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THE INTRINSIC SHAPE OF cD GALAXIES

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ABSTRACT

We show that the variation of isophotal diameter with observed ellipticity can be used to discriminate oblate from prolate galaxies as long as the observed light distribution is different from an inverse square (Hubble) law. The feasibility of this test is demonstrated by applying it to a sample of cD galaxies in Abell clusters. We find that if pole-on surface brightness is uncorrelated with intrinsic flatness, the sample is consistent with the oblate hypothesis, but that there exists a correlation between these quantities that will make the sample consistent with the prolate hypothesis. We also show that cD's should appear relatively smaller with increasing redshift when their diameters are compared to those of other cluster members. We find that the cD's from richer clusters show a dispersion in diameter, corrected to pole-on, that is consistent with that expected from measurement errors alone, implying a very small intrinsic dispersion in these galaxies.

Subject headings: galaxies: clustering — galaxies: structure

I. INTRODUCTION

Photographic surface photometry of cD galaxies in rich (i.e., Abell 1958) clusters (Oemler 1976 [OM]; Dressler 1979 [DR]; Carter 1977 [CA]) has shown that they are characterized by an extended, shallow, power-law falloff of surface brightness ($I \propto r^{-n}, n \sim 1.5$–$1.7$) often without a visible cutoff (i.e., steepening of the light profile) to the limits of the photometry. CA has argued that were the shallow profiles to be extrapolated to the limits of a typical cluster, the total luminosity of these galaxies could be 10 times that of the visible regions. This behavior is in sharp contrast to that of "cD's" in poor clusters (Morgan, Kayser, and White 1975 [MKW]; Albert, White and Morgan 1977 [AWM]), where recently published photometric profiles (Thuan and Romanishin 1981 [TR]) show close adherence to a de Vaucouleurs (1948, 1953) law, suggesting that these galaxies are more properly designated as giant ellipticals. In the present work we show that the shallow photometric profiles of cD's in rich clusters motivate a simple test capable of discriminating their intrinsic shape. To demonstrate the feasibility of our approach we have applied it to a sample of cD galaxies and present several arguments showing them to be intrinsically oblate.

II. POWER-LAW GALAXY MODELS

Richstone (1979), in considering various approaches to the problem of the intrinsic shape of elliptical galaxies (Binney 1978), exploited the projection invariants of oblate and prolate ellipsoids as shape discriminants. Thus the minor axis of a prolate ellipsoid is independent of orientation, etc.

Richstone's test cannot be directly applied to cD systems, however, because their essentially featureless power law profiles (OM, CA, DR) preclude defining a fiducal ellipsoid such as that defining the effective radius of a normal elliptical galaxy as used by Richstone. Any turning over of the profile at small radii which would serve to define such a radius seems to take place at or near the seeing disk for all but the nearest cD's. Richstone's idea, however, can be generalized to apply to isophotal diameters. To effect this generalization, we model elliptical systems as an extinction-free family of similar ellipsoids of revolution giving (Contopoulos 1956; Stark 1977)

$$F(r) = QF_0(r), \tag{1}$$

where $F(r)$ is the observed surface brightness distribution, $F_0(r)$ is $F(r)$ observed when the galaxy is seen as round, and

$$Q = f^{-1} = \left(\cos^2\theta + f_0^2 \sin^2\theta\right)^{-1/2} \quad \text{(prolate)},$$

$$Q = f = \left(\cos^2\theta + f_0^2 \sin^2\theta\right)^{1/2} \quad \text{(oblate)}, \tag{2}$$

where the flatness $f(\geq 1)$ is the observed ratio of major to minor axes, f_0 is the intrinsic flatness of the ellipsoids, and θ is the angle between the observer's line of sight and symmetry axis of the ellipsoids. The galaxy isophotes are described by

$$r^2 = Q^2 z^2 + y^2 , \tag{3}$$

where z and y are coordinates in the observer's sky. It is evident that keeping r constant and varying θ is equivalent to a line of sight that is always tangent to the same physical ellipsoid.

We further assume that

$$F(r) \propto r^{-n} \tag{4}$$

over a suitably defined region. It follows from equation (1) that n is itself a projection invariant. Combining equations (1)–(4) and assuming that the galaxy is always observed at a fixed isophotal level on the sky, we obtain

$$a = a_0 f^{1/n}, \quad b = a_0 f^{1/n-1} \quad \text{(oblate)}, \tag{5a}$$

$$a = a_0 f^{1-1/n}, \quad b = a_0 f^{-1/n} \quad \text{(prolate)}, \tag{5b}$$

for the variation of observed major (a) and minor (b) axis with flatness, where a_0 is the isophotal diameter that would have been observed at $f = 1$ (round). These equations imply the following consequences for elliptical systems:

1. The axes a, b, and consequently $f = a/b$ are observed quantities, as is n, so that the face-on diameter a_0 can be calculated if the intrinsic shape is known. This is pointed out for oblate galaxies in the Reference Catalog (de Vaucouleurs, de Vaucouleurs, and Corwin 1976).

2. As $n \to \infty$, galaxies take on a "hard edge" and their behavior approaches that of their solid geometrical counterparts (e.g., $a \to$ constant for oblate galaxies).

3. As $n \to 1$, there is an exact reversal and intrinsically oblate galaxies appear to behave like "hard" prolate objects (e.g., $a \to$ constant for prolate galaxies).

4. At $n = 2$ (Hubble 1936 law), equations (5a) and (5b) are identical, and oblate and prolate shapes cannot be differentiated by this test.

As a general rule, for $n > 2$ intrinsic shape and observed geometrical behavior tend to agree, and for $n < 2$ they disagree. Because of this reversal, and the observed intrinsic profile variations of elliptical galaxies (Kormendy 1977), it is important that tests for intrinsic shape of elliptical galaxies that utilize isophotal properties give clear predictions over a range of n rather than assuming that all E galaxies are described by a single generic profile.

For $n \approx 2$, $a/a_0 \approx 2$ for the flattest galaxies so that we are assuming in equation (4) that $F(r)$ is well described by a power law over a range of 0.3 in $\log r$, which is not unreasonable given typical photometric profiles.

In order for the variation of isophotal diameter with flatness to be used as a test of intrinsic shape, it is necessary to generalize equations (5a) and (5b) to describe a set of galaxies. This essentially requires some assumption about the relationship of a_0 and f_0, and we assume (Merritt 1982)

$$I_0 \propto f_0^m , \tag{6}$$

where I_0 is the pole-on central surface brightness and $m = 0$ corresponds to no correlation between I_0 and f_0. Thus a_0 in equation (5) is assumed to be proportional to $(f_0)^{m/n}$. Since f_0 is also $f_{\max}$, equation (6) predicts a trajectory of extremal configurations in the ($\log a_0$, $\log f$)-plane of slope m/n. While it is apparent that this test is not easily applied to elliptical galaxies, some guidelines are suggested. Suppose a galaxy's photometric profile is approximately described by a de Vaucouleurs (1948, 1953) law,

$$\log\left(\frac{I}{I_e}\right) = -3.33\left[\left(\frac{r}{r_e}\right)^{1/4} - 1\right], \tag{7}$$

the effective power law index is

$$n = 1.92(r/r_e)^{1/4}, \tag{8}$$

so that for n to be much less than two requires brighter, more centrally located isophotes where seeing effects (Schweizer 1979; Djorgovski 1982) are worse and n itself varies most rapidly with r. On the other hand, $n = 2.5$ at only $3r_e$, so that the outer regions of elliptical galaxies would seem to hold more promise. In the outer regions, however, deviations from any given profile law are generally more pronounced (Kormendy 1977). This problem could be circumvented by requiring that any galaxy in the sample conform to a given profile shape. While this degree of freedom has yet to be specifically exercised (except in the broadest sense) in any test, it in no way biases the sample but may diminish the power of the result.

We utilize this degree of freedom in the current work by restricting our sample to cD galaxies (Matthews, Morgan, and Schmidt 1964 [MMS]; Morgan and Lesh 1965 [ML]). These galaxies are ideally suited to the test since $n \approx 1.5$–1.7 in cD's over extended regions of the galaxy (CA, DR). Thus they conform especially well to the assumptions of our model. Moreover, they can be expected to exhibit the uniformity of first ranked galaxies (Sandage 1972), implying a small dispersion in intrinsic diameter. Because cD's probably have very different dynamical histories than other ellipticals, our conclusions as to intrinsic shape should not be extended beyond this class.

III. THE DATA

In order to examine the feasibility of the test proposed in the previous section, we have assembled a collection of cD galaxies in Abell (1958) clusters. A list of candidate cD's was generated by taking all Abell clusters with redshifts that are classified as Bautz and Morgan (1970 [BM]) I or I–II in which the first ranked galaxy was classified as a cD in the catalogs of Struble and Rood (SRI 1982, SRII 1983). SRI and SRII cover Abell clusters in distance classes 1–4 and 5, respectively. We also examined all clusters in distance class 6 with redshifts for possible cD's. From this preliminary list, all those clusters were eliminated in which accurate measurement of the size of the cD candidate was rendered virtually impossible because of companions or stars (e.g., A2634). We also eliminated from our sample all those clusters with $z > 0.15$ and $|b^{\mathrm{II}}| < 30°$ in order to obtain clear measurements and prevent overly large corrections. Because of the nature of the BM classification, for several clusters there was widespread disagreement on the BM type usually arising from whether or not the suspected cD or other bright galaxies in the field were considered foreground superpositions. In these cases we used all existing redshift and magnitude data to arrive at a final classification. Even where there was no disagreement in the literature, we found several instances where we felt the BM classification to be in error. A2666 (richness class 0), or example, is classified BM I by Leir and van den Bergh (1977 [LVDB]), and its first ranked galaxy is typed as a cD in SRI. There are several other large bright galaxies in the field of the cluster that were generally considered as foreground, but redshifts by Hintzen (1980) show them to be cluster members. Thus the assumed cD, while bright (Hoessel *et al.* 1980 [HO]), is certainly not very much larger than other cluster members. Galaxy size in relation to other cluster members was used in SRI and II and by MMS, ML, MKW, and in White's (1978 [WH]) survey of nearby clusters to identify cD candidates. Generally a galaxy had to be 2–3 times the diameter of other clusters members to be considered a cD. This way of identifying cD's is certainly subject to error because it is richness (Geller and Peebles 1976), redshift, and aspect dependent. To illustrate the difficulties in our selection mode we consider some specific cases:

A779.—This richness class $R = 0$ cluster is classified as BM I–II, and its first ranked galaxy was designated as a cD in SRI. Photometry of the galaxy (OM) shows it to have a relatively steep profile and to conform quite closely to a de Vaucouleur's law in its outer regions (TR). For this reason we exclude it from our sample.

A1631.—This $R = 0$ cluster is classified as BM I, and its cD candidate was for a time the largest galaxy in our sample. Its redshift (Dressler, private communication) shows it to be foreground galaxy. This conclusion was reached independently by Sarazin, Rood, and Struble (1982 [SRS]) in recalibrating Abell's (1958) redshift predictions.

A1837.—This $R = 1$ cluster is classified as BM I–II, but its redshift ($z = 0.0376$) [HO]) is one of the smallest in distance class 4 (SRI) and about 60% of that predicted by LVDB or m_{10} (SRS). It is the faintest BM I–II galaxy in the sample of HO, and its diameter as measured by us is about half that of other cD's, and, since the only cluster redshift is that of the first-ranked galaxy, we eliminate it on the assumption that it is a probable foreground galaxy.

A1308.—The first-ranked galaxy in this cluster ($R = 0$) is virtually round, and although it appears relatively large, repeated measurements have shown it to be again about half the size of other cD's. It is located in the overlap region of two Palomar Observatory Sky Survey (POSS) fields, and its image is badly washed out on both. Like A1837, its measured z (0.048) is smaller than that predicted from both m_1 and m_{10} (0.065, SRS and LVDB), and the only redshift for the cluster is that of the cD candidate. It is classified BM II–III in LVDB but a possible BM I by Corwin (1974), who also suspected the galaxy was a foreground.

It is likely that the high incidence of relatively poor clusters on this list is due to their failure to provide a sufficiently rigorous size standard against which a cD candidate is to be judged. It is thus easier for a foreground or smaller cluster member to appear dominant.

Relevant data for the 22 clusters that remain in our sample are given in Table 1 with indicated column identifications.

Our raw diameters (mm) are the means of measurements by both authors on POSS E prints and are first adjusted to a fiducial isophote in the rest frame of the galaxy (Sandage 1972) according to

$$(a)_{\mathrm{corr}} = (a)_{\mathrm{obs}}\left[(1+z)^5 10^{0.4\Delta m}\right]^{1/n}, \qquad (9)$$

where a K correction of the form $2.5\log(1+z)$ has been assumed and

$$\Delta m = 0.1 \csc|b^{\mathrm{II}}|. \qquad (10)$$

Our results are insensitive to any constant shift in Δm. As in § II, n is the local power law at the point of measurement which, following CA, we have taken nominally to be 1.7. The major axis A (kpc) is then calculated assuming, for definiteness, $H_0 = 100$ km s^{-1} Mpc^{-1} and $q_0 = 0.2$. Using A and f, we calculate $(A_0)_O$ and $(A_0)_P$, which are the isophotal diameters under the oblate and prolate hypotheses, respectively, corrected to face-on according to equation (5) with $n = 1.7$. Our final results are relatively independent of n and q_0 and totally independent of H_0.

As a result of cosmological and K dimming, it follows that high-n galaxies decrease in isophotal diameter with redshift less rapidly than low-n galaxies. An E galaxy, for instance, observed at $3r_e$ (Tonry and Davis 1981) and equal in size to a cD at a given isophote (in their rest frames) will appear ~10% larger than the cD when

TABLE 1

cD GALAXIES IN ABELL CLUSTERS

Abell No. (1)	z (2)	BM (3)	Ref.[a] (4)	R (5)	$a \times b$ (mm) (6)	$\log A$ (kpc) (7)	$\log f$ (8)	$(\log A_0)_O$ (kpc) (9)	$(\log A_0)_P$ (kpc) (10)
85	0.0506	I	1	1	0.75×0.55	1.62	0.1367	1.54	1.57
133	0.053	I	1	0	0.75×0.50	1.64	0.1739	1.54	1.57
399	0.0714	I–II	1	1	0.5×0.4	1.62	0.097	1.56	1.58
401	0.0746	I	1	2	0.75×0.46	1.82	0.2123	1.69	1.73
403	0.107	I	3	2	0.475×0.2	1.79	0.3717	1.57	1.63
496	0.0326	I	1	1	1.1×0.85	1.6	0.112	1.54	1.56
655	0.1267	I–II (I)	2	3	0.5×0.2	1.9	0.398	1.67	1.74
1146	0.141	I–II	2	4	0.375×0.3	1.83	0.097	1.78	1.79
1413	0.1427	I	2	3	0.56×0.187	1.99	0.476	1.71	1.80
1651	0.0825	I–II	1	1	0.475×0.3	1.66	0.1996	1.54	1.57
1795	0.0620	I	1	2	0.75×0.65	1.72	0.0621	1.68	1.69
1890	0.084	I–II	1	0	0.65×0.425	1.80	0.1845	1.69	1.72
1918	0.14	I–II	3	3	0.65×0.15	2.06	0.6368	1.68	1.79
1991	0.0584	I	1	1	1.0×0.6	1.82	0.2218	1.69	1.73
2029	0.0767	I	1	2	0.9×0.45	1.90	0.301	1.73	1.78
2052	0.0345	I–II	1	0	1.0×0.7	1.58	0.155	1.49	1.51
2107	0.0421	I	1	1	0.85×0.65	1.60	0.1165	1.53	1.55
2124	0.0671	I	1	1	0.95×0.625	1.86	0.1818	1.75	1.79
2199	0.0303	I	1	2	1.5×1.1	1.70	0.1347	1.62	1.64
2589	0.0414	I	1	0	1.0×0.7	1.67	0.1549	1.57	1.60
2626	0.0562	I–II	1	0	0.8×0.5	1.71	0.2041	1.59	1.63
2670	0.078	I–II (I)	4,1	3	0.6×0.475	1.71	0.1015	1.65	1.67

NOTE—Column headings are: (1) Abell number; (2) redshift; (3) Bautz-Morgan type; (4) reference for redshift; (5) Abell richness; (6) major and minor axis measurements; (7) log of major axis in kpc; (8) log of flatness; (9) and (10) log of major axis (in kpc) if observed as round for oblate and prolate hypotheses, respectively.

[a]Reference for redshift: 1 = SRI. 2 = SRII. 3 = SRS. 4 = Thompson, private communication

they are at a redshift of 0.1 and will appear 17% larger at $z = 0.15$, the limit of our sample. All surface brightness dimming effects favor E's over cD's, and cD's become less outstanding in relative size with increasing redshift (an impression noted by LVDB). This effect is more pronounced when cD's are compared to steeper power law galaxies such as S0's. An upper limit can be found by comparing cD's to objects with infinite n, in which case the diameter ratio will scale as $(1+z)^{5/n}$ ($n \sim 1.7$). Thus a cD that is intrinsically three S0 diameters at a given isophotal level will appear to be only two S0 diameters at $z = 0.15$.

The f values in Table 1 show that cD galaxies can have a flatness equal to or exceeding that of normal ellipticals. In MMS, ML, AWM, and MKW one finds stated in various ways that cD galaxies are never highly flattened; and while it is evident in reading all of these papers that "flattening" meant the disklike appearance of some S0's, the multiplicity of these statements has led to the general impression that cD's are near-round galaxies. This notion was most recently reiterated by WH, whose results were derived by looking only at relatively nearby clusters ($D \leq 4$). LVDB found that the first ranked galaxies of BM I clusters tended to be flatter in the mean than in other BM types, perhaps reflecting the willingness to classify a galaxy as a cD when it is viewed at higher f.

In order to determine the degree to which our measurements are isophotal, we have measured six of the first ranked galaxies investigated by OM. Using our own f-values, we have evaluated $\Delta = \log(r_{SF}/r_{25})$, where r_{SF} is $(ab)^{1/2}$ measured by us and r_{25} is $(ab)^{1/2}$ at $\mu_v = 25$ mag arcsec^{-2}. For our six galaxies, $\Delta = -0.002$ with a dispersion of 0.06, implying that our measurements are at 25 ± 0.3 mag arcsec^{-2}. We find no significant trend in Δ with f. Since $(ab)^{1/2}$ is essentially A_0, these results imply an experimental uncertainty in $\log A_0$ of 0.06.

IV. THE LOG A–LOG f RELATION

In Figure 1 we have plotted $\log A$, $\log(A_0)_O$, and $\log(A_0)_P$ as a function of redshift. While the increase in A with z probably reflects standard biases characteristic of incomplete samples (Rubin *et al.* 1976), it is evident from examination of Figures 1*b* and 1*c* that a large part of the increase in A is due to a general flattening of the sample with increasing z. According to a Kolmogorov-Smirnov test, there is less than a 5% chance that the distant and near halves of our sample have been randomly drawn from the same parent distribution of f-val-

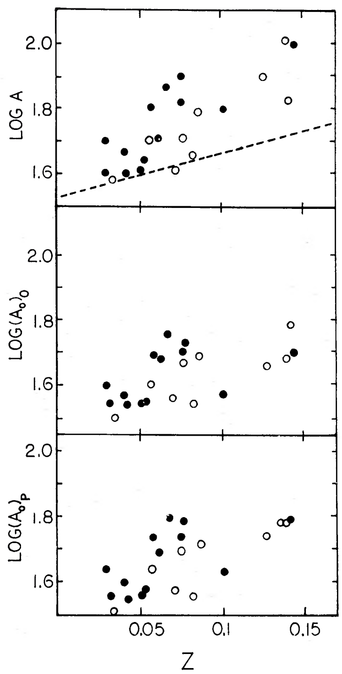

FIG. 1.—The distribution of A and A_0 (under oblate and prolate assumptions) with redshift z. Filled circles are galaxies in BM I clusters and open circles are galaxies in BM I–II clusters. The dotted line is the curve $\log A = (5/n)\log(1+z)+\text{constant}$ with $n \sim 1.7$, normalized to go through $\log A = 1.6$ at $z = 0.05$.

ues. This implies that the sample is significantly biased toward flatter galaxies with increasing z. While the chance of observing the flattest galaxies should improve with distance given the larger volume being sampled, their dominance over presumably more numerous rounder galaxies may be understood in terms of a combination of the increase in isophotal diameter with f acting together with the relative size effects discussed earlier. To illustrate this point, we have added to the plot of $\log A$ versus z in Figure 1 the curve $A = 3\log(1+z)+\text{const.}$ normalized to pass through $\log A = 1.6$ at $z = 0.05$. This curve gives the required increase in rest frame isophotal diameter to maintain a constant diameter ratio with respect to a galaxy whose light falls off infinitely fast.

According to Figure 1, there is a population of nearby cD's which, based on the discussion above and inspection of Table 1, is smaller and rounder in the mean than the sample in general, independent of either the oblate or prolate hypothesis. Moreover, it can be deduced from Table 1 that the mean cluster richness is also increasing with z (four of the five richness class 0 clusters have $z < 0.056$, and all of the seven richness class 1 clusters have $z < 0.083$). This is an important factor for two reasons. The first is that, as we have discussed, it is probably easier for a large elliptical or foreground galaxy to appear to be a cD in a less rich cluster. The second is that, as shown by OM, the brightness, and hence, in the mean, the diameter of the first ranked galaxy should increase with cluster luminosity, which in turn related to cluster richness. Thus it is reasonable to expect that the poorer nearby clusters would be smaller in the mean than the more distant richer clusters. Accordingly we apply a lower redshift cutoff of $z = 0.053$, leaving 15 more distant, larger cD's. The seven galaxies omitted by this cutoff have a mean $\log A_0$($= 1.55 \pm 0.02$ if oblate and 1.57 ± 0.02 if prolate) that is more than three standard deviations away from the mean of the total sample (1.63 ± 0.02 if oblate, 1.67 ± 0.02 if prolate) and a mean $f = 0.14 \pm 0.01$, which is more than two standard deviations away from that of the total sample (0.21 ± 0.03). For the 15 remaining clusters, relevant means and dispersions are given in Table 2. Inspection of Table 2 shows that flatness corrections can significantly improve the error in redshifts predicted from diameters. Using $A_0 = (AB)^{1/2}$ is probably a reasonable compromise between oblate and prolate predictions of A_0. Sandage's (1972) diameter measures of first ranked galaxies showed an intrinsic dispersion of 17% which is consistent with our dispersion in A_0, but smaller than our dispersion in A. In addition to superior plate material, it is likely that Sandage's sample was rounder in the mean and had a higher average n than ours, thus lowering the dispersion in A.

The $\log A$–$\log f$ relation for the 15 more distant clusters is shown in Figure 2. Its slope is 0.61 ± 0.12 with intercept 1.66 ± 0.04 (the cutoff at $z = 0.053$ is such that the slope is essentially unaffected by any cutoff at $z > 0.053$). The dispersion about the regression line for these points is 0.072, which exceeds that expected from measurement errors alone and requires some intrinsic source of scatter. This conclusion also follows from a comparison of the dispersion in intrinsic diameter under either the oblate or prolate assumption with the value of 0.06 expected from measurement errors alone. If all 22 points in the sample are included, then the relation is steepened to a slope of 0.75 ± 0.11 because of the addition of the smaller, rounder population.

It remains to evaluate the $\log A$–$\log f$ relation in the light of equation (6). One possibility is easily analyzed,

TABLE 2

MEAN SAMPLE DIAMETERS FOR GALAXIES WITH $z > 0.053$

Quantity	Mean	Dispersion
A (Major axis).......	67.2 kpc	20.1 kpc
B (Minor axis).......	37.5 kpc	8.1 kpc
$(A_0)_O$...............	47.0 kpc	7.4 kpc
$(A_0)_P$...............	52.1 kpc	8.8 kpc
$\text{Log}(A_0)_O$..........	1.67	0.071
$\text{Log}(A_0)_P$..........	1.71	0.078

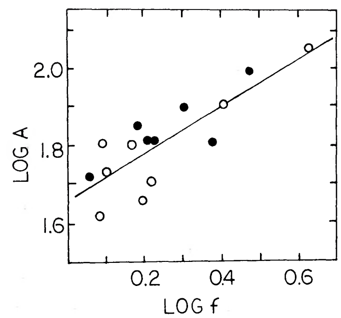

FIG. 2.—The variation of major axis A with flatness f. The solid line is $\log A = 0.6 \log f + 1.66$. Filled and open circles are as in Fig. 1.

and this is to assume $m = 0$ (f_0 and pole-on surface brightness are uncorrelated) and infer a value of n from the observed $\log A$–$\log f$ relation. Using 0.61 for the slope, this assumption gives $n = 1.66$ if the sample is intrinsically oblate and $n = 2.51$ if the sample is intrinsically prolate. While we have been assuming $n = 1.7$ throughout, this value was recommended by CA on the basis of two profiles. In Table 3 we give n values for the published profiles of DR and CA as determined by them, or by OM as determined by us. The entries of Table 3 would suggest a value of $n = 1.6$–1.65. Thus, independent of any redshift cutoff, our results are consistent with the $m = 0$ oblate hypothesis.

It is still possible that m could be zero and that cD's could be intrinsically prolate. A bias increasing $(A_0)_O$ with f sufficiently to make galaxies appear to be consistent with the $m = 0$ oblate hypothesis could have arisen from an accidental inclusion of smaller, rounder ellipticals, a systematic error in our determination of f (Holmberg 1946; Rood and Baum 1967), or the overlooking of smaller flatter galaxies. All of these effects can be evaluated by a program of surface photometry of cD and near cD candidates.

One further possibility can be checked immediately. Since the galaxies in the sample get flatter and come from richer clusters with increasing z, it is possible that the resulting increase in richness with f could produce the observed slope. This would not seem to be the case, since, if we limit the sample to just those clusters of richness classes 2 and 3, we find the slope is 0.63 ± 0.09 (correlation coefficient = 0.93) for nine clusters independent of any redshift cutoff, and 0.56 ± 0.10 (correlation coefficient = 0.89) if the one richness class 4 cluster (A1146) is included. MMS excluded richness class 0 and 1 clusters in their search for cD candidates. One interesting result of restricting the sample to richer clusters is that the relevant dispersions in $\log A_0$, and about the regression lines, for this sample of nine or ten clusters are all well within one standard deviation of the dispersion expected from measurement errors alone, and in some cases are as small as 0.05, suggesting that some of the residual dispersion in $\log A_0$ may come from mixing different richness groups. Finally we note that Abell (1958) did not consider richness class 0 clusters to be part of his homogeneous catalog. Excluding just these clusters from the total sample gives a slope of 0.72 ± 0.13, again consistent with the $m = 0$ oblate prediction independent of any redshift cutoff.

TABLE 3

POWER-LAW INDEX, n, FOR SOME cD GALAXIES

Cluster	Source	n
A2029........	DR	1.5
A2199........	CA	1.65–1.71
A2199........	OM	1.6–1.65
A2670........	OM	1.65–1.7
A1413........	OM	1.6–1.65

One possibility that is harder to evaluate is that cD's are intrinsically prolate but that m in equation (6) is not zero. This would imply an intrinsic increase in A_0 with f_0 producing the observed slope. In order to evaluate this possibility we have tried to model our test. For the sake of definiteness we chose the results obtained using the 15 more distant clusters. Using the observed distribution of $\log f$, we obtain an intrinsic one under the prolate assumption. We find it to be adequately described by a Gaussian of mean 0.2 and dispersion 0.33. As we have pointed out, our sample is probably not randomly selected, but our results are not too sensitive to the shape of the assumed distribution as long as it permits the realistic probability of very flat galaxies. Once f_0 is determined, a value of A_0 is selected from a Gaussian of mean $(m/n)\log f_0$ and dispersion σ_m/n (n the power law). Thus σ_m measures the total dispersion in the coupling of the pole-on surface brightness to intrinsic flatness and includes intrinsic as well as observational scatter. Finally, the galaxy is randomly projected and its major axis is determined assuming $n = 1.65$. For each set of 15 galaxies a slope and dispersion about the best fit regression line in the $\log A$–$\log f$ plane are determined. If the calculated slope and dispersion fall within the standard error of our observed values, a success is counted, and the percent of successes, α, in 5000 tries is determined as a function of m and σ_m. We should note that these were the only system diagnostics that we could find that were not strongly correlated with each other, and that the method of counting successes automatically takes care of any possible correlation in the diagnostics.

Contours of constant α in the $m - \sigma_m$ plane are shown in Figure 3; an inspection shows that the most probable value of m is 0.4 and that of σ_m is 0.105. If we believe that the dispersion in our ability to measure A_0 is actually 0.06, this requires $\sigma_m > 0.1$, so that the most

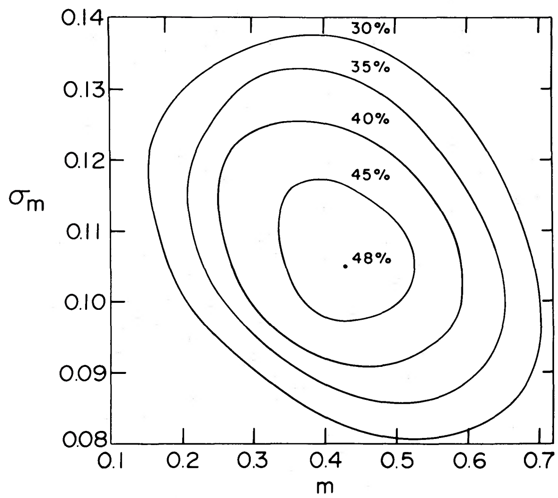

FIG. 3.—Contours of constant α in the m-σ_m plane. The point labeled 48% corresponds to the maximum value of α achieved in any run.

probable value of σ_m is very close to that expected from measurement uncertainties alone, implying a very tight correlation between A_0 and f_0. We can see from Figure 3 that $\sigma_m = 0.14$ is certainly possible, so that a residual dispersion of 0.25 mag in the coupling of pole-on surface brightness to intrinsic flattening cannot be ruled out. One could have estimated the value of m necessary to explain our results. Since prolate galaxies tend to project near maximum flatness, m had to be about n times the difference between the $m = 0$ oblate and prolate predictions, or $2 - n \sim 0.35$. In the case of ellipticals, oblate and prolate predictions for $m = 0$ actually reverse relative slopes as n is greater or less than 2, so that if A_0 increases with f_0, galaxies should appear to change intrinsic shape as n goes through 2 (i.e., m cannot simultaneously make galaxies appear larger and smaller). While, as we have cautioned, cD galaxies are probably not like normal ellipticals, our results are very different from the values of $m = 2$ required by Olson and de Vaucouleurs (1981) and $m = 1$ obtained by Merritt (1982) for a subset of the same data.

As discussed above, the uncertainties in defining the sample can be minimized by a program of surface photometry, but the fundamental ambiguity implicit in equation (6) is somewhat more complex. At the most basic level, it is clear that the bulk of the dispersion about the regression line and consequently the error in the slope is due to measurement errors, so that surface photometry should permit the placement of rather precise bounds on m, and should permit testing the prediction of equation (6) that the dispersion in $\log A$ necessarily decreases with f (basically because only the largest galaxies can appear to be the flattest, but these galaxies can increase the dispersion at all $f < f_0$).

A more fundamental question about the assumptions of equation (6) can be raised, however. Assume a sample of intrinsically prolate galaxies with m not equal to zero and consider the set of all possible tests of intrinsic shape based on $m = 0$ that can be performed on the sample. For some of these tests such as those of Olson and de Vaucouleurs (1981) or Richstone (1979), m must necessarily be large to mimic $m = 0$ oblate predictions. For other tests, such as those implicit in equation (5), the value of m necessary to mimic $m = 0$ oblate results varies in magnitude and sign with the choice of isophote at which the test is conducted. It follows then that unless m is very close to zero, a test randomly selected from the set of all tests will produce a result that, in general, is inconsistent with both the oblate and prolate $m = 0$ hypotheses. In this sense, the test that we have presented is "atypical" if m is not zero. In this case one must include as also being "atypical" the tests of Olson and de Vaucouleurs (1981) and Marchant and Olson (1979). Of the two tests conducted by Richstone (1979) that involved the assumptions of equation (6), one was identical in concept to that of Olson and de Vaucouleurs and should not be counted again; for the other, while its results were marginally consistent with the $m = 0$ oblate hypothesis, the degree of significance was never explicitly evaluated, so it cannot be counted as a clear result. The results of Merritt (1982), who analyzed a subset of the Olson and de Vaucouleurs sample, might seem at first sight to be a "typical" test since the measured slope of the variation of effective brightness with flatness is near zero, midway between the oblate and prolate predictions for $m = 0$. In this case, however, the rank correlation coefficient is zero, implying a very large formal error in the slope so that these results are probably consistent with both the oblate and prolate predictions for $m = 0$, and this test also should not be counted. Thus all of the tests of equation (6) done to date are either inconclusive in that they do not have sufficient resolution to accept or reject the $m = 0$ hypotheses, or they are "atypical." It is possible that tests which reject both the $m = 0$ oblate and prolate hypotheses at some reasonable confidence level simply do not get published since they do not seem to resolve the issue. However, in the light of preceding arguments, these tests, if they exist, are important in the determination of effects like those implied by equation (6), independent of the issue of intrinsic shape.

V. DISCUSSION

We have shown that the shallow photometric profiles of cD galaxies can be used to infer their intrinsic shape from the observed variation of A with f. The test is possible because cD's can be found that span a sufficient range of f, and because the dispersion of intrinsic cD diameters (A_0) is small ($\sim 16\%$). Our results establish that these conditions are met almost independent of any errors in our measurements. The two major sources of error in our work, (1) the inclusion of non–cD galaxies in the sample and (2) systematic errors in our determination of b/a (Holmberg 1946; Rood and Baum 1967), can both be eliminated by detailed surface photometry of the objects in our sample.

Measurements made from Sky Survey prints are consistent with the oblate hypothesis if pole-on surface brightness and flatness are uncorrelated. We find this result for a variety of ways of defining the sample. First ranked cluster galaxies provide a natural sample of small intrinsic dispersion that should prove very useful in tests of this kind. It is possible that the sample is intrinsically prolate but that intrinsically flatter galaxies have a higher pole-on surface brightness causing the oblate result. We find that for m as defined in equation (6), m in the range 0.4 ± 0.2 could have a reasonable chance of reproducing our observations.

The work of TR, Malmuth and Kirshner (1981), DR, and OM has led to a picture of merger-dominated, self-similar growth (Ostriker and Hausman 1977; Hausman and Ostriker 1978) in poor clusters and stripping-dominated (Richstone 1976) envelope accumulation in rich clusters. Thus "cD" galaxies in poor clusters remain photometrically ellipticals as they grow, and cD galaxies in rich clusters develop an extended envelope characteristic of the cluster potential. Supporting this view are numerous studies (Sastry 1968; Carter and Metcalfe 1981; Binggeli 1982) showing strong correlation between the position angles and ellipticities of first ranked galaxies and the clusters in which they are found, suggesting that our results refer also to the shape of the cluster potential well. Oblate clusters would be consistent with the pancake formation hypothesis (Zel'dovich 1970; Doroshkevich, Shandarin, and Saar 1978). On the other hand, White's (1979) N-body experiments indicate that merger products are more likely to be oblate, independent of cluster potential, so that cluster and cD shapes may not be so strongly coupled.

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REFERENCES

Abell, G. O. 1958, *Ap. J. Suppl.*, **3**, 211.
Albert, C., White, R., and Morgan, W. W. 1977, *Ap. J.*, **211**, 309 (AWM).
Bautz, L., and Morgan, W. W. 1970, *Ap. J. (Letters)*, **162**, L149 (BM).
Binggeli, B. 1982, *Astr. Ap.*, **107**, 338.
Binney, J. 1978, *Comm. Ap.*, **8**, 27.
Carter, D. 1977, *M.N.R.A.S.*, **178**, 137 (CA).
Carter, D., and Metcalfe, N. 1981, *M.N.R.A.S.*, **191**, 325.
Contopoulos, G. 1956, *Z. Astr.*, **39**, 126.
Corwin, H. 1974, *A.J.*, **79**, 1356.
de Vaucouleurs, G. 1948, *Ann. d'Ap.*, **11**, 247.
______. 1953, *M.N.R.A.S.*, **113**, 134.
de Vaucouleurs, G., de Vaucouleurs, A., and Corwin, H. 1976, *Second Reference Catalog of Bright Galaxies* (Austin: University of Texas Press).
Djorgovski, S. 1982, *Bull. AAS*, **13**, 796.
Doroshkevich, A. G., Shandarin, S., and Saar, E. 1981, *M.N.R.A.S.*, **184**, 643.
Dressler, A. 1979, *Ap. J.*, **231**, 659 (DR).
Geller, M., and Peebles, P. J. 1976, *Ap. J.*, **206**, 939.
Hausman, M., and Ostriker, J. P. 1978, *Ap. J.*, **224**, 320.
Hintzen, P. 1980, *A.J.*, **85**, 626.
Hoessel, J., Gunn, J., and Thuan, T. X. 1980, *Ap. J.*, **241**, 486 (HO).
Holmberg, E. 1946, *Medd. Lund. Obs.* (11), No. 117.
Hubble, E. 1936, *Ap. J.*, **71**, 231.
Kormendy, J. 1977, *Ap. J.*, **218**, 333.
Leir, A., and van den Bergh, S. 1977, *Ap. J. Suppl.*, **34**, 381 (LVDB).
Malumuth, E., and Kirshner, R. 1981, *Ap. J.*, **251**, 508.
Marchant, A. B., and Olson, D. W. 1979, *Ap. J. (Letters)*, **230**, L157.
Matthews, T., Morgan, W. W., and Schmidt, M. 1964, *Ap. J.*, **140**, 35 (MMS).
Merritt, D. 1982, *A.J.*, **87**, 1279.
Morgan, W. W., Kayser, S., and White, R. 1975, *Ap. J.*, **199**, 545 (MKW).
Morgan, W. W., and Lesh, J. 1965, *Ap. J.*, **142**, 1364 (ML).
Oemler, A. 1976, *Ap. J.*, **209**, 693 (OM).
Olson, D. W., and de Vaucouleurs, G. 1981, *Ap. J.*, **249**, 68.
Ostriker, J. P., and Hausman, M. 1977, *Ap. J. (Letters)*, **127**, L125.
Richstone, D. 1976, *Ap. J.*, **204**, 642.
______. 1979, *Ap. J.*, **234**, 825.
Rood, H. J., and Baum, W. A. 1967, *A.J.*, **72**, 398.
Rubin, V., Thonnard, N., Ford, W., and Roberts, M. S. 1976, *A.J.*, **81**, 719.
Sandage, A. 1972, *Ap. J.*, **173**, 485.
Sarazin, C., Rood, H. J., and Struble, M. F. 1982, *Astr. Ap.*, **108**, L7 (SRS).
Sastry, G. 1968, *Pub. A.S.P.*, **80**, 252.
Schweizer, F. 1979, *Ap. J.*, **233**, 23.
Stark, A. A. 1977, *Ap. J.*, **213**, 318.
Struble, M. F., and Rood, H. J. 1982, *A.J.*, **87**, 7 (SRI).
______. 1983, in preparation (SRII).
Tonry, J., and Davis, M. 1981, *Ap. J.*, **246**, 666.
Thuan, T. X., and Romanishin, W. 1981, *Ap. J.*, **248**, 439 (TR).
White, R. 1978, *Ap. J.*, **226**, 591 (WH).
White, S. 1979, *M.N.R.A.S.*, **189**, 831.
Zel'dovich, Ya. B. 1970, *Astrofizika*, **6**, 319.

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