THE ASTROPHYSICAL JOURNAL, 277:355–360, 1984 February 1

DOUBLE WHITE DWARFS AS PROGENITORS OF R CORONAE BOREALIS STARS AND TYPE I SUPERNOVAE[1]

R. F. WEBBINK
Department of Astronomy, University of Illinois
Received 1983 June 13; accepted 1983 July 27

ABSTRACT

Close double white dwarfs should arise from the second phase of mass exchange in close binaries which first encountered mass exchange while the more massive star was crossing the Hertzsprung gap. Tidal mass transfer in these double degenerate systems is explored. The sequence of double white dwarfs divides naturally into three segments. (1) Low-mass helium/helium pairs are unstable to dynamical time-scale mass transfer and probably coalesce to form helium-burning sdO stars. (2) In helium/carbon-oxygen pairs, mass transfer occurs on the time scale for gravitational radiation losses ($\sim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$); the accreted helium is quickly ignited, and the accretor expands to dimensions characteristic of R CrB stars, engulfing its companion star. (3) Carbon-oxygen/carbon-oxygen pairs are again unstable to dynamical time-scale mass transfer and, since their total masses exceed the Chandrasekhar limit, are destined to become supernovae. Inactive lifetimes in these latter systems between creation and interaction can exceed 10^{10} years. Birthrates of R CrB stars and Type I supernovae by evolution of double white dwarfs are in reasonable agreement with observational estimates.

Subject headings: stars: R Coronae Borealis — stars: supernovae — stars: white dwarfs

I. INTRODUCTION

Type I supernovae (SN I) differ strikingly from those of Type II (SN II) not only in the regularity of their outburst light curves, but also in their apparent lack of hydrogen and in their occurrence in elliptical galaxies, where SN II, which are believed to arise from massive stars, are not observed (see, e.g., the recent review by Trimble 1982*b*). The fact that SN I are seen in elliptical galaxies, whose luminosity is derived entirely from old low-mass stars ($M \lesssim 1.0\ M_\odot$) with ages of approximately 10^{10} years (e.g., Tammann 1982), has therefore led to the widespread belief that they must originate in binary systems, where one can at a very late date assemble a Chandrasekhar mass out of material from two stars of smaller individual masses. Oemler and Tinsley (1979) have pointed out, however, that the frequency of SN I per unit galaxy luminosity is much greater in late-type than in elliptical galaxies and in fact appears to be proportional to the star formation rates in those galaxies. They interpret this as implying that most progenitor stars have masses $\gtrsim 2\ M_\odot$.

One possible resolution of the apparent contradiction in progenitor masses is to suppose that SN I are triggered in cataclysmic binaries, where a relatively massive white dwarf (whose parent may well have exceeded $2\ M_\odot$) is ultimately pushed beyond the Chandrasekhar limit by accreting material from a low-mass companion (Warner 1974; see also Whelan and Iben 1973). Numerous studies of white dwarfs accreting material at rates of the same order of magnitude as in cataclysmic binaries have shown that, under various conditions, either helium or carbon detonation, or carbon deflagration, may occur, depending on the mass of the white dwarf and the accretion rate (Taam 1980; Woosley, Weaver, and Taam 1980; Fujimoto and Taam 1982; Nomoto 1982*a*, *b*; see Wheeler 1982 for a review). These models have met with varying degrees of success in producing SN I–like outbursts. But serious objections remain to the hypothesized connection between cataclysmic binaries and SN I. (1) The SN I rate in galaxies like the Milky Way evidently exceeds the birthrate of normal cataclysmic binaries by roughly an order of magnitude (Trimble 1982*a*). (2) Nova outbursts in the course of evolution of a cataclysmic binary evidently eject most of the mass accreted by the white dwarf, and indeed the large abundances of CNO isotopes observed in their ejecta imply that the white dwarfs may actually *decrease* in mass with time (Ritter 1983; MacDonald 1983). To circumvent these objections, one might appeal to nova-like systems which transfer matter at such a rapid rate that hydrogen shell flashes on the white dwarf are suppressed (Paczyński and Żytkow 1978; Iben 1982). Severe restrictions on allowable accretion rates must be imposed, however, and these objects should be detectable as extremely bright UV sources.

In this paper, we briefly survey an alternative suggestion (Webbink 1979*b*; see also Iben and Tutukov 1984) for SN I progenitors: close double white dwarf (CDWD) binaries. The origins of such systems are outlined in the following section, followed by discussions of the salient features of mass transfer in CDWD systems, and finally by a brief discussion of the probable fates of interacting systems, their birthrates, and possible relationships to subdwarf O stars, R Coronae Borealis variables, and Type I supernovae.

II. ORIGINS

In low-mass and intermediate-mass binary systems in which the more massive (primary) component first fills its Roche lobe while traversing the Hertzsprung gap in the Hertzsprung-

[1] This research was supported by National Science Foundation grant AST 80-18859.

Russell (H-R) diagram, this first mass transfer episode proceeds with relatively modest systemic losses of mass and angular momentum. At low initial primary masses ($\lesssim 3.5\ M_\odot$), the rapid initial reversal of the mass ratio is followed by a longer phase in which mass transfer is driven by evolutionary expansion of the lobe-filling star as its core grows in mass. These objects are observed as Algol-type binaries (see, e.g., Giuricin, Mardirossian, and Mezzetti 1983 for a discussion of their properties). This phase ends with the contraction of the exhausted giant as a low-mass helium (He) white dwarf. In more massive systems, the slow latter phase of mass transfer is greatly abbreviated, and the lobe-filling star leaves a nuclear burning helium star as a remnant (e.g., KS Per, υ Sgr—Plavec 1973; Schönberner and Drilling 1983). If this helium star is sufficiently massive ($\sim$1.0–2.6 $M_\odot$), it may undergo a further episode of mass loss during shell burning (De Grève, de Loore, and van Dessel 1978; Delgado and Thomas 1981). In either case, the initial primary leaves a carbon-oxygen (CO) white dwarf as a remnant.

When the original secondaries of these systems fill their lobes, they will as a rule already have evolved to the giant branch (Webbink 1979*b*). The combination of a deep convective envelope plus an extreme mass ratio (the mass-losing star much more massive than the accretor) is then extremely unstable, and mass transfer proceeds on a dynamical time scale (Paczyński and Sienkiewicz 1972; Plavec, Ulrich, and Polidan 1973). As a result, the accretor is swallowed up in the envelope of the giant (Webbink 1979*a*) and spirals inward through the common envelope (Paczyński 1976). This situation parallels the double-core common envelope evolution believed to give rise to cataclysmic binaries (e.g., Meyer and Meyer-Hofmeister 1979), except that both cores are degenerate in this case.

Aided by the fact that the remnant mass left by a star of given mass varies only slowly with orbital period for mass transfer in the Hertzsprung gap or in early giant branch evolution (see, e.g., De Grève and Vanbeveren 1980), we can make the following crude estimate of the final system parameters as functions of initial masses and orbital separations.

We assume that the first mass transfer episode may be treated conservatively, i.e., with negligible systemic losses of mass and angular momentum. The mass of the remnant, M_{1f}, left by a primary of mass M_{1o} is, to a good approximation, a function of M_{1o} alone, $M_f(M_o)$. This function, as illustrated in Figure 1, was constructed from the results of numerous published and unpublished theoretical evolutionary calculations of close binary and single stars, and it is in reasonable accord with the analytic approximations quoted by Giuricin, Mardirossian, and Mezzetti (1983). For conservative mass transfer, the mass of the companion star at the close of this episode is then

$$M_{2a} = M_{1o}(1 + q_o) - M_{1f}\,, \qquad (1)$$

where $q_o = M_{2o}/M_{1o}$ is the initial mass ratio, and the binary separation is

$$A_1 = A_0\left(\frac{q_o M_{1o}^2}{M_{1f} M_{2f}}\right)^2\,, \qquad (2)$$

where A_0 is the initial separation.

In the common envelope phase, we assume that the envelope is dispersed when the decrease in orbital energy equals the

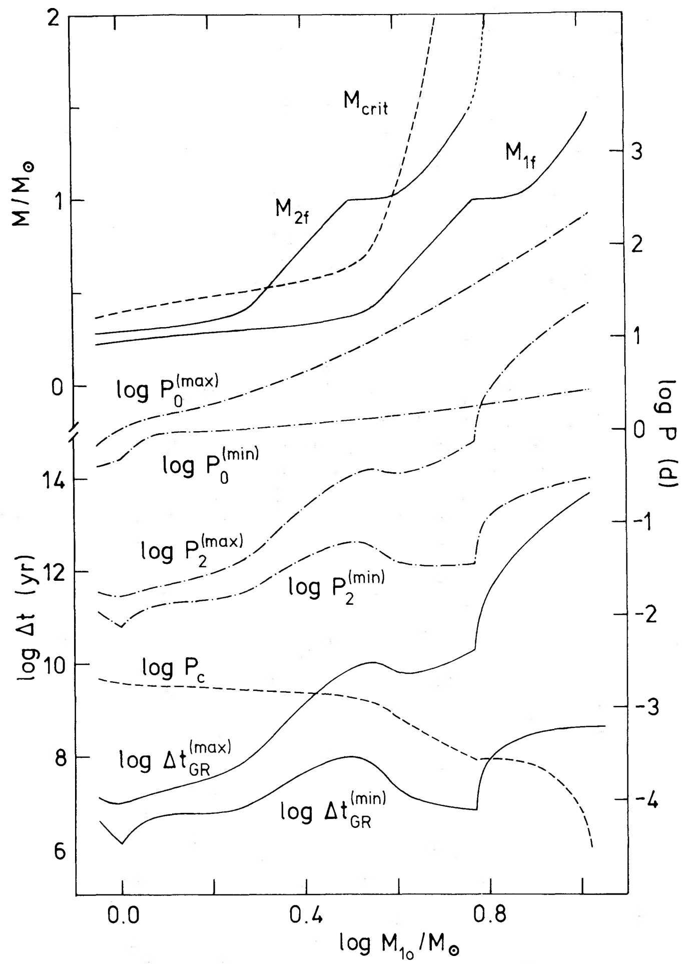

FIG. 1.—System parameters of CDWDs as functions of the initial mass, M_{1o}, of the primary component in the primordial binary: M_{1f}, mass of the white dwarf remnant of the original primary; M_{2f}, mass of the white dwarf remnant of the original secondary; $M_{\rm crit}$, mass which M_{2f} must exceed to stabilize the binary against dynamical time-scale mass transfer; $P_0^{(\rm min)}$ and $P_0^{(\rm max)}$, the range of primordial orbital periods leading to evolution of the type described here; $P_2^{(\rm min)}$ and $P_2^{(\rm max)}$, the corresponding range of initial periods upon arriving at the CDWD state; P_c, the orbital period at which the less massive white dwarf first fills its Roche lobe; $\Delta t_{\rm GR}^{(\rm min)}$ and $\Delta t_{\rm GR}^{(\rm max)}$, the range of gravitational radiation collapse times from P_2 to P_c.

original binding energy of the envelope. A rough estimate of that binding energy is

$$E_{\rm env}^{(0)} = -\frac{G(M_{2f} + M_{2e})M_{2e}}{A_1 r_L}\,, \qquad (3)$$

where $M_{2f} = M_f(M_{2a})$ is the core mass of the original secondary when it now fills its Roche lobe, $M_{2e} = M_{2a} - M_{2f}$ is the mass of its envelope, and $r_L = r_L(M_{2a}/M_{1f})$ is the fractional radius of that star (in terms of the binary separation) at that point. Equating equation (3) to the difference between final and initial orbital energies, we then obtain

$$A_2 = A_1 \frac{M_{1f} M_{2f}}{M_{2f} + M_{2e}}\left(M_{1f} + \frac{2}{r_L} M_{2e}\right)^{-1}\,, \qquad (4)$$

where A_2 is the final binary separation between the white dwarfs of masses M_{1f} and M_{2f}.

The resulting values of M_{1f} and M_{2f} and the upper and lower extremes of the estimated range of orbital periods of the post–common envelope binaries, $P_2^{(\max)}$ and $P_2^{(\min)}$, are illustrated in Figure 1. This range in periods reflects the allowed range of initial periods, $P_0^{(\max)}$ to $P_0^{(\min)}$, for primaries transferring mass in the Hertzsprung gap. (Systems with initial periods longer than $P_0^{(\max)}$ do not encounter mass transfer until the primary has become a red giant or supergiant, with a deep convective envelope. In this case, the first mass transfer episode proceeds on a dynamical time scale, with a common envelope phase likely to ensue, and the system follows quite a different evolutionary path from that outlined above.) An additional simplifying assumption has been made here, that the initial mass ratios of the binaries did not differ sensibly from unity, i.e., that $q_o = 1$. We justify this assumption on the grounds that studies of unevolved binaries (Abt and Levy 1976, 1978; Lucy and Ricco 1979; Trimble 1974, 1978) show a strong peak to the distribution of initial mass ratios at or near $q_o = 1$. Values of $q_o < 1$ generally produce somewhat more tightly bound systems with smaller values of M_{2f}.

We see in Figure 1 that the second white dwarf formed in these systems is normally the more massive one. For initial primary masses $M_{1o} \lesssim 2\ M_\odot$, double helium white dwarfs (He-He CDWDs) are produced; in the range $2\ M_\odot \lesssim M_{1o} \lesssim 3.7\ M_\odot$, the more massive component is a carbon-oxygen white dwarf (He-CO CDWD); and finally, for $3.7\ M_\odot \lesssim M_{1o} \lesssim 6\ M_\odot$, both white dwarfs are of the carbon-oxygen variety (CO-CO CDWD). This last mass interval also coincides with those systems having total masses exceeding the Chandrasekhar limit. For larger masses yet, $M_{1o} \gtrsim 6\,M_\odot$, the helium star exposed at the close of common evolution is sufficiently massive to ignite carbon non-degenerately and evolve to core collapse with minimal further losses to its white dwarf companion. We would therefore anticipate formation of a neutron star–CO white dwarf binary within $\lesssim 3 \times 10^6$ years of the close of the common envelope phase.

III. MASS TRANSFER

In view of the short orbital periods anticipated among newly formed CDWDs and the absence of plausible competing mechanisms, the evolution of these systems to an interacting stage is almost certainly driven by general relativistic gravitational radiation. This extracts angular momentum from the binary at a rate

$$\left(\frac{\partial \ln J}{\partial t}\right)_{\rm GR} \equiv -\tau_{\rm GR}^{-1} = -\frac{32}{5}\frac{G^{5/3}}{c^5}\frac{M_{1f}M_{2f}}{(M_{1f}+M_{2f})^{1/3}}\left(\frac{2\pi}{P}\right)^{8/3} \tag{5}$$

(see Landau and Lifschitz 1962). The resulting range of collapse times, $\Delta t_{\rm GR}$, for the orbit to reach the critical period, P_c, at which the less massive white dwarf begins tidal mass loss is shown in Figure 1 for our standard CDWD sequence. Note in particular that the duration of this inactive phase of binary evolution may last anywhere from $\lesssim 10^7$ years to $\gtrsim 10^{10}$ years, even in systems which run their normal course of nuclear evolution in less than 10^8 years.

The direction of evolution taken by the system when the less massive (and hence lower density) white dwarf ultimately fills its lobe depends critically on the mass of that lobe-filling star and the mass ratio of the binary. These will determine whether the lobe-filling white dwarf expands more rapidly or less rapidly than its Roche lobe in response to mass loss. The crucial quantities are $\xi_s \equiv (d \ln R_1/d \ln M_1)$, the radius-mass exponent of the lobe-filling star; and $\xi_L \equiv (\partial \ln R_L/\partial \ln M_1)_J$, the corresponding exponent for its Roche lobe (at constant total angular momentum). If $\xi_s < \xi_L$, the star grossly overfills its Roche lobe, and the mass loss rate quickly accelerates to a dynamical time scale, whereas if $\xi_s \geq \xi_L$, mass transfer is driven by angular momentum losses due in this case to gravitational radiation.

For fully degenerate dwarfs, ξ_s is essentially a function only of M_1/M_3, where M_3 is the Chandrasekhar mass limit. The parameter ξ_L can be readily calculated (see, e.g., Paczyński 1971*b*) as a function of the mass ratio $q = M_2/M_1$ alone in the (doubtful) limit of conservative mass transfer. We can therefore define a critical mass (a function of M_{1f} alone) which the accreting star must equal or exceed if the binary is to be stable against dynamic time-scale mass loss:

$$M_{\rm crit} = q_{\rm crit} M_{1f} ,$$

where

$$\xi_L(q_{\rm crit}) = \xi_s(M_{1f}/M_3) . \tag{6}$$

The limiting mass $M_{\rm crit}$ so defined is also plotted in Figure 1. We see that our CDWD sequence is divided into stable and unstable branches rather neatly according to the compositions of the white dwarfs, with systems having like compositions for both components being unstable and the mixed He-CO pairs being stable (at least in the conservative limit).

When the less massive white dwarf first fills its Roche lobe, the margin by which it overfills its lobe grows until its mass-loss time scale becomes comparable with the gravitational radiation time scale, $\tau_{\rm GR}$. This interval is typically extremely brief, of duration $\Delta t \sim (P\tau_{\rm GR}^2)^{1/3}$, where P is the orbital period of the system (typically a few minutes), and is characterized by a mass transfer rate increasing with time t since contact as t^3.

Because of the large fractional radius of the accreting white dwarfs in CDWDs throughout our sequence, the accretion stream strikes the mass-gaining star directly, and normal accretion disks do not form, except when the accretor is very near the Chandrasekhar limit (and hence of very small radius). At the high infall velocities appropriate to these systems ($v_{\rm ff} \gtrsim 10^8$ cm s^{-1}), the accretion shock which forms where the stream strikes the star is radiation-dominated in the postshock region, except in systems in the deepest throes of dynamical time-scale mass transfer. The incoming stream, on the other hand, has extremely low specific entropy.

If the binary is stable against dynamical time-scale mass transfer, the mass loss rate will saturate at the rate dictated by $\tau_{\rm GR}$:

$$\dot{M}_1 = -\frac{2M_1}{(\xi_s - \xi_L)\tau_{\rm GR}} . \tag{7}$$

As can be seen in Figure 2, this transfer rate is exceedingly high, typically of the order of $10^{-4}\ M_\odot$ yr^{-1} or more. Nevertheless, except near the limits of the stable regime, it

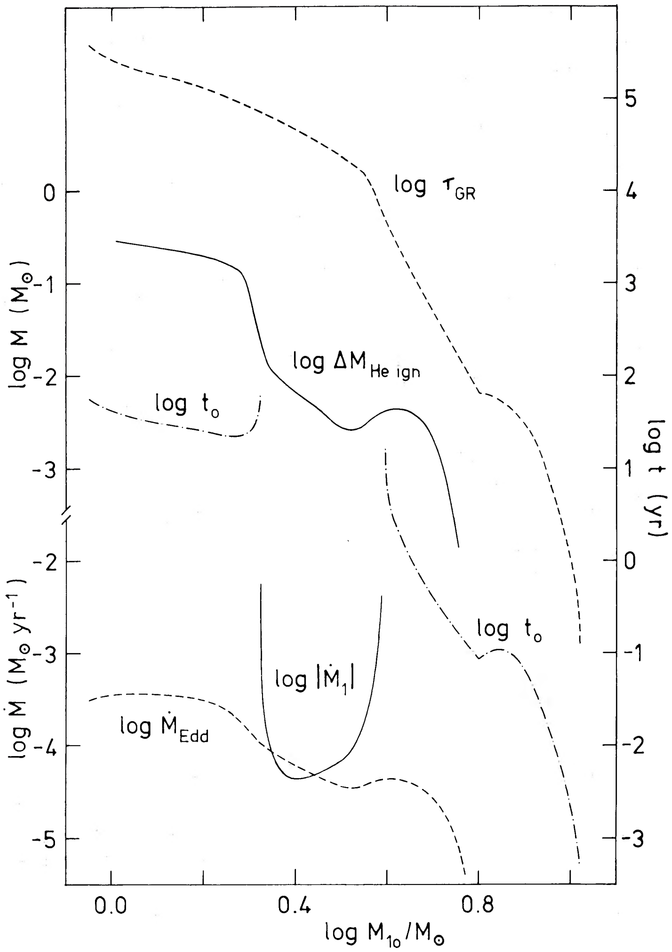

FIG. 2.—Parameters characterizing tidal interaction in the CDWD sequence illustrated in Fig. 1: τ_{GR}, the angular momentum decay time scale due to gravitational radiation at the onset of mass transfer; t_o, the tidal coalescence time scale for dynamically unstable systems; $\dot{M}_1$, the initial equilibrium mass transfer rate for dynamically stable systems; $\dot{M}_{Edd}$, the Eddington-limited accretion rate at the onset of mass transfer; $\Delta M_{He\ ign}$, mass of an accreted helium envelope at the instant at which nuclear burning consumes helium at a rate equal to the accretion rate.

never much exceeds the Eddington-limited (spherical) accretion rate in these systems ($\dot{M}_{Edd}$ in Fig. 2). This condition largely results from (*a*) the higher electron mean molecular weight in the material being transferred and, equally as important, (*b*) the fact that the systems are so compact at this stage of evolution ($P \sim 2$ minutes) that the accreting star fills up a significant fraction of its own Roche lobe [$R_2 \sim (0.3\text{–}0.5)R_{L_2}$]. With the mass-losing companion already so deep in the potential well of the accreting star, the accretion luminosity is significantly smaller than the usual estimate, $L_{acc} = GM\dot{M}/R$. We therefore anticipate that systemic mass losses for these systems should normally be fairly moderate.

In systems which are unstable to dynamical time-scale mass transfer, the rate accelerates rapidly once their orbital evolution is dominated by mass exchange rather than by gravitational radiation. With the approximation of the outer envelope of the white dwarf as a nonrelativistically degenerate gas of electron mean molecular weight μ_e', the mass loss rate from the lobe-filling star may be found in the Jedrzejec approximation (Paczyński and Sienkiewicz 1972):

$$\dot{M}_1 = -F(q)E(M_1/M_3)\left(\frac{\mu_e'}{\mu_e}\right)^{5/2}\frac{M_1}{P}\left(\frac{\Delta R_1}{R_1}\right)^3 , \qquad (8)$$

where

$$E(M/M_3) = \left(\frac{5\eta_1}{y_0}\right)^{3/2}\left(-\eta^2\frac{d\phi}{d\eta}\right)_{\eta_1}^{1/2} \qquad (9)$$

in Chandrasekhar's (1957, p. 412) notation; and

$$F(q) = \frac{\pi}{12}\left(\frac{3}{5}\frac{\mu_1}{r_L}\right)^{3/2}[a_2(a_2-1)]^{-1/2} , \qquad (10)$$

where $\mu_1 \equiv M_1/(M_1 + M_2)$, $r_L = R_L/A$, and a_2 is a function of μ_1 (see Webbink 1977). In equation (8), μ_e is the electron mean molecular weight in the interior of the lobe-filling star, and $\Delta R_1/R_1 = (R_1 - R_L)/R_1$ is the fractional radius excess of that star. Typical values of F lie in the range 0.5–0.6, and those of E in the range 20–40, but with $E \rightarrow 0$ as $M \rightarrow M_3$.

The outcome of dynamical time-scale mass transfer in these systems is almost certainly coalescence. It can readily be shown that equation (8) implies a mass transfer rate varying with time (at least initially) as $\dot{M} \sim (t_o - t)^{3/2}$, where t_o can be regarded as the *tidal coalescence time scale* of the binary, measured as is t from the onset of mass transfer in the CDWD. Estimates of t_o are plotted in Figure 2 for the unstable branches of our CDWD sequence. In principle, peak mass transfer rates of the order of M_1/P ($\sim 10^{31}$ g s^{-1}) are attainable; thus, while these rates may formally exceed the Eddington-limited accretion rate by many orders of magnitude, such a comparison is undoubtedly conceptually misleading because the mass transfer process itself amounts to true dynamical coalescence of the binary components. The amount of mass ejected from the system is then more likely dictated by angular momentum constraints on the coalesced star than by problems of energy transport. Since the potential energy per unit mass at the surface of the lobe-filling star is typically 30%–80% that at the surface of the accreting star, gross energetics requires that, in the absence of nongravitational energy sources, at least as high a fraction of the material lost by the secondary is retained within the system.

IV. FATE OF THE ACCRETING STAR

We have seen that the sequence of CDWDs expected to arise from close binary evolution is naturally divided into three segments. At low mass, He-He CDWDs occur which become unstable to dynamical time-scale mass transfer. At intermediate masses, He-CO pairs occur with somewhat more disparate component masses; they are stable against dynamical time-scale mass transfer and exchange mass on a gravitational radiation time scale. At higher masses, CO-CO CDWDs are created which are again unstable to dynamical time-scale mass transfer, in this case primarily because of the steep, negative radius-mass relationship for massive white dwarfs. We speculate now on the likely outcome of this final phase of mass transfer.

a) He-He *Systems*

These binaries have total masses in the range 0.50–0.75 $M_\odot$, i.e., greater than the core mass for degenerate helium ignition in normal low-mass giants, and it therefore seems likely that they undergo helium burning in the coalesced state. A large part of the energy released in coalescence probably ultimately goes into lifting the degeneracy of material accreted onto the more massive core, and the most promising site of ignition would therefore appear to be at the base of this envelope, rather than under fully degenerate conditions in its core (see Mazurek 1973). With degeneracy ultimately lifted throughout, the star will reside on the helium main sequence. Helium stars in this mass range spend nearly their entire nuclear burning lifetimes near the helium main sequence (Paczyński 1971*a*), in a region of the H-R diagram occupied observationally by the sdO stars (Greenstein and Sargent 1974; Wesemael *et al.* 1982). Adoption of the initial mass function of Miller and Scalo (1979) and a binary frequency of 0.14 systems per decade in orbital period (Webbink 1979*b*) leads to an estimated birthrate for He-He systems of approximately 2.9×10^{-11} pc^{-2} yr^{-1} in the local galactic disk.

b) He-CO *Systems*

In these more massive systems ($\sim$0.75–1.45 $M_\odot$), nondegenerate helium ignition at the base of the accreted envelope may be anticipated early in the progress of mass exchange. In Figure 2 are shown the results of a semianalytic estimate of the mass of this envelope at ignition, defined here as the instant at which the rate of helium consumption in the accreted envelope equals the mass accretion rate. For simplicity, adiabatic compression beneath a spherically symmetric accretion shock was assumed. With the ignition of a helium-burning shell and growth of a helium envelope surrounding the CO component, the engulfment and probable destruction of the He companion in a second phase of common envelope evolution appear inescapable. Shell helium-burning stars in this mass range expand to radii as great as $\sim$400 $R_\odot$ as CO core masses approach or exceed a solar mass. They evolve through the region occupied by R CrB stars in the H-R diagram and remain the most promising models for these objects (Paczyński 1971*a*; Zhilyaev *et al.* 1978). The estimated birthrate for He-CO systems is approximately 1.9×10^{-11} pc^{-2} yr^{-1}, which is comparable to that of the CO-CO systems discussed below as SN I progenitors, in agreement with Wheeler's (1978) estimates of their relative birthrates.

c) CO-CO *Systems*

These binaries have total masses of approximately 1.45–2.4 $M_\odot$, i.e., upwards of a Chandrasekhar mass. Since they are unstable to dynamical time-scale mass transfer, rapid coalescence as in the He-He systems seems the most likely outcome. Shock heating of the accreted CO envelopes is likely to be quite strong in these systems, producing an extended, nondegenerate CO envelope. Carbon ignition would appear most likely to occur at or near the base of this envelope, as argued above in the case of helium ignition in the He-He systems. Whether this develops immediately into a supernova-like event or not, such an outcome seems almost certain to occur eventually. The birthrate estimate for CO-CO systems is approximately 1.2×10^{-11} pc^{-2} yr^{-1}, which is at least comparable to Tammann's (1982) estimate for galactic SN I: approximately 2.9×10^{-11} pc^{-2} yr^{-1}.

V. CONCLUSION

In a brief outline such as this, it is obviously necessary to gloss over many interesting problems which appear along the way. (We note, for example, that, given the estimated birthrates quoted above, the corresponding surface brightness of the galactic disk from CDWDs *in gravitational radiation alone* is approximately 10^{-2} $L_\odot$ pc^{-2}.) It is only with some trepidation that I have ventured a guess as to the final states of these systems. Nevertheless, the obvious potentialities of evolution in CDWDs merit further investigation.

We note in closing that, ironically, the two known mass-transferring CDWDs, AM CVn (Smak 1967; Faulkner, Flannery, and Warner 1972; Patterson *et al.* 1979) and GP Com (Nather, Robinson, and Stover 1981), do not fit comfortably anywhere within the context of the above discussion—it seems highly unlikely in any of the scenarios outlined here that the mass-losing stars could have survived until eroded to the very small masses ($\sim$0.04 $M_\odot$ and $\sim$0.015 $M_\odot$ respectively) which they must have in AM CVn and GP Com. The possibility remains that they have evolved by a different route, for example, from fossil long-period cataclysmic binaries, as discussed by Warner (1978) and by Nather, Robinson, and Stover (1981).

REFERENCES

Abt, H. A., and Levy, S. G. 1976, *Ap. J. Suppl.*, **30**, 273.
———. 1978, *Ap. J. Suppl.*, **36**, 241.
Chandrasekhar, S. 1957, *An Introduction to the Study of Stellar Structure* (New York: Dover).
De Grève, J. P., de Loore, C., and van Dessel, E. L. 1978, *Ap. Space Sci.*, **53**, 105.
De Grève, J. P., and Vanbeveren, D. 1980, *Ap. Space Sci.*, **68**, 433.
Delgado, A., and Thomas, H. C. 1981, *Astr. Ap.*, **96**, 142.
Faulkner, J., Flannery, B., and Warner, B. 1972, *Ap. J. (Letters)*, **175**, L79.
Fujimoto, M. Y., and Taam, R. E. 1982, *Ap. J.*, **260**, 249.
Giuricin, G., Mardirossian, F., and Mezzetti, M. 1983, *Ap. J. Suppl.*, **52**, 35.
Greenstein, J. L., and Sargent, A. I. 1974, *Ap. J. Suppl.*, **28**, 157.
Iben, I., Jr. 1982, *Ap. J.*, **259**, 244.
Iben, I., Jr., and Tutukov, A. V. 1984, *Ap. J. Suppl.*, **54**, in press.
Landau, L., and Lifschitz, E. 1962, *The Classical Theory of Fields* (Reading, Massachusetts: Addison-Wesley).
Lucy, L. B., and Ricco, E. 1979, *A.J.*, **84**, 401.
MacDonald, J. 1983, in *IAU Colloquium 72, Cataclysmic Variables and Related Objects*, ed. M. Livio and G. Shaviv (Dordrecht: Reidel), p. 77.
Mazurek, T. J. 1973, *Ap. Space Sci.*, **23**, 365.
Meyer, F., and Meyer-Hofmeister, E. 1979, *Astr. Ap.*, **78**, 167.
Miller, G. E., and Scalo, J. M. 1979, *Ap. J. Suppl.*, **41**, 513.
Nather, R. E., Robinson, E. L., and Stover, R. J. 1981, *Ap. J.*, **244**, 269.
Nomoto, K. 1982*a*, *Ap. J.*, **253**, 798.
———. 1982*b*, *Ap. J.*, **257**, 780.
Oemler, A., Jr., and Tinsley, B. M. 1979, *A.J.*, **84**, 985.
Paczyński, B. 1971*a*, *Acta Astr.*, **21**, 1.
———. 1971*b*, *Ann. Rev. Astr. Ap.*, **9**, 183.
———. 1976, in *IAU Symposium 73, Structure and Evolution of Close Binary Systems*, ed. P. Eggleton, S. Mitton, and J. Whelan (Dordrecht: Reidel), p. 75.
Paczyński, B., and Sienkiewicz, R. 1972, *Acta Astr.*, **22**, 73.
Paczyński, B., and Żytkow, A. N. 1978, *Ap. J.*, **222**, 604.
Patterson, J., Nather, R. E., Robinson, E. L., and Handler, F. 1979, *Ap. J.*, **232**, 819.
Plavec, M. 1973, in *IAU Symposium 51, Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems*, ed. A. H. Batten (Dordrecht: Reidel), p. 216.
Plavec, M., Ulrich, R. K., and Polidan, R. S. 1973, *Pub. ASP*, **85**, 769.
Ritter, H. 1983, in *Proc. Workshop on High Energy Astrophysics, Nanjing (China)*, ed. Gong S.-M. (Kexue-Chubanshe: Science Press), in press.
Schönberner, D., and Drilling, J. S. 1983, *Ap. J.*, **268**, 225.

Smak, J. 1967, *Acta Astr.*, **17**, 255.
Taam, R. E. 1980, *Ap. J.*, **237**, 142.
Tammann, G. A. 1982, in *Supernovae: A Survey of Current Research*, ed. M. J. Rees and R. J. Stoneham (Dordrecht: Reidel), p. 371.
Trimble, V. 1974, *A.J.*, **79**, 967.
———. 1978, *Observatory*, **98**, 163.
———. 1982*a*, *Observatory*, **102**, 133.
———. 1982*b*, *Rev. Mod. Phys.*, **54**, 1183.
Warner, B. 1974, *M.N.R.A.S.*, **167**, 61P.
———. 1978, *Acta Astr.*, **28**, 303.
Webbink, R. F. 1977, *Ap. J.*, **211**, 486.
———. 1979*a*, in *IAU Colloquium 46, Changing Trends in Variable Star Research*, ed. F. M. Bateson, J. Smak, and I. H. Urch (Hamilton, New Zealand: University of Waikato), p. 102.
Webbink, R. F. 1979*b*, in *IAU Colloquium 53, White Dwarfs and Variable Degenerate Stars*, ed. H. M. Van Horn and V. Weidemann (Rochester: University of Rochester), p. 426.
Wesemael, F., Winget, D. E., Cabot, W., Van Horn, H. M., and Fontaine, G. 1982, *Ap. J.*, **254**, 221.
Wheeler, J. C. 1978, *Ap. J.*, **225**, 212.
———. 1982, in *Supernovae: A Survey of Current Research*, ed. M. J. Rees and R. J. Stoneham (Dordrecht: Reidel), p. 167.
Whelan, J., and Iben, I., Jr. 1973, *Ap. J.*, **186**, 1007.
Woosley, S. E., Weaver, T. A., and Taam, R. E. 1980, in *Type I Supernovae*, ed. J. C. Wheeler (Austin: University of Texas), p. 96.
Zhilyaev, B. E., Orlov, M. Ya., Pugach, A. F., Rodriguez, M. H., and Totochava, A. G. 1978, *Stars of Type R Coronae Borealis* (Kiev: Naukova Dumka).

RONALD F. WEBBINK: Department of Astronomy, University of Illinois, 1011 West Springfield Avenue, Urbana, IL 61801