Mon. Not. R. astr. Soc. (1983) **202**, 1025–1047

New light on faint stars – III. Galactic structure towards the South Pole and the Galactic thick disc

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Received 1982 June 11; in original form 1982 March 11

Summary. We have derived absolute magnitudes from photometric parallaxes for a complete sample of ~ 12 500 stars brighter than $I = 18$ in 18.24 square degrees towards the South Galactic Pole. From these data we derive the stellar luminosity function in absolute visual magnitude, bolometric magnitude and mass in the solar neighbourhood for all absolute magnitudes above the thermonuclear burning limit. The total mass in main-sequence stars in the solar neighbourhood is $0.038\, m_{\odot}\,\mathrm{pc}^{-3}$, and the total mass-to-light ratio is $1.2\,(m_{\odot}/L_{V\odot})$.

We have also derived the density laws with distance from the Galactic plane for each absolute magnitude. The stellar luminosity function shows a systematic change in shape, with stars with $M_V \lesssim +4$ substantially more concentrated to the plane than fainter stars. The density laws for stars with $M_V \gtrsim +4$ follow a single exponential with scale height ~ 300 pc for $100 \lesssim z \lesssim 1000$ pc, and a second exponential with scale height ~ 1450 pc for z distances from ~ 1000 to at least 5000 pc. This second exponential contains ~ 2 per cent of the stars in the solar neighbourhood.

We identify the 300 pc scale height component as old disc, and the 1350 pc scale height component as a Galactic 'thick disc'. The luminosity function and density law of the 'thick disc' are consistent with substantial flattening of the spheroid isodensity contours by the gravitational potential of the Galactic disc, in agreement with recent observations of several external galaxies.

1 Introduction

The stellar mass density in the solar neighbourhood, and its variation with distance from the Galactic plane, is a fundamental aspect of high latitude Galactic structure which is amenable to direct observational determination. For main-sequence stars, the appropriate technique is that of photometric parallax. This involves the determination of a suitable colour for every star in a complete sample, and the consequent derivation of the intrinsic absolute magnitude from the relevant absolute magnitude–colour relation. Calculation of the space density for each absolute magnitude near the Sun (the luminosity function), and its variation with distance from the Galactic plane (the density law) is then straightforward.

In Paper I (Reid 1982) we applied this technique to a complete sample of low-luminosity dwarfs identified by their objective prism spectra. This enabled the calculation of the stellar luminosity function for a complete sample of intrinsically faint main-sequence dwarfs, independently of their kinematic properties. The resulting space densities were found to be in excellent agreement with those derived from kinematic (proper motion) studies, and provided no evidence for an intrinsically low-luminosity, low-velocity dispersion population of main-sequence stars to explain the local missing mass. This sample was, however, restricted only to stars with $+7 \leqslant M_V \leqslant +12$, within 50 pc of the Sun.

Extension of the absolute magnitude limit to $M_V = +19$ was achieved in Paper II (Reid & Gilmore 1982), by automatic measurement of the complete sample of stars brighter than $I = 17.0$ in 18.24 square degrees on UK Schmidt telescope plates of the South Galactic Pole. Again, the derived stellar luminosity function from our purely photometric sample was consistent with that derived by kinematic techniques. This allows us to conclude that any missing mass in the solar neighbourhood does not reside in main-sequence stars more massive than the theoretical minimum value for thermonuclear burning (Grossman, Hays & Graboske 1974).

In this paper we extend the results of Paper II to derive absolute magnitudes for the ~ 12 500 stars in our complete sample which do not lie within 100 pc of the Sun (Section 2). We are then able to derive directly the variations in the shape of the stellar luminosity function (Section 3), and the Galactic stellar number density gradient (Sections 4 and 5), to ~ 5 kpc from the Galactic plane. From this we deduce the existence of a Galactic 'thick disc' (Section 6).

2 Observations

2.1 PHOTOGRAPHIC RESULTS

The observations, their measurement, calibration and reduction are described in detail in Paper II. Only a very brief summary is given here.

Our primary data are three V (IIa-D + GG 495) and three I (IVN + RG 715) UK Schmidt telescope plates centred on RA $00^h 53^m$, Dec $-28° 03'$ (1950.0), very near the South Galactic Pole (SGP). These plates were measured on the COSMOS machine at ROE, and were carefully calibrated using some 250 photoelectric and ~ 1500 photographic standards. Considerable care was taken to ensure that:

(i) the standards define a single, uniform photometric system;

(ii) no significant position- or colour-dependent effects exist in the derived magnitudes; and

(iii) star–galaxy separation and galaxy removal with a reliability of $\gtrsim 98$ per cent was attained.

Absolute magnitude–colour relations for all 202 stars with both photometry on our photometric system and trigonometric parallaxes with an accuracy of $\lesssim 10$ per cent were then derived. The necessary Lutz–Kelker corrections to the absolute magnitudes to correct for the parallax uncertainties were applied. A subset of these stars, covering the full range in colour, was observed by us at the same time as our observations of the SGP standards, and ensure that no systematic differences between the photographic and photoelectric magnitude and colour systems exist. Full details are given in Paper II.

In Paper II we were specifically concerned with the derivation of the stellar luminosity function for very low mass stars. To do this we defined the complete sample of stars with $I \leqslant 17.0$ and $V\text{–}I \geqslant 2.4$ ($M_V \geqslant 14.5$), and observed each star individually in the infrared *JHK* system. The limit at $I = 17.0$ was set to allow a sample sufficiently small to be observable, and because our I magnitude photoelectric standards are all brighter than this value. As discussed in Paper II we also have secondary standards derived from Pickering prism plates, which we believe to be reliable for $I \lesssim 18$. For derivation of the Galactic density laws we have therefore defined the complete sample of ~ 12 500 stars in 18.24 square degrees with $V \leqslant 19.0, I \leqslant 18.0$.

Thus we have reliable $V\text{–}I$ colours for a complete sample of ~ 12 500 stars towards the South Galactic Pole. We also have an $M_V/V\text{–}I$ relation, with the two photometric colour systems known to be in very good agreement. This $M_V/V\text{–}I$ relation is, however, appropriate only to unreddened nearby main-sequence dwarfs with therefore approximately solar metallicity. The significance of this constraint is discussed below.

2.2 DERIVATION OF SPACE DENSITIES

It is well known that metal-poor stars are systematically sub-luminous relative to solar abundance stars with the same visual colour. If there is then a correlation of the mean metallicity of the stars in a sample with some other parameter of that sample, a systematic, absolute magnitude dependent error will ensue. The specific correlation relevant here is a gradient of mean metallicity with distance from the Galactic plane. The neglect of such a gradient will lead to a distance dependent over-estimate of the absolute luminosity of a star, and a consequent systematic error in the derived space densities.

The local value of the vertical abundance gradient $\partial\,[\mathrm{Fe/H}]/\partial z$ is not well determined. Current estimates cover most values from $-0.225\,\mathrm{kpc}^{-1}$ (Janes 1979) to $-0.8\,\mathrm{kpc}^{-1}$ (McClure & Crawford 1971; Jennens 1975; Yoss & Hartkopf 1979). A value of $\sim -0.30\,\mathrm{kpc}^{-1}$ is typical of recent determinations near the plane. Recent studies of field RR Lyrae (Butler, Kinman & Kraft 1979; Butler *et al.* 1982) and globular cluster (Zinn 1980; da Costa, Ortolani & Mould 1982) abundances show little if any systematic trend in [Fe/H] within 1–2 kpc of the Galactic plane, a dominance of objects with $[\mathrm{Fe/H}] \approx -1.5$ at 5 kpc, but little or no further systematic decrease beyond this distance (*cf.* also fig. 5 of Eggen, Lynden-Bell & Sandage 1962).

This evidence for a smooth gradient from ~ 1 to ~ 5 kpc is complicated by the results of Rodgers, Harding & Sadler (1981) who derived solar abundances for main-sequence A stars up to ~ 4.5 kpc from the plane, and of Yoss & Hartkopf (1979) who also derived near solar CN abundances for K giants up to 3 kpc. Similarly Butler *et al.* (1979) have shown that only moderately metal-poor ($[\mathrm{Fe/H}] \sim -1$) stars exist up to 20 kpc from the plane.

In view of these uncertainties, two models of the vertical metallicity gradient have been adopted, as indicative rather than conclusive estimates of the true gradient.

$$\frac{\partial\,[\mathrm{Fe/H}]}{\partial z} = 0.00 \text{ for all } z; \tag{1}$$

and

$$\begin{aligned}\frac{\partial\,[\mathrm{Fe/H}]}{\partial z} &= -0.30\,\mathrm{kpc}^{-1}, \quad 0 \leqslant z \leqslant 5\,\mathrm{kpc}\\ &= 0.00 \qquad\qquad\quad z > 5\,\mathrm{kpc}.\end{aligned} \tag{2}$$

The second gradient is linear to 5 kpc, then assumes $[\mathrm{Fe/H}] = -1.5$ for all larger distances.

It is now necessary to calculate the effect of this metallicity gradient on the absolute magnitude–colour relation. Hanson (1979) has reanalysed available sub-dwarf parallaxes, and derived the following relation between ΔM, the distance a star lies below the (solar abundance) main sequence, and $\delta(U-B)_{0.6}$, its ultraviolet excess normalized to $B-V=0.6$:

$$\Delta M_V = -0.35 + 5.41\,\delta(U-B)_{0.6}.$$

To relate ultraviolet excess to metallicity, [Fe/H], we adopt the calibration of Carney (1979). Because this is non-linear, the final $\Delta M_V/[\mathrm{Fe/H}]$ relation becomes:

$$\begin{aligned}\Delta M_V &= 0.23\,z && 0 \leqslant z \leqslant 2\,\mathrm{kpc}\\ &= 0.46 + 0.13\,(z-2), && 2 < z \leqslant 5\,\mathrm{kpc}\\ &= 0.85 && z > 5\,\mathrm{kpc}.\end{aligned}$$

The absolute magnitudes calculated from the $M_V/V-I$ relation were therefore corrected by these amounts. In practice, a new distance is calculated, a revised ΔM_V correction applied, and so on to convergence to 0.03 mag.

This distance modulus and absolute magnitude are not corrected for reddening and extinction. Several authors have measured the interstellar absorption towards the SGP. Typical values in this field are $E(B-V) \lesssim 0.03$ mag (Bond, Perry & Bidelman 1971; Rodgers 1971; Philip & Tifft 1971; Appenzeller 1975; Erikkson 1978; Albrecht & Maitzen 1980), with a patchy distribution being apparent. This then leads to an uncertain reddening in $V-I$ of $\lesssim 0.02$ mag, and extinction in V of $\lesssim 0.10$ mag. Inspection of fig. 1(a) of Albrecht & Maitzen (1980) shows that the area covered in this study is in fact consistent with zero reddening. Similarly, a detailed study of the two-point correlation function for the stars in this field with derived distances greater than 1000 pc from the Sun (Hewett, Reid & Gilmore 1982, in preparation) gives no evidence for variable obscuration of more than ~ 0.02 mag.

A systematic reddening of 0.02 mag in $V-I$ leads to a systematic error in derived absolute magnitude of $\lesssim +0.1$ mag, in the sense that the derived distance modulus is too small. The extinction in V however (~ 0.1 mag) leads to an approximately compensating error in the opposite sense. There is therefore no significant error in the derived space densities. As the maximum systematic error of ~ 0.1 mag in M_V is small by comparison with the systematic uncertainties due to our crude knowledge of the metallicity gradient, we have therefore made no corrections for reddening or extinction.

Finally, we have an absolute magnitude, corrected for both the metallicity gradient and Malmquist bias (*cf.* Paper II for details), and a corresponding distance modulus, for every star in the sample. The consequent calculation of the stellar space density, as a function of both absolute magnitude and distance from the Galactic plane is straightforward.

The final distributions with absolute magnitude and distance are presented in Tables 1 and 2 for our two adopted metallicity gradients. In these tables, the density law with height for a given absolute magnitude is read across a row, while the luminosity function at a given distance from the Galactic plane is read down a column. These two representations of the data are discussed in turn below.

3 The stellar luminosity function

The stellar luminosity function, or number of stars per unit volume of space as a function of absolute magnitude, is the primary observational material for study of the initial mass function in the solar neighbourhood, the local mass density of luminous stars, and the age and structure of the various stellar populations in the galaxy (Miller & Scalo 1979, henceforth MS). The initial mass function is derived, together with estimates of the stellar creation

Table 1. Number of stars with $V \leqq 19, I \leqq 18$ in 18.24 square degrees towards the South Galactic Pole, assuming no vertical metallicity gradient. The data near $M_V = 6.5$ may be contaminated by K giants, and should be used with caution.

M_V	log z																		
	2.05	2.15	2.25	2.35	2.45	2.55	2.65	2.75	2.85	2.95	3.05	3.15	3.25	3.35	3.45	3.55	3.65	3.75	3.85
$\leqq$ 3.0	1	–	2	–	4	6	1	6	2	6	5	6	8	6	5	17	31	35	52
3.5	3	1	3	7	16	17	32	41	65	68	60	73	70	109	139	196	232	252	291
4.5	3	7	19	22	38	50	79	107	135	149	197	207	218	233	265	301	322		
5.5	4	14	22	23	45	67	104	126	158	175	170	208	204	242	264	320			
6.5	11	25	34	42	49	91	116	141	174	192	201	250	348	523					
7.5	3	4	11	24	43	55	113	116	130	197	250	340	469						
8.5	5	7	14	15	32	55	73	117	169	203	312								
9.5	8	11	16	23	53	76	118	191	239										
10.5	7	14	28	36	60	99	149												
11.5	17	16	32	37	52														
12.5	11	16																	

Table 2. Number of stars with $V \leqq 19, I \leqq 18$ in 18.24 square degrees towards the South Galactic Pole, assuming a metallicity gradient $d\,[\mathrm{Fe/H}]/dz = -0.30\,\mathrm{kpc}^{-1}$. The data for $M_V = 6.5$ may be contaminated by K giants, and should be used with caution.

M_V	log z																		
	2.05	2.15	2.25	2.35	2.45	2.55	2.65	2.75	2.85	2.95	3.05	3.15	3.25	3.35	3.45	3.55	3.65	3.75	3.85
$\leqq$ 3.0	1	–	2	–	3	6	2	2	2	2	4	4	3	1	3	5	5	8	8
3.5	3	2	3	6	14	21	29	42	52	52	52	26	39	54	67	90	109	92	125
4.5	3	8	20	23	46	53	76	121	149	166	196	209	253	269	329	370	394		
5.5	4	13	25	28	47	66	120	151	146	209	227	245	249	305	377				
6.5	13	24	34	45	51	108	128	165	176	202	220	279	382	460					
7.5	4	6	12	27	47	63	113	116	159	243	327	465							
8.5	5	7	14	16	33	59	82	140	194	285									
9.5	7	11	17	24	58	90	121	195											
10.5	8	14	28	45	64	104													
11.5	18	18	33	41															
12.5	10	19																	

rate, directly from the observed luminosity function and the mass luminosity relation (*cf.* MS for details), while an age estimate of a stellar population is possible from determination of the main-sequence turn-off point. One or both of these properties is likely to vary with distance from the Galactic plane, as current observations and Galactic evolutionary models both suggest that age, mean metallicity, and distance from the Galactic plane are correlated for a given population. In addition, the local stellar mass density is not well defined, with consequent important effects for the reality and extent of any local 'missing mass' (*cf.* Paper I for details and a review). It is therefore of interest to determine the solar neighbourhood luminosity function over the widest possible range, the resulting mass density, and the variation of these with height above the Galactic plane.

3.1 THE SOLAR NEIGHBOURHOOD FUNCTION

3.1.1 Correction to the Galactic plane

The main-sequence stellar luminosity function for $9 \leqslant M_V \leqslant 19$ was derived and discussed in Paper II for a complete sample of stars within 100 pc of the Sun. For comparison with earlier results, and to derive a total local mass density, it is convenient to correct this derived function to its value at $z = 0$. This is possible using the results of Section 4.2 below.

Anticipating those results, we know that main-sequence stars with $3.0 \leqslant M_V \lesssim 11.0$ are distributed above the Galactic plane between $z = 100$ pc and $z = 1000$ pc according to an exponential with scale height ~ 300 pc, independently of the assumed mean metallicity gradient over this interval. If we then *assume* that this density law may be extrapolated to $z = 0$ and to $M_V = +19$, we can easily correct the luminosity function of Paper II to $z = 0$. It must be emphasized, however, that we have no direct evidence for the density law followed by very low mass stars (*cf.* Section 4.1 for a discussion). The assumption of a single scale height for stars with $M_V \geqslant +11$, identical to that for brighter stars, is equivalent to the

Table 3. The stellar luminosity function in the plane.

M_V	log $\Phi^{\star}$ (stars mag^{-1}) pc^{-3}	log Φ (Wielen)	log Φ (Luyten)
9.0	$-2.56^{+0.24}_{-0.38}$	−2.32	−2.09
10.0	$-2.16^{+0.12}_{-0.17}$	−2.14	−2.02
11.0	$-2.01^{+0.10}_{-0.14}$	−1.99	−1.95
12.0	$-1.70^{+0.08}_{-0.09}$	−1.82	−1.90
13.0	$-2.04^{+0.11}_{-0.14}$	−1.75	−1.85
14.0	$-2.16^{+0.12}_{-0.17}$	> -1.99	−1.86
15.0	$-2.46^{+0.16}_{-0.26}$		
16.0	−2.8:	> -2.20	−1.90
17.0	< −2.5:		−2.02
18.0	< −2.3:	$\geqslant -3.23$	−2.1
19.0	−1.7:		−2.2

* Corrected to the Galactic plane as discussed in the text.

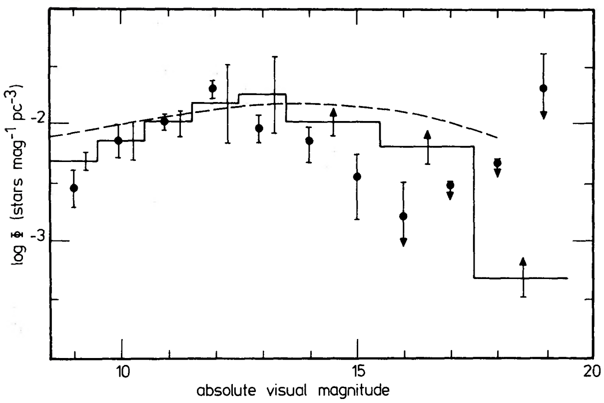


Figure 1. The final luminosity function, corrected to the solar neighbourhood as described in the text (solid points, Poisson error bars). Also shown are the functions derived by Wielen (1974; solid line, Poisson error bars) and Luyten (1968; broken line).

assumption that the initial mass function has not changed significantly for stars above and below this limit during the lifetime of the galactic disc. It is therefore physically plausible.

The assumption of a single exponential extrapolation to $z = 0$ is more questionable. We know (Section 4.1) that stars with $M_V \lesssim +3$ follow an exponential density decrease with scale height ~ 100 pc and commonly identify this population as 'young disc'. This density law must apply to an entire luminosity function, of which only the high mass component is not greatly outnumbered by older stars with a larger scale height. Some (unknown) fraction of lower mass stars must therefore follow this density law. Our neglect of this fraction will lead to an underestimate of the true space density at $z = 0$. This underestimate will be constant for all M_V, unless the young disc initial mass function is significantly different than that for older stars. The amplitude of this underestimate may be estimated by comparing the absolute value of our luminosity function, corrected to $z = 0$, with those derived by Wielen (1974) and Luyten (1968) (fig. 1) who have sampled many more stars near the Sun than are in our function. No significant difference is detectable.

Our corrected luminosity function, together with those of Wielen and Luyten (as tabulated by MS) is presented in Table 3 and Fig. 1. No metallicity gradient for $z \leqslant 100$ pc was adopted. A gradient of $-0.30\,\mathrm{kpc}^{-1}$ changes the derived space densities by < 0.01 in $\log \phi$, which is negligible by comparison with Poisson statistics.

The uncertainties in this function have been considered by MS, by comparison of all the available determinations. They derive final uncertainties of ± 0.15 in $\log \phi$ for $M_V \lesssim -2$, and ± 0.10 in $\log \phi$ for $-2 \lesssim M_V \lesssim +10$. For fainter magnitudes, the formal Poisson sampling errors are shown in Table 3.

3.1.2 The bolometric luminosity function

Bolometric corrections are tabulated by Allen (1973) for stars with $M_V \leqslant +9$, with an accuracy adequate for this discussion. For fainter stars the absolute visual magnitude–bolometric correction relation may be derived from the results of Veeder (1974) and Greenstein, Neugebauer & Becklin (1970). Their results, together with an approximate polynomial fit to the data, are shown in Fig. 2(a).

To derive the corresponding bolometric luminosity function, we have adopted the visual magnitude function for $-6 \leqslant M_V < +2$ tabulated by MS, that for $2 \leqslant M_V \leqslant +10$ tabulated by Wielen, and that of Section 3.1.1 for $M_V > +10$. The resulting total luminosity function for

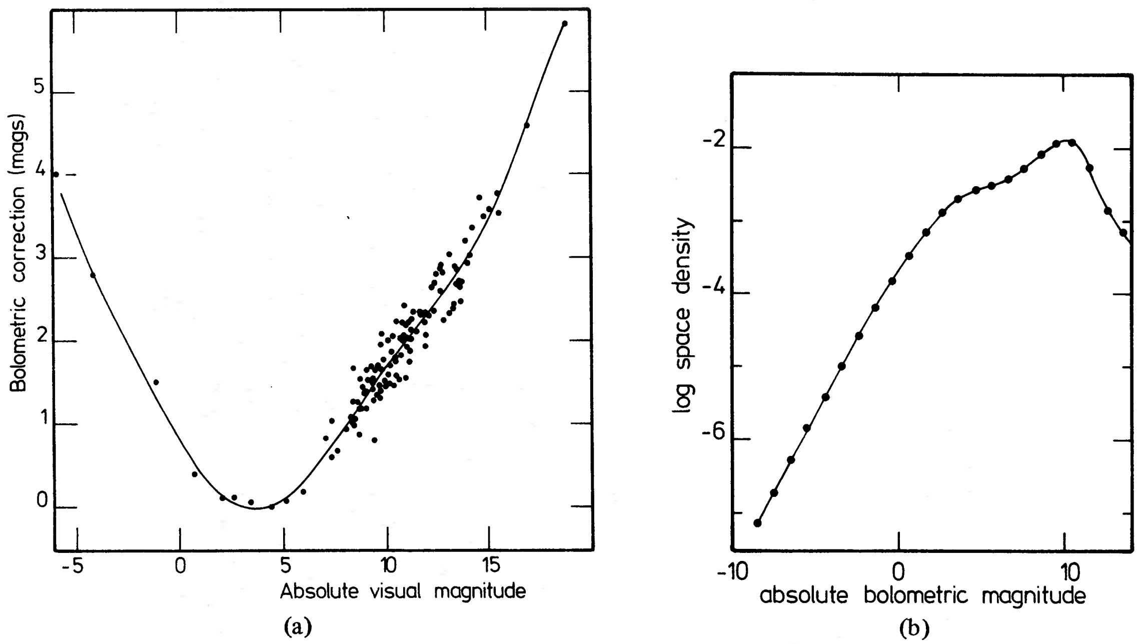


Figure 2. (a) The adopted relation between absolute magnitude and bolometric correction. (b) The luminosity function in absolute bolometric magnitude resulting from Figs 1 and 2(a).

$-6 \leqslant M_V \leqslant +19$ is presented in Table 4(a), along with the resulting bolometric luminosity function in Table 4(b). The bolometric function is also shown in Fig. 2(b). The formal uncertainties in log ϕ (M_{BOL}) are ~ 0.2 for $M_{BOL} \lesssim +10$, and increase rapidly for fainter stars.

3.1.3 The stellar mass function and total mass density

The main-sequence mass–luminosity relation has been recently reviewed by Popper (1980), who summarizes all available reliable estimates of stellar masses. These, together with our adopted mass–luminosity relation, are shown in Fig. 3(a). The resulting stellar mass function, again using the luminosity function of Table 4(a), is presented in Table 4(c) and Fig. 3(b).

The uncertainties in these values have again been estimated by MS, whose (independently derived) stellar luminosity function, main-sequence mass–luminosity relation, and absolute magnitude–scale height relation (Section 4) are all in very good agreement with those of this paper. The uncertainties quoted by MS are +0.4/–0.2 in the present day mass function for low mass stars. The agreement of our results with those of MS supports both these estimated confidence limits, and the resulting conclusions about the history of the solar neighbourhood stellar creation rate.

Integration of this function provides the total mass in main-sequence stars in unit volume of space in the solar neighbourhood with masses in the range $-1 \leqslant \log m/m_\odot \leqslant 1.5$. The derived value is $0.038\,m_\odot\,\mathrm{pc}^{-3}$ with a corresponding mass to light ratio of 1.2. The formal error of these numbers is difficult to evaluate, as the dominant contribution to the luminosity is from young stars $M_V \lesssim +3$, while the mass is predominantly in stars near $M_V \sim +12$. Neither region of the luminosity function is well determined in the solar neighbourhood. As the luminosity function used to derive this limit includes all stars above the theoretical minimum mass for thermonuclear burning, a significant ($\gtrsim 10$ per cent) revision of these values is possible only if the stellar population represented in the cone sampled in this project is atypical of the solar neighbourhood (*cf.* also Tinsley 1981). We are currently completing analysis of a similar study of several other fields in order to test this possibility.

Table 4. The stellar luminosity function.

(a)

M_V	$\Phi^{\star}$
−6	1.78 E−8
−5	7.94 E−8
−4	3.55 E−7
−3	1.58 E−6
−2	7.08 E−6
−1	3.16 E−5
0	1.20 E−4
1	3.31 E−4
2	7.24 E−4
3	1.28 E−3
4	2.32 E−3
5	3.22 E−3
6	3.61 E−3
7	3.04 E−3
8	3.94 E−3
9	3.98 E−3
10	7.08 E−3
11	1.00 E−2
12	1.58 E−2
13	1.38 E−2
14	7.94 E−2
15	4.47 E−3
16	2.82 E−3
17	1.58 E−3
18	1.00 E−3
19	6.3 E−4
20	3.5 E−4

(b)

M_{bol}	$\Phi^{\star}$
−8.5	7.16 E−8
−7.5	1.87 E−7
−6.5	5.11 E−7
−5.5	1.41 E−6
−4.5	3.82 E−6
−3.5	1.01 E−5
−2.5	2.60 E−5
−1.5	6.37 E−5
−0.5	1.50 E−4
0.5	3.29 E−4
1.5	6.63 E−4
2.5	1.20 E−3
3.5	1.89 E−3
4.5	2.54 E−3
5.5	3.00 E−3
6.5	3.51 E−3
7.5	4.86 E−3
8.5	7.85 E−3
9.5	1.16 E−2
10.5	1.16 E−2
11.5	5.6 E−3
12.5	1.3 E−3
13.5	7.5 E−4

*Unit = stars mag^{-1} pc^{-3}.

(c). The stellar mass function.

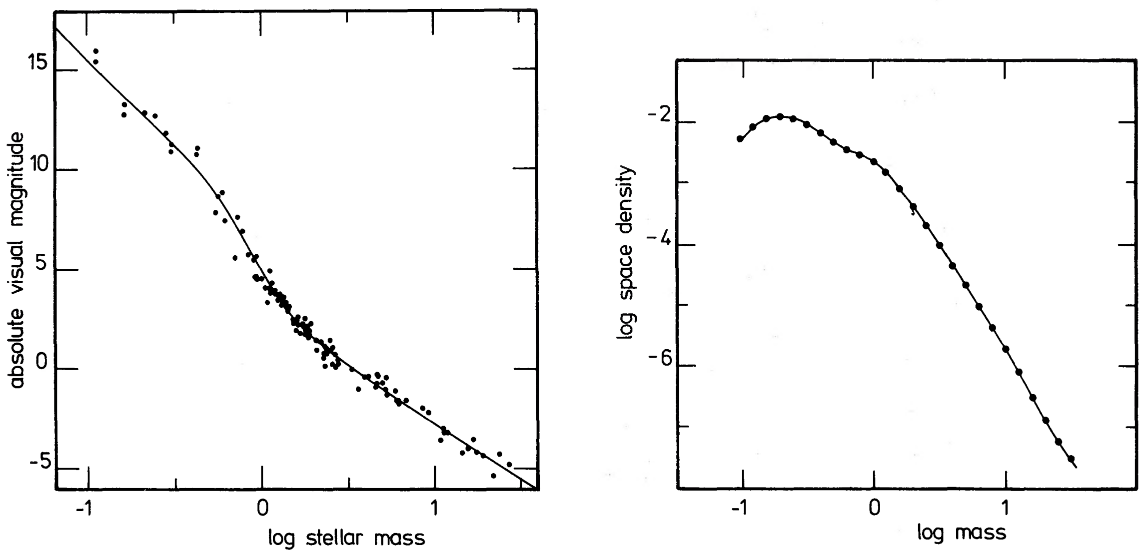
$\log\left(\frac{m}{m_0}\right)^{\dagger}$	Φ (stars/pc^3/0.1 log m_0)
−1.0	5.21 E−3
−0.9	8.48 E−3
−0.8	1.14 E−2
−0.7	1.26 E−2
−0.6	1.18 E−2
−0.5	9.52 E−3
−0.4	6.94 E−3
−0.3	4.82 E−3
−0.2	3.55 E−3
−0.1	2.96 E−3
0.0	2.33 E−3
0.1	1.50 E−3
0.2	8.29 E−4
0.3	4.17 E−4
0.4	2.01 E−4
0.5	9.54 E−5
0.6	4.47 E−5
0.7	2.09 E−5
0.8	9.65 E−6
0.9	4.40 E−6
1.0	1.96 E−6
1.1	8.21 E−7
1.2	3.20 E−7
1.3	1.33 E−7
1.4	6.17 E−8
1.5	2.94 E−8

† Quoted value is lower limit of mass bin.

3.2 VARIATION OF THE LUMINOSITY FUNCTION WITH *z* DISTANCE

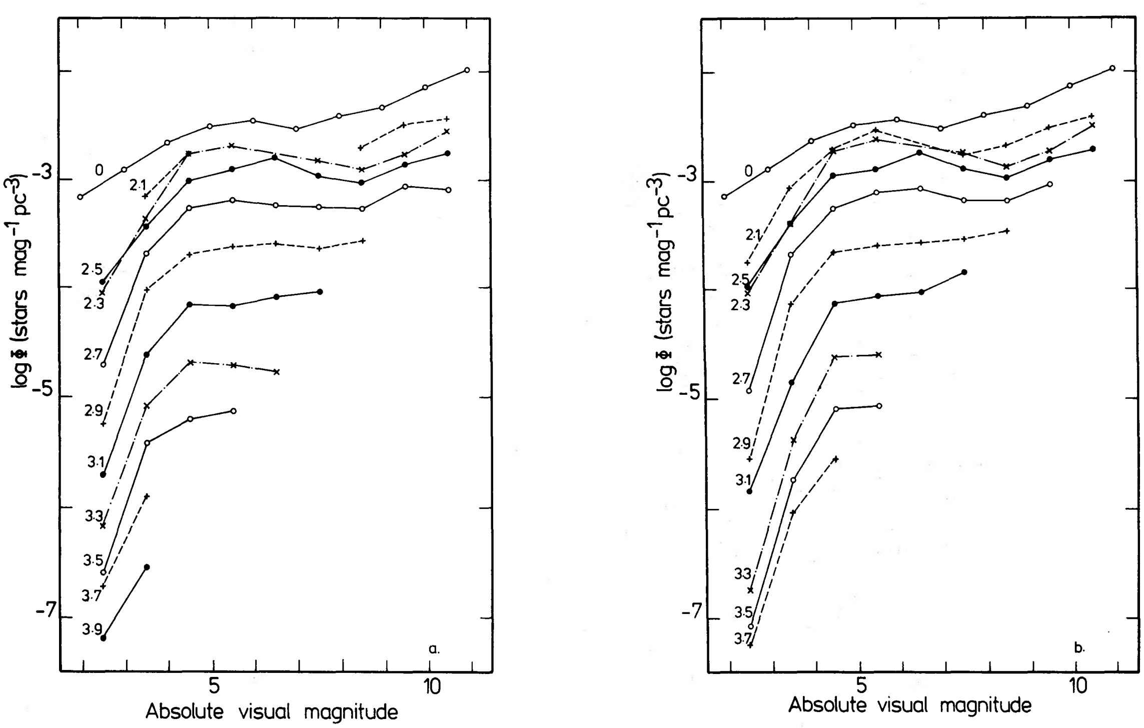
It has been known for many years that the stellar luminosity function changes shape with distance from the Galactic plane, with intrinsically luminous stars ($M_V \lesssim +3$) becoming less common, relative to less luminous stars (Bok & MacRae 1941; Luyten 1960; Upgren 1963; Chiu 1980a). There have, however, been very few programmes with a sufficient number of stars studied to directly determine the systematic changes of the luminosity function with height, to a sufficiently large distance that the spheroid luminosity function becomes predominant. The luminosity functions derived in this project are capable of illustrating this change, and are shown, for 10 different distance intervals from the plane ($\log z = 2.1$ to 3.9 in steps of 0.2) in Fig. 4(a) and (b) for the two adopted metallicity gradients.

The most obvious feature of the diagram is the rapid steepening of the luminosity function for $M_V \lesssim +4$ with z distance. This steepening is more clearly seen in Table 5, which presents the ratios of the space density of stars with $4.0 \leqslant M_V < 5.0$ to those with $1.0 \leqslant M_V < 3.0$ and $3.0 \leqslant M_V < 4.0$ as a function of z distance. The systematic relative decrease in the space density of stars with $M_V \lesssim +3$ is evident, continuing to at least 2 kpc. Also apparent is the significant change in the relative numbers of stars brighter and fainter than $M_V \sim +4$ at $z = 1$ kpc. This change, together with the density law data discussed in Section 5.2, is evidence for a significant change in the dominant stellar population at this distance from the plane. The significance of this is discussed below (Section 6).



(a) (b)

Figure 3. (a) The adopted mass–luminosity relation, from data in Popper (1980). (b) The stellar present day mass function in solar mass units. An indication of the uncertainties at the low mass end may be derived by assuming the luminosity function of Fig. 1 is constant for all absolute magnitudes below $M_V = +12$. In this case the mass function is constant near $\Phi = 1.5 \times 10^{-2}$ for $\log m \leqslant -0.8\, m_\odot$.



(a) (b)

Figure 4. (a) Variation of the stellar luminosity function with distance from the Galactic plane. The logarithmic distances from the plane are marked for each function. The Wielen function in the plane is also shown. This figure assumes no metallicity gradient with distance from the plane. (b) As for Fig. 4(a), but assuming a vertical metallicity gradient $\partial\,[\mathrm{Fe/H}]/\partial z = -0.3\ \mathrm{kpc}^{-1}$.

Table 5. Log (z distance).

	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5
$\dfrac{\phi(4.5)}{\phi(2.5)}$	1.05 ± 0.09	1.33 ± 0.08	1.04 ± 0.03	1.69 ± 0.06	1.52 ± 0.05	1.70 ± 0.04	2.12 ± 0.06	1.96 ± 0.04
$\dfrac{\phi(4.5)}{\phi(3.5)}$	0.35 ± 0.02	0.68 ± 0.03	0.45 ± 0.01	0.44 ± 0.01	0.48 ± 0.01	0.71 ± 0.01	0.75 ± 0.01	0.64 ± 0.01

4 Density law near the Galactic plane

4.1 AVAILABLE EVIDENCE

It is commonly assumed that the stellar space density distribution perpendicular to the Galactic plane is well approximated by an exponential density law. The evidence for this is summarized in fig. 2 of Bahcall & Soneira (1980; henceforth BS) and fig. 3 of MS. Two features of this data are immediately apparent – the consistent agreement from such a large number of determinations, and the wide range in absolute magnitude sampled ($-6 \leqslant M_V \leqslant +16$).

Many of these data points are, however, reanalyses of the same primary data, often including corrections for systematic errors, and are therefore not independent. Fortunately, Schmidt (1963) has reanalysed all earlier density analyses and published scale heights for stars with $-5 \leqslant M_V \leqslant +9$. (Both BS and MS plot data from this reference to $M_V = +16$. The source of this fainter data is unclear.) An extensive review is given by Elvius (1965) of other pre-1965 data, primarily from his own work and that by Upgren (1962, 1963), while more recent work has concentrated on determining the density law for specific classes of luminous stars, usually G–K giants or A–F dwarfs.

For A–F dwarfs, Hill, Hilditch & Barnes (1979) identified both a systematic error in the magnitude scale of the data analysed by Upgren (1963) and a recalibration necessary in his absolute magnitude–spectral type relation. With this recalibration they derived density laws for A and F dwarfs towards the NGP. These results are *not* well fitted by an exponential, but may be approximated as such over a short distance interval. Specifically, while the F stars are roughly consistent with a single exponential with scale height ~ 125 pc to $z \sim 500$ pc, the A stars follow an exponential with scale height ~ 90 pc to only $z \sim 300$ pc, and then require a second exponential with scale height ~ 170 pc thereafter.

At larger distances above the plane the status of A stars is even less clear, with analyses by Greenstein & Sargent (1974), Rodgers (1971), Chromey (1980) Rodgers *et al.* (1981) and Tobin & Kilkenny (1981) identifying many metal rich main-sequence OBA stars of solar metallicity at very large distances from the Galactic plane.

A similar result was derived by Woolley & Stewart (1967) in their analysis of several earlier studies of A dwarfs. While they emphasize (as do Hill *et al.*) that a model similar to those of Camm (1950) was required to explain both the density and radial velocity data, the density law they derive is reasonably well fit by a single exponential of scale height ~ 90 pc to $z \sim 300$ pc, then a second component with scale height $z \sim 150$ pc to at least 500 pc. Eriksson (1978) derived density laws in the SGP for giants and dwarfs with $M_V \lesssim +8$. Again he required a three-component Gaussian fit to model his density and velocity data. (The need for several Gaussian components when velocity data are available is a universal conclusion of attempts to derive the Galactic force law K_z.) A reasonable approximation for stars nearer than ~ 1 kpc is a single exponential, while for larger distances the data require a second exponential with scale height ~ 700–1000 pc. The third exponential component in the density data does not dominate at least until 3 kpc. These results are in reasonable agreement with an earlier study of the same field by Bok & Basinki (1964) and those presented in this paper.

For intrinsically faint ($M_V \gtrsim +5$) stars, only the analyses of Faber *et al.* (1976) Sandage & Katem (1977) and Chiu (1980a, b) are relevant. Faber *et al.* (1976) reanalysed the data of Weistrop (1972), by modelling her counts to simultaneously derive both the shape of the luminosity function and the scale height of the disc, for an assumed single exponential. They derive a scale height of ~ 300 pc at $M_V \sim +17$. This value is however very uncertain, as only 27 stars in Weistrop's sample are fainter than $M_V \sim 12.5$, and only another 23 fainter than $M_V \sim +11$.

Sandage & Katem (1977) discuss the implications for the scale height of the disc of counts of M dwarf field stars projected on the field of the cluster M15. From this they derive a scale height of ~ 470 pc for $+8 \leqslant M_V \leqslant +12$, and comment that the disc dominates the Galactic density at least to 2 kpc. As this result is based on a reliable sample of only 11 stars at $V = 19$–20, the uncertainties are large.

Chiu (1980a, b), in an extensive study of faint proper motion stars was forced to model the disc density law rather than derive it. For stars with $+4 \leqslant M_V \leqslant +13$, a scale height in the range 300 to 500 pc was adopted. As Chiu emphasizes, however, he assumed an exponential density law, and did not determine it directly.

4.2 DENSITY LAWS FROM OUR DATA

Using the results of Table 2 we are able to derive the exponential scale height of the stellar density distribution over the range for which it is a reasonable approximation. In deriving this, we exclude our density law data for all stars with $V \geqslant 18.5$, for all K stars with $V \leqslant 14$, and for all distances outside the interval $100\,\text{pc} \leqslant z \leqslant 1000\,\text{pc}$. These precautions are necessary to minimize galaxian contamination of the sample, minimize contamination of the GK dwarf sample by more distant giants, and to restrict the distance interval to the interval where a single exponential is a good fit. The 100 pc limit then arises both because we have very few stars nearer than this and because of possible contamination by young disc stars. The outer distance limit of $z = 1000$ pc arises from inspection of Fig. 6 in which it is apparent that the exponential fit is no longer valid beyond this distance. Weighted least-squares exponential scale heights were derived, and are presented in Table 6 both for the

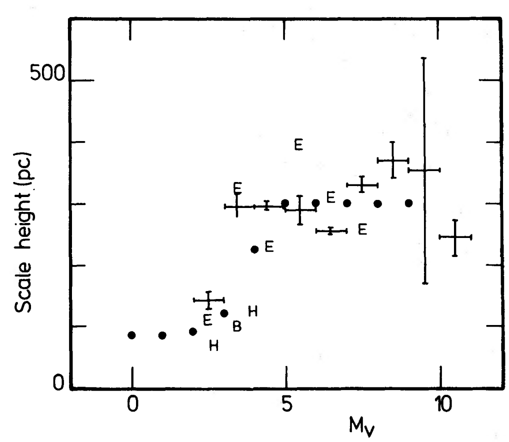


Figure 5. Variation of the disc scale height for $2.0 \lesssim \log z \lesssim 3.0$ with absolute magnitude. Solid points are from Schmidt (1963), letters are as follows: E – Eriksson (1978); B – Bok & Basinski (1964); H – Hill *et al.* (1979); and points with error bars are from this paper. These results are not sensitive to the adopted metallicity gradient.

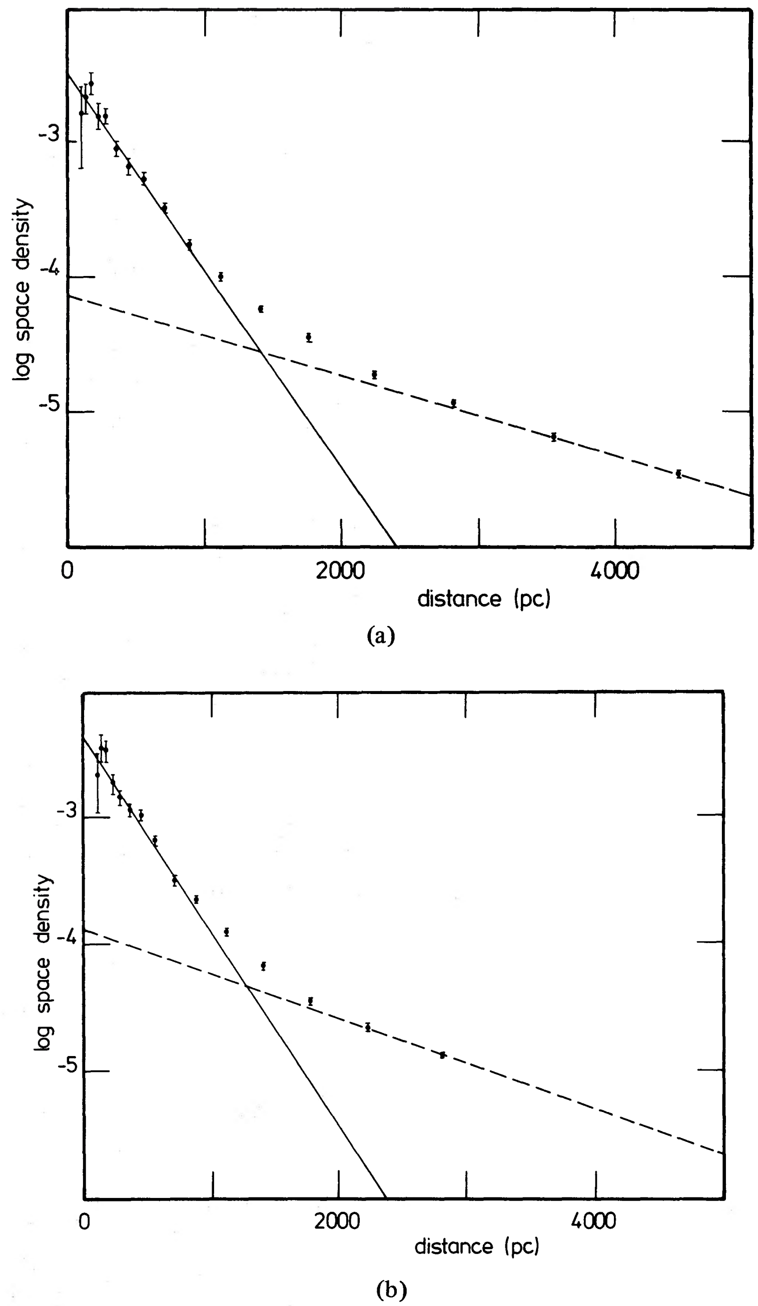


Figure 6. (a) The density distribution for stars with $4 \leqslant M_V \leqslant 5$ with distance from the Galactic plane. On this scale, a straight line represents an exponential decrease. The fitted lines correspond to exponentials with scale height 300 pc (solid line) and 1350 pc (broken line), and correspond to the 'old disc' and 'thick disc' respectively. (b) As for Fig. 6(a), for stars with $5 < M_V \leqslant 6$.

case of no metallicity gradient perpendicular to the plane, and for

$$\frac{\partial\,[\mathrm{Fe/H}]}{\partial z} = -\,0.30\,\mathrm{kpc}^{-1}.$$

Fig. 5 presents the final data for the variation of scale height with absolute magnitude for main-sequence stars. The weighted mean value for all unevolved stars with $M_V \gtrsim +3$ is 302 ± 6 pc for no metallicity gradient, and 303 ± 55 pc with our adopted value, excluding the value at $M_V = 6.5$ because of possible giant contamination. These values are appropriate

Table 6. Disc exponential scale heights.

$\frac{\partial\,[\mathrm{Fe/H}]}{\partial z} = 0.0$		$\frac{\partial\,[\mathrm{Fe/H}]}{\partial z} = -0.3\,\mathrm{kpc}^{-1}$	
M_V	Scale height	M_V	Scale height
2.5	175 ± 32	2.5	147 ± 24
3.5	356 ± 14	3.5	296 ± 19
4.5	283 ± 14	4.5	294 ± 12
5.5	285 ± 6	5.5	288 ± 23
6.5	272 ± 48	6.5	253 ± 6
7.5	307 ± 3	7.5	331 ± 15
8.5	310 ± 24	8.5	369 ± 35
9.5	284 ± 124	9.5	352 ± 179
10.5	251 ± 12	10.5	244 ± 29
mean*	302 ± 6	mean*	303 ± 55

*Weighted mean excluding $M_V = 2.5$ and $M_V = 6.5$ bins.

to the stellar population commonly referred to as 'old disc', and adequately describe the density decrease of the stellar population in the Galaxy from ~ 100 to ~ 1000 pc from the plane. The situation at greater distances is discussed below.

5 The density law beyond $z = 1$ kpc

As the results of Section 5.2 below indicate that the dominant component of the vertical structure of the Galaxy for 1 kpc $\leqslant z \leqslant$ 5 kpc is *not* well described by the commonly adopted two component Galactic models, it is important to discuss existing evidence in favour of a simple disc-plus-spheroid model of the Galaxy.

5.1 EXISTING DETERMINATIONS

5.1.1 Globular clusters as tracers

The globular cluster distribution in the Galaxy has been shown to follow the density law

$$\rho(r) \propto r^{-3.26 \pm 0.23}$$

(Harris 1976; Frenk & White 1980), with r the galactocentric distance, while de Vaucouleurs & Buta (1978) note that their distribution is also consistent with the $r^{1/4}$ law, when seen in projection. Some caution must be applied before assuming that this value also applies to all field Population II objects, however. Forte, Strom & Strom (1981) have shown that the globular cluster distributions in four Virgo cluster ellipticals are significantly more extended than the Galactic (stellar) luminosity, and are also significantly bluer at all galactocentric distances. They interpret this to mean that the globular clusters form a component which is both chemically and dynamically distinct from the underlying galactic spheroid, although in our Galaxy the available evidence is consistent with similar distributions of metallicity for both globular clusters and field stars (compare fig. 9 of Searle & Zinn 1978 with fig. 3 of Butler *et al.* 1979).

5.1.2 RR Lyrae stars as tracers

Major investigations of the Galactic z density gradient using field RR Lyraes have been reviewed by Plaut (1965) and presented by Kinman, Wirtanen & Janes (1966), Kinman, Mahaffey & Wirtanen (1982) and Oort & Plaut (1975).

Plaut (1965, fig. 7b) plots the space density of RR Lyraes perpendicular to the Galactic plane for various ranges of distance from the Galactic Centre. These data are quite well represented by the law

$$\frac{\partial \log \rho(z)}{\partial z} \approx -0.18z; \qquad z \leqslant 15\,\text{kpc}$$

which corresponds to an exponential decrease with scale height ~ 2.3 kpc.

Kinman *et al.* (1966, 1982) derived a complete sample of 54 large amplitude variables in 74 square degrees near the North Galactic Pole, and 62 variables in three other fields, and derived the density law

$$\frac{\partial \log \rho(z)}{\partial z} \approx -0.071z$$

which is valid for $5 \lesssim z \lesssim 20$ kpc. The data are also equally well fitted by a power law decrease $\rho \propto r^{-3.4}$, with roughly spherical isodensity contours with galactocentric distance r. Their first density law above coresponds to an exponential with scale height ~ 6 kpc.

Oort & Plaut (1975) also derived a galactocentric power-law density decrease with exponent ~ – 3.0. This value is very uncertain, as their range and distribution of z distances is very limited.

5.1.3 Field star determinations

The only extensive surveys of the stellar population in the Galactic halo, using complete samples of unevolved stars away from the Galactic plane, with machine measured magnitudes, are those by Weistrop (1972), Chiu (1980a, b) and the Basle group (Becker 1980, and references therein).

Weistrop (1972) carried out photographic *UBV* photometry of ~ 14 000 stars in 13.5 square degrees near the North Galactic Pole. Her data were not sufficiently accurate to allow direct determination of the Population II density law, but were found to be in agreement with a model consisting of the luminosity function of the globular cluster M92, a density law $\rho(r) \propto r^{-3}$, and spherical isodensity contours. In view of the uncertainties in this fit to her data, and possible systematic errors even for quite blue stars (Faber *et al.* 1976) these results must be given low weight.

Chiu (1980a, b) has discussed the Berkeley results of photometric and proper motion studies of three fields totalling ~ 0.3 square degrees to $V \sim 20.5$. Unfortunately, due to the very small area surveyed, too few stars were measured (~ 180 in total in his north polar field) to allow a direct determination of both the luminosity function and the density law for the Galactic spheroid. A density law was therefore adopted, and a luminosity function derived.

Becker (1980) and Fenkart (1980) have summarized the results of RGU photographic photometry in eight fields totalling 10 square degrees by the Basle group. This study isolates stars by ultraviolet excess in the two-colour *U–G/G–R* diagram, and specifically studies only metal-poor stars. They do not apply an explicit metallicity gradient with distance from the Galactic Centre or the Galactic plane, but rather assume a discrete metal-poor population, with constant mean metallicity and dispersion about this mean. Consequently, it is not clear how their results are to be related to populations defined by kinematic criteria, or by position in the Galaxy. For example, Becker (1980) identifies a density *minimum* at the Galactic Centre for his metal-poor stars, whereas other spheroid tracers, such as RR Lyrae stars or globular clusters, are consistent with a density *maximum*.

The density laws derived by Fenkart (1969) for the South Galactic Pole (SA141) and the North Galactic Pole (SA57), while based on relatively few stars (~ 300 of each of metal rich and metal poor), and with the caveats above, are consistent with exponential density decreases with z distance for both the disc (up to ~ 2 kpc) and spheroid (up to ~ 5 kpc) components.

In summary, therefore, the previously available data for the decrease in stellar number density with distance from the Galactic plane, for distances greater than ~ 1 kpc, do not uniquely define the density law. Existing results are consistent with both an exponential perpendicular to the Galactic plane, with scale height ~ 4 ± 2 kpc, and a power law from the Galactic Centre, with exponent ~ − 3.3 ± 0.3.

5.2 THE DENSITY LAW BEYOND $z = 1$ kpc

In Fig. 6(a), (b), we present the density gradient, on a log–linear scale, for two different absolute magnitude ranges, and in Fig. 7(a), (b), the same data on a log–log scale. Also shown are best fitting curves. Within ~ 1 kpc of the plane, the data are well described by a single exponential with scale height ~ 300 pc (Section 4). Beyond this distance two functions are equally well fitted to the density law. That shown in Fig. 6 is an exponential decrease perpendicular to the Galactic plane, with scale height ~ 1450 pc (this value is not well determined), and containing ~ 2 per cent of the solar neighbourhood stars at $z = 0$. The

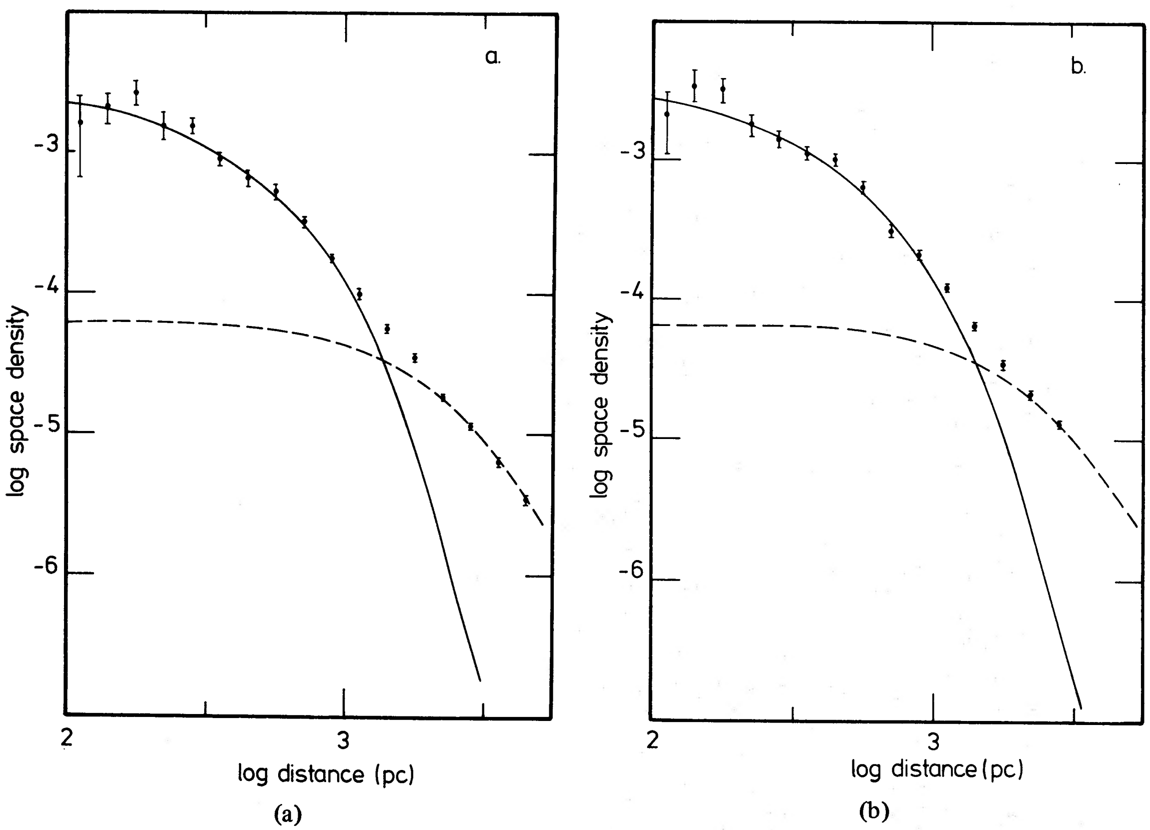


Figure 7. (a) The density distribution for stars with $4 \leq M_V \leq 5$, with distance from the Galactic plane. The solid curve is an exponential with scale height 300 pc from the Galactic plane. The broken curve is a power law from the Galactic Centre, with index − 3.5 and axial ratio 4:1. Comparison with Fig. 6(a) shows that this density distribution fits the data as well as does an exponential from the plane. (b) As for (a), for $5 < M_V \leq 6$.

curves of Fig. 7, while an equally good fit to the data, are defined by

$$\rho(z) = \rho(z = 0)\left[\frac{D_0^2 + K^2 z^2}{D_0^2}\right]^{-n/2}$$

and represent the stellar number density decrease as a power law of index n, centred on the Galactic nucleus (which is D_0 distant from the Sun), and with axial ratio $K = a/c$. The parameters used in Fig. 7 are: $D_0 = 9$ kpc, $n = 3.5, K = 4$. Many other combinations are possible, though a rather oblate spheroid is necessary. Again, ~ 2 per cent of stars at $z = 0$ are in this component.

Over the interval $1000 \leqslant z \leqslant 5000$ pc, we are therefore not able to distinguish between an exponential decrease perpendicular to the Galactic plane, and a power law decrease from the Galactic Centre in a moderately oblate spheroid (axial ratio ~ 4:1).

Those stars with $M_V \lesssim +4$, however, are significantly better fitted, for $z \gtrsim 1200$ pc, by an exponential with scale height ~ 1650 pc or by a power-law spheroid with $K \approx 2$, than by the values of 1450 pc or $K \approx 4$ appropriate for fainter stars. (Inspection of Fig. 7 shows that the density data for $2.75 \leqslant \log z \leqslant 3.75$ are in fact exceptionally well described by a power law, of index -2.45, relative to the Galactic plane. As the data points for $2.75 \leqslant \log z \leqslant 2.95$ are part of the 300 pc scale height exponential fit, this is presumably fortuitous.)

In summary, those stars fainter than the break in the stellar luminosity function at $M_V \sim +4$, and beyond $z \sim 1000$ pc, are well described either by an exponential perpendicular to the plane with scale height ~ 1450 pc, or by a galactocentric power law of index ~ -3.5 and spheroid axial ratio ~ 4:1. Brighter stars are better described by a larger scale height (~ 1650 pc) or less flattened spheroid (~ 2:1). In all cases, about 2.0 ± 0.3 per cent of the stars in the solar neighbourhood are required to belong to this population. These values are only very weakly dependent on the assumed metallicity gradient. The extrapolation to $z = 0$ changes the normalization by less than 0.5 per cent, and the gradient of the exponential beyond ~ 1 kpc is reduced by less than 50 pc with no metallicity gradient. The only exception to this is, as expected, for the bluest stars ($M_V \lesssim +4$) whose numbers are not replaced by intrinsically brighter stars when the metallicity correction reduces their absolute magnitudes. There the large scale height exponential has a best fit value of ~ 1800 pc with no metallicity gradient, and ~ 1650 pc with our adopted value.

6 Discussion

6.1 THE STELLAR MASS FUNCTION

There are two features of the stellar mass function which have recently been shown to be statistically significant. The change in slope for $m \lesssim m_\odot$ appears in our data for all distances within ~ 300 pc of the Sun, and has also been shown to be significant in the immediate solar neighbourhood (Upgren & Armandroff 1981). For $m \lesssim 0.2 m_\odot$ the mass function flattens, and may turn over.

These features are not consistent with star formation models in which a single physical process is invoked for all stellar masses, but do support theories in which low mass star formation is predominately controlled by different physical processes than high mass star formation (for recent discussions *cf.* MS; Silk 1980; Hunter & Fleck, 1982).

6.2 COMPARISON WITH THE BAHCALL–SONEIRA MODEL GALAXY

Bahcall & Soneira (1980, BS) have described in detail a model of the Galaxy specifically designed to be consistent with existing stellar number–magnitude–colour distributions.

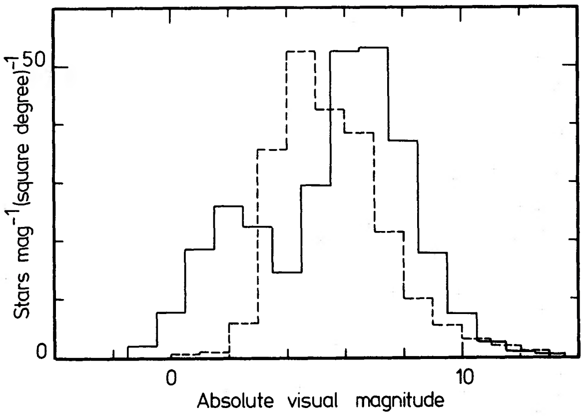


Figure 8. The distribution in absolute magnitude for all stars with apparent magnitude $15 \leqslant V \leqslant 17$ in one square degree towards the South Galactic Pole (broken line), and the predicted distribution from the model of Bahcall & Soneira (1980). The reasons for the manifest disagreement are discussed in the text.

Their model is primarily constrained by high-latitude data with $20 \leqslant B \lesssim 22.5$, and $B \leqslant 16$. In view of the considerable uncertainties in some of the input parameters to this model (Gilmore 1981) it is of interest to compare its detailed predictions with the results presented here. Bahcall & Soneira (1981) have tabulated the detailed distribution of stars in terms of both apparent and absolute magnitude expected in several fields. A comparison of their predicted absolute magnitude distribution for the apparent magnitude interval $15 \leqslant V \leqslant 17$, with the results of this survey is presented in Fig. 8. There is evidently very little agreement between the two distributions. The same conclusion has been reached by Tritton & Morton (1982) in an independent study of a second field.

The principal origin of this discrepancy, which is particularly pronounced for $M_V \lesssim +3$, is the shape of the spheroid luminosity function adopted by BS. As discussed by Gilmore (1981) BS adopted the luminosity function of the young disc for the spheroid component, and did not allow any steepening below $M_V \sim +4$, even though this is to be expected as an age effect and is shown in detail in their fig. 2. The pronounced steepening of the luminosity function with distance from the Galactic plane discussed in Section 3 (Fig. 4) shows the limitations of their choice of luminosity function. Those spheroid stars with $M_V = 0$ which contribute to the model counts at $V = 17$, are ~ 40 kpc from the plane. No unevolved main-sequence stars of this absolute magnitude are seen at such large galactocentric distances, although some few unrecognized K giants may be present. The blue star peak in the absolute magnitude distribution of Fig. 8 is therefore simply due to the neglect of the difference between the young disc and spheroid luminosity functions (*cf.* fig. 3 of Chiu 1980b).

Similarly, BS adopted a simple two-component density profile of the stellar distribution perpendicular to the Galactic plane. While their chosen old disc parameters are in very good agreement with those derived here, their model is not consistent with our derived density laws for $z \gtrsim 1$ kpc. In particular, for $1\,\text{kpc} \lesssim z \lesssim 5$ kpc, we find the dominant stellar population to follow either an exponential or a power law in a highly flattened spheroid. The Bahcall–Soneira model adopts a spherical component with a power law decrease, but no significant flattening.

The combination of an incomplete density description and an inappropriate spheroid luminosity function jointly lead to the substantial discrepancies between the stellar distribution derived here, and that predicted from the model. The absolute magnitude distribution of Fig. 8 does not, however, imply that the bimodal number–magnitude–colour distribution derived by Kron (1980), and modelled by BS, for $B \gtrsim 20$ is not apparent at brighter magnitudes. In fact, the distribution in the number–magnitude–colour diagram for our data is significantly bimodal for $V \gtrsim 16.5$, with peaks near $B-V = 0.6$ and $B-V = 1.5$ (Paper II, table 4). As discussed by Gilmore (1981), this is due to the shape of the absolute magnitude–colour relations, and not the distribution of absolute magnitudes. Fig. 2 of Paper I presents our absolute magnitude–colour relations, and amply illustrates this effect, with $B-V$ colour becoming only very weakly absolute magnitude dependent for $B-V \gtrsim 1.4$.

6.3 THE GALACTIC THICK DISC

Within ~1000 pc of the plane, the smoothly steepening luminosity function, and the exponential stellar number density decrease are consistent with the identification of the dominant stellar population as 'old disc'. The extrapolation of the density law determined in the interval $1000 \lesssim z \lesssim 5000$ pc to $z = 0$, however, requires the population which dominates in this interval to comprise ~2 per cent of the solar neighbourhood stellar number density. It is of interest to identify this component in the solar neighbourhood, and to test its identification with, for example, extreme Population II, as defined by Schmidt (1975).

The solar neighbourhood number density of extreme Population II stars has been discussed by many authors, with derived ratios to the disc number density being 1.25×10^{-3} (Schmidt 1975), $1 \pm 7 \times 10^{-3}$ (Harris 1976), $5 \pm 2 \times 10^{-3}$ (Fenkart 1980) and 5×10^{-3} (Chiu 1980a). While these values are all severely limited by small sample statistics, and all employ somewhat different definitions of extreme Population II, they are inconsistent with the value $\sim 20 \times 10^{-3}$ derived here. We therefore conclude that this population is *not* extreme Population II.

An obvious identification with the intermediate Population II (IPII) is, however, possible. Strömgren (1964, 1976) and Crawford, Mavridis & Strömgren (1979) note that ~7 per cent of main-sequence F stars in the solar neighbourhood form an intermediate population using a purely photometric metallicity parameter ($0.045 \leqslant \delta M_1 \leqslant 0.09$) as definition but also have intermediate kinematics (*cf.* also Oort 1965, section 3). IPII stars may then be identified with the sum of the metal-poor end of the old disc metallicity distribution and the population discussed here. This identification is further supported by the results of Section 3, Table 5, which show a significant steepening of the luminosity function for $M_V \lesssim +4$ occurring at $z \sim 1000$ pc. This is just the distance at which the density law data indicate a significant change of slope, and support the suggestion of a non-disc population becoming dominant.

The steepening of the luminosity function, and probable mild metal deficiency of this population, are also consistent with this population being older than the old disc. It is then straightforward to interpret the intermediate Population II as the local component of the Galactic spheroid, whose kinematics are modified by the gravitational potential of the locally massive Galactic disc (Freeman 1980; Monet, Richstone & Schecter 1981). This interpretation is supported by the recent discovery of a component with projected spatial and colour distribution consistent with the Galactic intermediate Population II in several edge-on disc galaxies (Burstein 1979; van der Kruit & Searle 1981a, b). We therefore identify the component which dominates the stellar number density in the Galaxy between ~1000 pc and at least 5000 pc above the Galactic plane as a 'thick disc' of Galactic spheroid stars.

The vertical density variation of a self-gravitating, isothermal population, such as a massive disc, follows the relation:

$$\rho(z) = \rho_{z=0} \operatorname{sech}^{2P}(z/z_0).$$

This is approximately an exponential with scale height $z_0/2P$ for $z \gg z_0$, where $z_0/2$ is the scale height of the disc (here 300 pc) and P is the ratio of the mean square velocity dispersions of disc and test population (Spitzer 1942; van der Kruit & Searle 1981a, b). Our measured scale height of ~ 1450 pc, and an adopted disc velocity dispersion of ~ 20 km s^{-1} (Wielen 1974), implies a velocity dispersion of ~ 60 km s^{-1} for IPII. This value is in agreement with the result of Hartkopf (1981), who derived z velocity dispersions increasing from ~ 20 km s^{-1} for stars with [Fe/H] ~ 0.0 to ~ 50 km s^{-1} for [Fe/H] ~ − 1.0; Oort (1965, table 5) who quotes dispersions of ~ 50 km s^{-1} for RR Lyraes with [Fe/H] $\gtrsim -1$; and Wielen (1974) who derived 65 km s^{-1} for nearby subdwarfs. Similarly, Eriksson (1978) was forced to adopt a component comprising ~ 3 per cent of solar neighbourhood stars with a vertical velocity dispersion of ~ 70 km s^{-1}, to model his SGP velocity distribution data. The adoption of a component with a local number density and velocity dispersion of this order is a common feature of analyses of the Galactic K_z force, and in fact requires a density law similar to that derived here (compare fig. 4 of Oort 1960 with Fig. 6 of Section 5, this paper).

Our identification of the Galactic thick disc with a moderately metal poor, old population of spheroid stars responding to the gravitational potential of the Galactic disc is therefore consistent with available information on both the metallicity dispersion and the velocity dispersion of these stars in the solar vicinity. The relation of the thick disc to the main body of the Galactic spheroid is less clear. Recent studies of the kinematics of the bulge components of several disc galaxies (at least one of which, NGC 4565, has itself a thick disc) have shown them to be flattened by rapid rotation, with a relatively small contribution from an anisotropic velocity dispersion tensor (Whitmore & Kirshner 1981; Illingworth & Schecter 1982; Kormendy & Illingworth 1982). The density profile of the thick disc is, however, consistent with flattening due entirely to the gravitational potential of the massive disc, with rotation and velocity anisotropy being relatively unimportant. The observation that thick disc structure is seen only in galaxies with detectable bulges (van der Kruit & Searle 1981a) implies that the stars in the thick disc may be related to those in the bulge, and presumably therefore are rapidly rotating about the Galactic nucleus. Metal-rich RR Lyraes ($\Delta S < 5$; [Fe/H] $\gtrsim -1$) and long-period variables were assigned by Oort (1965) to the intermediate Population II, and also have rotation which dominates their velocity dispersion, in agreement with this. Metal-poor RR Lyraes (Oort 1965; Woolley 1978), extreme subdwarfs (Oort 1965) and globular clusters (Frenk & White 1982), however, have relatively slow rotation, large and anisotropic velocity dispersions and small ellipticity, and do not seem to form thick discs. They therefore define a distinct component of the Galaxy which is not the 'normal' bulge/ spheroid component. The metal-rich A stars discussed by Rodgers *et al.* (1981) are a further complication. They do not obviously fit either the thick disc or the spheroid metallicity distributions, even though their kinematics are comparable, while they are kinematically distinct from the disc, whose metallicity distribution is, however, similar.

7 Conclusion

We have applied the technique of photometric parallax to a complete sample of ~ 12 500 stars brighter than $I = 18.0$ towards the South Galactic Pole to derive the solar neighbourhood luminosity function, and its variation with distance from the Galactic plane.

Extrapolation of the derived luminosity function to the Galactic plane provides the first photometrically based luminosity function which is complete to $M_V \sim +19$, comparable to the minimum mass for thermonuclear burning. Adopting appropriate bolometric corrections and a mass–luminosity relation then provides the bolometric luminosity function and present day mass function in the solar neighbourhood. Both show a significant decrease for low mass stars, thereby eliminating main-sequence stars with mass $\lesssim 0.1$ solar masses as a candidate for any local missing mass. Neither function is well approximated by a single power law over a wide dynamic range.

From these relations, we derive the total mass in main-sequence stars in the solar neighbourhood to be $0.038\,m_0\,\mathrm{pc}^{-3}$, and a total mass-to-light ratio, in solar V magnitude units, of 1.2.

The shape of the stellar luminosity function shows a smooth variation with distance from the Galactic plane, with stars with $M_V \lesssim +4$ being significantly more concentrated to the Galactic plane. This is consistent with the standard picture of stellar velocity dispersion increasing with age. The luminosity function shows a significant steepening for $M_V \lesssim +4$ at $z \sim 1000$ pc below the plane, indicating a significant change in the dominant stellar population beyond this distance, with an older population becoming dominant. The mass-to-light ratio of this population cannot be directly measured from our data, as the most luminous stars (K giants) are not specifically identified. An estimate is possible by smoothly extrapolating our function at $z > 1$ kpc to $M_V = -1$, cutting off for $M_V \leqslant -1$, and assuming the disc function for fainter stars. The derived value is then approximately 4, in solar visual units.

A significant change also occurs in the density laws. For $z \lesssim 1$ kpc the data are very well fit by a single exponential with scale height ~ 100 pc for stars with $M_V \lesssim +4$, and ~ 300 pc for fainter stars. Beyond $z \sim 1$ kpc to at least $z = 5$ kpc, an exponential decrease with scale height ~ 1450 pc is apparent. We identify these populations as young disc, old disc and thick disc respectively. The density law and luminosity function of the thick disc are most simply interpreted as evidence for the spheroidal intermediate Population II stars responding to the gravitational potential of the massive disc.

Comparison with the 'standard model' of high latitude star counts due to Bahcall & Soneira (1980) indicates substantial disagreements. These are caused by the differences in both the spheroid luminosity function and the density laws between those assumed in the model and those derived here.

Our picture of the vertical structure of the Galaxy then becomes: the old disc, with scale height ~ 300 pc, dominates the density laws to $z \sim 1$ kpc. Beyond this, an older, more metal-poor population is seen. This population is equally well described by an exponential decrease from the Galactic plane with scale height ~ 1450 pc, and by a power law decrease from the Galactic Centre with exponent ~ -3.5 and axial ratio $\sim 4{:}1$. By analogy with external galaxies, the former density law is preferred and this population is identified as a Galactic 'thick disc'. About 2 per cent of solar neighbourhood stars belong to this population. Locally, they may be identified with the intermediate Population II, and are in fact simply the stars in the Galactic spheroid responding to the gravitational potential of the disc. A third component of the Galaxy is represented by extreme Population II and the globular clusters. This presumably dominates the far outer regions of the Galaxy, but is not a significant ($\gtrsim$ few per cent) contribution to the stellar population within 5 kpc of the Sun. A simple two-component model of the stellar distribution within the Galaxy is not consistent with these results.

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