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ANALYSIS METHODS FOR RESULTS IN GAMMA-RAY ASTRONOMY

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ABSTRACT

The current procedures for analyzing results of γ-ray astronomy experiments are examined critically. We propose two formulae to estimate the significance of positive observations in searching γ-ray sources or lines. The correctness of the formulae are tested by Monte Carlo simulations.

Subject headings: gamma-rays: general — numerical methods

I. INTRODUCTION

Evaluation of the statistical reliability of positive results in searching discrete γ-ray sources or lines is an important problem in γ-ray astronomy. Since both the signal-to-background ratio and detector sensitivity are generally limited in this energy range, one must carefully analyze the observed data to determine the confidence level of a candidate source or line, that is, the probability that the count rate excess is due to a genuine source or line rather than to a spurious background fluctuation, even though all systematic effects are believed to have been removed.

Figure 1 shows a typical observation in γ-ray astronomy. A photon detector points in the direction of a suspected source for a certain time t_{on} and counts N_{on} photons, and then it turns for background measurement for a time interval t_{off} and counts N_{off} photons. The quantity α is the ratio of the on-source time to the off-source time, $\alpha = t_{\text{on}}/t_{\text{off}}$ (in some cases of searching for lines, N_{on} is the number of counts under a peak in an energy spectrum, and the peak is taken to be n_S channels wide; N_{off} is the number of counts in n_b channels adjacent to the peak; then $\alpha = n_S/n_b$). Then we can estimate the number of background photons included in the on-source counts N_{on}:

$$\hat{N}_B = \alpha N_{\text{off}} \, . \tag{1}$$

The observed signal, the probable number of photons contributed by the source, is

$$N_S = N_{\text{on}} - \hat{N}_B = N_{\text{on}} - \alpha N_{\text{off}} \, . \tag{2}$$

For a positive observation of an emission source, the excess counts $N_{\text{on}} - \hat{N}_B$ may have been caused only by a statistical fluctuation in the background rate. That the background is not known exactly in a γ-ray astronomy experiment generally and can be inferred only from the limited background counts is a basic difficulty in evaluating the statistical reliability of an observational result.

There have been various procedures adopted by different experimenters to estimate statistical reliability. The significances of the published positive results have often been overestimated by the observers because of the incorrectness of their methods of analysis. Hearn (1969) has suggested a relative likelihood method for consistent analysis of γ-ray astronomy experiments. O'Mongain (1973) applied this approach to the early observations of high-energy γ-ray sources and found that a very large number of reported γ-ray sources could reasonably be explained as background fluctuations. Cherry *et al.* (1980) improved the evaluation of relative likelihood and used it to reanalyze the reported γ-ray lines with similar results. But from the point of view of mathematical

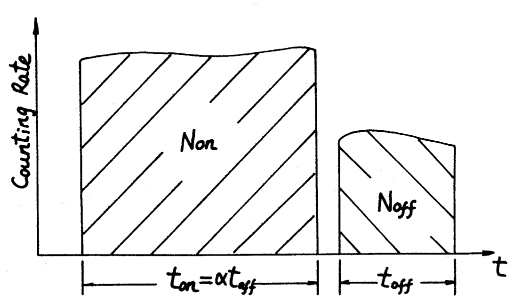

FIG. 1.—A typical observation in γ-ray astronomy

statistics, this relative likelihood method is also incorrect. It underestimates the significances of observations systematically in most actual cases. In other words, it unreasonably reduces the estimate of the statistical reliability of experimental results.

In this paper we make a critical examination of the current methods in analyses of γ-ray astronomy experiments, and we propose two formulae to evaluate the statistical significance of an observational result: one is derived by immediately estimating the standard deviation of the observed signal N_S (§ II), and the other one by applying the method of statistical hypotheses test (§ III). In § IV we test the correctness of the formulae by Monte Carlo simulations, and in § V we discuss how to determine the confidence level of a positive result which is obtained from many independent observations.

II. STANDARD DEVIATION OF SIGNAL

Since on-source counts N_{on} and background counts N_{off} are results of two independent measurements, we can calculate the variance of the signal N_S defined by equation (2):

$$\sigma^2(N_S) = \sigma^2(N_{\text{on}}) + \sigma^2(\alpha N_{\text{off}}) = \sigma^2(N_{\text{on}}) + \alpha^2\sigma^2(N_{\text{off}}) . \tag{3}$$

Then the estimate of the standard deviation of N_S is

$$\hat{\sigma}(N_S) = \sqrt{\hat{\sigma}^2(N_{\text{on}}) + \alpha^2\hat{\sigma}^2(N_{\text{off}})} = \sqrt{N_{\text{on}} + \alpha^2 N_{\text{off}}} . \tag{4}$$

Defining the significance S as a ratio of the excess counts above background to its standard deviation, we have

$$S = \frac{N_S}{\hat{\sigma}(N_S)} = \frac{N_{\text{on}} - \alpha N_{\text{off}}}{\sqrt{N_{\text{on}} + \alpha^2 N_{\text{off}}}} . \tag{5}$$

The formula above is simply from the Poisson law of the counts N_{on} and N_{off}. Considering the fact that the discrepancies between the distribution of the significances computed by equation (5) for Monte Carlo simulation samples and the expected normal distribution are considerable in the case $\alpha \neq 1$ (see § IV, Fig. 2), it is necessary to improve the above estimate for the standard deviation of N_S further (Ma and Li 1983).

When one evaluates the statistical reliability of an observational result, that is, estimates the probability that the observed signal was due only to the background, it should be assumed that there was no extra source, and all the observed counts, not only N_{off} but also N_{on}, were due to the background. Under this assumption on-source counts N_{on} would follow a Poisson distribution with expectation and variance $\langle N_B \rangle$, and off-source counts N_{off} would do the same but with $\langle N_B \rangle/\alpha$ instead of $\langle N_B \rangle$, where $\langle N_B \rangle$ is the expectation of background counts in on-source time t_{on}. Then equation (3) can be written as

$$\sigma^2(N_S) = \sigma^2(N_{\text{on}}) + \alpha^2\sigma^2(N_{\text{off}}) = (1 + \alpha)\langle N_B \rangle ,$$

and the standard deviation of N_S,

$$\sigma(N_S) = \sqrt{(1 + \alpha)\langle N_B \rangle} . \tag{6}$$

Usually equation (1) is used to estimate background. But in the case of the assumption that all the recorded photons are due to background, we can get a more accurate estimate of $\langle N_B \rangle$, the expected number of the background photons in t_{on}, by using all the observed data (N_{on}, N_{off}):

$$\langle \hat{N}_B \rangle = \frac{N_{\text{on}} + N_{\text{off}}}{t_{\text{on}} + t_{\text{off}}} t_{\text{on}} = \frac{\alpha}{1 + \alpha} (N_{\text{on}} + N_{\text{off}}) . \tag{7}$$

Then the estimate of the standard deviation of N_S is

$$\hat{\sigma}(N_S) = \sqrt{(1 + \alpha)\langle \hat{N}_B \rangle} = \sqrt{\alpha(N_{\text{on}} + N_{\text{off}})} , \tag{8}$$

and the significance is

$$S = \frac{N_S}{\hat{\sigma}(N_S)} = \frac{N_{\text{on}} - \alpha N_{\text{off}}}{\sqrt{\alpha(N_{\text{on}} + N_{\text{off}})}} . \tag{9}$$

An observational result with significance S can be called an "S standard deviation result." In the case that the numbers of photons counted are not too few (say $N_{\text{on}} \gtrsim 10$, $N_{\text{off}} \gtrsim 10$), counts N_{on} and N_{off}, then N_S, are approximately normally distributed. Under the assumption mentioned above that no extra sources exists, $\langle N_S \rangle = 0$, significance S will approximate a standard normal variable with zero mean and unit variance, and we can take the Gaussian probability of S as the confidence level of the observation result. Our Monte Carlo simulation results (see § IV, Fig. 2) show that the distributions of significances evaluated by equation (9) are closer to a standard normal curve than those evaluated by equation (5).

Some variant methods have been adopted to estimate the significances when experimenters reported their positive results of γ-ray sources or lines. They often overestimate the significance since they do not consider all the statistical factors in an observational result. For example, many experimenters use the standard deviation of the number of background photons N_B as a measure of the statistical error of the observed signal N_S, and define the significance as

$$S = \frac{N_S}{\sqrt{\hat{N}_B}} = \frac{N_S}{\sqrt{\alpha N_{\text{off}}}} . \tag{10a}$$

But in the general case of $\alpha \neq 1$, $\hat{N}_B$ does not simply follow a Poisson distribution, and its variance should be evaluated by the equation $\sigma^2(\hat{N}_B) = \alpha^2 N_{\text{off}}$. Then equation (10a) should be rewritten as

$$S = \frac{N_S}{\sigma(\hat{N}_B)} = \frac{N_S}{\alpha\sqrt{N_{\text{off}}}} . \tag{10b}$$

In equations (10a) and (10b), just the statistical fluctuation of the background counts N_{off} has been considered, and not that of N_{on}. Obviously they underestimate the statistical error of the signal N_S and, therefore, overestimate its significance.

Considering the statistical error of on-source counts N_{on} as the error of an observational result, some other experimenters let

$$S = \frac{N_S}{\sqrt{N_{\text{on}}}} . \tag{11}$$

Equation (11) also overestimates the significance because the statistical fluctuation of the background counts N_{off} has not been considered in it.

Finally, more than a few experimenters use the square root of the number of signal photons as the standard deviation of the signal; then the significance

$$S = \frac{N_S}{\sqrt{N_S}} . \tag{12}$$

But N_S is a quantity derived from the directly observed values (N_{on}, N_{off}) by equation (2); it does not simply follow a Poisson distribution with variance N_S. Hence equation (12) is not correct either.

III. LIKELIHOOD RATIO METHOD

Another way of estimating the significance is by use of the method of hypotheses test in mathematical statistics. In the present problem there are two unknown parameters: the expectation of the number of source photons, $\langle N_S \rangle$, and the expectation of the number of background photons, $\langle N_B \rangle$. The statistical hypothesis tested here, called "null hypothesis," is: no extra source exists, and all observed photons are due to background, that is, $\langle N_S \rangle = 0$. This is a test problem of a composite hypothesis where just partial parameters are involved. There is a theorem in statistics (see Wilks 1962, § 13.8; Eadie *et al.* 1971, § 10.5.2; Li 1980, § 6.3.4) which can be used to solve this sort of problem:

Theorem.—Letting observed data $X = (x_1, x_2, \ldots, x_N)$, unknown parameters $\Theta = (E, T) = (\epsilon_1, \epsilon_2, \ldots, \epsilon_r, \tau_1, \tau_2, \ldots, \tau_S)$, and statistical hypotheses:

Null hypothesis: $E = E_0 = (\epsilon_{10}, \epsilon_{20}, \ldots, \epsilon_{r0})$,

Alternative hypothesis: $E \neq E_0$,

define the maximum likelihood ratio

$$\lambda = \frac{L(X \mid E_0, \hat{T}_c)}{L(X \mid \hat{E}, \hat{T})} = \frac{P_r(X \mid E_0, \hat{T}_c)}{P_r(X \mid \hat{E}, \hat{T})} , \tag{13}$$

where $L(X \mid \Theta')$ is the likelihood function of N observed values X given parameters $\Theta = \Theta'$, that is, the probability of experimental results X given $\Theta = \Theta'$; $\hat{E}$ and $\hat{T}$ are the maximum likelihood estimates of parameters E and T; $\hat{T}_c$ are the conditional maximum likelihood estimates given $E = E_0$. On condition of null hypothesis $E = E_0$ being true, variable $-2 \ln \lambda$ will asymptotically follow a χ^2 distribution with r degrees of freedom, while $N \to \infty$, as denoted by

$$-2 \ln \lambda \sim \chi^2(r) .$$

In our case, the observed data $X = (N_{\text{on}}, N_{\text{off}})$, estimated unknown parameters $\Theta = (\langle N_S \rangle, \langle N_B \rangle)$, and

Null hypothesis: $\langle N_S \rangle = 0$,

Alternative hypothesis: $\langle N_S \rangle \neq 0$.

On general condition the maximum likelihood estimates of $\langle N_B \rangle$ and $\langle N_S \rangle$ can be computed by equations (1) and (2), respectively; they are $\langle \hat{N}_B \rangle = \hat{N}_B = \alpha N_{\mathrm{off}}$ and $\langle \hat{N}_S \rangle = N_S = N_{\mathrm{on}} - \alpha N_{\mathrm{off}}$. On the other hand, if null hypothesis is true, or $\langle N_S \rangle = 0$, the conditional maximum likelihood estimate of $\langle N_B \rangle$ should be computed by equation (7); that is, $\langle \hat{N}_B \rangle = [\alpha/(1+\alpha)](N_{\mathrm{on}} + N_{\mathrm{off}})$. Then we can express the likelihood functions as follows:

$$
\begin{aligned}
L(X \mid E_0, \hat{T}_c) &= P_r\left[N_{\mathrm{on}}, N_{\mathrm{off}} \mid \langle N_S \rangle = 0, \langle N_B \rangle = \frac{\alpha}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right] \\
&= P_r\left[N_{\mathrm{on}} \mid \langle N_{\mathrm{on}} \rangle = \frac{\alpha}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right] P_r\left[N_{\mathrm{off}} \mid \langle N_{\mathrm{off}} \rangle = \frac{1}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right] \\
&= \left\{\left[\frac{\alpha}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right]^{N_{\mathrm{on}}} \Big/ N_{\mathrm{on}}!\right\} \exp\left[-\frac{\alpha}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right] \left\{\left[\frac{1}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right]^{N_{\mathrm{off}}} \Big/ N_{\mathrm{off}}!\right\} \\
&\quad \times \exp\left[-\frac{1}{1+\alpha}(N_{\mathrm{on}} + N_{\mathrm{off}})\right], \\
L(X \mid \hat{E}, \hat{T}) &= P_r(N_{\mathrm{on}}, N_{\mathrm{off}} \mid \langle N_S \rangle = N_{\mathrm{on}} - \alpha N_{\mathrm{off}}, \langle N_B \rangle = \alpha N_{\mathrm{off}}) \\
&= P_r(N_{\mathrm{on}} \mid \langle N_{\mathrm{on}} \rangle = N_{\mathrm{on}}) P_r(N_{\mathrm{off}} \mid \langle N_{\mathrm{off}} \rangle = N_{\mathrm{off}}) \\
&= \frac{N_{\mathrm{on}}^{N_{\mathrm{on}}}}{N_{\mathrm{on}}!} \exp(-N_{\mathrm{on}}) \frac{N_{\mathrm{off}}^{N_{\mathrm{off}}}}{N_{\mathrm{off}}!} \exp(-N_{\mathrm{off}})\,;
\end{aligned}
$$

and the maximum likelihood ratio

$$
\lambda = \frac{L(X \mid E_0, \hat{T}_c)}{L(X \mid \hat{E}, \hat{T})} = \left[\frac{\alpha}{1+\alpha}\left(\frac{N_{\mathrm{on}} + N_{\mathrm{off}}}{N_{\mathrm{on}}}\right)\right]^{N_{\mathrm{on}}} \left[\frac{1}{1+\alpha}\left(\frac{N_{\mathrm{on}} + N_{\mathrm{off}}}{N_{\mathrm{off}}}\right)\right]^{N_{\mathrm{off}}} . \tag{14}
$$

In this case only one parameter, $\langle N_S \rangle$, is involved in null hypothesis; thus $r = 1$. According to the theorem above, if the null hypothesis is true and both N_{on} and N_{off} are not too few, $-2 \ln \lambda$ will approximately follow a χ^2 distribution with 1 degree of freedom:

$$
-2 \ln \lambda \sim \chi^2(1)\,, \qquad \text{or} \qquad \sqrt{-2 \ln \lambda} \sim \chi(1)\,. \tag{15}
$$

As we know, if u is a standard normal variable, then u^2 will follow a χ^2 distribution with 1 degree of freedom:

$$
u^2 \sim \chi^2(1)\,, \qquad \text{or} \qquad |u| \sim \chi(1)\,. \tag{16}
$$

Comparing equation (15) with equation (16), we can see that if the null hypothesis $\langle N_S \rangle = 0$ is true, in other words, if all counts come from the background, the variable $(-2 \ln \lambda)^{1/2}$ will be equivalent to the absolute value of a standard normal variable; hence, we can directly take the value of $(-2 \ln \lambda)^{1/2}$ as the significance of the observed result

$$
S = \sqrt{-2 \ln \lambda} = \sqrt{2}\left\{N_{\mathrm{on}} \ln\left[\frac{1+\alpha}{\alpha}\left(\frac{N_{\mathrm{on}}}{N_{\mathrm{on}} + N_{\mathrm{off}}}\right)\right] + N_{\mathrm{off}} \ln\left[(1+\alpha)\left(\frac{N_{\mathrm{off}}}{N_{\mathrm{on}} + N_{\mathrm{off}}}\right)\right]\right\}^{1/2} . \tag{17}
$$

If an event $(N_{\mathrm{on}}, N_{\mathrm{off}})$ was obtained by a single observation where N_{on} and N_{off} are not too few, and the value of the significance of this event evaluated by equation (17) (or by eq. [9]) is S, then one can say that an "S standard deviation event" has been observed. In the case that only emission sources or lines are interesting, that is, just the case of $N_S > 0$ is considered (or for an absorption case, $N_S < 0$), the significance level of the event, or the probability that an event with a significance which is not less than S is produced by background, denoted by p, can be evaluated by the Gaussian probability

$$
p = N(u = S; 0, 1)\,, \tag{18}
$$

where $N(u;\, 0,\, 1)$ is the standard normal distribution function, that is, the distribution function of the normal variable u with zero mean and unit variance. Then the probability that a real source exists, that is, the confidence level, denoted by ξ, is

$$
\xi = 1 - p\,. \tag{19}
$$

The confidence level for an event obtained from many observations will be discussed in § V.

In the relative likelihood method used by Hearn (1969), O'Mongain (1973), and Cherry *et al.* (1980), the confidence level of a single observation is evaluated by

$$
\xi = 1 - \lambda'\,, \tag{20}
$$

where the likelihood ratio λ' is defined as

$$\lambda' = \frac{P_r(N_{on}|B)}{P_r(N_{on}|S+B)}, \tag{21}$$

where $P_r(N_{on}|B)$ is the probability that the counts N_{on} are due only to background, and $P_r(N_{on}|S+B)$ is the probability that N_{on} are due to a hypothetical source plus background.

According to mathematical statistics, the confidence level can be evaluated from λ' by equation (20) only if the variable $-2 \ln \lambda'$ follows a χ^2 distribution with 2 degrees of freedom. But $-2 \ln \lambda'$ is not distributed as $\chi^2(2)$, so equation (20) is not correct. As has been stated, variable $-2 \ln \lambda$ approximately follows the distribution of $\chi^2(1)$ where the maximum likelihood ratio λ is defined by equation (14), and the significance can be evaluated from λ by equation (17); but generally the relative likelihood λ' defined by equation (21) does not follow the distribution of $\chi^2(1)$, and therefore, we cannot take a procedure similar to equation (17) to evaluate the significance from λ'. The foregoing discussion illustrates that the relative likelihood method mentioned above has no correct foundation in mathematical statistics. The author has pointed out (Li 1980) that the attempt to simply calculate the confidence probability directly from the value of the likelihood function comes from misinterpreting the meaning of likelihood function.

IV. MONTE CARLO SIMULATIONS

For the purpose of checking the methods of estimating statistical significance, we have done Monte Carlo simulations by means of computer. First we assumed a certain expected value of background counts, $\langle N_{off} \rangle$, and the ratio of on-source observation time to off-source time, α. Under the hypothesis that only background photons have been detected, we have $\langle N_{on} \rangle = \alpha \langle N_{off} \rangle$, where $\langle N_{on} \rangle$ is the expected value of the counts in the on-source time. Then we obtained a random sample of observations, (N_{on}, N_{off}), by generating the random number N_{on} from the Poisson distribution with expectation $\langle N_{on} \rangle$ and the random number N_{off} from the other Poisson distribution with expectation $\langle N_{off} \rangle$, and evaluated the signal of the sample by $N_S = N_{on} - \alpha N_{off}$ and its significance S by equations (5), (9), and (17), respectively. The procedures of sampling and evaluating described above were repeated about 10^5 times for each set of assumed values of $\langle N_{off} \rangle$ and α. The integral frequency distributions of the significances of all apparent source events ($N_S > 0$) in our Monte Carlo simulation samples are shown in Figures 2*a*–2*f*. The lines indicate the standard normal distribution. It can be seen from the simulation results shown in Figure 2 that compared with the Gaussian probability, equation (5) systematically underestimates the significances for the case $\alpha < 1$ and overestimates for $\alpha > 1$. (In contrast, for an absorption source, $N_S < 0$, eq. [5] overestimates the significances for $\alpha < 1$ and underestimates for $\alpha > 1$.) Equation (9) is better than equation (5), but the distributions of the significances calculated by equation (17) from the maximum likelihood ratio method are generally most consistent with the expected Gaussian probabilities.

Figure 3 shows the scattering of the significances of some reported positive observations of γ-ray lines, where the data are taken from the summary of Cherry *et al.* (1980, Table 1), the abscissae of the points are the significance values evaluated by equation (17), and their ordinates are the significance values given by experimenters (for the points indicated by pluses) or by Cherry *et al.* (indicated by open circles) or by equation (17) of this paper (indicated by filled circles). The correctness of equation (17) has been illustrated by our Monte Carlo simulations; therefore, the significance values indicated by closed circles should represent the real significance distribution of the observational results. It is clear in Figure 3 that the experimenters overestimated the significances considerably, whereas Cherry *et al.* (1980) underestimated them (except for a few cases with significance $S \gtrsim 7$). In connection with the latter, the upper limit fluxes obtained by the relative likelihood method of Hearn (1969) are often overestimated.

V. CONFIDENCE LEVEL

Equation (19) for evaluating the confidence level is correct if only a single observation is made. In practice, however, many observations could be made in an experiment by repeating the measurement many times, or by scanning an area of the sky or a range of energies. In the case of getting a candidate source or line event with a significance S after M attempts being made, the confidence level of this event is not only dependent on its significance, S, but also on the total number of analyzed samples, M. The probability, p, that an event with a significance not being less than S is produced by the background in a single observation can be evaluated by equation (18). Then the probability of producing k such events by the background is

$$p_k = C_M^k p^k (1-p)^{M-k},$$

where C_M^k is the binomial coefficient. From this equation it is easy to compute the probability that none of such apparent source events ($k = 0$) is produced by the background in all M observations, or the probability that such an event is due to a real source, that is, the confidence level, as follows:

$$\xi = p_0 = (1-p)^M. \tag{22}$$

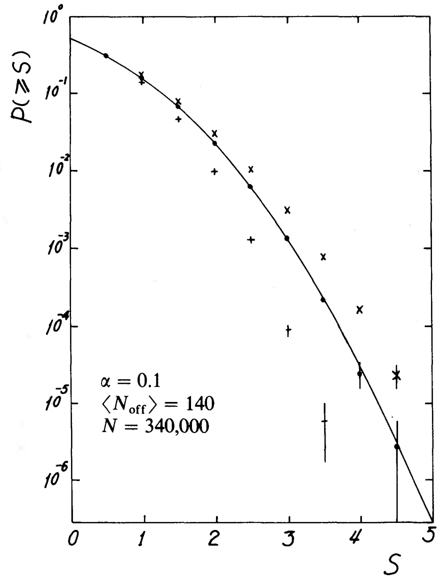

FIG. 2*a*

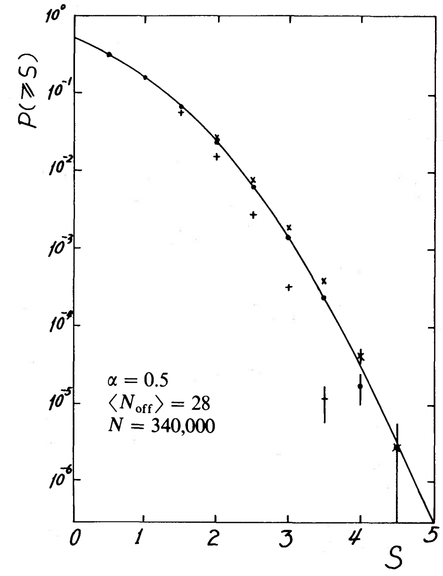

FIG. 2*b*

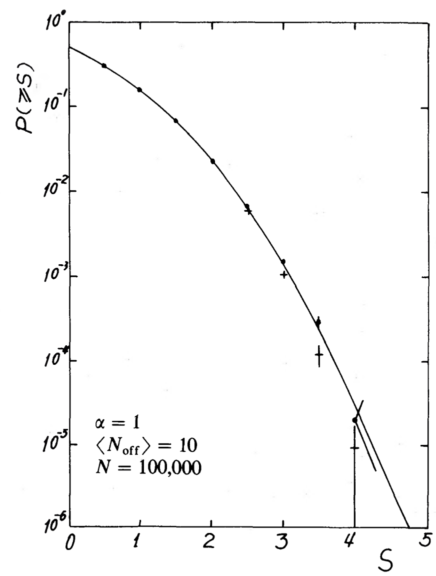

FIG. 2*c*

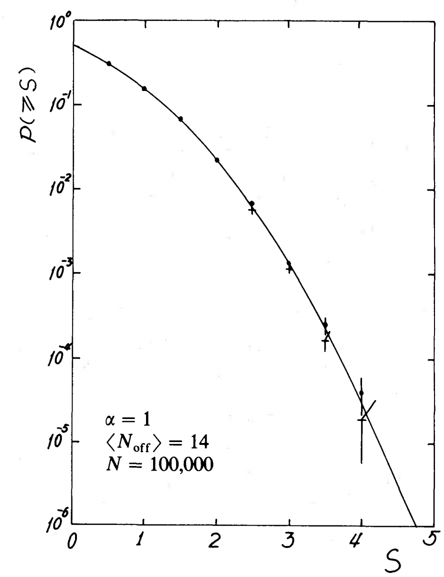

FIG. 2*d*

FIG. 2.—Integral frequency distributions of the significances of the Monte Carlo samples.

Pluses, from eq. (5):
$$S = \frac{N_{on} - \alpha N_{off}}{(N_{on} + \alpha^2 N_{off})^{1/2}} .$$

Crosses, from eq. (9):
$$S = \frac{N_{on} - \alpha N_{off}}{[\alpha(N_{on} + N_{off})]^{1/2}} .$$

Filled Circles, from eq. (17):
$$S = 2^{1/2}\left\{N_{on} \ln\left[\frac{1+\alpha}{\alpha}\left(\frac{N_{on}}{N_{on}+N_{off}}\right)\right] + N_{off} \ln\left[(1+\alpha)\left(\frac{N_{off}}{N_{on}+N_{off}}\right)\right]\right\}^{1/2} .$$

N is the number of samples for the Monte Carlo procedure. The curves indicate the standard normal distribution.

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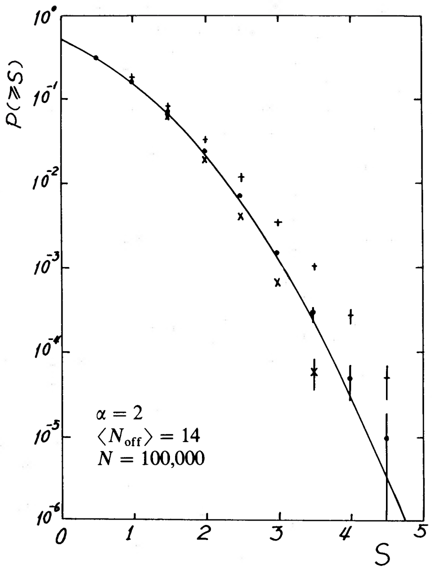

FIG. 2*e*

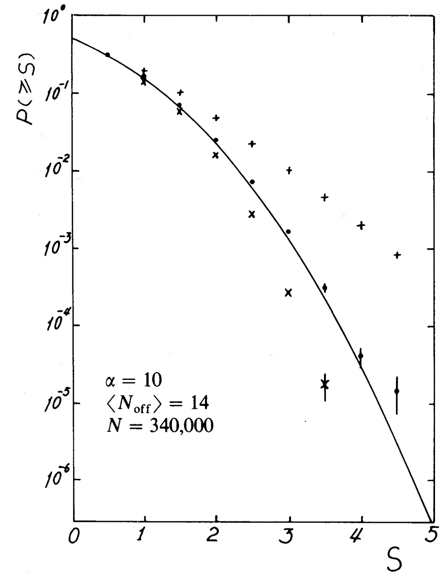

FIG. 2*f*

Hearn (1969) and Cherry *et al.* (1980) suggested using the following formula to compute the confidence level:

$$\xi = 1 - M\lambda' .$$

This formula is not based on a correct statistical foundation. One can even get a negative confidence level when M is large enough, which is obviously unreasonable.

VI. CONCLUSIONS

For the observation of the kind illustrated by Figure 1, we derived a formula, equation (17), to evaluate the significance of an observed event (N_{on}, N_{off}) by using the method of maximum likelihood ratio test. Our Monte Carlo simulation results show that in the case that the observed counts are not too few (say $N_{\text{on}} \gtrsim 10$, $N_{\text{off}} \gtrsim 10$), the significance distributions evaluated by equation (17) are reasonably consistent with the Gaussian

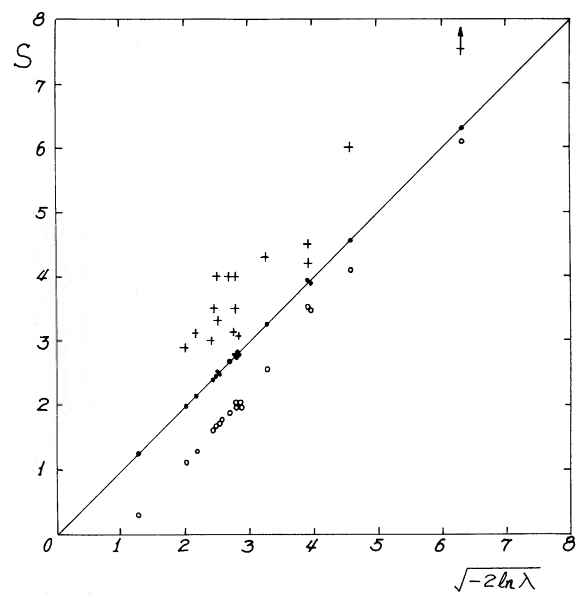

FIG. 3.—Significances of some reported γ-ray lines. *Pluses*, significance S by experimenters; *open circles*, significance S by Cherry *et al.* (1980); *filled circles*, significance S by eq. (17) of this work.

probabilities. Compared with equations (5) and (9), which are derived simply from the Poisson-Gaussian distribution of counts, equation (17) can be applied to the case with fewer observed counts (see Figs. 2*c* and 2*d*). The other important advantage of this formula is that it can be applied to the general case of $\alpha \neq 1$. In many practical experiments, the on-source observation time is not equal to background time. In some observations, including a few for which important results are reported, the ratios of on-source time to background time, α, are quite far from unity because of the limitations of the objective conditions of the experiments. The Monte Carlo results shown in Figure 2 illustrate that equation (17) is satisfactory at least in the wide range $\alpha = 0.1$–10. For the case $\alpha \sim 1$, say $0.5 \lesssim \alpha \lesssim 1.5$, one can use the simpler formula of equation (9) to evaluate the significance. Finally, after the significance of an event has been determined, the confidence level to which one may claim the presence of a source or a line should be computed by equation (22).

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REFERENCES

Cherry, M. L., Chupp, E. L., Dunphy, P. P., Forrest, D. J., and Ryan, J. M. 1980, *Ap. J.*, **242**, 1257.
Eadie, W., Drijard, D., James, F., Roos, M., and Sadoulet, B. 1971, *Statistic Methods in Experimental Physics* (Amsterdam: North-Holland).
Hearn, D. 1969, *Nucl. Instr. Methods*, **70**, 200.
Li, T. P. 1980, *Mathematical Treatment in Experiments* (Beijing: Science Press).
Ma, Y. Q., and Li, T. P. 1983, *Acta Ap. Sinica*, in press.
O'Mongain, E. 1973, *Nature*, **241**, 376.
Wilks, S. 1962, *Mathematical Statistics* (New York: Wiley).

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