov 16.21	3753	1	24.2±0.4*	10.87	4.3	13.55
1982 Dec 10.29	3758	1	24.7±0.3	10.72	2.7	14.29
1982 Dec 13.20	3776	1	24.0±0.2	10.70	2.4	13.64
1983 Jan 14.26	3770	1	23.5±0.3	10.51	1.7	13.29
1983 Feb 13.2	3873	1	24.5±0.3	10.33	4.0	14.12
1983 Dec 31.4	3912	1	23.2	8.21	1.6	14.14
1984 Jan 27 – Jan 30	***	21	22.86±0.07	8.01	4.1	13.65
1984 Feb 4.27 – Feb 4.40	3928	4	23.05*	7.97	4.9	13.80
1984 Mar 4.25	3934	1	23.6±1.0	7.75	7.1	14.24

* Using B − V = 0.4 (see IAU Circ 3873)

** Using equation 1 and assuming that $\phi(\beta)$ follows the mean relationship for C type asteroids, i.e. for β's of 1°, 3° and 5°, $\phi(\beta)$ has values of 0.125, 0.315 and 0.45 respectively (see Bowell and Lumme, 1979, fig. 8).

*** West and Pedersen (1984).

would have a phase function, at that angle, of about +0.46 mag. Substituting the above values into equation (1) gives a $V(1,0)$ for Halley's comet of 13.33. A more general approach is shown in Table 1.

The heliocentric distance of the comet has changed from 11.05 to 7.75 AU during the time interval covered by Table 1. If the cometary coma is produced by H_2O sublimation it should start to form around 3.5 AU. Comae formed by more volatile substances commence further out. The data in Table 1 have been divided into two sets. The mean value of $V(1,0)$ is 13.60±0.14 for the winter 1982/83 observations ($11.05 \geqslant r \geqslant 10.33$), this value being obtained by averaging the brightnesses. The values of $V(1,0)$ obtained for winter 1983/84 ($8.21 \geqslant r \geqslant 7.75$) are dominated by the 21 photometric magnitudes obtained by West & Pedersen (1984) with the Danish 1.5-m telescope at La Silla. Each observation had an rms error of ±0.2 mag and night-to-night variability was found. Similar variability was detected by Lecacheux et al (*IAU Circ. No. 3928*, Le Fevre 1984). Averaging all the 1983/84 values gives a $V(1,0)$ of 13.71±0.07. Within the errors the values for the two years are the same leading to the important conclusion that no coma has yet developed. It is also extremely unlikely that a constant level of coma activity has occurred while the heliocentric distance has decreased from 11.05 to 7.75 AU. We are seeing a bare cometary nucleus. The mean $V(1,0)$ value of the whole data set is 13.68±0.06.

The brightness of an asteroid, and a bare cometary nucleus, is proportional to the surface area visible to the observer and the albedo of that surface. This was quantified by Zellner & Bowell (1977) who quoted the formula

$$\log\left(\frac{D^2 p_v}{4}\right)=5.642-0.4\,V(1,0). \tag{2}$$

The diameter of the body, D, is in kilometres and p_v is the geometric albedo in the visual band. For Halley's comet $D^2 p_v$ is equal to $5.90 \pm 0.33\,\mathrm{km}^2$.

What is the albedo of the surface of Halley's comet? The short answer is that we do not know. The meteoroids leaving the surface of cometary nuclei and migrating into the meteoroid stream have been found, from the spectroscopy of meteors, to have a composition similar to carbonaceous chondrites. It thus seems reasonable to suppose that the nucleus has an albedo similar to that of C-type (carbonaceous) asteroids and these are found to have $p_v \leqslant 0.065$. The albedo of the Moon is about 0.067. If, following Öpik (1963), this is *assumed* to be the value for the nucleus of Halley's comet the figure quoted above for $D^2 p_v$ leads to a diameter of 9.4 ± 0.3 km. Note that we have assumed that the comet has a uniform albedo. Newburn & Reinhard (1981) adopted an albedo value of 0.2 for Halley which corresponded to 25 per cent of the surface being covered with ice of albedo 0.6 and 75 per cent of the surface covered with dust of albedo 0.05. This leads to a diameter of 5.4 km.

Our use of the word 'diameter' should not be taken as indicating that we think that the nucleus is spherical although it must be admitted that comets probably 'round themselves off' as they decay. Comet Halley is thought to spin with its rotational axis perpendicular to its orbit plane. Comets also have a tendency to spin about the axis of maximum moment of inertia. Thus if the comet is non-spherical its equatorial diameter will be larger than its polar diameter. Opposing this is the fact that surface loss will occur preferentially around the hot equator thus tending to decrease the differences in the diameters. This process could continue until the comet took on the shape of an apple core but before reaching this shape the moment of inertia decrease would cause the comet to flip over and once again spin about a new axis of maximum moment of inertia.

R. M. West (1984) and West & Pedersen (1984) found that the brightness of Halley's comet during 1984 January 27–30 showed night-to-night variations of about 1 mag. This brightness change could be interpreted by presupposing that Halley's comet was spinning and was lemon shaped, the long axis being 1.6 (i.e. $\sqrt{2.51}$) times the shorter axis. Another (but I think less likely) possibility is that there is a considerable change in albedo around the nucleus.

The use of the mean value of $V(1,0)$ in the analysis above has been assumed to be equivalent to calculating the mean $D^2 p_v$. Unfortunately at the moment we cannot be more precise than this. At present (1984 October) no one has been able to interpret the fluctuations in cometary brightness in terms of a specific spin period. Measurement of the spin axis orientation seems to be even further off.

Cometary mass is one of the most awkward quantities to measure. The direct method – of observing the perturbation that a comet induces in the orbit of a celestial body of known mass – has as yet yielded no results. Comets have passed through the satellite system of Jupiter and within six Moon-orbit radii of the Earth without producing measurable changes. It would help if we knew the density of a comet. Hughes (1974a) argued that, if the interstices of meteroids were originally full of ice, the density of the comet would be around $1.1\,\mathrm{g\,cm^{-3}}$. Comet modellists (see Newburn & Reinhard 1981) usually assume a density of $1.0\,\mathrm{g\,cm^{-3}}$. Wallis & Macpherson (1981) concluded that non-gravitational forces can only in general be reconciled with H_2O outgassing if the mean comet density is below $0.7\,\mathrm{g\,cm^{-3}}$.

If we assume that the density of the nucleus of Halley's comet is $0.5\,\mathrm{g\,cm^{-3}}$ this, coupled with a diameter of 9.4 km, yields a mass of 2.2×10^{17} g. Notice that in obtaining this quantity we have had to assume both an albedo and a density. Maybe it would be more helpful to quote the mass as being $7.5 \times 10^{15} \varrho p_v^{-1.5}$ g.

3 Decay

First let us try and estimate how much mass Halley's comet lost at its 1910 apparition. Newburn (1981) gives (in his table 9) the total gas production rate of Halley's comet in 1910 as a function of heliocentric distance. These distances can be easily converted into times with respect to perihelion passage and a crude averaging of rates for each interval leads to a total gas loss of 5.1×10^{36} molecules during the 1910 apparition. Newburn & Reinhard (1981) concluded from the 1910 spectra that the parent molecules of Halley were 83.4 per cent H_2O and 16.6 per cent other molecules, these others having a mean molecular mass of 44 amu. They concluded that the mean molecular mass of all the molecules leaving the comet was 22.3 amu, i.e. 3.7×10^{-23} g. So the total gaseous mass loss during the 1910 apparition comes to 1.9×10^{14} g. If one assumes that the ratio of the dust to gas production rates (by mass) is 0.5 then the total mass lost by the comet during its last appearance was 2.8×10^{14} g.

If the comet has a diameter of 9.4 km and a mean density of $0.5\,\mathrm{g\,cm^{-3}}$ this is equivalent to it loosing a layer of thickness 200 cm from its surface.

Each time Halley's comet passes perihelion it loses mass, the nucleus decreases in size and the absolute brightness decreases. Hughes (1983b) tried to estimate the change in the absolute magnitude, H_{10}, of Halley's comet over the last 2000 yr. The absolute magnitude can be obtained by fitting observations of apparent magnitude, m, to the formula

$$m=H_0+5\log\Delta+2.5\,n\log r. \tag{3}$$

The coefficient n was assumed to be 4.0 and H_0 was thus replaced by H_{10}. [Note that H_{10} is the absolute magnitude of the active comet whereas $V(1,\,0)$ was the absolute magnitude of the inactive nucleus.] It has long been realized that the estimation of cometary magnitude depends to a large extent on the instrument being used to view the comet. To overcome this problem Hughes followed Broughton (1979) and used results from only one type of detector, the naked eye, and one group of observers, those who were the first to pick out the comet against the starry background on each of its returns to the Sun. Only observations prior to 1759 were used, so the comet was unexpected. The results indicated that the comet was getting fainter by 0.0205 ± 0.0210 mag per apparition. So two thousand years ago the comet was about 0.5 ± 0.5 mag brighter. The fact that the error is as large as the quoted value underlines the uncertainty of the result.

Ferrin (1984) reanalysed the data and took into account the effect of the elongation angle on the ability to recover the comet. He concluded that the comet was getting fainter by 0.055 mag per apparition. Using the diagram given in Ferrin's paper the uncertainty in this value has been estimated as being about ±0.015.

If one assumes that the brightness of a comet is proportional to the surface area of its nucleus it can easily be shown that

$$\Delta H_{10}\simeq-0.724\,\frac{\Delta M}{M} \tag{4}$$

where $\Delta M/M$ is the fractional mass loss per apparition and ΔH_{10} is the change in absolute magnitude per apparition (see Hughes & Daniels 1983).

Substituting 0.0205 into equation (4) gives $\Delta M/M=0.028$ and equating ΔM to 2.8×10^{14} g leads to a 1910 cometary mass of 10^{16} g. The fact that this is way below our previous estimate of 2.2×10^{17} g points to the fact that 0.02 mag per apparition is probably a considerable overestimate. The value should be closer to 9×10^{-4} mag per apparition indicating that the change in the magnitude of the comet during recorded history has been negligible. If one has more faith in the ΔH_{10} value of 0.02, the substitution into equation (4) of the ΔM and M values obtained previously yields the result that $\varrho p_v^{-1.5}=1.32\,\mathrm{g\,cm^{-3}}$. Here a density of $0.5\,\mathrm{g\,cm^{-3}}$ yields the unfashionably high albedo of 0.52.

A low value of magnitude change and fractional mass loss is supported by the work of Yeomans & Kiang (1981) who find that the orbital evolution of Halley's comet over the last 2000 yr is consistent with the non-gravitational parameters A_1 and A_2 remaining essentially constant over that period. This indicates that the size and mass of the comet has changed little during that time.

4 The meteoroid stream

Decaying comets produce meteoroid streams and Halley's comet is no exception. The Earth intersects Halley's stream twice during the year. In May the Earth passes to within 0.065 AU of the cometary orbit the result being the Eta Aquarid shower and observations of this shower have been used by McIntosh & Hajduk (1983) to estimate the spatial density of the cometary dust in this region of the annulus around Halley's comet. They found that the density was $3\times10^{-24}\,\mathrm{g\,cm^{-3}}$. In October the Earth gets to within 0.154 AU, the Orionids are observed and the spatial density is about $10^{-24}\,\mathrm{g\,cm^{-3}}$. Extrapolating to 0.01 AU the authors concluded that the dust density near the cometary orbit was about $5\times10^{-24}\,\mathrm{g\,cm^{-3}}$. By integrating these dust densities throughout the stream they concluded that the total mass of dust in the stream was about 5×10^{17} g. This value can be compared with Hughes (1974b) who estimated that the mass of dust in the Quadrantid, Geminid and Perseid streams was 4.6×10^{13}, 2.0×10^{15} and 8.8×10^{14} g respectively and with Lovell (1954) who calculated values of 5×10^{14}, 5.7×10^{15} and 2.0×10^{15} g respectively for the same three streams. Halley's meteoroid stream seems to be considerably more massive than these other three streams.

The relationship between the mass of a comet and the mass of the associated meteoroid stream at any one time is rather complicated. Usually stream formation is gentle and most people hypothesize that the cometary nucleus loses a constant thickness of material at each perihelion passage. A rough, but telling, illustration of this decay process is given in Table 2. It can be seen that the stream gains mass very quickly and is about 75 per cent of its final mass by the time the comet has changed in magnitude by unity. This indicates that, in the majority of cases, dense streams should still be associated with bright parent comets. Also if the mass of Halley's stream is 5×10^{17} g it is unlikely that the original mass of the comet was (accounting for gas loss) more than

Table 2. This table illustrates the decay of a comet which loses a constant thickness of nucleus material at each perihelion passage. The comet starts out with a mass of M, a radius of R and an absolute magnitude of H. A fraction a of its mass goes into the meteoroid stream ($a\sim0.3$). It has been assumed that the brightness of the comet is proportional to its surface area.

Percentage of cometary life that has passed	Radius of nucleus	Mass of comet	Mass of stream	Magnitude of comet
0	R	M	0	H
10	0.9R	0.729M	0.271Ma	H+0.23
20	0.8R	0.512M	0.488Ma	H+0.48
30	0.7R	0.343M	0.657Ma	H+0.77
40	0.6R	0.216M	0.784Ma	H+1.1
50	0.5R	0.125M	0.876Ma	H+1.5
60	0.4R	0.064M	0.936Ma	H+2.0
70	0.3R	0.027M	0.973Ma	H+2.6
80	0.2R	0.008M	0.992Ma	H+3.5
90	0.1R	0.001M	0.999Ma	H+5.0
100	0	0	Ma	∞

1985MNRAS.213..103H

about three times larger. There is nothing shown in Table 2 that is inconsistent with the statement that Halley's comet has a mass of 2.2×10^{17} g, the meteoroid stream has a mass of 5×10^{17} g and the magnitude of the comet changes undetectably from one perihelion passage to the next.

It is reasonable to propose that in terms of cometary life Halley's comet is middle-aged. We can argue as follows. If the comet has already lost 5×10^{17} g of dust this is equivalent to a total mass loss of 15×10^{17} g. The original mass of the comet was thus 1.72×10^{18} g, and, assuming the density is $0.5\,\mathrm{g\,cm^{-3}}$, the diameter of the nucleus was 18.7 km.

The radius decreases at about 200 cm per perihelion passage. So to slim from a diameter of 18.7 km to one of 9.4 km assuming a constant rate of radius decrease takes about 2300 passages. At this rate the comet will have disappeared altogether in a further 2300 perihelion passages.

5 Conclusions

Whipple (1978) wrote 'there is no satisfactory correlation between cometary radius or mass and absolute magnitude, although various attempts have been made to write such formulae'. Whipple (1976) made this attempt himself and had suggested

$$\log M(g)=19.39-0.6\,H_{10}. \tag{5}$$

This derives directly from the assumption that brightness is proportional to the surface area of the nucleus which, converting to absolute magnitude, yields $D\propto10^{-0.2H}$. Changing to mass gives $M=C\,10^{-0.6H}$ where C is the mass of a zero-magnitude comet [which Whipple took to be 2.5×10^{19} g in equation (5)]. The constant in Whipple's equation (5) came solely from observations of Comet Bennett 1970 II. He took the Sekanina & Miller (1973) value of 6 km for the diameter of the nucleus of Bennett, the Beyer (1972) value of 3.6 for the absolute magnitude and then assumed a density of $1.5\,\mathrm{g\,cm^{-3}}$.

Morris & Green (1982) found that Halley's comet in 1910 had a preperihelion absolute magnitude of 5.49 ± 0.07 and a post-perihelion absolute magnitude of 5.44 ± 0.05.

If we use the mean of these values with our estimation that comet Halley has a mass of 2.2×10^{17} g the Hughes version of equation (5) turns out to be

$$\log M(g)=20.61-0.6\,H_{10}. \tag{6}$$

Obviously this equation assumes that Halley's comet is a typical comet and contains assumptions as to the density of the nucleus and its albedo. Removing the former of these leaves us with

$$D(\mathrm{km})=116\times10^{-0.2H_{10}}. \tag{7}$$

Kresák (1984) cautioned against the use of single mass and diameter relationships and noted that the relationship might vary depending on whether one is considering new or old comets, i.e. those fresh from the Oort cloud as opposed to those which have already passed close by the Sun a few times. He suggested that the diameters of the zeroth magnitude comets in these two groups were 32 and 150 km respectively. Opposition to this suggestion can be gleaned from the work of Donn (1977) who compared spectroscopically the composition of new and evolved comets and concluded that there was no readily apparent difference.

Another attempt has been made to measure the size of Halley's comet. On 1910 May 19 the comet transitted the solar disc ingressing at 03.40 GMT and egressing at 04.38 (see Bobrovnikoff 1931). An expedition was sent to Hawaii to try and observe this. Ellerman, using a 6-inch telescope 'was able during the critical interval to see small sun-spots most clearly but absolutely no trace of the comet's nucleus could be detected. Had the latter been a single solid mass as much as 200 miles in diameter, it should have been seen without difficulty' [see Olivier (1930) and also Barnard (1911) and Ellerman (1910)]. The probability that the nucleus is only 9.4 km in diameter easily explains why the observers were disappointed.

Halley's comet has often been regarded as a prominent source of material to the Solar System dust cloud. Whipple (1967) quoted its average mass loss as $5\times10^6\,\mathrm{g\,s^{-1}}$ equivalent to an amazing 1.2×10^{16} g per apparition. The values given in this paper lead to a mass influx of on average $4\times10^4\,\mathrm{g\,s^{-1}}$, a factor of 100 lower.

Ferrin (1984) calculated that Halley's comet had only another 39 revolutions left before disappearing. His approach is best illustrated by reference to equation (7). If Halley's comet at present has an absolute magnitude of 5.466 which changes (according to Ferrin) by +0.055 mag per aparition the diameter changes by 0.23 km per apparition and in $9.4/0.23\sim39$ revolutions there will be nothing left. The much longer lifetime quoted in this paper results from the conclusion that the analysis of ancient observations undertaken by Hughes (1983b) and Ferrin (1984) both lead to an overestimate of rate of change of Halley's absolute magnitude. This is probably close to 9×10^{-4} mag per apparition.

Acknowledgments

I would like to thank Max Wallis for his most helpful comments and Uwe Keller for his encouragement.

References

Barnard, E. E., 1911. *Science,* N.S. XXXIV, 531.
Beyer, M., 1972. *Astr. Nach.,* **293,** 241.
Bobrovnikoff, N. K., 1931. *Publs Lick Obs. No. 17,* Pt. 2, 374.
Bowell, E. & Lumme, K., 1979. *Asteroids,* p. 132, ed. Gehrels, Tom, University of Arizona Press.
Broughton, R. P., 1979. *J. R. astr. Soc. Can.,* **73,** 24.
Donn, B., 1977. *Comets, Asteroids, Meteorites,* p. 15, ed. Delsemme, A. H., University of Toledo Press.
Ellerman, F., 1910. *Publs astr. Soc. Pacif.,* **22,** 165.
Ferrin, I., 1984. *Astr. Astrophys.,* **135,** L7.
Hughes, D. W., 1974a. *J. Br. astr. Ass.,* **84,** 272.
Hughes, D. W., 1974b. *Space Res.,* **14,** 709.
Hughes, D. W., 1983a. *Asteroids, Comets and Meteors,* p. 239, eds Lagerkvist, C.-I. & Rickman, H., University of Uppsala Press.
Hughes, D. W., 1983b. *Mon. Not. R. astr. Soc.,* **204,** 1291.
Hughes, D. W. & Daniels, P. A., 1983. *Icarus,* **53,** 444.
Jewitt, D. C., Danielson, G. E., Gunn, J. E., Westphal, J. A., Schneider, D. P., Dressler, A., Schmidt, M. & Zimmerman, B. A., 1982. *IAU Circ. No. 3737.*
Kresák, L., 1984. *Dynamics of Comets, IAU Colloq. No. 83,* Reidel, in press.
Le Fevre, O., Lecacheux, J., Mathez, G., Lelièvre, G., Bàudrand, J. & Lemonnier, J. P., 1984. *Astr. Astrophys.,* **138,** L1.
Lovell, A. C. B., 1954. *Meteor Astronomy,* Oxford University Press.
McIntosh, B. A. & Hajduk, A., 1983. *Mon. Not. R. astr. Soc.,* **205,** 931.
Morris, C. S. & Green, D. W. E., 1982. *Astr. J.,* **87,** 918.
Newburn, R. L. & Reinhard, R., 1981. *The Comet Halley Dust and Gas Environment,* p. 19, ESA SP-174.
Newburn, R. L., 1981. *The Comet Halley Dust and Gas Environment,* p. 3, ESA SP-174.
Olivier, C. P., 1930. *Comets,* p. 117, Bailliers, Tindall and Cox, London.
Öpik, E. J., 1963. *Ir. Astr. J.,* **6,** 93.
Sekanina, Z. & Miller, F. D., 1973. *Science,* **179,** 565.
Wallis, M. K. & Macpherson, A. K., 1981. *Astr. Astrophys.,* **98,** 45.
West, R. M., 1984. *IAU Circ. No. 3943.*
West, R. M. & Pedersen, H., 1984. *Astr. Astrophys.,* **138,** L9.
Whipple, F. L., 1967. *The Zodiacal Light and the Interplanetary Medium,* p. 409, NASA SP-150.
Whipple, F. L., 1976. *Mem. Soc. R. Sci. Liège, Ser. 6,* **9,** 101.
Whipple, F. L., 1978. *Cosmic Dust,* p. 1, ed. McDonnell, J. A. M., Wiley, New York.
Yeomans, D. K. & Kiang, T., 1981. *Mon. Not. R. astr. Soc.,* **197,** 633.
Zellner, B & Bowell, E., 1977. *Comets, Asteroids, Meteorites,* p. 185, ed. Delsemme, A. H., University of Toledo.