Correcting Predictions of Solar Eclipse Contact Times for the Effects of Lunar Limb Irregularities

DAVID HERALD
PO Box 254, Woden, ACT 2606, Australia

Conventionally, predictions of second and third contacts at solar eclipses are computed assuming a smooth lunar limb. However, the irregularities of the lunar limb may introduce sizeable errors in such predictions, which are dependent upon the observer's location within the path of central eclipse. The correction to be applied to a smooth limb contact time to incorporate the effect of the irregular limb can be ascertained from an appropriately constructed chart, and precepts are given for the construction of such a chart. Additionally, the limiting magnitude of both a total and an annular eclipse is investigated.

INTRODUCTION

Traditionally, the predicted times of second and third contacts at a solar eclipse are computed on the assumption that the Moon is a smooth circular body. However, this is not strictly true and, in particular, the irregular limb of the Moon introduces an error in the thus predicted contact times which may potentially reach tens of seconds at locations well away from the central line. To reduce the effect of the irregular lunar limb, the national almanacs have for some time applied a correction of $-0''\!\cdot\!6$ to the lunar latitude ($-0''\!\cdot\!5$ before 1964) prior to computing the Besselian elements, this correction being the difference between the centre of mass and the mean centre of figure of the Moon (see *Explanatory Supplement*[1] and Watts[2]); since 1981, however, this correction has not been applied to the predictions given in the *Astronomical Almanac.* Additionally, the Central Line Duration of Total Phase (but *not* annular phase) as published in the national almanacs, including the *Astronomical Almanac*, is computed using a reduced lunar radius "in lieu of approximate limb corrections". Apart from the above, no allowance for the effects of the irregular lunar limb is made in any of the predicted data given in national almanacs, nor in the detailed predictions of local circumstances as published for appropriate eclipses in the United States Naval Observatory *Circulars.*

Duncombe[3] presents selected lunar limb profiles which "allow the eclipse observer to predict the effects of the lunar limb irregularities on the time of second and third contacts". These profiles were derived from the charts prepared by Watts[2], and plot the height of the actual limb above the mean limb on a highly magnified radial scale. Although precepts are given for determining the position on the lunar limb of the contact points, no explanation is given for determining anything other than the fact that an appreciable correction may exist; furthermore, the precepts may lead one to a misleading estimate of the effect of the lunar limb, particularly when the contact point is in the vicinity of a deep valley or high mountain.

The charts of Duncombe[3] may, by suitable graphical construction dependent upon the eclipse circumstances, be converted into a convenient form so as to enable an accurate determination of the corrections to the second and third contact times, for any location within the eclipse path. If the solar limb is plotted relative to the mean lunar limb, on the same radial scale, at one of the contact times, then an epicyclic curve will be generated which is tangential to the mean lunar limb at the contact point. Unless the eclipse magnitude is close to 1·0 the epicycle rapidly deviates from the lunar limb at angles displaced from the contact point. Because of the highly magnified radial scale of the profiles (≈70:1), the movement of the solar-limb curve in small time intervals from the time of contact is essentially perpendicular to the mean lunar limb at the contact point (the exception being near the points which determine the north and south limits of the eclipse). By definition, the second and third contacts are those times at which the solar limb is tangential to the lunar limb, but does not intersect it at any point. Thus, to correct an eclipse prediction for the effect of the lunar limb it is only necessary to relate the displacement of the solar limb from the mean lunar limb to a correction in time, which can be effected through a computation of the relative radial rate of motion of the Sun and Moon.

This concept can be extended to represent the radial displacement of the solar limb at the time of contact for any position angle of contact. Referring to figure 1, if the line CD is a line connecting the centre of the Moon to the point of contact X (assuming a smooth limb), and the curve AXB represents the

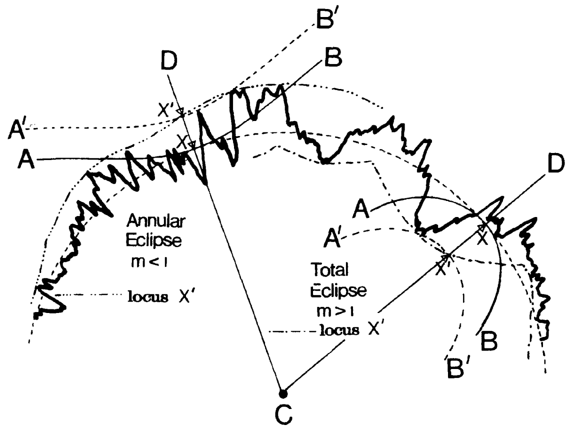

Figure 1. The relationship between the solar and lunar limbs at second or third contact during a solar eclipse is shown. The distance XX′ represents the distance the solar limb is from the mean lunar limb at the time of true contact. The figure shows the relationship for both total and annular eclipses. The radial scale is magnified approximately 70 times.

solar limb on the same radial scale as the lunar limb, then the actual time of contact will occur when the solar limb is displaced radially along CD to the point X′ so as to be tangential to the lunar limb. If the line CD is rotated to a new position angle, and the procedure repeated, a different location of X′ is obtained. By plotting the locus of X′ for all position angles a curve representing the displacement of the solar limb from the mean lunar limb at the actual time of contact will be obtained for any nominated position angle of contact.

To use this displacement curve to correct predicted contact times, it is necessary to relate the displacement to a time interval. This is most conveniently achieved by plotting with the displacement curve a curve representing the location of the solar limb at that position angle at some specified time interval from the smooth-limb contact time (*e.g.* 5, or 10, seconds). The location of the displacement curve relative to the mean lunar limb and the time-displaced solar limb then gives directly the correction to be applied to the contact time for the effect of the lunar limb.

CONSTRUCTION OF THE LIMB CORRECTION CHART

Construction of the Displacement Curve

If M is the magnitude of the eclipse, then relative to the mean lunar limb the height of the solar limb h is given by the approximate expression

$$h = 960'' (M-1)(1-\cos P) \qquad (1)$$

where P is the angle from the point of nominal contact. The family of curves of h for different eclipse magnitudes are shown in figure 2.

Equation (1) assumes a mean solar radius; the error in h thus introduced is no more than 2%, which is entirely negligible. Also, the magnitude of an eclipse will vary along the eclipse path due to the varying topocentric and geocentric distances of the Moon. This variation may reach 2% in M, or $20'' (1-\cos P)$ in h. The greatest variation in h will occur near the very ends of the eclipse path, where the observer's distance from the fundamental plane along the eclipse path changes most rapidly. By choosing for the value of M the magnitude of the eclipse for a site located approximately midway along the path, the displacement curve constructed will be valid for most of the eclipse path, with small errors near the ends of the path.

The curve generated by equation (1) is drawn on the same scale as the profiles of Duncombe[3], conveniently on tracing paper, with a hole at X so as to be able to mark the points X′. The line CD is also marked so that the displacement of the curve from the lunar limb can be maintained in a radial direction. The point X′ (figure 1) is plotted for a series of arbitrary position angles around the limb. Connecting the points X′ generates the displacement curve.

Constructing the Time-displaced Solar Limb

To a reasonable approximation the apparent motion, r, of the Moon relative to the Sun is given by

$$r = \frac{\pi n}{3600} \text{ ''/sec} \qquad (2)$$

where π is the lunar parallax (topocentric) in arc-seconds, and n is the motion of the eclipse shadow relative to the observer on the fundamental plane, in Earth radii per hour. Relation (2) may with further approximations be reduced to

$$r = 0{\cdot}97Mn \text{ ''/sec} \qquad (3)$$

where M is the magnitude of the eclipse.

The quantity n is usually computed in the course of accurate eclipse predictions. If it is not readily avail-

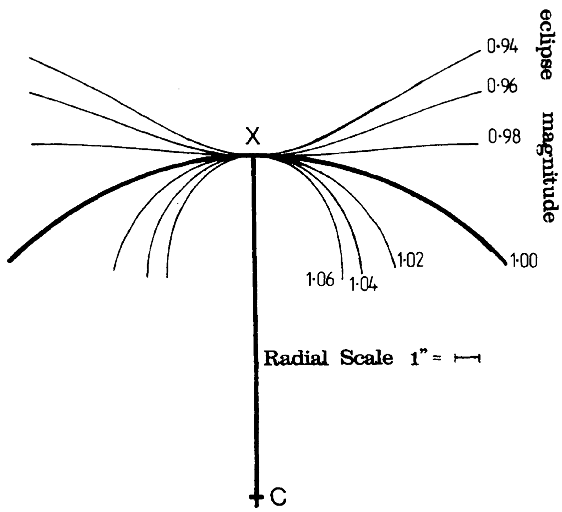

Figure 2. During an eclipse, the form of the solar limb, relative to the lunar, is related directly to the magnitude of the eclipse. C represents the centre of the Moon; the radial scale is magnified approximately 70 times.

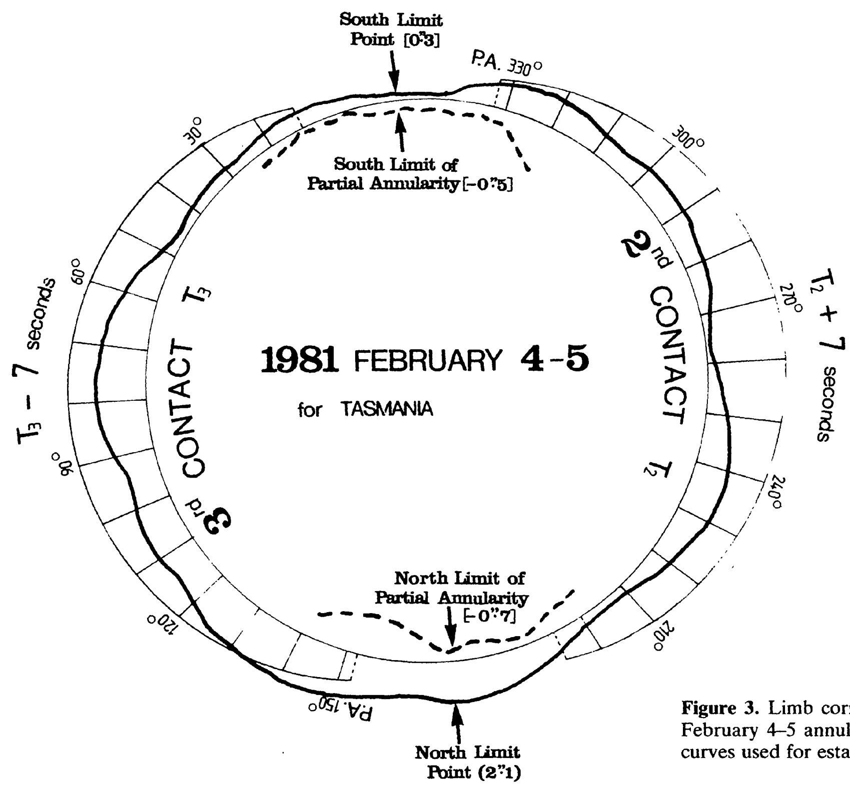

Figure 3. Limb correction chart constructed for the 1981 February 4–5 annular eclipse, for sites in Tasmania. The curves used for establishing the limits of partial annularity are also shown.

able, it may be determined approximately from the central line duration of central phase, d (secs), for a nearby location from

$$n = \pm \frac{3900\,(1-M)}{d\,(1+M)} \qquad (4)$$

where the sign is taken such that n is positive.

The value of r represents the maximum relative radial motion that is present, and this occurs in the direction of the apparent relative motion, N. This direction can be determined during the course of computing accurate local eclipse predictions from the Besselian elements (as arctan u'/v', where u' and v' are the rates of motion of the shadow relative to the observer on the fundamental plane), or by taking the arithmetic mean of the position angles of second and third contacts computed for a nearby location, and adding or subtracting 90° to obtain a figure near 90°. The radial motion of the solar limb relative to the lunar limb is then given by

$$r \cos\,(\mathrm{PA}-N)\ ''/\mathrm{sec} \qquad (5)$$

where PA is the position angle of the direction of interest, measured through east from the north point.

To plot the position of the time-displaced solar limb, one first follows the precepts provided by Duncombe[3] to identify for the site location the north point on the profile, and thus on the displacement curve. Then the solar limb displaced by some convenient interval of time can be plotted using equation (5). Finally, to give a ready indication of both position angles and radial directions, radial lines at convenient intervals of position angle are drawn between the mean lunar limb and the displaced solar limb. In figures 3 and 4, examples of the final form of the charts are given.

To use the chart, it should be remembered that second contact occurs at the east limb for total eclipses, and the west limb for annular eclipses; similarly, the northern limit is defined by the northern limb for total eclipses, and by the southern limb for annular eclipses.

Unlike the displacement curve, the curve representing the time-displaced solar limb depends considerably on the site location, both for the value of r and the direction N. Consequently, although for any given eclipse it is satisfactory to generate only one displacement curve, it is, in general, necessary to generate several time-displaced solar-limb curves to overlay the displacement curve to enable accurate corrections to be determined along the whole of the eclipse path.

The Limits of the Eclipse Path

From the displacement curve, it is possible to determine the locations of the eclipse limits relative to the smooth-limb limits, by measuring the distance that the displacement curve differs from the mean limb at position angles $N \pm 90°$, in arcseconds. This distance can be converted directly to a displacement on the Earth's surface perpendicular to the eclipse path by multiplying it by the factor

$$(1{\cdot}863\ \mathrm{km}/'')\sqrt{\frac{\sin^2 D}{\sin^2 a} + \cos^2 D} \qquad (6)$$

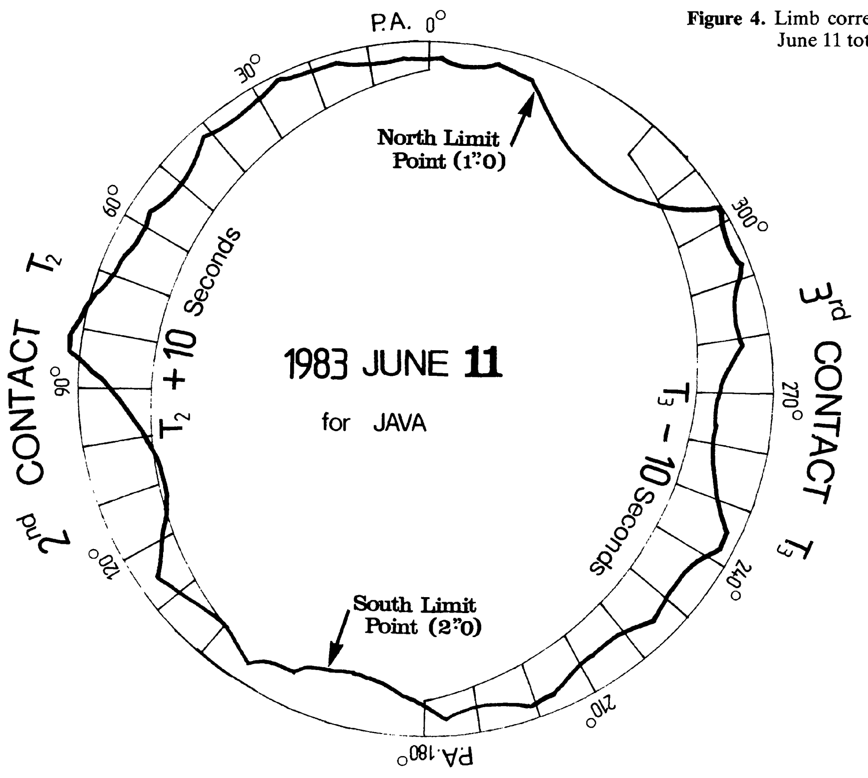

Figure 4. Limb correction chart constructed for the 1983 June 11 total eclipse, for sites in Java.

where *a* is the Sun's altitude, and *D* is the difference between the Sun's azimuth and the azimuth (bearing) of the northern or southern limit[4].

For an annular eclipse, the limits and contact times refer to the commencement and end of the appearance of a complete ring of light. (Hence, in constructing the displacement curve for an annular eclipse, it is the top of the mountains, rather than the bottom of the valleys, to which the solar limb is made tangential.) Between these times and the phase normally understood as a partial eclipse, there is a brief phase which I shall call 'partial annularity', which is characterized by having a substantially continuous string of Baily beads stretched between the cusps of the solar crescent so as to give the appearance of a complete ring of light broken intermittently by lunar peaks. The duration of this phase can be determined by constructing, in addition to the displacement curve (which is an exterior locus for an annular eclipse), an interior locus as for an eclipse of magnitude $2-M$. The difference between the interior and exterior locus, converted to a time difference by means of the time-displaced solar limb, represents the duration of this phase at each contact.

The main relevance of the partial annularity phase is in determining by observation the location of the limits of the eclipse, leading to a measurement of the solar radius[4,5]. Just outside each true limit of an annular eclipse, there is a narrow region having a width determined by applying equation (6) to the distance of the interior locus from the displacement curve (in arcsecs) in position angles $N \pm 90°$, where, at maximum eclipse, the phase will be partial annular. Within these regions, the number of Baily beads present at maximum eclipse will be very sensitive to the location of the observer relative to the shadow axis, thus providing a sensitive measurement of the location of the eclipse limit. (Observations to test the practical utility of this method were attempted at the 1981 February 4–5 annular eclipse, both in Tasmania and New Zealand. Although all observers within the zones of partial annularity were completely clouded out, observations near to, but inside, the eclipse limit generally confirm the viability of this approach.)

Annular–Total Eclipses

Approximately every nine years (half the period of the Saros) eclipses occur having a magnitude close to 1·0, and these often give rise to annular–total eclipses. For the eclipse magnitude to be close to 1·0, the apparent lunar semi-diameter must appear to be almost the same as that of the Sun, and this places some constraints on the position of the Moon in its orbit relative to its perigee, with corresponding constraints on the longitude libration. Calculation of the longitude libration for eclipses between 1970 and 2040 having a magnitude close to 1·0 shows that the longitude libration generally falls within the ranges $-5°$ to $-4°$, and $+5°$ to $+4°$. Using the profiles of Duncombe[3] for librations $-5°$ and $+5°$, I have fitted a circle as either an interior or exterior tangent to the profile so as to derive the limiting magnitudes of total and annular eclipses. Table I presents the results. As can be seen

Table I

Longitude libration −5°		Longitude libration +5°	
Total eclipse	1·0016	Total eclipse	1·0027
Partial annular		Partial annular	
Annular eclipse	0·9981	Annular eclipse	0·9979

from this table, any eclipse with magnitude greater than 0·998 cannot be a true annular eclipse, as the solar crescent cannot form an unbroken ring. Similarly, a magnitude of at least 1·0016 (or 1·0027 if the longitude libration is positive) must exist before a truly total eclipse occurs.

SOME COMMENTS UPON THE ACCURACY OF ECLIPSE PREDICTIONS

Use of the Limb Correction Chart will substantially remove the greatest source of error in predictions of the times of second and third contacts of an eclipse. Thus it is perhaps appropriate here to discuss briefly the remaining causes of an observed difference between an observed and predicted contact time. In approximate order of importance, they are as follows:

(*a*) Uncertainty in the ephemeris positions of the Sun and Moon; the *Astronomical Almanac* for 1982 states that errors of up to 0″·5 in longitude exist in the solar ephemeris, whilst errors of up to 0″·5 in longitude and 0″·2 in latitude exist in the lunar ephemeris. The combined effect of the longitude errors provide for a possible time shift of up to 3 seconds, although the probable error in a predicted time from this cause is of the order of ±1 second. The latitude uncertainty in the lunar ephemeris provides for an uncertainty in the location of the eclipse limits of about 0·5 km and significant uncertainties in the contact times for sites very close to the actual limit.

(*b*) Uncertainties in the data concerning the lunar limb and the construction of the Limb Correction Charts. There are four separate effects here:

(i) The approximations used in the expressions used to derive the Limb Correction Charts, and the fact that those charts are constructed graphically. In practice, the errors due to the approximations made will be much less than 1 second, whilst the error in reading the chart and its construction will be of the order of ±1 second.

(ii) The errors in the datum to which the Watts Charts were drawn (see references 6, 7 and 8). Whilst the error in the datum has been substantially determined, and thus in principle could be allowed for in the preparation of the Limb Correction Charts, the small magnitude of the effect renders it of importance only in the analysis of accurate eclipse contact timings.

(iii) Uncertainties in the location of the actual lunar limb relative to the datum of the Watts Charts at any given point on the limb[6], which is of the order of ±0″·2.

(iv) Errors in the Watts Charts, and in particular the absence of significant features[6,9,10].

The combined effects of (i), (ii) and (iii) give rise to an uncertainty on the central line of slightly more than ±1 second, increasing by the factor

$$\sec(\mathrm{PA}-N) \qquad (7)$$

towards the path limits, where PA and N are as for equation (5). Likewise, the corresponding uncertainty in the location of the path limit is of the order of ±0·6 km. The effect of (iv) is more difficult to ascertain, as not only must an error exist, it must be so located and of such a size as to be more prominent than the feature which would otherwise be at the point of actual contact. Generally speaking, the number of significant errors in the Watts Charts are too small to produce anything other than the occasional, localized error in the contact times.

(*c*) Uncertainty in the size of the Sun, due to secular changes. This is a small effect at best, current evidence not indicating any historical value that differs from the present value of the solar radius by more than 0″·3, equivalent to a central line change in the contact times of 1 second, increasing by the factor (7) towards the limits.

(*d*) Uncertainty in the definition of the actual surface of the Sun. As the Sun is not a solid body, there is not a precisely definable surface. Mori and Kubo[11] measured the maximum radial rate of change of magnitude at the solar limb as being 7·0 magnitudes per arcsec. Thus, any differences in the brightness of the last portion of the Sun at the time at which it is observed to disappear (as might be caused by differences in background illumination by the chromosphere or corona, or by scattering effects of cloud) will result in a change in the observed effective radius of the Sun. In clear skies, the magnitude uncertainty is not large as the contact is readily discernible against the background of the red chromosphere which provides an essentially constant reference illumination. The probable error from this source is estimated to be less than 1 magnitude, which converts to less than 0·3 second on the central line, increasing by the factor (7) towards the limits[4].

(*e*) Uncertainty in the location of the observer. So long as an observer is able to determine his site co-ordinates to ±30 metres, uncertainties in the location of the observer will not give rise to any significant change in the contact times. However, if site co-ordinates cannot be determined to this accuracy, as by having to rely upon old or large-scale maps, errors may be introduced. Appropriate expressions relating changes of site co-ordinates to contact times are given in Section 9*D* of the *Explanatory Supplement*[1] under the subheading 'differential corrections'.

The combined effect of (*a*) to (*e*) is a probable error in the predicted time of an eclipse somewhat less than ±2 seconds, increasing by the factor (7) towards the limits, and similarly an uncertainty in the location of the actual limit of an eclipse path of the order of ±1 km.

I would, finally, note that, provided sufficient timings of eclipse phenomena are made (as by many scattered observers recording contact times, or by recording Baily-bead phenomena), the uncertainties discussed under (*b*) (iii) and (iv) can be substantially eliminated, and specific values of each of the other 'uncertain' quantities at the time of the eclipse can be determined. To aid in such analyses, I would urge all observers who record the times of second and third contacts at an eclipse to forward their observations of the contact times, together with their site co-ordinates, to the Nautical Almanac Office, US Naval Observatory, Washington, DC 20390, USA.

ACKNOWLEDGEMENT

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Additional Notices

PUBLIC LECTURE ARRANGED BY THE ASSOCIATION

On the evening of Tuesday 1983 December 20 a public lecture organized by the Association is being given at the London Planetarium, Marylebone Road, London, NW1, by the Astronomer Royal, Professor Graham Smith, FRS. There will be an an admission fee, payable at the door.

For further details see the *Newsletter* enclosed with this issue of the *Journal*.

ERRATUM

In the August issue of the *Journal* [**93** (5)] on page 233 in the Letter to the Editor from Mr Kenneth Medway, the focal ratio of the 50 mm refractor should read f/25, not f/2.

TERRESTRIAL PLANETS SECTION—CHANGE OF OFFICER

Until the appointment of a new Venus Co-ordinator would members kindly forward all observations to the Director at 25 Whitchurch Road, Chester, CH3 5QA.

Richard Baum, *Director*

THE ASTRONOMER, 1983 MEETING

This year's meeting of *The Astronomer* will be held at the Angel Hotel, Bridge Street, Northampton, on Saturday November 12.

This is an all-day meeting catering specifically for active observers. There will be an extensive exhibition of equipment, photographs and other observational material. During the day many experienced observers will give talks on a variety of topics linked to observational astronomy and illustrate these with their own equipment. Full programme details will be released very shortly.

A cordial invitation is extended to any member of the Association, whether a subscriber to *The Astronomer* or not, to attend. Admission is £3 per person for the all-day event and this also includes coffee and tea. Luncheon can also be booked for an additional £4 per person. Bookings must be made well in advance. Please send your name, address and telephone number together with appropriate remittance to: Guy M. Hurst, Editor, 1 Whernside, Manor Park, Wellingborough, Northamptonshire, NN8 3QQ.