DISCOVERY OF A SHELL AROUND ALPHA LYRAE¹

H. H. AUMANN, F. C. GILLETT, C. A. BEICHMAN, T. DE JONG, J. R. HOUCK, F. J. LOW, G. NEUGEBAUER, R. G. WALKER, AND P. R. WESSELIUS Received 1983 September 22; accepted 1983 November 18

ABSTRACT

IRAS observations of α Lyrae reveal a large infrared excess beyond 12 μ m. The excess over an extrapolation of a 10,000 K blackbody is a factor of 1.3 at 25 μ m, 7 at 60 μ m, and 16 at 100 μ m. The source of 60 μ m emission has a diameter of about 20". This is the first detection of a large infrared excess from a main-sequence star without significant mass loss. The most likely origin of the excess is thermal radiation from solid particles more than a millimeter in radius, located approximately 85 AU from α Lyr and heated by the star to an equilibrium temperature of 85 K. These results provide the first direct evidence outside of the solar system for the growth of large particles from the residual of the prenatal cloud of gas and dust.

Subject headings: infrared: sources - stars: circumstellar shells - stars: individual

I. INTRODUCTION

The bright A0 V star α Lyrae (Vega, HR 7001) is the primary spectrophotometric reference star in the northern hemisphere. It has been extensively observed from the ground in the wavelength range from 0.3 μ m to 20 μ m and in the ultraviolet using balloons and satellites. Absolute flux measurements have been carried out at visible wavelengths (Hayes 1970; Oke and Schild 1970), at 1.04 μ m (Hayes, Latham, and Hayes 1975), and at 2.2 and 3.8 μ m (Selby *et al.* 1983).

Recent model calculations for α Lyr, based on a wealth of ultraviolet, optical, and infrared data (Kurucz 1979; Dreiling and Bell 1980) reproduce the observed energy distribution and many spectral details over the range from 0.12 μ m to at least 2 μ m. At longer wavelengths, α Lyr has generally been used as a standard candle, assuming its spectrum is that of a 10,000 K blackbody beyond 2 μ m. The absolute measurements at 3.8 μ m (Selby *et al.* 1983) are consistent with this extrapolation but indicate an $8\% \pm 4\%$ excess relative to the model preferred by Dreiling and Bell. Morrison and Simon (1973) reported a 0.3 mag excess at 25 μ m, but more recent observations (Gehrz, Hackwell, and Jones 1974) found the colors of α Lyr consistent with other normal A0 and A1 stars in the 2.3–12.6 μ m range, and Tokunaga (1984) found no evidence for an unusual 10.6–20 μ m flux density ratio.

Alpha Lyrae and a number of other standard stars were observed using the *IRAS* detector array in a special calibration mode, which uses the very accurate pointing and scanning capability of the spacecraft to gather data on specific sources with much higher photometric accuracy and spatial resolution than is achievable in the survey mode.

II. OBSERVATIONS

a) Photometric

The calibration mode used to achieve high photometric accuracy scans a star across the survey array at the standard rate along the five "best" tracks through the focal plane. These tracks can be repeated to within a few arc seconds, thus reducing the effects of cross-scan response variations across a detector mask to a few percent. Results are presented in Table 1A for six stars observed in this manner.

The typical probable error of the data given in Table 1A is 1%, 2.5%, 2%, and 3% at 12, 25, 60, and 100 μ m, respectively, except for the cases noted. The absolute calibration of the *IRAS* fluxes is discussed by Neugebauer *et al.* (1984, hereafter Paper I). The band-to-band relative calibration uncertainty of the data in Table 1A is estimated to be less than 15%.

Table 1B includes a comparison with ground-based observations in the 10 μ m window, where the ground-based observations have been adjusted to correspond to a 12 μ m effective wavelength, assuming a blackbody spectrum at the effective temperature of the star. Also shown is the ratio of observed flux in the 25, 60, and 100 μ m bands compared to that expected for a blackbody at the effective temperature of the star, normalized to the observed 12 μ m flux. The 12 μ m flux from α Lyr and the other stars is in excellent agreement with the ground-based observations, while at the longer wavelengths, α Lyr is the only star in Table 1B that shows a significant deviation from a blackbody at its effective temperature.

b) Spatial Observations

Several additional calibration observations were analyzed to measure the size of the emitting region and to investigate the possibility that a physically unrelated source, such as a background galaxy, is near the line of sight to α Lyr and is responsible for the observed long-wavelength emission. For

¹The Infrared Astronomical Satellite (IRAS) used in these observations was developed and is operated by the Netherlands Agency for Aerospace Programs (NIVR), the US National Aeronautics and Space Administration (NASA), and the UK Science and Engineering Research Council (SERC).

L24

these tests, α Bootis and β Gruis were used as comparison sources. They are bright and show no significant deviation from their expected blackbody spectra in all four bands. They are therefore assumed to be pointlike sources.

Scans of α Lyr and β Gru made at 1/16 of the standard rate across detectors in all four wavelength bands show that the in-scan positions of the centroids of the emission from α Lyr at 25, 60, and 100 μ m are the same as that of the 12 μ m source to within the measurement uncertainty of 2". At 60 μ m, α Lyr is extended relative to β Gru. The signal-to-noise ratio in a 3 Hz bandwidth present in scans of both sources is in excess of 100, and a significant formal deconvolution is possible. A first-order estimate of the 60 μ m in-scan size of α Lyr, based on the widths of the scan profiles at one-fourth and three-fourths of the peak, is 15" \pm 4" (full width at half-maximum, FWHM).

In the cross-scan direction, it is possible to use the narrow region of overlap between adjacent detectors in a given color band (Paper I) to enhance the positional accuracy and spatial resolution of a measurement. The basic approach is to map the overlap areas with high cross-scan resolution and numerically to synthesize detectors of widths equal to the detector overlaps of 13", 19", 25", and 43" at 12, 25, 60, and 100 μ m, respectively, by adding the time-shifted signal of two overlapping detectors. Alpha Lyrae and α Bootis were scanned across the overlap areas at 12 and 60 μ m, using 6"-12" cross-scan steps. These scans showed that the difference between the cross-scan positions of the centroids of the emission from α Lyr at these wavelengths is less than 3". At 12 μ m, the

apparent FWHM of the synthesized detector is the same for α Boo and α Lyr, indicating that both sources are pointlike. At 60 μ m, the observed FWHM of α Boo is $25'' \pm 2''$, while for α Lyr it is $34'' \pm 3''$. The FWHM of the 60 μ m emitting region of α Lyr, relative to α Boo, obtained from Gaussian deconvolution, is $23'' \pm 6''$.

The coincidence of the centroids in all four bands in the in-scan direction and of the 12 and 60 μ m centroids in cross-scan to within a few arc seconds makes the possibility of chance alignment of an unassociated, cool, 60–100 μ m source with α Lyr vanishingly small; the density of infrared-bright field galaxies is less than 0.5 deg⁻² (Soifer *et al.* 1984). It is concluded that the long-wavelength excess is physically associated with α Lyr and that the diameter of the emitting region is approximately 20" at 60 μ m.

III. INTERPRETATION

The spectral distribution of the observed excess from α Lyr, obtained by subtracting the flux corresponding to a 10,000 K blackbody normalized to 0.0 mag at 2.2 μ m from the observed flux at all wavelengths, is shown in Figure 1. The upper limit at 12 μ m includes the effect of a 5% absolute calibration uncertainty. An acceptable model for the infrared excess must reproduce the observed spectrum and be compatible with both the size of the 60 μ m source and its near coincidence with the star. These conditions can only be met by a shell or ring of relatively large particles distributed around α Lyr at a distance of about 85 AU. In the following discussion, α Lyr is assumed

A .																
	(DBSERVED FLU														
BS No.	12 μm	25 µ m	60 µ m	100 µ m	Sp. Type ^a	$T_{\rm eff}({\rm K})^{\rm b}$										
7001	28.6	8.7	8.9	7.0	A0 V	9850										
3982	6.6	1.52	0.3 ^c	< 0.4 ^c	B7 V	12200										
2491	102	25	4.0	2.0 ^d	A1 V	9400										
2326	106	23.4	4.1	1.5	F0 II	7500										
5340	500	110	19.7	6.8	K2 III	4460										
8636	630	147	28.0	10.2	M5 III	2950										
	BS No. 7001 3982 2491 2326 5340 8636	BS No. 12 μm 7001 28.6 3982 6.6 2491 102 2326 106 5340 500 8636 630	OBSERVED FLU BS No. 12 µm 25 µm 7001 28.6 8.7 3982 6.6 1.52 2491 102 25 2326 106 23.4 5340 500 110 8636 630 147	A. OBSERVED FLUX DENSITY (J BS No. 12 μm 25 μm 60 μm 7001 28.6 8.7 8.9 3982 6.6 1.52 0.3 ^c 2491 102 25 4.0 2326 106 23.4 4.1 5340 500 110 19.7 8636 630 147 28.0	A. OBSERVED FLUX DENSITY (Jy) BS No. 12 μm 25 μm 60 μm 100 μm 7001 28.6 8.7 8.9 7.0 3982 6.6 1.52 0.3 ^c < 0.4 ^c 2491 102 25 4.0 2.0 ^d 2326 106 23.4 4.1 1.5 5340 500 110 19.7 6.8 8636 630 147 28.0 10.2	A. OBSERVED FLUX DENSITY (Jy) BS No. 12 μm 25 μm 60 μm 100 μm SP. TYPE ^a 7001 28.6 8.7 8.9 7.0 A0 V 3982 6.6 1.52 0.3 ^c < 0.4 ^c B7 V 2491 102 25 4.0 2.0 ^d A1 V 2326 106 23.4 4.1 1.5 F0 II 5340 500 110 19.7 6.8 K2 III 8636 630 147 28.0 10.2 M5 III										

TABLE 1	
PHOTOMETRIC OBSERVATIONS	

		F(obs)/F(and)	$F_{\nu}(\text{obs})/B_{\nu}(T_{\text{eff}})^{\text{e}}$				
	SOURCE	12 μm	25 µ m	60 µ m	100 µ m	Reference	
	α Lyr	0.97	1.28	7.4	16.1	1	
	α Leo	1.04	0.98	1.10 ^c	< 4 ^c	1	
	α CMa	1.00	1.03	0.93	1.29 ^d	1	
	α Car	0.99	0.92	0.90	0.91	2	
	α Βοο	0.95	0.88	0.87	0.83	1	
	β Gru	1.00	0.89	0.91	0.90	2	

B.

^aHoffleit 1964.

^bJohnson 1966.

^cLow signal-to-noise ratio measurement, based on inspection of raw data.

^dSource confusion with position-dependent background.

^eNormalized to 12 μ m.

REFERENCES.—(1) Gehrz, Hackwell, and Jones 1974. (2) Thomas, Hyland, and Robinson 1973.



FIG. 1.—Energy distribution of the infrared excess from α Lyr. The error bars represent the 10% calibration uncertainty. The 12 μ m upper limit indicates the effect of the 5% uncertainty in the absolute calibration at 12 μ m. The solid line represents a 85 K blackbody spectrum with a solid angle of 7×10^{-13} sr fitted to the excess. The dashed line represents a 500 K blackbody spectrum with a solid angle of 6.3×10^{-16} sr arbitrarily fitted to the 12 μ m upper limit.

to be at a distance of 8.1 pc (Jenkins 1963), to have an angular diameter $\theta_* = 3.2 \times 10^{-3}$ arcsec (Hanbury-Brown, Davis, and Allen 1974), an effective temperature $T_{\rm eff} = 9700$ K, and a mass of 2 M_{\odot} (Dreiling and Bell 1980).

Free-free emission as an explanation for the excess radiation from α Lyr can be ruled out. Such emission is associated with a number of early-type stars (e.g., Gehrz, Hackwell, and Jones 1974; Wright and Barlow 1975). However, the observed slope of the excess from α Lyr between 12 and 60 μ m, $f(\nu) \propto \nu^{-1}$ to ν^{-2} , is incompatible with either an optically thin free-free slope, $f(\nu)$ constant, or a partially optically thick free-free slope of $f(\nu)$ constant to $f(\nu) \propto \nu^2$ (Wright and Barlow).

The observed excess beyond 12 μ m can be fitted by an 85 K blackbody spectrum with an effective radiating solid angle of 7×10^{-13} sr. While the uncertainty associated with the preliminary calibration allows reasonable fits for blackbodies in the range 75–95 K, an 85 K blackbody will be used for illustration. The luminosity of the 85 K object is 1.4×10^{-3} L_{\odot} , i.e., 2.5×10^{-5} L_{*} . If the object were interpreted as an optically thin shell of material around α Lyr, its mean optical depth in the visible would be 2.5×10^{-5} . If the object were interpreted as an optically thick blackbody, its diameter would be 0".2, i.e., 62 times larger than the star.

The angular diameter of a thin shell of material in thermal equilibrium with the central star can be expressed as

$$\theta_d/\theta_* = 0.5(\epsilon_{\rm vis}/\epsilon_{\rm IR})^{1/2}(T_*/T_d)^2,$$

where (θ_*, T_*) and (θ_d, T_d) are the angular diameter and effective temperature of the star, and the angular diameter of

the shell as seen from Earth and the temperature of the particles in the shell, assuming the particles are perfectly conducting and spherical. The parameters $\varepsilon_{\rm vis}$ and $\varepsilon_{\rm IR}$ are the mean absorptivity and emissivity of the particles at wavelengths where they absorb stellar radiation and emit infrared. With $T_d = 85$ K,

$$\theta_d = 20.7 (\epsilon_{\rm vis}/\epsilon_{\rm IR})^{1/2}$$
 arcsec.

The dust shell model is consistent with the observed shape of the excess and the observed size of the 60 μ m source, provided that ($\varepsilon_{vis}/\varepsilon_{IR}$) is near unity. Since ε_{vis} for most material is near unity (Jones and Merrill 1976), it follows that ε_{IR} also has to be near unity. This condition will normally be satisfied if the particle radius, *a*, is comparable to or larger than $\lambda_p/2\pi$, where λ_p is the wavelength of the peak emission. For $\lambda_p = 60 \ \mu$ m, it follows that $a > 10 \ \mu$ m.

With $\varepsilon_{\rm vis}/\varepsilon_{\rm IR} = 1$, the equilibrium distance for 85 K material is 85 AU. This value will be used in what follows, although observationally it is uncertain by about 30%, and it is likely that the radiating material is distributed over a range of distances from the star. The upper limit to the observed excess at 12 μ m is consistent with an 85 K shell, but a component of hotter material cannot be ruled out. Such material must, however, have a much smaller surface area than that of the 85 K shell. Figure 1 illustrates the case for an arbitrarily chosen 500 K blackbody, fitted to the 12 μ m upper limit. The corresponding solid angle of radiating material is 7×10^{-16} sr, 0.001 of the surface area of 85 K radiating material. Further IRAS observations, reduction of the calibration uncertainties, and observations over a wider range of wavelengths are needed to investigate the radial distribution of material in more detail.

Alternatives to the optically thin ring or shell of large particles around α Lyr can be ruled out. A single blackbody heated by α Lyr or a blackbody stellar companion are ruled out because the observed diameter of the source is about 20" while the diameter of such objects would be only 0".2.

Models incorporating emission from typical circumstellar grains with an emissivity proportional to ν or ν^2 can also be ruled out. With $\epsilon_{\rm vis}/\epsilon_{\rm IR} = 0.0055$, representative of 0.1 μ m dirty silicate grains at 40 μ m (Jones and Merrill 1976), the diameter of the shell would have to be larger than 200".

More complex models, involving one or more low-luminosity sources properly distributed around α Lyr, could fit the observations, but the contrived nature of such models makes them very unlikely. For example, α Lyr could have a lowluminosity companion surrounded by a dust shell. In order that heating from α Lyr not be important, the companion would have to be substantially farther away than 85 AU and located behind the star, very close to the line of sight. With $\varepsilon_{vis}/\varepsilon_{IR} = 0.0055$, the luminosity of the companion giving the observed dust temperature at 85 AU would be $0.35 L_{\odot}$, and the optical depth of the cloud would be 0.004. Such a companion should be detectable as an optical double. Thus, it is concluded that the excess observed is due to radiation from a collection of particles larger than 10 μ m in thermal equilibrium with the star at a distance of about 85 AU. L26

1984ApJ...278L..23A

IV. DISCUSSION

There are three alternatives to the origin of the material in the shell around α Lyr: (1) it is continuously produced by mass loss from α Lyr; (2) it was at some time produced through mass loss from α Lyr which is no longer active or it was captured from the interstellar medium; (3) it was left over from the original cloud of dust and gas which formed α Lyr.

No observational evidence for mass outflow from α Lyr has been presented to date. The most sensitive indicator of mass outflow in early-type stars is the presence in the ultraviolet spectrum of emission lines or distortions of absorption-line profiles, such as blueshifted absorption components or P Cygni line profiles. None of these effects are seen toward α Lyr (Lamers, Stalio, and Kongo 1978), and an upper limit to the mass loss rate, set by observations of Fe II absorption lines near 0.25 μ m, is $10^{-12} M_{\odot}$ yr⁻¹ (Lamers and Waters 1983).

The origin of the particles in the shell cannot be the capture of interstellar grains or the remnants of recent, but currently no longer active, mass loss. The sizes inferred for particles in the interstellar medium (0.01–0.25 μ m), in regions of current star formation (0.01–0.25 μ m, Mathis and Wallerhorst 1981), or in circumstellar dust shells (0.05-0.1 µm, Jones and Merrill 1976) are so small that they would be ejected from the vicinity of α Lyr because of radiation pressure. Radiation pressure is greater than gravitational force for black spheres with radius $a < (0.6 L_{\star}/\rho M_{\star})$, where a is the particle radius in microns, L_{\star} and M_{\star} are the luminosity and mass of the star in units of the Sun, and ρ is the density in g cm⁻³. Thus, for $\rho =$ 2 g cm⁻³, particles smaller than 9 μ m in radius would have been blown away from α Lyr by radiation pressure.

We conclude that the radiating solid material is in orbit around the star and has been since the time of formation of the star. This implies an important further constraint on the size of the radiating material. The Poynting-Robertson effect causes particles to spiral into α Lyr over a long period of time. The time t in years required for a particle of radius a (cm) and density ρ (g cm⁻³) to spiral into a star of luminosity L_* from an initial distance of A (AU) is (modified from Lang 1980) $t = 7 \times 10^6 a \rho A^2 / L_*$. The main-sequence lifetime of α Lyr is roughly 3×10^8 years, and its age, while less than this, is highly uncertain. For illustrative purposes, take $t = 10^8$ years. For $\rho = 2$, $L_* = 58 L_{\odot}$, and A = 85 AU, particles of a radius less than $a_{\min} = 0.06$ cm will have been swept away. Thus, particles remaining in the 85 K shell must be much larger than 0.12 cm in diameter.

The mass of an optically thin shell is given by

$$M = \frac{4}{3} \left(\Omega D^2 \rho \langle a^3 \rangle / \langle a^2 \rangle \right) / \epsilon_{\rm IR},$$

where $\langle a^3 \rangle$ and $\langle a^2 \rangle$ are weighted by the particle size distribution. The minimum mass of the shell is obtained by letting $\langle a^3 \rangle / \langle a^2 \rangle = a_{\min} = 0.06$ cm. With $\epsilon_{IR} = 1$, this results in $M_{\min} = 1.2 \times 10^{-2} M_{Earth}$. Assuming a power-law size distribution function of the type $dN(a) = \text{const.} \times a^{-3.5} da$, deduced by Dohnanyi (1969) for the asteroids of the solar system, $\langle a^3 \rangle / \langle a^2 \rangle = (a_{\min} \times a_{\max})^{1/2}$. The *IRAS* observations provide no information on a_{\max} , the maximum size of the bodies in the shell. Arbitrarily taking $a_{\text{max}} = 5 \times 10^7$ cm (the radius of Ceres), the mass of the shell is $M_{\text{shell}} = 2 \times 10^{30}$ $g = 300 M_{Earth}$. The mass of the shell could therefore be much larger than the mass of the asteroid belt and may be comparable to the mass of the solar system excluding the Sun.

A discussion of the implications of the shell around Vega on theories dealing with stellar nebulae and their evolution into solid bodies of substantial size is beyond the scope of this Letter. However, three aspects should be pointed out:

1. The particles in the shell around α Lyr are on the order of or larger than 0.12 cm in size, while the size of dust grains in the interstellar medium or regions of current star formation ranges from 0.01 to 0.5 μ m (Mathis and Wallerhorst 1981). This implies a very significant growth in particle size since the time of formation of α Lyr.

2. The main-sequence lifetime of α Lyr is about 3×10^8 yr, while the age of the solar system is about 4.5×10^9 yr (Allen 1973). The shell around α Lyr thus must represent an evolutionary stage intermediate between the final phases of star formation and the present state of our solar system.

3. The time scale of active planet formation, $10^7 - 10^9$ years (Safronov 1980), brackets the main-sequence lifetime of α Lyr.

V. SUMMARY

IRAS observations have revealed the presence of a strong infrared excess beyond 12 μ m associated with the nearby bright star α Lyr, a main-sequence star with a mass outflow rate of less than $10^{-12} M_{\odot}$ yr⁻¹. The most likely source of this excess is thermal radiation by solid material in thermal equilibrium with the central star at a distance of roughly 85 AU and temperature of about 85 K.

It is concluded that this material is a remnant of the cloud out of which α Lyr formed and that the particles must have diameters of the order of more than 0.12 cm. These observations thus represent the first evidence outside the solar system for the growth of particles much larger than a millimeter in size from the residual of the prenatal cloud of gas and dust.

Drs. H. Lamers and R. Waters kindly provided the upper limit on the mass loss from α Lyr, and the Telescope Operations Support Team (TOST), in particular Barbara Brown, Peter Doms, Don Langford, and Priscilla Shutie, helped with the development, scheduling, and analysis of the many additional observations used for this study.

REFERENCES

Allen, C. W. 1973, Astrophysical Quantities (London: Athlone). Dohnanyi, J. S. 1969, *J. Geophys. Res.*, **74**, 2531. Dreiling, L. A., and Bell, R. A. 1980, *Ap. J.*, **241**, 736. Gehrz, R. D., Hackwell, J. A., and Jones, J. 1974, *Ap. J.*, **191**, 675. Hanbury-Brown, R., Davis, J., and Allen, L. R. 1974, M.N.R.A.S., 167,

Hayes, D. S. 1970, Ap. J., **159**, 165. Hayes, D. S., Latham, D. W., and Hayes, S. A. 1975, Ap. J., **197**, 587.

1984ApJ...278L..23A

No. 1, 1984

- Hoffleit, D. 1964, Yale Catalogue of Bright Stars (3d rev. ed.; New Haven:
- Hoffleit, D. 1964, Yale Catalogue of Bright Stars (3d rev. ed.; New Haven: Yale University Observatory).
 Jenkins, L. F. 1963, General Catalogue of Trigonometric Stellar Parallaxes (New Haven: Yale University Observatory).
 Johnson, H. L. 1966, Ann. Rev. Astr. Ap., 4, 193.
 Jones, T. W., and Merrill, K. M. 1976, Ap. J., 209, 509.
 Kurucz, R. L. 1979, Ap. J. Suppl., 40, 1.
 Lamers, H., Stalio, R., and Kongo, Y. 1978, Ap. J., 223, 207.
 Lamers, H., and Waters, R. 1983, private communication.
 Lang, K. R. 1980, Astrophysical Formulae (New York: Springer-Verlag).
 Mathis, J. S., and Wallenhorst, S. G. 1981, Ap. J., 244, 483.
 Morrison, D., and Simon, T. 1973, Ap. J., 186, 193.
 Neugebauer, G., et al. 1984, Ap. J. (Letters), 278, L1 (Paper I).

- Oke, J. B., and Schild, R. E. 1970, Ap. J., 161, 1015. Safronov, V. S. 1980, Accumulation of the Protoplanetary Bodies, Early Solar System Processes and the Present Solar System, Proceedings of the International School of Physics "Enrico Fermi," Course 73, (New York: North Holland), p. 58.
- North Holland, p. 58.
 Selby, M. J., Montain, C. W., Blackwell, D. E., Petford, A. D., and Leggett, S. K. 1983, *M.N.R.A.S.*, **203**, 795.
 Soifer, B. T., *et al.* 1984, *Ap. J. (Letters)*, **278**, L71.
 Thomas, F. A., Hyland, A. R., and Robinson, G. 1973, *M.N.R.A.S.*, **165**, 2001.
- 201
- Tokunaga, A. T. 1984, A.J. submitted.
- Wright, A. E., and Barlow, M. J. 1975, M.N.R.A.S., 170, 41.

Note added in proof.—The flux densities quoted in this Letter should be multiplied by 1.00, 1.16, 1.20, and 1.26 for bands 1 through 4, respectively, to make the results consistent with the preliminary calibration used in the other Letters in this issue. This adjustment does not change the basic conclusions of this Letter.