MUTUAL PHENOMENA OF THE GALILEAN AND SATURNIAN SATELLITES IN 1973 AND 1979/1980

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ABSTRACT

This paper presents astrometric data derived from photometric observations of mutual occultations and eclipses of the Galilean satellites in 1973 and 1979 and four Saturnian satellites in 1980. The results augment and revise an earlier compilation inasmuch as we have (a) obtained new data, the first by this means in the Saturnian system, (b) used radii measured by Voyager to reduce all light curves, and (c) tabulated heliocentric positions for the mutual eclipses rather than the less well-defined geocentric ones. We argue that the accuracy approaches 0.01 arcsec. Orbital parameters based on these data are generally in very good agreement with other recent analyses.

I. INTRODUCTION

Several years ago in Paper III (Aksnes and Franklin 1976) we assembled and analyzed data obtained from mutual eclipses and occultations of the Galilean satellites observed during the passage of the Earth and Sun through the plane of Jupiter's equator in 1973. The present paper reports the results of a similar study that treats (a) 19 observations of 11 events involving pairs of the three Galilean satellites, J1–J3, obtained during the unfavorable 1979 apparition (maximum frequency of events occurred near the conjunction of Jupiter with the Sun) and (b) 14 of 13 in which the Saturnian satellites, Enceladus, Tethys, Dione, and Rhea (S2–S5), participated during 1979–1980. Because the Voyager missions now provide the most accurate radii for all satellites considered here, we concentrate on using the data exclusively as a source of accurate relative separations in right ascension, $\Delta\alpha \cos\delta$, and declination, $\Delta\delta$, at the observed midtime of an event. However, the three light curves showing the least scatter yield radii for Enceladus and Dione that are nicely in accord with determinations by Voyager. We have also reduced the 1973 material, adding some new data and using Voyager radii for J2–J4 to improve values of $\Delta\alpha \cos\delta$ and $\Delta\delta$. One other alteration in reduction procedure has been introduced and it applies only in the case of the mutual eclipses. As we shall discuss in the next section, the strictly observational positions provided by the eclipses are heliocentric values of $\Delta\alpha \cos\delta$ and $\Delta\delta$, rather than the geocentric ones given in Paper III. Thus astrometric quantities tabulated here are to be preferred over the earlier ones.

Observational material regarding the Galilean satellites is brought together in Sec. II and briefly discussed in the light of Lieske's studies (1978, 1980) that revise Sampson's theory. Section III presents observations of mutual events in the Saturnian system. The new material, although of limited amount, makes it clear that (a) reasonably accurate light curves can supply relative positions accurate to $\leqslant 85$ km ($0''.012$ at Saturn's mean distance) and (b) ephemerides based on the theories and observationally determined constants of G. Struve (1924–1933) now contain, for S3–S5, errors as large as ~ 900 km in longitude and about half as great in latitude. The mutual events provide a sufficient amount of data to revise a minimal number of the orbital constants of S3–S5. A comparison of our results with analyses since the time of Struve published by Kozai (1957) and Sinclair (1977) shows remarkably good agreement—especially with the longitudes of the latter paper.

In the Appendix we mention the observation in the Saturn system of a curious light curve with two well-separated minima that may have some cautionary relevance to other occultation studies among objects in the solar system.

II. PHENOMENA OF THE GALILEAN SATELLITES IN 1973 AND 1979

Since the Voyager missions have provided accurate radii for the Galilean satellites and described surface features and albedo variations, observations of mutual events can now best be used solely for astrometric purposes. In this area their contribution remains considerable, in precision if not abundance, as has most recently been shown by Lieske (1980). His paper, which derives new ephemerides for the Galilean satellites based upon Jovian eclipse, photographic, and mutual event data, reaffirms our earlier estimate of the accuracy of the latter: the relative positions in right ascension, $\Delta\alpha \cos\delta$, and declination, $\Delta\delta$, of a satellite pair at midevent show re-

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siduals near 0.03 arcsec, or ~100 km at Jupiter's mean distance. As we shall discuss in some detail toward the end of this section, we now suspect that this value is a clear overestimate of the residual for the $\Delta\alpha \cos\delta$'s. One reason is that Lieske's paper has aided our understanding of a systematic effect present in certain mutual event observations. In Fig. 2 of Paper III we drew attention to the curious fact that nearly simultaneous occultations and eclipses of Europa by Io provided slightly different longitude corrections to Sampson's theory and that this disparity became more severe—corresponding to a longitude difference, ΔX, of ~100 km—as the phase angle, α, approached its maximum of ~12°. Lieske's study not only confirmed the systematic difference, but also indicated that (a) values of $\Delta\alpha \cos\delta$ (which are measured in a direction only slightly inclined to the satellites' mean orbital plane) derived from mutual occultations are the ones that agree more closely with astrometric data from other sources and (b) no systematic effects are discernible in the $\Delta\delta$'s. In the right-hand one-third of Lieske's (1980) Fig. 5, where $\alpha \gtrsim 8\overset{\circ}{.}4$, the systematic nature of the $\Delta\alpha \cos\delta$ residuals, all but one of which correspond to an eclipse, is clearly apparent. We believe that, at least in part, the resolution of this problem is as follows. Consider first an occultation. It can provide two positional quantities: (a) the $(O-C)$ difference in the midtime corrects the predicted relative longitude in the mean orbital plane, while (b) the $(O-C)$ difference in the amplitude of the light curve supplies a latitude correction. Clearly, no information is available concerning any possible correction along the line of sight. But, since occultations are viewed from the Earth, the astrometric separations of two satellites, $\Delta\alpha \cos\delta$ and $\Delta\delta$, are well-defined quantities independent of any model or theory. For an eclipse the corrections lie in the two analogous directions, referred to the vector joining the Sun to the pair of satellites. Any attempted transformation of the two heliocentric positions, obtained from an eclipse light curve, to geocentric values of $\Delta\alpha \cos\delta$ and $\Delta\delta$ would require a knowledge of the unobservable correction to the satellite positions in the line of sight direction as seen from the Sun. The small uncertainty near opposition, introduced either by ignoring or modelling this unknown factor, grows with phase angle and principally affects $\Delta\alpha \cos\delta$ because that quantity lies in a direction close to the Earth-Sun-Jupiter plane. A new analysis of the 1973 mutual eclipses to derive *heliocentric* values of $\Delta\alpha \cos\delta$ and $\Delta\delta$ which, like their geocentric counterparts obtained from occultations, are strictly observed quantities untied to any the-

TABLE I. Astrometric data, 1973 events, Galilean satellites.

1973 mo. da	Date, UT; h	m	s	OBS	JED − 2440000	Event	Δα cos δ (arcsec)	Δδ (arcsec)	W
6/10	09	10	00	V	1843.85686	1O2	0.245	-0.734	2
6/10	09	09	58	L	.85684	1O2	0.241	-0.726	1.5
6/17	11	15	51	L	1850.94475	1O2	0.200	-0.620	1.5
6/17	11	15	53	M	.94478	1O2	0.196	-0.607	2
6/24	13	20	49	M	1858.03198	1O2	0.169	-0.514	2
6/28	02	22	42	SA	1861.57514	1O2	0.170	-0.532	2
7/8	17	27	52	N	1872.20421	1O2	0.121	-0.385	1
7/8	17	28	04	MT	.20435	1O2	0.099	-0.320	1
7/12	06	29	35	F	1875.74719	1O2	0.095	-0.305	1.5
7/12	06	29	23	C	.74705	1O2	0.106	-0.342	2
7/12	06	29	28	V	.74710	1O2	0.109	-0.353	1.5
7/12	06	29	24	MP	.74706	1O2	0.104	-0.336	2
7/19	08	32	22	L	1882.83265	1O2	0.085	-0.276	1.5
7/22	21	34	06	P	1886.37557	1O2	0.079	-0.261	1.5
7/22	21	34	02	R	.37553	1O2	0.081	-0.264	2
7/26	10	35	32	M	1889.91827	1O2	0.072	-0.238	2
8/2	12	39	25	P	1897.00432	1O2	0.071	-0.239	1.5
8/13	03	48	09	A1	1907.63525	1O2	0.057	-0.196	1.5
8/13	03	48	09	A2	.63524	1O2	0.048	-0.165	2
8/16	16	51	42	P	1911.17929	1O2	0.048	-0.166	2
8/20	05	56	28	F	1914.72416	1O2	0.040	-0.140	1.5
8/20	05	56	28	C	.72416	1O2	0.043	-0.151	2
8/27	08	07	47	C	1921.81508	1O2	0.042	-0.144	1
8/27	08	07	48	L	.81509	1O2	0.030	-0.102	1
8/27	08	07	50	M	.81511	1O2	0.036	-0.124	2
9/3	10	23	19	S	1928.90885	1O2	0.025	-0.084	2
9/6	23	32	28	C	1932.45667	1O2	0.019	-0.062[a]	1.5
9/6	23	32	35	PA	.45675	1O2	0.019	-0.062[a]	1.5
9/10	12	44	23	S	1936.00640	1O2	0.012	-0.037[a]	1.5
9/21	04	31	37	C	1946.66348	1O2	-0.006	+0.038	1.5
9/21	04	31	37	L	.66347	1O2	-0.014	0.069	1.5
9/21	04	31	43	F	.66355	1O2	-0.009	0.047	1
9/24	17	55	32	R	1950.22148	1O2	-0.030	0.136	2
9/28	07	25	08	M	1953.78342	1O2	-0.036	0.171	2
12/21	00	32	26	C	2037.48998	2O1	+0.155	-0.478	1.5
9/24	01	45	18	C	1949.54776	3O2	0.218	-0.879	2
9/24	01	45	05	MD	.54761	3O2	0.199	-0.829	1.5
10/1	06	06	18	L	1956.72844	3O2	0.247	-0.984	1
10/1	06	05	58	MW	.72822	3O2	0.252	-1.003	1.5
7/26	10	25	18	M	1889.91117	1E2	0.243	-0.828	1.5
8/2	12	47	15	P	1897.00976	1E2	0.171	-0.574	1.5
8/13	04	24	21	A1	1907.66038	1E2	0.095	-0.309	1
8/13	04	24	18	A2	.66035	1E2	0.093	-0.308	1.5
8/16	17	37	39	P	1911.21121	1E2	0.071	-0.227	2
8/20	06	52	26	F	1914.76303	1E2	0.045	-0.145	1
8/27	09	24	43	M	1921.86850	1E2	-0.002	+0.010	1.5
9/3	12	03	11	P	1928.97820	1E2	-0.043	0.145	2
9/3	12	03	13	S	.97822	1E2	-0.040	0.136	2
9/7	01	25	25	H	1932.53510	1E2	-0.070	0.234	1
9/10	14	52	30	S	1936.09536	1E2	-0.077	0.256	1.5
9/14	04	23	56	NM	1939.65863	1E2	-0.098	0.330	1
11/24	23	08	35	PA	2011.43362	1E2	+0.261	-0.736	1.5
12/2	01	26	35	L	2018.52890	1E2	0.213	-0.596	1
11/4	23	19	22	A1	1991.44283	2E1	0.041	-0.115	1
11/4	23	19	23	A2	.44284	2E1	0.059	-0.172	1
11/4	23	19	24	A3	.44285	2E1	0.047	-0.136	1
11/12	01	33	09	V	1998.53510	2E1	-0.018	+0.054	1
11/15	14	40	06	P	2002.08129	2E1	-0.028	0.080[a]	1
11/19	03	47	02	B	2005.62747	2E1	-0.062	0.178	1
11/29	19	08	11	CI	2016.26629	2E1	-0.183	0.514	1.5
12/13	23	37	08	F	2030.45203	2E1	-0.311	0.857	1.5
9/19	11	59	18	P	1944.97449	1E3	+0.299	-0.863	2
11/2	02	47	56	L	1988.58791	1E3	0.290	-0.840	1.5
11/2	02	47	50	F	.58785	1E3	0.296	-0.858	1
10/29	00	44	47	PA	1984.50275	3E1	0.161	-0.465	1.5
11/5	03	41	46	L	1991.62503	3E1	0.080	-0.227	1
11/20	00	40	18	H	2006.49773	3E1	0.065	-0.196	1
11/26	13	49	16	P	2013.04508	3E1	-0.110	+0.329	1
12/14	12	43	05	P	2030.99779	2E3	-0.005	0.005	1
9/9	21	54	03	W	1935.38816	3E2	+0.076	-0.277	1
9/17	02	53	25	V	1942.59557	3E2	0.078	-0.261	1.5
9/17	02	53	26	L	.59558	3E2	0.073	-0.246	1.5
10/1	11	14	43	P	1956.94261	3E2	0.045	-0.145	2
10/1	11	14	49	D	.94267	3E2	0.063	-0.203	1
10/8	15	05	39	P	1964.10238	3E2	0.029	-0.094	1.5
10/22	22	25	47	A	1978.40678	3E2	-0.030	+0.087	1
10/22	22	25	50	CI	.40681	3E2	-0.039	0.113	1
10/30	01	59	04	L	1985.55425	3E2	-0.062	0.181	1.5
10/30	01	59	07	H	.55428	3E2	-0.065	0.191	1.5
12/4	19	11	25	CI	2021.26816	3E2	-0.299	0.840	1
12/11	22	32	43	A	2028.40744	3E2	-0.363	1.008	1
10/28	23	16	14	PA	1984.44127	4E1	+0.384	-1.108	2
12/1	13	27	29	P	2018.02956	4E1	-0.145	+0.404	1
10/21	12	29	45	P	1976.99299	2E4	+0.320	-0.958	2
11/7	03	29	09	L	1993.61609	2E4	-0.044	+0.118[a]	1
10/30	03	57	40	L	1985.63660	4E2	+0.065	-0.190	1
10/30	03	57	45	H	.63666	4E2	0.057	-0.170	1
10/4	12	00	09	P	1959.97391	3E4	0.233	-0.725	2
11/25	01	29	34	MD	2011.53153	3E4	-0.119	+0.322	1.5
11/25	01	29	37	PA	.53155	3E4	-0.118	0.321	1
10/30	21	05	31	SA	1986.35033	4E3	+0.113	-0.345	1.5
11/15	18	09	50	CN	2002.22693	4E3	0.158	-0.450	1
12/17	00	34	31	PA	2033.49168	4E3	-0.094	+0.272	1

Note to Table I

[a]Case in which an observationally determined latitude correction is imposed to remove indeterminacy (see text).

ory, is the major reason for the revised astrometric results provided in Table I. (The revised constants given in Paper III were not affected by the small inaccuracies in $\Delta\alpha \cos\delta$ for the eclipses because we derived and used instead heliocentric longitude corrections to Sampson's theory.) We have also incorporated into the solution radii obtained by Voyager (Davies *et al.* 1979) for J2 (1563 km), J3 (2638 km), and J4 (2410 km) each with an uncertainty of ± 10 km, but have retained the mean value of 1820 km for J1. Because these radii differ by only $+30$, $+30$, and -35 km—or in the 1%–2% range—from those we determined from the 1973 mutual events, we can expect very small concomitant changes in $\Delta\alpha \cos\delta$ and $\Delta\delta$ and then principally in the latter because its determination chiefly rests on the depth of a light curve. In fact, the largest corrections to $\Delta\alpha \cos\delta$ and $\Delta\delta$ traceable just to the new radii (i.e., as given by the occultations) are only 0.01 and 0.02 arcsec.

Table I is an abridged version of Tables I and II of Paper III. Its first (compound) column lists the observed UT of midevent, followed by an abbreviation for the observing station, which is identified in Paper III. Six light curves (Abraham and Strauss 1979) obtained at Porto Alegre (PA), Brazil, are the only additions. The next column gives the Julian Ephemeris Day corresponding to Col. 1, but corrected for light time, i.e., it refers to the instant that light left the eclipsed or occulted satellite. Columns 5 and 6 provide midevent separations in right ascension and declination, referred to the mean equator and equinox of 1950, between the centers of the satellites as they appeared from the observing site listed in Col. 2 for occultations and the Sun's center for eclipses. The notation 2E3 or 2O3 (combined as 2E, O3 in Table II) used in Col. 4 stands for satellite J2 eclipsing or occulting satellite J3. The encounter geometry of a small number of events (e.g., an annular eclipse or occultation) is such that precise values of $\Delta\delta$ are not obtainable. The total number of observations in 1973 is sufficient to improve the constants of Sampson's theory and thereby calculate latitude corrections for the indeterminate events. For such observations, indicated by the superscript "a," $\Delta\delta$ and, to a much lesser extent, $\Delta\alpha \cos\delta$, are not strictly observed quantities. Since the light curves are of varying quality, it is useful to have some means of weighting each observation. For every event, we have calculated a raw weight, which is inversely proportional to the product of the air mass and the square root of the rms residual that results when a light curve is fitted to the observed points. On the basis of this value, one of the three weights listed in the final column is assigned.

After the large number of observations made in 1973, the 1979 material comes as a disappointment. The apparition was a poor one, requiring that most observations be made through large air masses so that uncertain sky corrections and approximate light curves resulted. This is partly our fault. Not properly heeding the $\sim 0\overset{\circ}{.}5$ inclination of Europa, we failed to predict a series of 23 events involving it and Io in March and April 1980. Table II, which contains results from the 1979 data, differs from the format of Table I only by providing in its final column the photometric constant, $K \equiv (1 + I_k/I_l)^{-1}$, required for the reduction of any observation in which light from both satellites is measured. The quantities I_k and I_l are the unobscured intensities at the time of an event in which satellite k eclipses or occults satellite l. If not directly measured before and/or after an event, the I's were obtained from the photometric studies described in Paper III. V filters were used except where otherwise indicated. New observatory designations are as follow: CH, NI, PM, and SM are in France, at Chiran, Nice, Pic du Midi, and

TABLE II. Astrometric data, 1979 events, Galilean satellites.

1979 mo. da	Date, UT; h	m	s	OBS	JED − 2440000	Event	Δα cos δ (arcsec)	Δδ (arcsec)	W	K; Filter
10/1	04	23	41	CH	4147.64834	1O2	−0.108	−0.250	1	0.4610
10/1	04	23	38	NI	.64832	1O2	−0.102	−0.237	1	0.4610
10/1	04	23	36	PM	.64830	1O2	−0.123	−0.285	1	0.4390 R
1/14	23	39	40	SM	3888.46159	2O3	+0.277	+1.182	1.5	0.6440 R
1/14	23	39	14	SM	.46129	2O3	0.256	1.123	1	0.6530 U
1/28	07	53	29	BC	3901.80457	2O3	0.077	0.305	1	0.6640
1/28	07	53	09	LS	.80434	2O3	0.080	0.309	1	0.6640
10/1	03	04	49	HA	4147.59358	1E2	0.071	0.179	1	
10/1	03	04	44	CN	.59352	1E2	0.072	0.182	1	
10/1	03	04	48	CH	.59356	1E2	0.057	0.146	1	
10/8	05	28	01	PM	4154.69344	1E2	0.007	0.048	1	
10/15	07	49	20	A	4161.79204	1E2	−0.023	−0.057	1	R
10/29	12	27	02	L	4175.98593	1E2	−0.152	−0.369	1.5	
11/2	01	35	56	CN	4179.53406	1E2	−0.160	−0.390	1	B
11/9	03	52	35	CN	4186.62955	1E2	−0.262	−0.635	1.5	
1/14	13	14	54	P	3888.02772	2E3	+0.083	+0.326	1	0.6450
1/14	13	13	24	D	.02667	2E3	0.094	0.321	1	0.6450
1/28	07	14	11	BC	3901.77728	2E3	0.104	0.434	1	0.6640
1/28	07	13	55	LS	.77709	2E3	0.104	0.411	1	0.6640

Note to Table II

The durations of the 2 E, O 3 events were $\gtrsim 10$ times longer than other events here and in Table I, hence the range in midtime determinations.

Saint Michel, respectively; BC and LS refer to the ESO Observatory at La Silla (Chile), the former to the 60-cm Bochum telescope; and HA stands for Harestua, Norway.

In Paper III we used the astrometric results to revise the constants of the most important terms in Sampson's theory. Given the amount of data, gathered over only six months by a previously untried technique, that analysis was designed much more to show that a self-consistent set of parameters describing the motions of all four satellites could be derived from a limited number of two-body events than it was to provide any definitive revision. On the basis of that analysis, we concluded that these revisions to the leading constants of Sampson's theory could provide positions of the Galilean satellites with an accuracy of ~ 0.03 arcsec, or ~ 100 km, at least over the duration in which the data were acquired. Since Paper III appeared, Lieske (1978, 1980) has completed his revision of Sampson's theory and reevaluation of its constants by use of positions from different sources that included the 1973 mutual events and that incorporated Jovian eclipse timings made as far back as 1878. Because Lieske's analysis—resulting in the two ephemerides labelled E-1 and E-2—provides the most accurate current representation of the motion of the Galilean satellites, a comparison even with an approximate solution but involving just the mutual events is still a useful means of assessing the latter's precision. Where possible, Table III makes this comparison and includes the earlier determinations of various constants by Sampson and de Sitter as well. The agreement is quite satisfactory save in the case of the free eccentricity, $\frac{1}{2}p_1$, of Io. It seems possible that several small terms appearing in Sampson's theory but not included in ours have combined to force the large value on p_1. In spite of this difficulty, we can conclude on the basis of the small standard errors in Table III, the large decrease of the latitude (σ_z) and longitude (σ_x) residuals as compared with earlier theories and the agreement with Lieske's parameters that mutual events yield astrometric results of good quality. In fact, we now believe that the actual rms residual σ_x would be considerably smaller than the 75 km given in Table III for the 1973 data had we used a more precise form of Sampson's theory. (The 1979 material is definitely of poorer quality.) We have two reasons for arguing that $\sigma_x = 75$ km represents an unknown mixture of model deficiencies and observational errors, so that to claim that the observations are completely responsible for this value is to be unrealistically critical of them. First, the formal standard errors in the midtime obtained by folding each light curve upon itself are typically 1–2 s and the corresponding errors in the longitudes, because they depend only on the midtime (and the well-known satellite relative velocities), lie in the neighborhood of ~ 25 km. Accurately measured light curves with proper attention to timing should be able to achieve this level of precision fairly routinely. An examination, among the higher-quality 1973 data, of 24 cases in which two or three observations were made of a single event

TABLE III. Mean longitudes (l), periJoves (π), free and forced eccentricities (p,q), inclinations (γ), and nodes (θ), with standard errors, on JD 2441920.5 ET for different ephemerides.

Elem.	S	D	E-1*	E-2*	RS(73)	RS(73+79)
ℓ_1	297°.212	297°.213	297°.343 ± 0.002	297°.310 ± 0.018	297°.343 ± 0.005	297°.348 ± 0.005
ℓ_2	130.298	130.282	130.337 ± 0.008	130.335 ± 0.004	130.332 ± 0.003	130.322 ± 0.002
ℓ_3	136.842	136.816	136.834 ± 0.010	(136.847)	136.829 ± 0.002	136.827 ± 0.002
ℓ_4	187.014	187.000	187.019 ± 0.007	187.001 ± 0.005	187.046 ± 0.001	187.044 ± 0.001
π_1	193.24	2.52	237.98 ± 26	267.56 ± 73.8	318.72 ± 17.1	327.61 ± 10.0
π_2	22.94	148.21	33.03 ± 25	76.82 ± 15.5	29.98 ± 28.2	65.86 ± 12.1
π_3	166.64	174.98	176.56 ± 2	179.32 ± 0.92	174.71 ± 0.78	173.96 ± 0.81
π_4	333.27	333.47	332.90 ± 0.3	332.59 ± 0.13	332.79 ± 0.07	332.91 ± 0.08
p_1	0.0053	0.0013	0.0049 ± 0.0021	0.0011 ± 0.0023	0.0397 ± 0.0088	0.0672 ± 0.0074
p_2	0.0095	0.0150	0.0123 ± 0.0049	0.0106 ± 0.0025	0.0133 ± 0.0036	0.0199 ± 0.0044
p_3	0.1738	0.1593	0.1665 ± 0.0043	0.1684 ± 0.0033	0.1756 ± 0.0029	0.1715 ± 0.0027
p_4	0.8449	0.8436	0.8404 ± 0.0064	0.8403 ± 0.0034	0.8328 ± 0.0032	0.8357 ± 0.0036
q_1	0.4715	0.4657	–	–	0.4742 ± 0.0102	0.4728 ± 0.0112
q_2	1.0702	1.0694	–	–	1.0638 ± 0.0053	1.0748 ± 0.0035
q_3	0.0738	0.0771	–	–	(0.0738)	(0.0738)
σ_x km	~757	~880	–	–	75	93
γ_1	0°.0272	0°.0317	0°.0276 ± 0.0289	0°.0403 ± 0.0111	0°.0374 ± 0.0117	0°.0290 ± 0.0083
γ_2	0.4669	0.4668	0.4790 ± 0.0112	0.4674 ± 0.0117	0.4685 ± 0.0083	0.4569 ± 0.0051
γ_3	0.1782	0.1788	0.1824 ± 0.0041	0.1856 ± 0.0087	0.1934 ± 0.0102	0.1813 ± 0.0064
γ_4	0.2719	0.2452	0.2639 ± 0.0071	0.2538 ± 0.0296	0.2382 ± 0.0111	0.2500 ± 0.0076
θ_1	26.78	110.07	97.29 ± 62	93.11 ± 18.5	116.20 ± 8.32	101.28 ± 11.36
θ_2	129.89	135.88	134.17 ± 1.7	135.80 ± 1.1	136.21 ± 0.39	136.35 ± 0.46
θ_3	131.98	129.27	127.21 ± 1.7	126.48 ± 2.5	127.80 ± 0.71	127.57 ± 0.58
θ_4	319.08	325.63	321.49 ± 1.5	324.70 ± 2.4	321.50 ± 0.32	321.19 ± 0.29
σ_z km	~357	~327	–	–	96	113

Notes to Table III

S, D, E-1, E-2, and RS refer to solutions by Sampson (1921), de Sitter (1931), Lieske (1978, 1980) and two ("revised Sampson") in this paper. Asterisks indicate that longitudes and periJoves have been "precessed" backwards (by subtracting 0°.698) from 1950.0 to agree with the 1900.0 epoch. Quantities in parentheses are assumed. The σ's are rms residuals, in longitude, σ_x, and latitude, σ_z, derived from astrometric results provided by mutual events (cf. Paper III).

yields a rms residual in longitude of 35 km, or not too far from the above estimate. (A portion of the 35-km value probably results from errors in recording time signals on the photometric outputs.) Second, both the latitude corrections and the satellite radii derived in Paper III ought to be more uncertain than the longitude corrections because the least-squares solution for the first two depends on the depth and width of a light curve and, for occultations, on the intensity ratios of the satellites. Largely because of sky corrections, these quantities are less well determined than the midtimes. Yet Voyager demonstrated that the errors in these radii were only 30–35 km and the near equality (cf. Table III) of σ_z to σ_x is surprising unless these residuals reflect model inadequacies in addition to observational errors.

We have already remarked that our approximate modelling (cf. Paper III) of the longitude corrections cannot exploit an observational accuracy much below ~ 100 km. We also suspect that a similar comment applies to the E1 and E2 ephemerides developed by Lieske. Although his important revision of Sampson's theory includes perturbations as small as ~ 10 km, the observations used to derive many of the constants are much too approximate to take advantage of this high accuracy. As Lieske (1980) has pointed out, one liability of the 1973 mutual event observations (even if they are as accurate as we believe) in a solution with other types of data is their relatively small number and limited time span of ~ 6 months. If the next very favorable apparition in 1985/1986 is well observed, the baseline will be extended to 12 yr and their influence relative to other observations will begin to achieve a balance that is more in line with what we have argued is an accuracy in longitude of several tens of kilometers. This translates into an uncertainty in the $\Delta\alpha \cos\delta$ separations approaching 0.01 arcsec.

III. PHENOMENA OF THE SATURNIAN SATELLITES IN 1979/1980

Table IV, an expanded version of Tables I and II, brings together parameters describing the better-quality light curves of mutual events among the Saturnian satellites, together with certain results derivable from them. Material of a type not contained in Tables I and II appears in the columns labelled ΔM, Δ_0, and Δ, which are, respectively, the observed light drop in magnitudes and the rms noise on an intensity scale from 0 to 1 obtained by folding the light curve about its midtime and by fitting a modelled curve (cf. Paper III) to the observed points. Our analysis, patterned closely after that of Paper III, represents the measured light curves in terms of the two known radii and the unobscured brightness ratio of the satellites and then determines from each curve an observed correction in longitude, $\Delta X(O-\mathrm{GS})$, Col. 8, and latitude, $\Delta Z(O-\mathrm{GS})$, Col. 10, to the relative satellite positions given by the theories and constants of G. Struve (1924–1933). The next step uses the pairs, $\Delta X(O-\mathrm{GS})$ and $\Delta Z(O-\mathrm{GS})$, to improve a minimum number of the orbital constants. We continue by inverting the process and obtain from the new constants a revised set, $\Delta X(O-R)$, Col. 9, and $\Delta Z(O-R)$, Col. 11, in order to assess the accuracy of the procedure. As in the case of the Galilean satellites, this technique identifies the constants most in need of revision and checks the consistency of the solution. A more thorough revision of all constants requires more data over a longer time. Values of the model-independent positions, $\Delta\alpha \cos\delta$, and $\Delta\delta$ are therefore provided for future use, but with one caveat. The solution of any light curve for a latitude correction is double valued—the center of one satellite can lie at equal distances N or S of the other.

TABLE IV. Astrometric data, Saturnian satellites.

1980 mo. da	Date, UT; h	m	s	OBS	JED −2440000	Event	ΔM	K	Δ_0	ΔX (O-GS) km	ΔX (O-R) km	ΔZ (O-GS) km	ΔZ (O-R) km	Δ	Δα cos δ arcsec	Δδ arcsec	W
4/18	05	24	18	L	4347.67594	2O3	0.18	0.715	0.0073	-75	17	-224	65 83	0.0076	0.003	-0.033	2
4/18	05	24	16	A	4347.67591	2O3	0.16	0.715	0.0221	-46	46	-366	-77 -59	0.0318	0.005	-0.056	1
11/28*	12	13	06	L	4205.95365	3O4	0.23	0.442	0.0054	-421	-134	107	-32 -119	0.0073	0.003	-0.069	2
1/22	05	58	24	A	4260.69846	3O4	0.29	0.441	0.0297	-325	-36	225	29 -38	0.0307	-0.002	+0.053	1
								(0.470		-322		260			0.000	0.058)	
2/6	11	08	59	NM	4275.91521	3O5	0.16	0.621	0.0047	-553	21	637	-29 +4	0.0103	0.007	0.113	2
								(0.630		-553		642			0.007	0.114)	
2/20	15	46	37	D	4290.10873	3E5	0.16	0.621	0.0317	-935	-174	128	44 -60	0.0443	-0.006	0.077	1
2/29	18	25	28	W	4299.21932	3E5	0.33	0.622	0.0232	-458	120	220	-134 -8	0.0262	-0.008	0.052	1
								(0.610		-458		203			-0.008	0.049)	
3/15	17	48	31	D	4314.19382	4E3	0.58	0.530	0.0241	-381	138	-199 +239	-117 61	0.0254	0.002 -0.003	-0.032 +0.032	1
4/23	16	11	10	D	4353.12481	4O3	0.56	0.558	0.0267	-331	-29	-203	-33 -60	0.0378	0.004	-0.051	1
2/28	22	48	07	W	4298.40170	4E5	0.15	0.478†	0.0192	-892	23	441	56 2	0.0175	0.009	-0.112	2
4/6	01	59	01	A	4335.53398	4E5	0.27	0.635	0.0135	-347	60	185	-107 -102	0.0170	0.006	-0.078	1.5
4/20	22	25	11	CN	4350.38472	5E3	0.10	0.378	0.0170	-581	5	623	-19 11	0.0199	0.006	-0.121	1.5
3/3	03	57	26	A	4301.61658	5O4	0.26	0.323	0.0280	-737	16	492	156 124	0.0264	-0.008	0.090	1.5
3/30	07	42	27	L	4328.77271	5E4	0.28	0.321	0.0074	-541	-59	336	43 38	0.0092	0.006	-0.071	2
									σ(km):	536	82	356	80 68				

Notes to Table IV

*Year of this observation is 1979.

†S3 also in measuring diaphragm so that the photometric parameter $K = [1 + I_4/I_5 + I_3/I_5]^{-1}$. Two sets of ΔZ(O-R) residuals are given because the double-valuedness of the ΔZ correction is not resolved for the March 15 event, 4E3. Details in text. Entries in parentheses show effect on Δδ of differences from best photometric estimates.

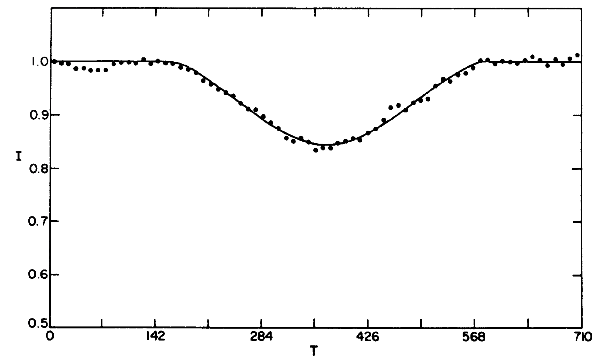

FIG. 1. Lowell Observatory light curve of the occultation of Dione by Tethys on 1979 November 28, as observed with a 72-in. telescope and a filter centered on 8500 Å. Time, T, is measured in seconds after $12^h07^m00^s$ UT. Dots result from 10-s integrations. The fitted curve corresponds to a radius, R, of 555 ± 10 km (SE) for Dione, assuming the Voyager value of 525 km for Tethys. Another event (5E4, March 30), also observed at Lowell, gives $R = 568 \pm 17$ km, assuming the Voyager value of 765 km for the radius of Rhea.

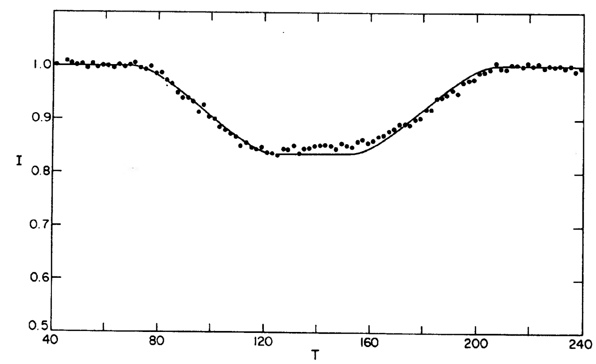

FIG. 2. Lowell observation of occultation of Tethys by Enceladus (S2). Time measured in seconds after 1980 April 18, $05^h22^m00^s$ UT. Derived radius for S2 is 243 ± 1 km, assuming the Voyager value of 525 km for Tethys.

Observations of mutual phenomena of the Galilean satellites have been numerous enough—and the satellites themselves large enough—so that the appropriate solution was always clear and there was never a need to comment on it. In the Saturnian case, the problem is more difficult because of both the limited data and the smaller satellites. Still, there is only one observation (March 15, 4E3) in which both of the two $\Delta Z\,(O - \mathrm{GS})$ residuals, when reduced with all the other unambiguous cases, yield rms residuals $\sigma_z < 150$ km and also acceptable values of the inclinations, γ_i, and nodal longitudes, θ_i. (These quantities are more precisely defined in the following paragraphs.) Thus, for this case alone we have supplied two values for both $\Delta\alpha \cos\delta$ and $\Delta\delta$ (although there is no essential difference in the first of these quantities between the two cases) and must call upon future observations or those obtained by Voyager (Synnott 1983, private communication) to decide which is correct.

All the light curves are well represented by radii measured by the Voyager mission and available photometry. Midtimes for the three light curves obtained at the Dodaira Observatory, kindly made available before publication, differ by less than 9 s from values given by Soma and Nakamura (1982). Adopted radii (Tyler *et al.* 1982) are: Enceladus (S2), 250 ± 10 km; Tethys (S3), 525 ± 10 km; Dione (S4), 560 ± 10 km; and Rhea (S5), 765 ± 10 km. From three light curves with the least scatter (cf. Figs. 1 and 2) we have derived radii for S2 and S4 in excellent agreement with the above values. Such agreement, although at least one of the formal standard errors is fortuitously small, illustrates the high precision and internal consistency of the mutual event technique. Obtaining brightness ratios presented more of a problem. For all cases in Table IV, even eclipses near quadrature, the measuring diaphragm contained both satellites, and in one instance a third of comparable brightness was unavoidably present. Since it was generally impossible to obtain brightnesses of individual satellites near the time of an event, values of $K \equiv (1 + I_k/I_l)^{-1}$ given in Table IV are drawn from three sources: Noland *et al.* (1974), Franz, and Millis (1975), and unpublished measures made on eight nights as part of this project. All three are in various ways incomplete, but the following reasonable assumptions can be inferred from them. First, the brightness dependence on solar phase angle, α, is the same for S3–S5. Second, for these three objects, the brightness variations with orbital phase are sinusoidal with extrema at the two elongations and with amplitudes that are independent of wavelength between $V\,(\lambda_{\mathrm{eff}} = 5540$ Å) and ~8500 Å. Thus for our purposes, i.e., for relative magnitudes, m, among S3–S5, we can use the relations

$$m(3) = 10.21 + 0.05 \sin(\phi - 180),$$
$$m(4) = 10.39 + 0.16 \sin(\phi - 180), \qquad (1)$$

and

$$m(5) = 9.67 + 0.095 \sin(\phi - 180),$$

where the orbital phase angle, ϕ, is measured from superior conjunction.

UBV observations by Franz and Millis (1983, private communication) made since the completion of our reductions show that these assumptions are slightly in error. Their work indicates that small differences exist between the phase curves of the leading and trailing hemispheres for both Tethys and Rhea (although the average curves for these two satellites and for Dione are quite similar) and that the amplitude of the brightness variations with orbital phase increases at wavelengths shorter than that corresponding to V. (There is no new evidence that it decreases on the longward side of V.) Replacement of the constants in the relations (1) by the newer, revised ones of Franz and Millis corrects the relative magnitude difference between two satellites by no more than 0.05, which, in the relevant cases, alters K (Table IV) by 0.012 or less. As is made clear by the parenthetical entries in Table IV, changes in K must be several times greater before ΔZ and $\Delta\delta$ are seriously affected. The brightness ratio for the single 2O3 event rests entirely on interpolations in the measures of Franz and Millis (1975) and, given the unusual and varied surface of S2 and its faintness, is less reliable. The accurate radius obtained for Enceladus from the better light curve of this event argues, however, that $K = 0.715$ cannot be far from correct.

Table V compiles values of the mean longitudes, l_i, inclinations, γ_i, to the ring plane, and nodal longitudes, θ_i, on the ring plane for Tethys, Dione, and Rhea near the mean epoch for the mutual events, JED 2444240.5 = 1980 January 2.0 ET. Since only two observations of one event (2O3) involve Enceladus, we have not attempted to revise its orbit but have used Struve's elements. For Tethys and Dione, the mean longitudes and nodal longitudes are measured from the mean equinox of date along the mean ecliptic of date to the ascending node of the ring plane, then along the ring plane to the ascending node of the satellite's orbit and finally, for l, along the satellite's orbit. For Rhea, the mean longitude, l_5, is measured from the mean equinox of date along the mean ecliptic of date and then along Rhea's orbit, but γ_5 and θ_5 are referred to the ring plane and γ_5, in contrast to γ_3 and γ_4, varies because of the action of Titan.

Since both Struve and Kozai measured l relative to the equinox of date, but θ relative to the equinox of 1889.25, we have applied a constant correction, $\Delta\dot{\theta} = +0^\circ.014$/yr to their nodal rates, $\dot{\theta}$, which we have then used to bring their values of θ for 1889.25 forward to our 1980 epoch. Similarly, l and θ as computed from the elements of Garcia (1972) and Sinclair (1977) have been precessed from the 1950 epoch to the 1980 one.

Table V shows an excellent agreement between the mean longitudes derived from the mutual events and those obtained by Sinclair from satellite positions (relative to Titan) made during the apparitions of 1972–1976. The agreement is very nearly within the standard errors of our results and well within the standard errors (that are some three times larger) of Sinclair's values. In view of the much earlier mean epoch of the observations, the agreement with Kozai's mean longitudes is also quite satisfactory.

With regard to the inclinations and nodes, the agreement between the different determinations in Table V is less satisfactory. Because Sinclair's values are rather discordant, perhaps owing to the short four-year baseline of his observations, and because of the ambiguity due to the two latitude solutions from the mutual events, it seems most advisable at this time to adopt Kozai's values for the inclinations and nodes. To date, all data reductions have assumed that γ_3 and γ_4 are constant. Given the nature of the Mimas-Tethys resonance, it is unlikely that γ_3 can have a small constant value (Kalnajs 1982, private communication). Thus part of the scatter in γ_3 may reflect a real variation, but the evidence at present is very weak.

For current ephemerides of Tethys, Dione, and Rhea, we recommended Kozai's (1957) orbital elements, except that the mean longitudes at the epoch (E_0 in Kozai's notation) should, based on mutual event results, receive corrections of $+0^\circ.030$, $-0^\circ.007$, and $-0^\circ.025$, respectively. Corresponding corrections to the longitudes given by Struve and Sinclair (for the case in which positions were determined relative to Titan) are $-0^\circ.028$, $-0^\circ.074$, $-0^\circ.080$; and $-0^\circ.016$, $-0^\circ.004$, $-0^\circ.004$.

IV. FINAL REMARKS

Although much of this paper may seem to consider revisions to orbital parameters, our chief interest really lies in the area of describing observations of mutual events among the Jovian and, for the first time, the Saturnian satellites and deriving from them astrometric quantities for future investigations. Thus all revisions discussed here are, to a great extent, directed toward assessing the accuracy and potential contribution of the observations. To complete this evaluation, we offer the following remarks, adding the hope that observations of mutual events will be continued. With regard to the Jovian satellites, we have argued that mutual events can provide longitude corrections that are more accurate than our earlier estimate of 0.03 arcsec by at least a factor of 2. Thus determinations with uncertainties of a few tens of kilometers are distinctly possible. Such accuracy means that continued observations may be able to detect, or set limits on, the effect of tidal dissipation on Io's orbit. Therefore, there is a definite need for careful photometry

TABLE V. Mean longitudes (l), inclinations (γ), and nodes (θ) for Tethys, Dione, and Rhea for JED 2444240.5.

Element	Struve	Kozai	Garcia	Sinclair*	Mutual events*
l_3	204°.769	204°.711	204°.978	204°.674 ± 0°.035 204.757 ± 0.035	204°.741 ± 0°.012
l_4	349.381	349.314	349.089	349.278 ± 0.025 349.303 ± 0.025	349.307 ± 0.009
l_5	205.590	205.535	205.405	205.507 ± 0.017 205.514 ± 0.017	205.510 ± 0.005
γ_3	1.093	1.094	1.073	1.009 ± 0.039 1.018 ± 0.036	1.051 ± 0.026 1.023 ± 0.021
γ_4	0.023	0.017	0.031	0.104 ± 0.035 0.050 ± 0.034	0.021 ± 0.021 0.034 ± 0.031
γ_5	0.361	0.369	0.362	0.381 ± 0.035 0.373 ± 0.034	0.339 ± 0.029 0.305 ± 0.023
θ_3	34.8	35.6	35.3	34.6 ± 2.1 37.1 ± 2.1	31.0 ± 2.0 34.6 ± 1.8
θ_4	268.9	158.2	326.8	128.6 ± 19.0 154.6 ± 41.0	305.0 ± 92.2 193.6 ± 21.3
θ_5	311.8	314.4	317.6	311.9 ± 5.3 318.0 ± 5.4	319.0 ± 3.4 314.4 ± 3.6

Notes to Table V

*Sinclair's two values of l, γ, and θ correspond to satellite positions relative to stars (upper values) and relative to Titan (lower values). For the mutual events the upper and lower values of γ and θ correspond to the upper and lower values of ΔZ (O–R) in Table IV. All errors are standard errors.

and exact timing of a reasonable number of the some 300 events, especially those involving Io, that are observable during 1985/1986 (see Aksnes and Franklin 1984 for predictions). Events having low-amplitude light curves (i.e., that are not total or annular) are of particular interest because they provide well-defined latitude as well as longitude corrections.

In the Saturnian system, we have found that successful observations of mutual events can readily be carried out with instruments of moderate aperture. Filters centered on the near-infrared methane bands are a decided asset, but good seeing is more important. The event of February 6 was measured with the 120-cm reflector at Cloudcroft, New Mexico, using a Johnson V filter, yet good seeing led to little scatter. On the other hand, seeing was poorer during all four of the Agassiz (A) observations (153-cm reflector) and the scatter considerable despite the use of a filter centered at 8000 Å with a 320-Å half-peak bandwidth. (A V filter under these conditions gave residuals more than half again as large as those listed in Table III.) Only clouds at critical times prevented the successful observation of a 3O4 event at Kitt Peak with one of the 40-cm instruments. Two events involving Mimas were recorded, but poor conditions lead to light curves too noisy for useful analysis. With better seeing, observations of events including that satellite seem possible. Is it worthwhile to pursue these observations subsequent to the Voyager encounters? Beyond the need for occasional accurate positions to preserve ephemerides, they are also required to provide precise masses for the inner resonant satellites. In both the Mimas-Tethys and Enceladus-Dione systems, the ratio of the amplitudes of the liberations in longitude determines the mass of the smaller body. Because amplitudes are small for Tethys ($\sim 2°$) and Dione ($\sim 0°.02$), these two quantities remain, once masses for Tethys and Dione are established, the principal sources of uncertainty in the masses of Mimas and Enceladus. An extensive series of accurate positions are clearly necessary for improvement. The $\sim$14-yr interval between apparitions providing mutual events is short compared to the $\sim$70 yr libration in the Mimas-Tethys system and comparable to the $\sim$11-yr period of Enceladus-Dione. Astrometric measures have also yielded seemingly accurate masses for Tethys and Dione. The former's mass, for example, follows once the two inclinations, γ_T and γ_M, and libration period, T, are known, viz., $M_T \propto (\gamma_T \gamma_M T^2)^{-1}$. Past studies (e.g., Kozai 1957) claimed that the standard error on this product is $\sim 2\%$. It was a matter of some surprise when a direct measure by Voyager (Tyler *et al.* 1982) gave $M_T = (1.33 \pm 0.16) \times 10^{-6}$, which differs by $\sim 20\%$ from the best "astrometric" value, $(1.095 \pm 0.022) \times 10^{-6}$. It seems unlikely that these orbital parameters can be sufficiently in error to resolve the discrepancy, but they should be checked by measurements precise enough to look also for variations in the inclinations, γ.

We are most grateful for the valuable assistance of R. E. McCrosky, G. Schwarz, P. Collins, D. Smith, and the staffs of the Wise and Sacramento Peak Observatories, especially R. Radick and J. Africano, and B. Andersen at HARESTUA.

APPENDIX

Occultations of stars—or other bodies—are increasingly being used to extend our knowledge of the sizes, shapes, and orbital motions of the occulting objects—usually smaller members of the solar system. Recently, there have been several occasions in which a predicted stellar occultation by a minor planet has been accompanied by secondary event(s) (Binzel and Van Flandern 1979; Williamon 1980). Such ocurrences provide the basic evidence favoring the binary nature of certain asteroids. Few, if any, secondary events have been observed under favorable conditions, a circumstance that is chiefly the consequence of the nature of the phenomenon. The purpose of this note is certainly not to detract from these important efforts; it is merely to record (as a sort of warning) a single instance of the observation of a secondary event, associated with a predicted occultation, which had, by chance, a clear and nonastronomical origin.

Figure 3 is a plot of the combined light, through a diaphragm 8 arcsec in diameter, of satellites S4 (Dione) and S5

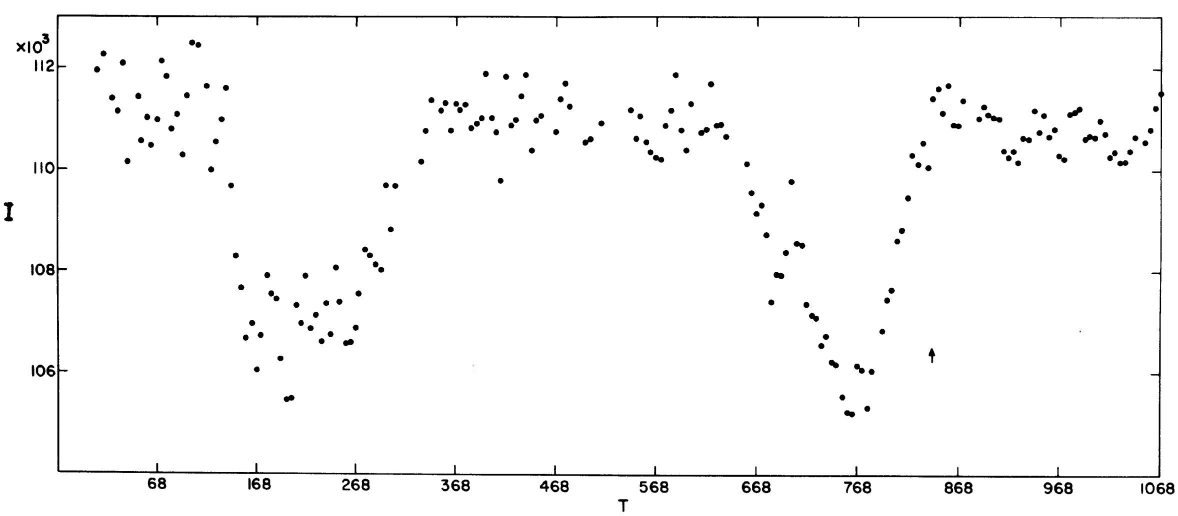

FIG. 3. Plot of the combined light of Saturn's satellites Dione and Rhea. Later minimum corresponds to the actual occultation; arrow marks the predicted midtime (cf. Aksnes and Franklin 1978). The earlier minimum was caused by the passage of a probable contrail whose identification was made possible only by bright moonlight. Time, T, is measured in seconds from the epoch: 1980 March 3, $03^h 44^m 56^s$ UT. Each point corresponds to the mean of five 1-s integrations using the 1.5-m telescope at Agassiz Station. Moonlit sky background of 87 800 has not been removed.

(Rhea) for some 20 min on 1980 March 3. The observations were made through an interference filter centered at 8000 Å when the satellites were 5.8 radii (56 arcsec) from Saturn's center. The second minimum whose midtime occurs ~90 s before the predicted time (arrow) corresponds to an occultation of S4 by S5. The earlier minimum is the anomalous one and might have remained a mystery but for the nearby presence of the 16-day-old Moon. Sky conditions were generally good, though a very thin, diminishing cirrus layer was visible in the moonlight. Part of the scatter in Fig. 3 is due to these clouds and indifferent seeing. Also visible to the observer was a faint discrete feature whose length and narrow fuzzy width suggested that it had originated as a persistent contrail from a jet aircraft. In any event, the passage of this extended linear feature over Saturn corresponded in midtime and duration to the first dip in Fig. 3. In the absence of moonlight, this definite identification would not have been possible. The long duration in this case may well have been unusual; normally a "contrail occultation" would be a good deal shorter.

What has prompted these remarks are the obvious but striking parallels between this observation and those supporting the existence of binary asteroids. It is unusual to have good words for the nearly full Moon, but in this instance it may have prevented bewilderment and/or a hasty discovery.

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