Heliostats, Siderostats, and Coelostats: A Review of Practical Instruments for Astronomical Applications

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Members of the 'stats family of astronomical instruments have in common the function of feeding light from a celestial object to a fixed telescope. Particular groups are known as heliostats, siderostats, uranostats and coelostats, considerable variations in design being possible within each group. This paper explains their general construction, differences, and respective advantages, concluding with survey of those two-mirror forms currently applied to solar astronomy.

INTRODUCTION

The abundance of light from the Sun has frequently prompted the desire to study it with a long-focus telescope producing an image of considerable size at its prime focus, thereby facilitating subsequent magnification and/or dispersion of a selected portion of that image. However, in the simpler optical designs this results in a structure of such length and weight as to be mechanically unwieldy and liable to flexure, so even in the seventeenth century thought was being given by Hooke and others to directing light into a fixed telescope by means of a plane mirror. This mirror must be moved in such a way that, looking into the exit beam, the Sun appears to stand still: the mirror and its drive constitute a *heliostat*.

The loss of light associated with reflection has tended to restrict the technique to work on the Sun, but nevertheless sufficient variations in design and application have arisen to cause troublesome anomalies in nomenclature[1–5]. The confusion is further compounded by the fact that, prior to the invention of the electric arc and filament lamp, sunlight provided the most intense source of illumination available. Projection or 'solar' microscopes, for example, employed the Sun simply as a source of light. The use of apparatus of this nature, or experiments on sunlight itself, were facilitated if a beam could be sent in a fixed direction to enter a room *via* a hole in a shutter throughout the day, independently of the motion of the Sun across the sky. Instruments designed to fulfil this function were therefore also known as heliostats.

It should be noted that all the designs to be discussed employ plane mirrors throughout, and so are non-imaging: their exit beam must feed a lens or concave mirror to produce a real image. It must also be remembered that any design employing one—or an odd number—of reflections will introduce left-to-right field inversion (F becoming ꟻ), necessitating the viewing of standard charts from the rear. (Remember that a simple refractor has no mirrors, whereas a Newtonian reflector has *two* mirrors.) Due to the finite angular diameter of the Sun, the beam leaving any heliostat will diverge within a cone of 0°·5 apex angle.

SIDEROSTATS

Any instrument which keeps a star centred in a given field of view independently of the passage of time might be termed a siderostat, so in a sense the ordinary clock-driven equatorial telescope is one such device. However, if—as above—we consider the telescope proper (*i.e.* that part subsequent to the image-forming element) to have the direction of its optical axis fixed with respect to the ground, then it is possible to narrow the definition. Single-mirror devices will be considered first.

The polar siderostat

The simplest siderostat-fed telescope is the polar refractor, where the telescope is physically positioned along the polar axis and light is reflected into it by a plane mirror adjustable in declination. The optical axis may be elevated or depressed, as shown in figures 1 and 2 respectively. Telescope and flat rotate together at the sidereal rate about the coincident polar and optical axes, in a direction such as to annul the effect of the Earth's rotation. The equatorially mounted plane mirror directing an exit beam along its polar axis constitutes a *polar siderostat*.

The main advantage of a polar telescope is that the observer can sit in a warm room. Its disadvantages include the need for an accurate flat of large diameter, that in the depressed version the flat is situated in turbulent air near the ground, and that (in its simple form) the instrument is unable to view

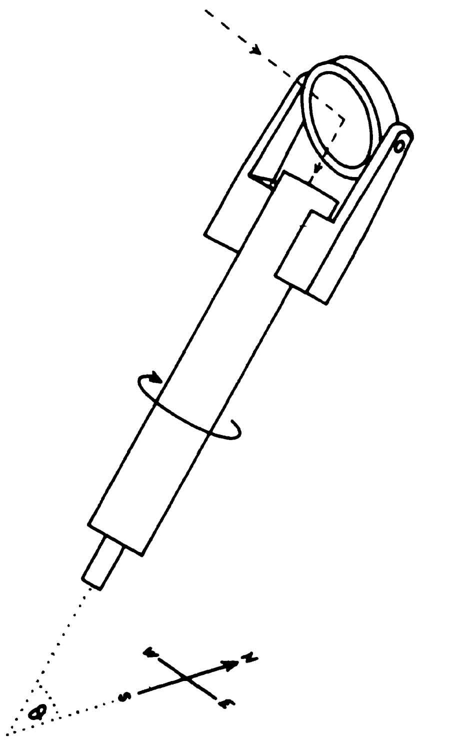

Figure 1. Polar telescope, elevated version. φ denotes latitude in this and subsequent diagrams, which have been constructed for φ = 52°.

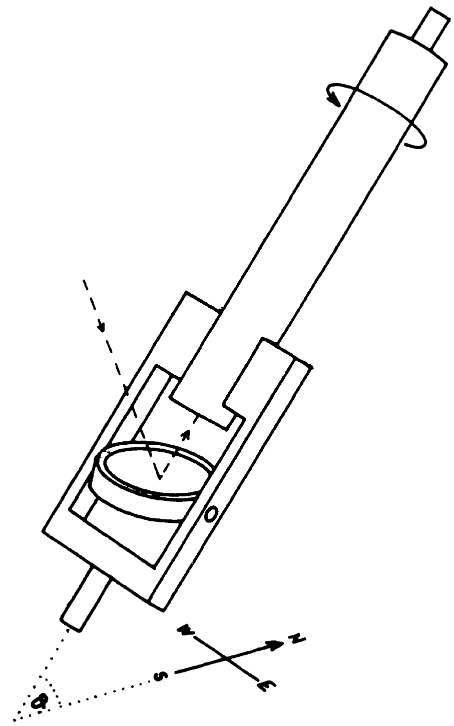

Figure 2. Polar telescope, depressed version.

the sky around the celestial pole. To these must be added the inconvenience of field reversal. For these reasons few polar telescopes have been constructed for general astronomical use[6–11].

It will be realized that if a marker is positioned at the eyepiece end of a polar telescope, but not mechanically coupled to it, then the field of view will appear to rotate at a uniform rate of one revolution per sidereal day with respect to that marker. This will not inconvenience the visual observer, but it will be clear that for long exposures a camera would need to be coupled to the moving telescope—which is of course the normal situation for astrophotography. This rotation of the field about its central point is a characteristic of *all* siderostats.

Early single-mirror siderostats and heliostats

It is obviously easier to mount a long-focus telescope or spectrograph if it is positioned horizontally at a given site. However, only on the equator would this coincide with the polar axis: elsewhere a more complicated *biaxial* motion of a single plane mirror is required. The underlying theory has recently been clearly explained by Dougherty[5], so will not be repeated here. A driven polar axis is always present in some form.

Mechanical realization of this theory as a clock-driven apparatus was first achieved by 's Gravesande in 1742 (ref. 12), but his 'heliostate' was intended more for general experimentation and illumination, although a simple lens of long focal length could be brought into the beam if desired. Refined portable heliostats for this purpose of illumination (driven at the solar rate) were designed by Gambey[13], Silbermann[14] and others. Their delicate clockwork mechanisms could support and drive only a rather small, thin mirror of speculum metal or front-silvered plate glass—quite unsuitable for giving an image of astronomical quality (figure 3). Apart from

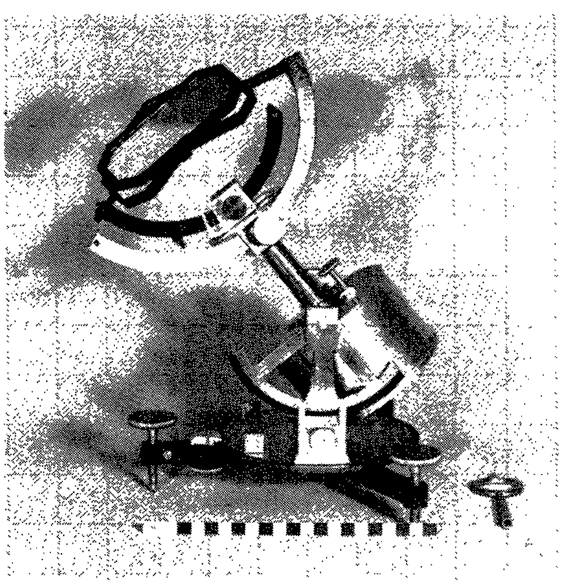

Figure 3. A Silbermann pattern portable heliostat. This particular example was originally used for the illumination of petrological microscopes, and is now in the Whipple Museum of the History of Science, Cambridge University.

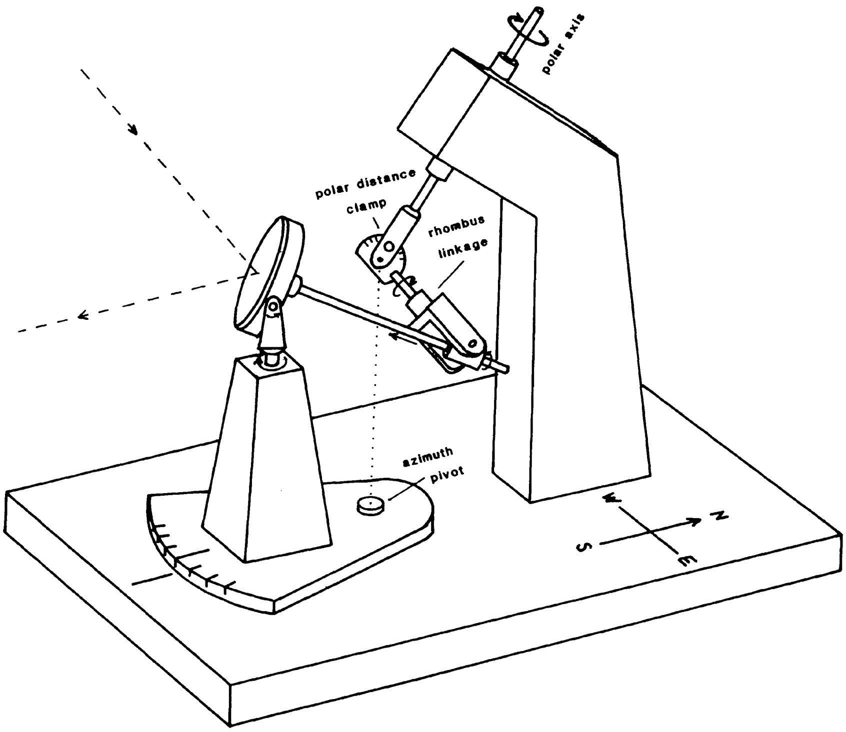

Figure 4. The Foucault siderostat. (Diagram only: not to scale.) The theory underlying its operation *via* the rhombus linkage is explained in reference 5.

this, the accumulated play in the multiple linkages involved would make examination of a magnified image most difficult, as would the non-uniform rotation of the field (see below). As previously stated, these small heliostats are best classified as *solar illumination devices*, their only serious application to astronomy being in the early days of solar spectroscopy[15]. Nevertheless, these rare and elegant examples of the instrument-maker's art deserve examination in their own right: a paper is in preparation.

The Foucault siderostat

The only really successful *astronomical* single-mirror siderostat is that invented by Foucault in 1862 (ref. 16 and figure 4), the single mirror being preferred to minimize loss of light in stellar applications. Foucault returned to the independent altazimuth mounting of the counterbalanced plane mirror first employed by 's Gravesande, thereby taking its weight off the drive. He improved the latter by designing a 'rhombus linkage'[5]. Engravings of Foucault siderostats are reproduced in a number of older works[2,3,17,18,19], but the diagram shown in Sidgwick[4] is incorrect.

It is not essential that the beam from a Foucault siderostat be horizontal or directed exactly south along the meridian; a limited variation in azimuth can be achieved by moving the mirror assembly as a whole about the 'azimuth pivot' marked in figure 4. However, a horizontal meridian beam was normally employed in permanent stations devoted to solar studies, so Foucault siderostats for this specific application might be set to run at the solar rate and omit the azimuth pivot (figures 5 and 6).

An alternative 'overdrive' arrangement[5,16] is said to be easier to align[20], but both designs are mechanically awkward and inefficient, the rotating/sliding joint on the mirror rod being particularly liable to binding and inaccuracy. An attempt was made to redesign the linkage[21], but appears never to have been put into practice.

Foucault siderostats do not appear to have made any significant contributions to *stellar* astronomy: indeed the largest, a 2m monster feeding the 1·25m objective of a 57m long horizontal telescope at the Paris Exhibition of 1900, was part of a notorious fiasco[22,23,24]. However, when driven at the solar rate, these instruments made possible some classic studies of the solar constant[25] and the composition and temperatures of the photosphere, sunspots, etc.[26] A few redundant examples still exist in professional observatories (figures 5 and 6), but must be considered museum pieces.

The uranostat

'Uranostat' is a general term for any single plane mirror provided with two perpendicular axes of rotation and angled to reflect a beam back into the hemisphere containing the Sun[1,2,27]. One axis may be made a polar axis, but rotation about the declination axis then allows the reflected beam to depart in directions other than along the polar axis. The penalty is drift (which may be counteracted over short periods by adjustments to the two driving rates) and non-uniform field rotation. These motions are minimized when the beam exits in the plane of the meridian, and for short-exposure photographs may not be significant. In practice, this restricts the uranostat to work with the Sun. Simple and com-

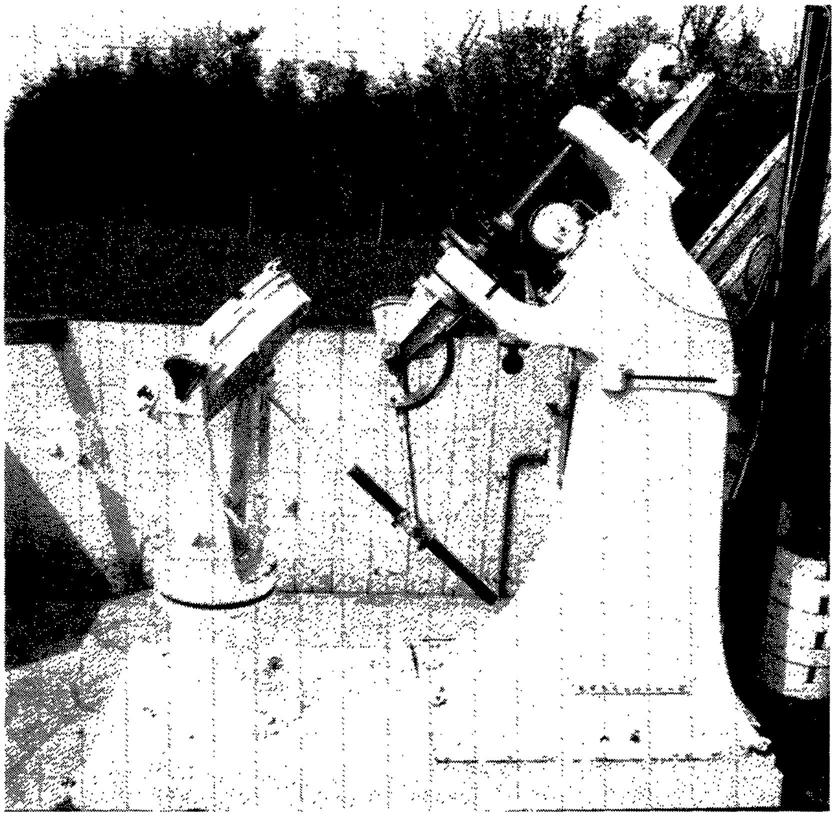

Figure 5. The 450mm Foucault siderostat at Cambridge University Observatory. Made by Cooke and Sons *c.* 1900, it once fed a spectroheliograph which has since been dismantled. (Photographed 1984 May 2.)

paratively cheap, it was at one time commonly combined with horizontal telescopes of southerly azimuth for routine solar photographic patrol[27].

The name is derived from the Greek ὀυρανός—'heaven', and would imply that the instrument maintains a celestial field stationary and without rotation. In this sense it is a misnomer.

Non-uniform field rotation

The price paid for using the single moving mirror in portable heliostats and in the Foucault siderostat to reflect the beam out of the polar axis is that the field will, in general, not only rotate but do so at a *non-uniform* rate[28,29,30]. The situation is analogous to the motion transmitted by Hooke's universal joint; one complete rotation will still be made in an entire sidereal day. Such rotation is, of course, irrelevant for illumination purposes.

It may be shown[28] that, with a Foucault siderostat set to produce a horizontal exit beam to the south, if ϕ denotes the latitude of the site and θ the polar distance of the centre of the chosen field, then:

The field rotates non-uniformly in a clockwise direction when $\theta < \phi$

The field rotates non-uniformly in an anti-clockwise direction when $\theta > \phi$

The latter is the usual condition with solar work at mid-latitude sites.

Many designs were published in 1901 (refs 31–34) to correct this field rotation by giving a compensating non-uniform rotation to a plateholder *via* further mechanical linkages, but these plans were still-born: the coelostat had already been invented.

THE SINGLE-MIRROR COELOSTAT

In 1895 Lippmann[35] showed that there is only one way in which a moving mirror can produce a non-rotating image of a sky field. The mirror must be fixed with a line element (usually a diameter) of its plane reflecting surface parallel to the polar axis, and rotated about that axis once in 48 sidereal hours in the same direction as the apparent diurnal motion of the heavens[5] (figure 7). This rotation period arises from the fact that a reflected beam moves through twice the angle described by the moving mirror. In practice, it is easier to design and set-up a coelostat (and the mirror is a minimum size) if a diameter of the reflecting surface is actually *coincident* with the polar axis.

The mirror of the Foucault siderostat achieves the coelostat condition for the special case of $\theta = \phi$.

Lippmann's publication immediately attracted the attention of both professional and amateur astronomers[28,29,36–43], although according to Hartmann[1] it was a re-invention of a design originally produced by E. E. August before 1839.

No motion of the coelostat mirror itself in declination is allowed, so the polar distance of the image of a star is equal to that of the star itself in the opposite

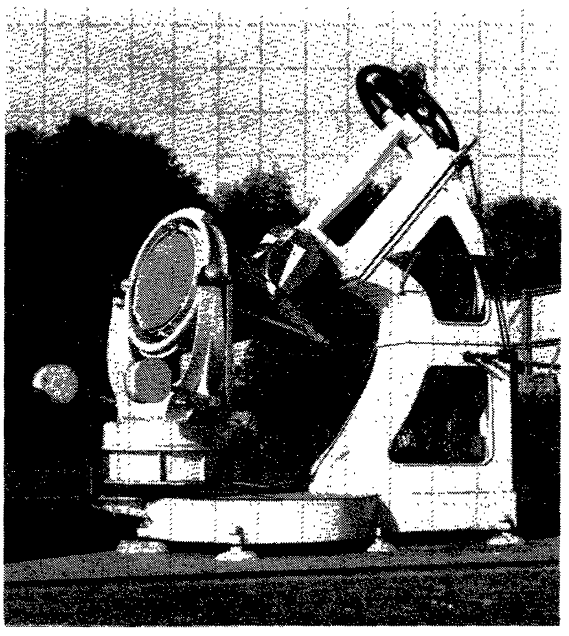

Figure 6. The large Foucault siderostat at Meudon Observatory. *Photograph courtesy of Th. Weimer.*

hemisphere[5,44]. To study any given star within the range of the instrument when installed at a particular latitude, a horizontal receiving telescope must therefore be adjustable in azimuth[39,42] (figure 7). It follows that manual rotation of the coelostat mirror around its polar axis will result only in those stars sharing the same parallel of declination appearing successively in a fixed horizontal telescope directed towards the coelostat from a pre-set azimuth. Turning on the clock drive will then hold a star selected from amongst these steady at the centre of a non-rotating field. Note that a circumpolar star cannot be reflected in a horizontal direction at all.

The above situation is acceptable for temporary stations set up to record total solar eclipses, where

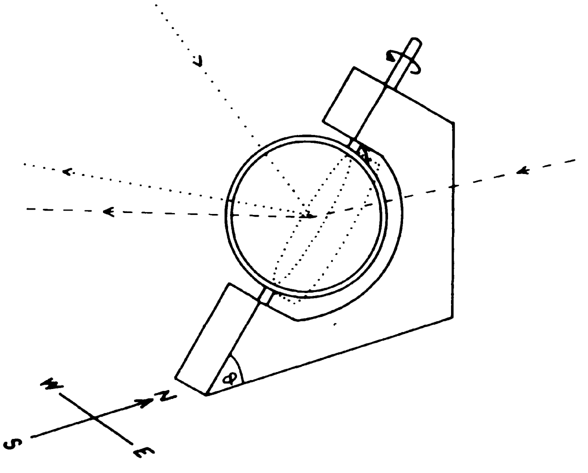

Figure 7. Principle of the coelostat. A ray from a star at an arbitrary (non-circumpolar) declination is drawn for two different times of day. At the later hour the exit beam is directed horizontally towards a particular azimuth.

the Sun is the star at an accessible declination and the horizontal telescope may be set at a pre-calculated azimuth. The coelostat (driven at one rotation per 48 solar hours) soon replaced the siderostat for this purpose. If it is not essential that the telescope be horizontal, then inclined and/or meridian positions are possible[44]: these may be more convenient, less susceptible to driving errors, or need a smaller coelostat mirror[36].

HELIOSTATS

In my opinion a heliostat should be exactly what it claims to be—an instrument which reflects a beam of sunlight in a fixed direction. This beam may or may not be directed into the hemisphere containing the Sun, and may or may not rotate. This specialization means that:

(*a*) An abundance of light is available, so two (or even more) reflections are acceptable.

(*b*) There is no need to provide for a movement in declination exceeding 24° either side of the celestial equator.

(*c*) The main drive motor will operate at the mean solar rate, permitting use of ordinary mains-driven synchronous motors. It is accepted that (apart from errors of construction or siting) the effects of refraction, the changing declination of the Sun, and the equation of time, will necessitate fine adjustments *via* a manual control.

It is obvious that the Moon and planets could be held steady by such equipment: no special nomenclature is justified. In practice, varieties of the polar siderostat and coelostat adapted to solar work are the only forms in which these instruments are to be found in modern astronomical usage.

Two-mirror heliostats

Permanent solar observatories generally require a horizontal or vertical exit beam; the latter became the preferred form when the superiority of the tower telescope against the disturbing effects of near-ground convection currents and turbulence was demonstrated by Hale in 1907 (refs 45 and 46). The only practicable way to achieve these fixed orientations as the Sun's declination changes through the year is by the use of a second mirror.

The Two-mirror Polar Heliostat

The direct descendant of the polar siderostat, this is much the simplest instrument to construct[47]. Horizontal and vertical beam versions are shown in figures 8 and 9. In middle latitudes, the first mirror must be about twice the diameter of the objective lens to accommodate the entire beam when the Sun is at its lowest declination (Winter solstice).

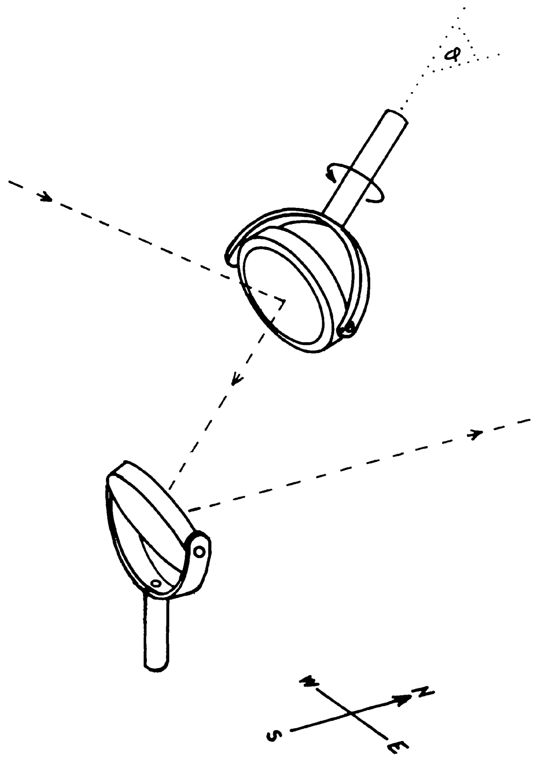

Figure 8. A two-mirror polar heliostat producing a horizontal beam.

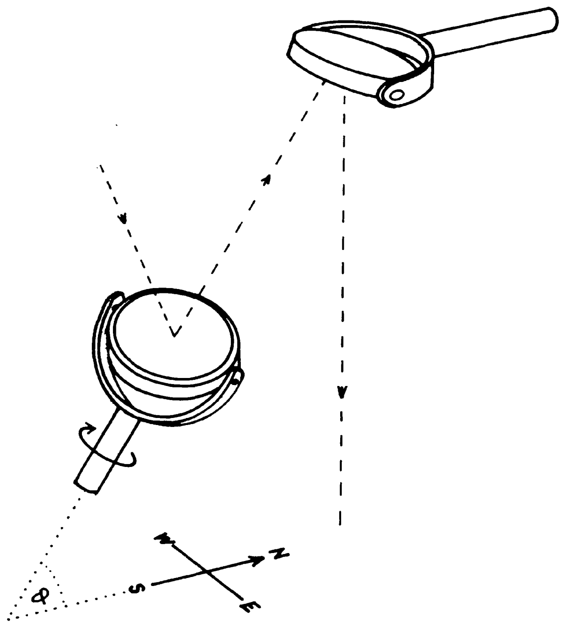

Figure 9. A two-mirror polar heliostat producing a vertical beam.

It will be remembered that, so long as the beam leaving the moving mirror always departs along the polar axis, then the Sun's image will not drift, but will rotate at a uniform rate of 15° per hour. The final image may therefore be rendered stationary by a Dove prism[48,49] rotating in the opposite sense at exactly half this rate[50]. A limitation is set by the aperture of this prism and the desirability that the beam traversing it should not be steeply converging[49]. The Dove prism also adds its own left-to-right inversion, but this is rarely significant in solar work and can always be corrected by 'flipping' the negative when printing photographs.

Sunlight reflected from a mirror is always polarized to a greater or lesser degree, and the directions of the axes of polarization change with the angle of incidence. The two-mirror polar heliostat maintains a fixed angle of incidence throughout a day's observations, and is therefore the best system for studies involving polarization[27,51,52].

The Two-mirror Coelostat

This is the only practicable way of achieving a large, non-rotating image of the Sun by all-reflection optics, using the output beam to feed a Cassegrain or off-axis paraboloid system. This is essential for complete achromatism and studies in the ultraviolet, but many investigations may be equally well pursued with a good achromatic lens as the image-forming element. A coelostat generally requires a smaller mirror than the corresponding polar heliostat.

Nowadays the entire two-mirror system is frequently called 'a coelostat', the mirror rotating on the polar axis being referred to as the 'primary flat', and the stationary mirror required to redirect its beam into the fixed telescope becoming the 'secondary flat'. Horizontal and vertical exit beam versions are possible. The former was used in the earliest observatories dedicated to solar research[27,53–55], but with few exceptions (for example reference 56, and the solar telescope built at Cambridge in the 1950s by Von Klüber) was soon supplanted by the superior vertical orientation incorporated in solar towers[27,57–62]. The Mt Wilson, CalTech and McMath-Hulbert solar observatories are well-known examples of the latter type, and are featured in many textbooks and reviews[27,63–65].

No adjustment in declination of the beam produced by the primary mirror is available: the latter *must not* be inclined out of the polar axis. At the same time, the secondary flat *must* remain in front of the fixed vertical telescope, accurately positioned along an imaginary extension of its optic axis. Therefore, to accommodate the limited changing declination of the Sun through the year, options available are:

(*a*) *Adjustable primary mirror*
The primary mirror is moved as a whole, maintaining its polar inclination, whilst the secondary is kept at a fixed height above the vertical telescope. This is the original method[57], the primary assembly (the coelostat proper) being supported upon a carriage capable of motion along rails laid north-south (figure 10). If these rails rise at the polar

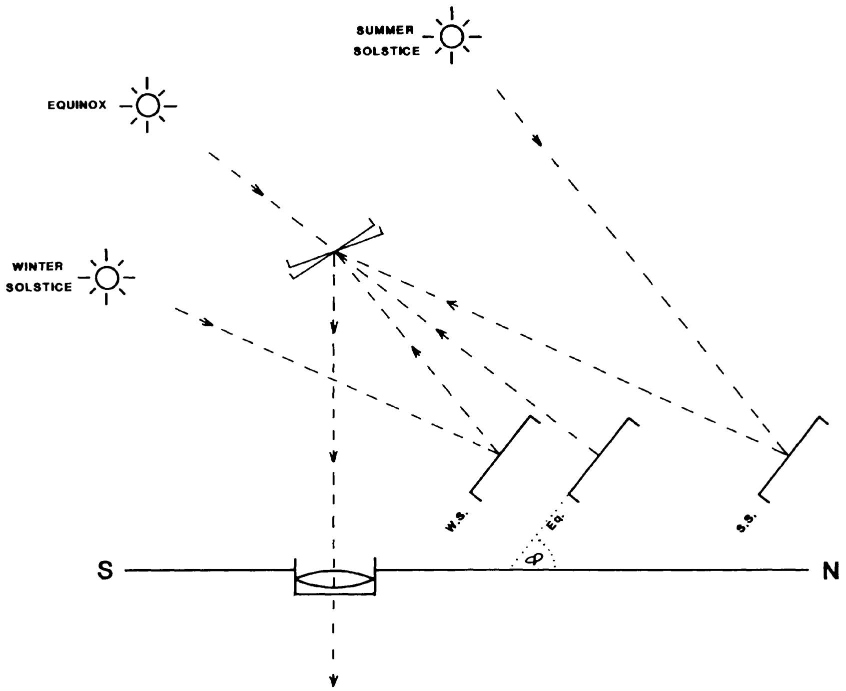

Figure 10. Two-mirror coelostat, vertical beam, type (*a*). The coelostat proper moves along rails laid N–S to accommodate the changing declination of the Sun through the year. The symbols W.S., Eq., and S.S. denote winter solstice, equinox, and summer solstice positions of the coelostat mirror.

elevation to form an inclined ramp (figure 11) the assembly can be enclosed by a smaller protective dome[58–61].

(*b*) *Adjustable secondary mirror*
The secondary assembly is moved as a whole up or down the imaginary optic axis extending above the fixed vertical telescope (figure 12).

There remains a further complication: when the Sun is near the meridian the secondary mirror assembly of the arrangements illustrated in figures 10–12 may (as shown) block the primary with its shadow! It is therefore necessary to have a second, underlying, set of rails positioned east-west; and to work with the primary carriage to the east in the mornings (when critical work is best undertaken)

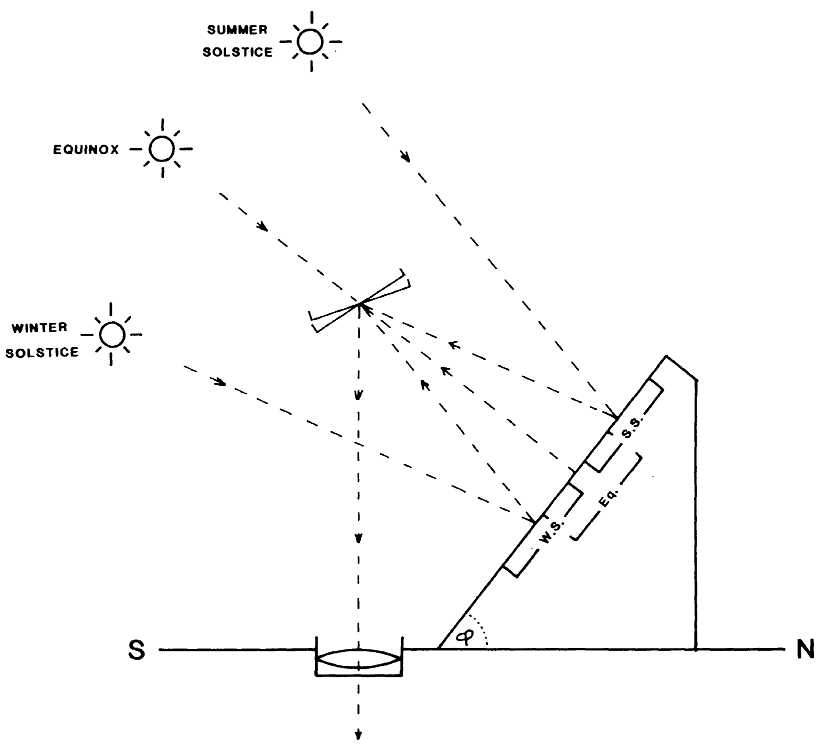

Figure 11. As figure 10, but with the N–S rails rising at the polar elevation.

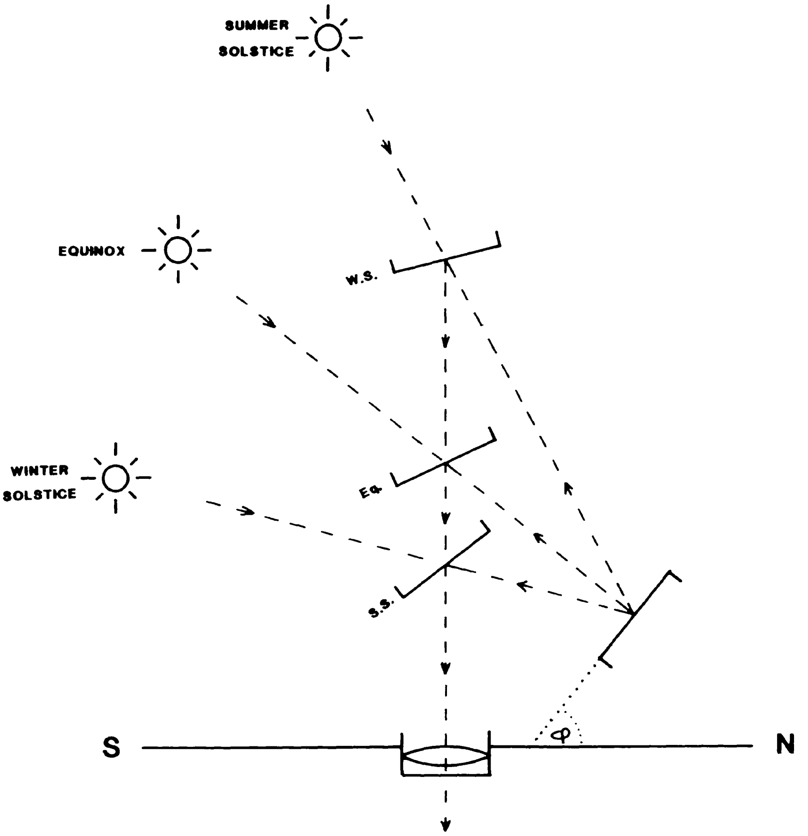

Figure 12. Two-mirror coelostat, vertical beam, type (*b*). The secondary mirror is moved along the optic axis of the fixed vertical telescope.

and then quickly shift it over to the west just before noon on each working day* (figure 13). This demands that the secondary mirror be adjustable about two axes: altazimuth is possible, but an equatorial mounting facilitates fine adjustment of the image[58,61].

It will be appreciated that a two-mirror coelostat is not an easy thing to make, especially with the accuracy and rigidity demanded of astronomical mountings. It would certainly be foolhardy to omit making a scale model in wood of any proposed instrument. Helpful ideas for small coelostats are given in references 47, 66 and 67. Anyone contemplating a larger instrument is recommended to study the published plans of the coelostat built by Grubb Parsons in 1934 for the vertical solar telescope at Oxford University[3,60,68] (figure 14). Professional research in solar astronomy is no longer supported at UK sites, so this instrument is rarely used nowadays. For the same reason, the 450mm (18-inch) coelostat of the horizontal Cambridge telescope (figure 15) has been partially dismantled and its optics placed in storage. A modern, working, coelostat type (*b*) at Meudon is illustrated in figure 16.

*At some installations[51,57] it is possible to reverse these orientations to give more acute angles of incidence and reflection, or even to keep the coelostat set to the north of the secondary mirror from April to September[61].

CHOICE OF INSTRUMENT

A potential constructor should first bear in mind that, due to differential heating, turbulence, and the addition of errors consequent upon multiple reflections, the resolution finally achieved with any mirror-fed system is unlikely to match that expected of an equatorially-mounted refractor of the same aperture directed towards stars. For high resolution studies of the Sun, there is an obvious move towards dedicated telescopes of the equatorial type ('solar spars'), preferably located on islands or peninsulas[69,70].

However, assuming that a fixed telescope has been decided upon, then the choice of instrument to feed it is essentially between the two-mirror forms of the polar heliostat and coelostat. Professional observatories invariably choose the latter—with the exception of the world's largest solar telescope at Kitt Peak, Arizona, where a 2m (78-inch) single-mirror heliostat feeds an elevated polar telescope[65,71]. Amateurs[50,72], educational institutions[73], and planetaria[74] generally find one- or two-mirror polar heliostats perfectly adequate. This is especially so when a large 'military surplus' Dove prism is available for insertion in the optical path when image rotation is a nuisance[50,73].

A wide range of excellent polar heliostats (including an ingenious and compact weather-sealed design known as a 'Sun Capsule') is available com-

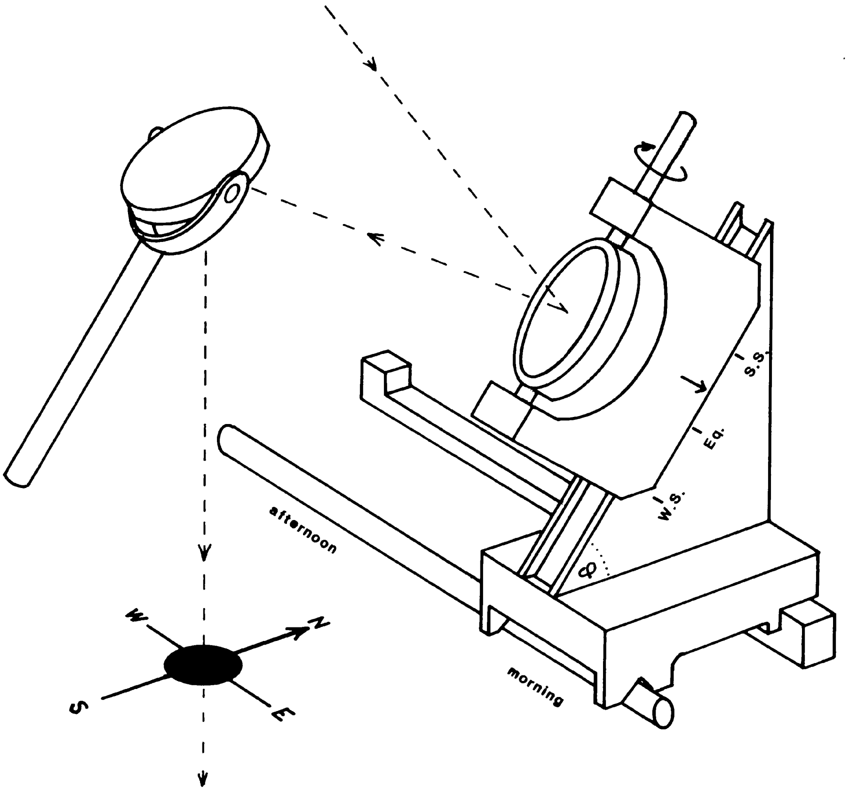

Figure 13. A complete two-mirror, vertical beam, type (*a*) coelostat for solar work. The most common inclined-track pattern (figure 11) is represented.

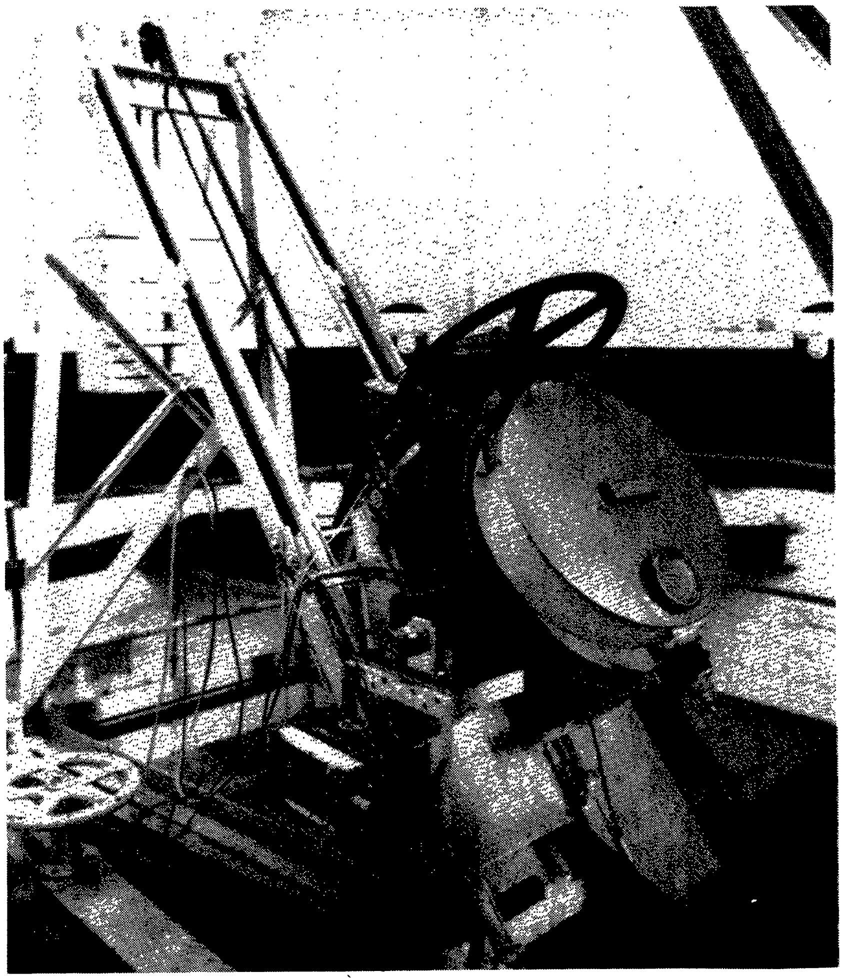

Figure 14. The 400mm coelostat feeding an all-reflecting vertical solar telescope at Oxford University Observatory. (Photographed 1984 May 4.)

mercially in America[75], but it is hoped that a later article will deal with the design and construction of our own 200m polar heliostat/vertical solar telescope at the University of Leicester[76].

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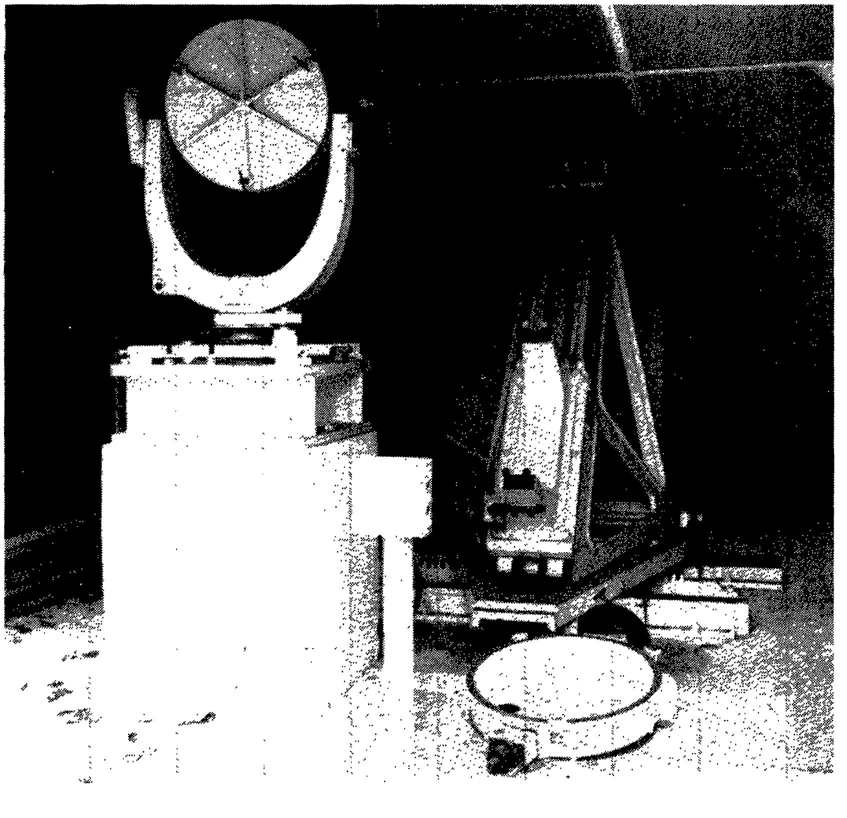

Figure 15. The 450mm coelostat which once fed the horizontal solar telescope at Cambridge University Observatory. (Photographed 1984 May 2.)

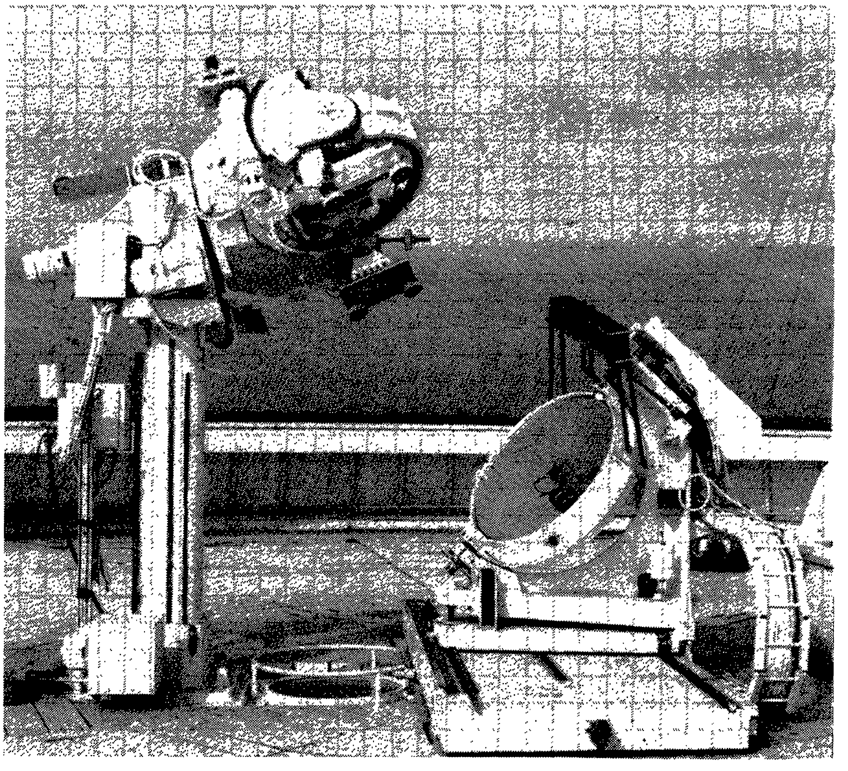

Figure 16. The coelostat on the solar tower at Meudon Observatory. Built about 1967, this is a two-mirror type (*b*) instrument. The whole secondary assembly is moved along the extension of the optic axis of the fixed vertical telescope by a rack attached to the massive column. *Photograph courtesy of Th. Weimer.*

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